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**Research Paper
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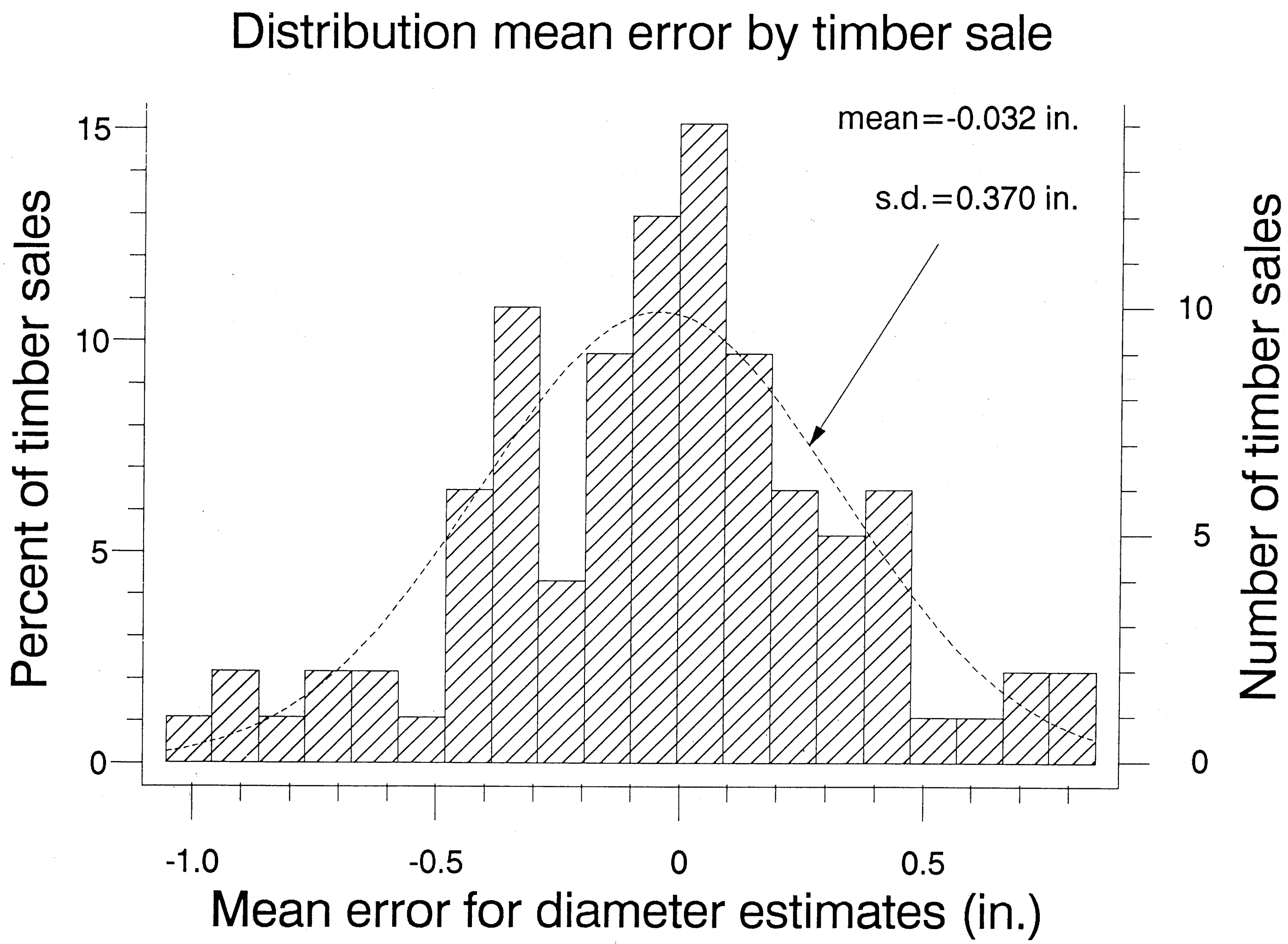
Profile Models for Estimating Log End Diameters in the Rocky Mountain Region

Raymond L. Czaplewski, Amy S. Brown, and Raymond C. Walker



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Abstract

Merchantable volume equations do not exist for some current merchantability standards, and frequent development of new volume equations is often impractical. Stem profile models provide a flexible alternative to merchantable volume equations. Profile models can estimate end diameters for all logs in a main stem using two standing tree measurements: diameter at breast height and total tree height. Predictions of log end diameters are used in scaling algorithms to estimate merchantable volume under different scaling rules and merchantability criteria. The stem profile model of Max and Burkhart was applied to seven tree species in the Rocky Mountain Region of the Forest Service. This regional model overestimated log end diameters by an average of 0.03 to 0.20 inch. These biases were reduced by use of second-stage models that correct for bias and explain weak patterns in the residual diameter predictions that are correlated with diameter at breast height. In extreme cases, mean error for specific timber sales exceeded 0.8 inch, even though the models were unbiased for the entire region.

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Introduction

Estimates of merchantable wood volume are needed for timber sale preparation, and forest inventory and planning. For decades, these estimates were obtained with volume equations that used standing tree measurements of diameter at breast height and total height. Merchantable volume equations are developed for specific utilization standards, such as inside bark Scribner board foot volume between a 1-foot stump and a 5-inch top diameter outside bark, subject to a minimum 16-foot log length, with diameters rounded to the lower 0.1 inch. A merchantable volume equation is useful for many years, assuming utilization standards do not change. However, utilization standards can evolve rapidly, often in response to local market and economic conditions. The ability to assess the effects of proposed changes to current standards is also needed. Continuous development of volume equations for new merchantability standards is becoming less practical given the rate at which these standards are changing.

Stem profile models (i.e., taper models) can also be used to predict merchantable volume. Stem profile models predict diameters at any height along the main stem using standing tree measurements of diameter at breast height and total tree height. They can, therefore, estimate the number of merchantable logs in each tree using predicted diameter at standard log ends. Log length and end diameters, which are common merchantability criteria, are also inputs to algorithms that compute volume according to cubic or board foot scaling rules. Therefore, a single stem profile model can be used to predict volume using a wide variety of current and future merchantability standards, under different rules for scaling wood volume.

Although many stem profile models have been developed (Sterba 1980), no one model is consistently best for all estimated tree dimensions, all geographic regions, and all tree species. However, the segmented polynomial model of Max and Burkhart (1976) is consistently one of the better models. Cao et al. (1980) found that the Max and Burkhart model was the best predictor of upper stem diameters for loblolly pine (*Pinus taeda*) among the six models evaluated. Martin (1981) compared five stem profile models for Appalachian hardwoods and recommended the Max and Burkhart model as the most accurate and precise predictor of diameters, heights, and cubic volumes, especially for the lower bole. Gordon (1983) found that the Max and Burkhart model was a less biased predictor of diameter than his compatible, fifth-order polynomial model. Amidon (1984) evaluated diameter predictions for eight stem profile models using five mixed-conifer species in California; the Max and Burkhart model was "reasonably precise and unbiased" and ranked higher than most other models. The Alberta Forest Service (1987), which evaluated 15 stem profile models for seven species groups, recommended the Max and Burkhart model because of its generally superior ability to predict diameters, heights, and cubic volumes. There have been no direct comparisons of stem profile models for trees from the Rocky Mountain Region. However, based on the results in the reports cited, we applied the segmented polynomial stem profile model of Max and Burkhart to seven tree species in the Rocky Mountain Region of the Forest Service.

Data

The seven tree species included white fir (*Abies concolor*), subalpine fir (*Abies lasiocarpa*), Engelmann spruce (*Picea engelmannii*), lodgepole pine (*Pinus contorta*), ponderosa pine (*Pinus ponderosa*), Douglas-fir (*Pseudotsuga menziesii*), and aspen (*Populus tremuloides*). Those trees measured between 1981 and 1987 were from active timber sales on the following national forests (NF's): Bighorn and Shoshone NF's in northern Wyoming; the Black Hills NF in northeastern Wyoming and western South Dakota; the Medicine Bow NF in southern Wyoming; the Arapaho-Roosevelt and Pike-San Isabel NF's in eastern Colorado; the Routt, White River, and Grand Mesa NF's in northwestern Colorado; and the Rio Grand and San Juan NF's in southwestern Colorado. The original purpose was to construct local volume equations for each sale. These same data were used in this study to develop stem profile models.

Trees were felled and bucked into nominal 16-foot logs. Diameters were measured at stump height, breast height (4.5 feet), and at the ends of all logs to a 4- to 6-inch top diameter (inside bark). Diameter inside bark (d_{ib}) was cross-sectionally measured twice at 90° angles with a tape to the nearest 0.1 inch at the stump and log ends and then averaged. A bark gauge was used to estimate d_{ib} at breast height for 62% of the trees. Log lengths were measured with tapes to the nearest 0.1 foot. Trees with forks, recently broken tops, or obvious measurement errors were excluded. Table 1 gives the number of accepted trees by species, NF, and d.b.h. groups.

Table 1.—Number of trees used to develop stem profile models for inside bark diameters, by species, national forest, and d.b.h. group.

Species National Forest	Diameter at breast height (inches) 3.5–9.0	9.1–11.0	11.1–12.5	12.6–14.0	14.1–16.0	16.1–19.0	19.1–40.0	Total
White fir (d.b.h. range: 8.4–28.1 inches)								
Pike-San Isabel	2	6	4	11	18	23	23	87
Rio Grande				3	1	2	1	7
Total	2	6	4	14	19	25	24	94
Subalpine fir (d.b.h. range: 6.8–26.3 inches)								
Grand Mesa	9	16	11	10	8	13	3	70
Rio Grande	1	6	8	7	11	10	6	49
Shoshone	2	6	9	4	7	6	1	35
Routt	3	8	1	2	5	3		22
Medicine Bow	4	4	3	1				12
Arapaho-Roosevelt	2	7	1	1				11
Pike-San Isabel	1	2	1	1	1		1	7
White River		1	5	1				7
San Juan			1	1			2	4
Bighorn	2	1						3
Total	24	51	40	28	32	32	13	220
Engelmann spruce (d.b.h. range: 6.9–38.8 inches)								
Rio Grande	3	36	45	50	64	74	99	371
Grand Mesa	24	21	27	20	28	33	47	200
Pike-San Isabel	4	17	27	31	22	21	14	136
Shoshone	2	3	4	2	7	4	27	49
Routt	1	2	1	4	5	5	25	43
Sân Juan		4	5	2	5	6	13	35
Arapaho-Roosevelt	2	6	2	3	5	5	6	29
Medicine Bow	2	4	3	1	2	4	6	22
White River			7	2	6	3	4	22
Bighorn	2		2	1	1	3	2	11
Total	40	93	123	116	145	158	243	918
Lodgepole pine (d.b.h. range: 3.8–27.2 inches)								
Medicine Bow	124	73	36	24	15	7	1	280
Routt	85	51	42	51	20	8	2	259
Shoshone	29	47	39	22	18	17	9	181
Grand Mesa	68	39	26	28	16	3		180
Arapaho-Roosevelt	69	42	32	13	14	3		173
Bighorn	16	34	14	11	8	5	2	90
Pike-San Isabel	3	10	5	12	10	1		41
White River	7	8	4	9	3		2	33
Total	401	304	198	170	104	44	16	1,237
Ponderosa pine (d.b.h. range: 5.1–40.6 inches)								
Pike-San Isabel	21	75	61	87	108	94	64	510
Black Hills	62	72	59	71	74	70	34	442
Arapaho-Roosevelt	35	27	29	14	29	17	12	163
San Juan		1	4	3	7	12	42	69
Medicine Bow	11	10	5	4	5	2		37
Rio Grande	1	5		1	1	1	2	11
Routt					1			1
Total	130	190	158	180	225	196	154	1,233
Douglas-fir (d.b.h. range: 8.6–34.2 inches)								
Pike-San Isabel		25	22	23	33	32	28	163
Rio Grande		6	9	7	9	14	22	67
Grand Mesa						2	2	4
White River						1	1	2
Bighorn	1							1
Total	1	31	31	30	42	49	53	237
Aspen (d.b.h. range: 4.9–21.6 inches)								
Grand Mesa	2	9	18	18	29	18	6	100
San Juan	10	15	17	9	15	18	7	91
Rio Grande		1		3	1			5
Total	12	25	35	30	45	36	13	196

Stem Profile Model

The Max and Burkhart model is used to estimate upper stem diameters $(\hat{d})_1$:

$$\hat{d}_1 = D\sqrt{b_1(h/H-1)+b_2(h^2/H^2-1)+b_3(a_1-h/H)^2 I_1+b_4(a_2-h/H)^2 I_2}$$

where
- $\hat{d}_1$ = stem diameter estimate of d_{ib} at height h;
- D = d.b.h. (inches at 4.5 feet);
- h = height at the upper stem diameter prediction (feet);
- H = total tree height (feet);
- b_i = linear regression parameters (table 2); and
- a_i = join points (table 2); upper join point is $i = 1$; lower is $i = 2$.

$$I_i = \begin{cases} 1, \text{ if } h/H < a_i \\ 0, \text{ otherwise} \end{cases}$$

The join points (a_i) are nonlinear in the regression model, which uses $(d_{ib}/D)^2$ as the response variable to estimate b_i; a_i were estimated using scatter plots of the empirical first derivative of stem taper (Czaplewski, in press). Graphical analysis of scatter plots led us to conclude that variance of the regression residuals, $(d_{ib}/D)^2 - (\hat{d}_1/D)^2$, was approximately homogeneous within stem segments bounded by the join points relative to (h/H).

Reducing Bias in Predicting Diameter Inside Bark

Stem profile models that minimize residuals of $(d_{ib}/D)^2$ will systematically overestimate diameters (d_{ib}), and this overestimation is termed a bias. Inside bark diameters were overestimated by an average of 0.03 to 0.20 inch, which were large relative to their standard errors (table 3). The ratio of the mean to its standard error is the *t*-statistic, and it is used to judge the relative magnitude of the mean bias. (The *t*-tests assume a normal distribution of residuals and independence of errors for measurements from a single tree, which proved to be poor assumptions in our study. However, *t*-statistics substantially larger than 3 are reasonably interpreted as indicating the presence of bias.)

Biased diameter predictions might be expected because the response variable in the regression model is $(d_{ib}/D)^2$, which is used to stabilize variance and reduce the nonlinear structure of the model. Transformation bias is detected when the model is subsequently retransformed to estimate d_{ib} rather than $(d_{ib}/D)^2$. The magnitude of transformation bias can be predicted by a second-stage model. Less ad hoc methods are available to correct for transformation bias for power transformations, e.g., Flewelling and Pienaar (1981) and Taylor (1986). However, there are no analogous procedures described directly in the literature for the transformation $(d_{ib}/D)^2$ of d_{ib}.

Using the trees in our data set, the following second-stage model produced approximately unbiased predictions of diameter inside bark $\hat{d}_2$ using a biased prediction $\hat{d}_1$ from the Max and Burkhart model:

$$\hat{d}_2 = \hat{d}_1[c_1 + c_2 D + c_3 h^2]$$

where c_1 to c_3 are regression parameters. The bracketed term is a multiplier that corrects for systematic retransformation bias. It ranges from 0.79 to 1.08 for trees in table 1, although 90% of its values range from 0.93 to 1.04, and half of its values range from 0.98 to 1.01.

The form of this model is based on exploratory data analyses of the transformed residual errors $(d_{ib} - \hat{d}_1)/\hat{d}_1$. This transformation was selected to stabilize variance, avoid discontinuous changes at join points, produce a diameter estimate of zero at the top of the tree (i.e., $\hat{d}_2 = 0$ for $h = H$ because $\hat{d}_1 = 0$ for $h = H$) and permit estimation of c_1 to c_3 using linear regression. Residual errors were weakly correlated with the following independent variables: tree size (represented by D), tree form (measured by D/H), and a curvilinear function of height (h) of the upper stem diameter prediction.

The practical significance of these correlations with independent variables varied by tree species. Independent variables in each second-stage regression model were selected using backward elimination stepwise regres-

Table 2.—Regression statistics[1] and coefficients[2] for Max and Burkhart (1976) stem profile models used for estimating inside bark diameters.

Species	RMSE	b_1	b_2	b_3	b_4	a_1	a_2
White fir	0.1371	−2.91187	1.26772	−3.76391	58.596	0.50	0.13
Subalpine fir	0.1230	−3.11638	1.46021	−2.63725	105.472	0.55	0.09
Engelmann spruce	0.1286	−2.26300	0.92540	−0.80682	382.694	0.65	0.05
Lodgepole pine	0.1185	−3.65010	1.45492	−2.20082	52.058	0.77	0.11
Ponderosa pine							
Black Hills	0.1268	−2.59737	0.96927	−1.43195	50.867	0.75	0.11
Other NF's	0.1242	−3.80739	1.75784	−3.56366	55.776	0.62	0.13
Douglas-fir	0.1408	−5.86345	2.98778	−4.12919	82.838	0.72	0.12
Aspen	0.1225	−5.18995	2.57262	−3.85160	117.934	0.69	0.09

[1]*Root mean square error (RMSE) in* $(d/D)^2$ *units.* R^2 *statistics range from 0.97 to 0.98.*
[2]*All coefficients are dimensionless and are valid with both English and metric units.*

Table 3.—Stem profile and second-stage models for inside bark diameters (d_{ib}) and a summary of error[1] in predicting stump, breast height, and upper stem diameters.

	Mean residual error from stem profile model		Second-stage model for d_{ib}							
			Mean residual error		Regression statistics[2]			Regression coefficients[3]		
n	inches	t	inches	t	F	R^2	RMSE	c_1	c_2	c_3
White fir										
472	-0.102	-1.86	-0.102	-1.86	—	—	—	1.0	0.0	0.0
Subalpine fir										
1,083	-0.056	-1.88	-0.056	-1.88	—	—	—	1.0	0.0	0.0
Engelmann spruce										
5,078	-0.198	-11.26	0.086	4.74	748	0.13	.1164	1.0	-0.0080396	0.063127
Lodgepole pine										
5,902	-0.079	-7.27	0.004	0.39	306	0.05	.1131	1.0876	-0.0080764	0.0
Ponderosa pine										
Black Hills										
1,973	-0.124	-5.24	-0.023	-0.98	179	0.08	.1231	1.1331	-0.0095335	0.0
Other NF's										
3,914	-0.153	-8.03	0.008	0.45	434	0.01	.1296	1.1251	-0.0082315	0.0
Douglas-fir										
1,249	-0.099	-2.69	-0.099	-2.69	—	—	—	1.0	0.0	0.0
Aspen										
1,207	-0.027	-0.91	-0.027	-0.91	—	—	—	1.0	0.0	0.0

[1] *Mean residual error is measured minus predicted* d_{ib} *(at stump, d.b.h., and end of merchantable 16-foot logs);* t-*statistic is the mean residual divided by its standard error of the mean (dimensionless).*

[2] *The degrees of freedom for the* F *statistic are (*k-1, n-k-1*), where* k *is the number of non-zero regression coefficients (including* c_1*), and* n *is the number of observations.*

[3] *Coefficient units are:* c_1, *dimensionless;* c_2, *inches;* c_3, *inches/feet;* c_4, *feet;* c_5, *feet*^{2}.

sion. The transformed residual error, described above, was the response variable. The least significant regression parameters were eliminated, one at a time, until the partial *F*-statistic for each regression parameter exceeded its critical value by a factor of 6, with $\alpha = 0.05$. This type of criterion is recommended by Draper and Smith (1981) to "distinguish statistically significant and worthwhile prediction equations from statistically significant equations of limited practical value." The regression parameters associated with D/H and h were eliminated from all models. Other regression parameters that were occasionally eliminated have values of zero in table 3. The second-stage model greatly reduces the bias from diameter predictions, but the standard errors of the residuals for diameter predictions decreased less than 6%.

The effect of the second-stage model on the representation of tree form was not obvious. To check suspicions that the second-stage model could produce illogical predictions of stem shape, we closely scrutinized the second-stage estimates of inside bark diameter ($\hat{d}_2$) for all trees in table 1. For each tree, $\hat{d}_2$ was always positive and became smaller as height h increased. There were no unusual changes in the rate of stem taper for any tree, as evaluated using the second derivative of $\hat{d}_2$ with respect to h (Czaplewski, in press); there were no stair-step patterns, i.e., sign reversals in the second derivative as h increased for any tree. Therefore, the suspicions were not realized; the second-stage model produced logical diameter estimates for all trees in table 1. However, it is conceivable that the second-stage model could make illogical predictions of stem diameters for a tree if its d.b.h. and total height are not represented by the trees in table 1, and this potential, although unlikely, problem should be considered when the second-stage model is applied.

Hypotheses on Model Specificity

We hypothesized that white fir could be combined with Douglas-fir and a single model used, because these species have traditionally been combined in past volume equations. Stem profile and second-stage models were fit to this combined data set, and residuals for second-stage model predictions of d_{ib} from the combined model were summarized separately for each species. White fir had a mean error of 0.44 inch, and Douglas-fir had a mean error of −0.18 inch; these were large compared to the standard error of the mean (0.03 inch). Therefore, these species were separated into two models to better fit the data. A similar hypothesis, for similar reasons, was posed for Engelmann spruce and subalpine fir. The mean error for d_{ib} predictions from the second-stage combined model was nearly zero for Engelmann spruce, but was −0.38 inch for subalpine fir; this was large compared to the standard error of 0.02 inch. Therefore, these species were separated into different models.

We also hypothesized, based on perceived differences in tree form, that a separate stem profile model would be required for ponderosa pine in the Black Hills NF. Stem profile and second-stage models were fit using all ponderosa pine trees in table 1. The mean residual error in predicting d_{ib} was reduced from 0.13 inch (for all ponderosa pine) to 0.01 and 0.08 inch (table 3) when separate models were estimated for the Black Hills NF and the other NF's of the Rocky Mountain Region. This reduction was large relative to the standard error of the mean (0.02 inch). We further hypothesized that separate models are needed for northern and southern portions of the Black Hills because of perceived differences in tree form. The improvement in mean error was less than one standard error of the mean (i.e., $t<1$) and was not considered important.

The above decisions regarding model specificity were based on *t*-statistics. These decisions might be more formally tested by use of a conditional error statistic for nonlinear models (Milliken 1982); Draper and Smith (1981) describe this as the "extra sum of squares" principle for prediction residuals. However, repeated measures of diameter on each tree violate the assumption of independence, and such tests were not used.

Local Applications

The models in this paper were developed using data from a large geographic region. In practice, they are typically applied to much smaller, local sites (e.g., a timber sale or inventory compartment). The mean error, when averaged across many sites, is expected to be very close to zero; however, the mean error for any one site can be larger because of local differences in tree form or size distribution. This problem occurs with many regional models, including regional volume equations.

Figure 1 portrays the distribution of errors in predicting d_{ib} for individual timber sales. Characteristics of a local site, such as lightning frequency, can effect the average form of trees from that site. Variations in characteristics between sites can cause systematic differences in tree form between sites, even when sites are located near each other. Therefore, it is not surprising that an unbiased regional model, which averages differences among many sites, can be biased when applied to a given local site.

Five percent of the most extreme errors in estimating d_{ib} are individually plotted in figure 1. Nearly one-half of these extreme errors are for stump diameters, which are highly variable relative to upper stem diameters. The remaining extreme errors were studied for associations with other independent variables, including species, NF, d.b.h., form (D/H), upper stem diameter, and height of the predicted diameter. Extreme errors tended to occur more frequently near tree top, and for trees with a large d.b.h., especially when the D/H ratio was large. Extreme overestimates more frequently occurred for small diameters, while extreme underestimates occurred more often for large diameters.

Figure 2 gives the distribution of mean error for each of 96 timber sales in table 1. On average, the bias is near zero. However, the biases for specific timber sales are often larger. For 86% of the timber sales, mean error in predicting d_{ib} was within 0.5 inch of zero, but mean error was over 1 inch for one sale. This is an example of the high variability in tree form among local sites, even within the same NF. It is unlikely that additional, easily measured, site-specific predictor variables can be identified that greatly improve diameter estimates for local sites (David Bruce, personal communication).[2]

[2]*David Bruce, 1988, personal communication, on file with senior author.*

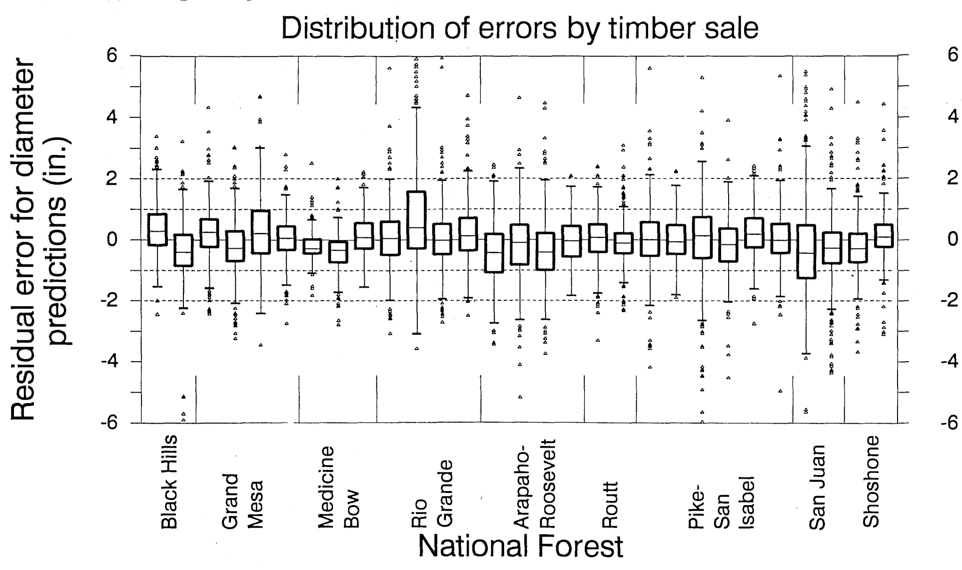

Figure 1.—Distribution of diameter prediction errors ($d_{ib}-\hat{d}_{2}$) by individual timber sales. Even though diameter predictions are unbiased for the regional data set, there can be substantial bias and variation for local timber sales. Included are all trees from table 1 from sales that had at least 50 trees measured for stem analysis, which includes 29 sales and 54% of the available stem data. All tree species are combined together, although most sales are predominantly one species. Negative residuals represent overestimates. Boxplots (Titus 1987) display the distributions using the 25th, 50th, and 75th percentiles to define the boxes. Lines emerging from each box are 1.5 times the interquartile range. More extreme residuals for individual diameter estimates are plotted as points.

Accuracy and Precision

As the precision of diameter predictions improves, individual diameter estimates from an unbiased estimator will be closer to their true values. The predictor variables in the second-stage model (i.e., D, D/H, h, h^2) improve precision; for all trees in table 1, the standard deviation of residuals from the second-stage model is 6% less than that from the stem profile models alone, even if the overall mean error for each profile model was subtracted from each diameter prediction.

The second-stage models can also improve accuracy of diameter estimates for local sites. Tree sizes can vary among local sites, and bias that is correlated with tree size can adversely impact local estimates. The standard deviation of mean error from each of the 96 timber sales (fig. 2) was reduced 12% using the second-stage models, compared to estimates from the stem profile models after their overall mean error was subtracted. Much of this improvement was caused by better estimates for three timber sales, but especially one ponderosa pine sale from the San Juan NF. This sale had unusually tall trees with large d.b.h.'s. Mean error for this sale from the stem profile model was −1.43 inch; mean error was −0.13 inch after applying the second-stage model. The second-stage model could be influenced by data from this one sale, which represented one-eighth of the data for ponderosa pine from NF's other than the Black Hills. Therefore, the second-stage model for ponderosa pine was refit without trees from this sale. The reduction in standard deviation of mean bias for each sale in figure 2 was 8%, rather than 12%, which indicates the second-stage model can produce useful improvements in diameter estimates for a few local sites.

Discussion

Emphasis has been placed on minimizing bias in predictions of upper stem diameters, even though merchantable volume estimation is the final goal. It is common practice in research studies to integrate stem profile models to estimate gross cubic volume (Martin 1981). However, forestry organizations use log length and end diameters to scale merchantable volume (Biging 1988). Also, merchantability standards are defined primarily by the upper end diameter estimate, not the integrated estimate of gross cubic volume. Even in research studies, short section lengths and end diameters are usually measured to indirectly estimate cubic volume (e.g., Smalian's formula); water displacement methods are seldom used to measure volume directly (Martin 1984). Therefore, accurate predictions of upper stem diameters can indirectly produce useful estimates of merchantable volume.

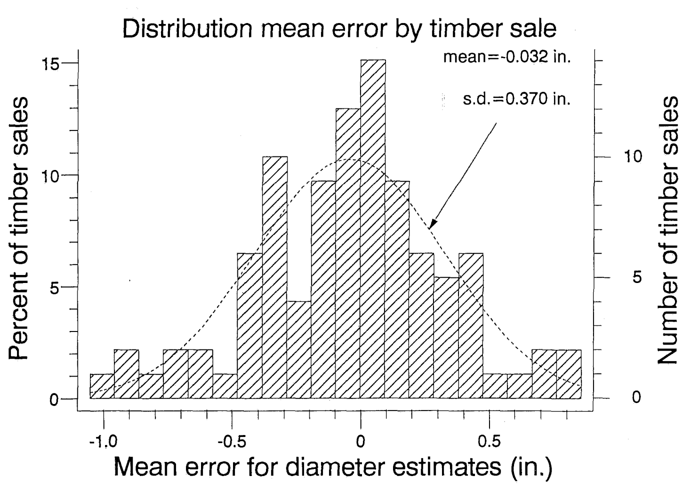

Figure 2.—Distribution of mean error (bias) in diameter predictions ($d_{ib}-\hat{d}_2$) for all species, for each of the 96 timber sales in table 1. This is an example of range in bias that can occur when regional stem profile models are applied to local sites. Negative errors result from overestimates from the model. The dashed line is the normal distribution with the same mean and standard deviation (s.d.) as the observed distribution.

Gross cubic volume estimates from section length and predicted end diameters (i.e., numerical integration) will differ slightly from gross cubic volume estimates obtained from analytically integrating the stem profile model. However, estimates of scaled merchantable volume, not gross cubic volume, are frequently needed for timber sales. Also, it is prudent to maintain consistency in merchantable volume estimation for timber sale preparation, log scaling, inventory, and planning. Therefore, the stem profile and second-stage models are used to predict both the number of merchantable logs in a tree and diameters at heights that correspond to log ends. These predictions are entered into existing algorithms used by forestry organizations to scale log volume. This increases the data processing load, but institutional consistency and flexibility are maintained.

Without the second-stage model, the Max and Burkhart stem profile model can readily estimate heights to given top diameters and cubic volume of stem sections, in addition to diameters at given heights. Although this provides considerable convenience, many of these predictions can be significantly biased. The Max and Burkhart model can produce unbiased estimates of $(d/D)^2$, but this does not guarantee that all transformations of the prediction equation (i.e., estimated diameter, height, and merchantable volume) are unbiased. However, the second-stage model can produce approximately unbiased estimates of stem diameter. Presumably, this will reduce bias in predictions of merchantable height (at the cost of increased computations) and in volume predictions that use diameter predictions from the second-stage model as input variables to scaling algorithms. Therefore, the second-stage model can provide both mensurational and institutional benefits.

The stem profile models in this paper are expected to provide useful predictions of merchantable volume given a variety of merchantability standards and scaling rules. These predictions are expected to be unbiased, when many local sites are averaged together, and when applied to the same population of trees used to estimate regression parameters (table 1). However, the errors evaluated in this paper used the same data that were used to estimate regression models; the range of errors could be larger when these models are applied to other data. It is not known how well these models will predict diameters for trees from other populations. The distribution of tree form and size can change over time, especially for second-growth stands, and new stem profile models will be required at some time in the future.

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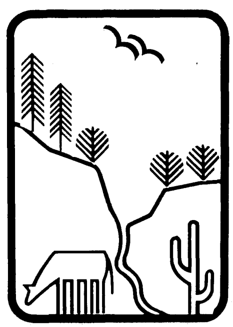
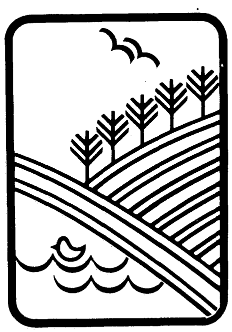
The segmented polynomial stem profile model of Max and Burkhart was applied to seven tree species in the Rocky Mountain Region of the Forest Service. Errors were reduced over the entire data set by use of second-stage models that adjust for transformation bias and explained weak patterns in the residual diameter predictions.

Keywords: Taper equations, merchantable volume estimates, bias correction, *Abies concolor, Abies lasiocarpa, Picea engelmannii, Pinus contorta, Pinus ponderosa, Pseudotsuga menziesii, Populus tremuloides*

Rocky
Mountains



Southwest



Great
Plains

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Forest Service

Rocky Mountain Forest and Range Experiment Station

The Rocky Mountain Station is one of eight regional experiment stations, plus the Forest Products Laboratory and the Washington Office Staff, that make up the Forest Service research organization.

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Research programs at the Rocky Mountain Station are coordinated with area universities and with other institutions. Many studies are conducted on a cooperative basis to accelerate solutions to problems involving range, water, wildlife and fish habitat, human and community development, timber, recreation, protection, and multiresource evaluation.

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Lincoln, Nebraska
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