
Volume II
SECTION 4

PARAMETERS FOR STABILITY ANALYSIS

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Section 4. Parameters for Slope Stability Analysis

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4A. Fundamental Stress-Strain Relationships

Cliff Denning, Geotechnical Engineer, Mt. Hood National Forest

4A.1 Introduction

In this section, the significant stress-strain parameters used in mechanical ("rational") slope stability analysis will be discussed. Included are parameter definitions and common methods for quantifying them. The intent of section 4A is to introduce some basic definitions and concepts concerning stress and strengths in soil and rock. Section 4A is followed by sections on soil weight/volume relationships, soil and rock shear strength, ground water, root strength, and tree surcharge. Figure 4A.1 illustrates that section 4 pertains to the level III data base within the three-level stability analysis process.

![Diagram](https://via.placeholder.com/150)

**Figure 4A.1.—**Section 4 pertains to the level III data base in slope stability analysis.

4A.2 Definitions

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<th>Force (L×M/T²)</th>
<th>load in pounds (lb), newtons (N), or kilonewtons (kN)</th>
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<td>1 N = 1 kg·m/s²</td>
<td>1 lb = 4.4482 N</td>
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Stress \((M/LX^2)\) force per unit area in pounds per square foot (psf), pounds per square inch (psi), or kilopascals (kPa)

1 psi \(=\) 144 psf
1 psi \(=\) 6.9 kPa
1 psf \(=\) 0.0479 kPa
1 kPa \(=\) 1 kN/m\(^2\)

Normal Stress, \(\sigma\) stress perpendicular to a plane

Shear Stress, \(\tau\) stress tangent to and within the plane

Pressure a stress acting uniformly in all directions, such as from a fluid (i.e., air or water). Force per unit area.

Strain, \(\varepsilon\) deformation per unit length. \(\varepsilon\) is dimensionless (e.g., in./in. or mm/mm). Deformation is normally in response to an applied stress. There are two types of strain: lateral ("bulging sideways"), \(\varepsilon_x\), and direct ("deformation under load"), \(\varepsilon_y\).

\[
\varepsilon_x = \frac{\Delta X}{X_0} \\
\varepsilon_y = \frac{\Delta Y}{Y_0}
\]

Stress-Strain Curve plot of stress against associated strain
Elastic Modulus, $E$ the slope of the stress-strain curve (Sowers, 1979), or (change in stress)/(change in strain).

$$E = \frac{\Delta \sigma}{\Delta \varepsilon}$$

Tangent Modulus, $Et$ the slope of a straight line drawn tangent to a stress-strain curve at a particular point on the curve. The tangent modulus at the initial point of the curve is the initial tangent modulus (Lambe and Whitman, 1969) as shown.

Secant Modulus, $Es$ the slope of a straight line connecting two separate points on the stress-strain curve as shown (figure reprinted with the permission of John Wiley and Sons from Soil Mechanics by T.W. Lambe and R.V. Whitman. Copyright ©1969 John Wiley and Sons).

Poisson’s Ratio, $\nu$ the ratio of lateral strain, $\varepsilon_r$, to direct strain, $\varepsilon_y$

$$\nu = \frac{\varepsilon_r}{\varepsilon_y}$$

4A.3 Stress at a Point

When a force is applied to a plane surface of a solid, the resulting stress can be resolved into three components. The normal stress component, $\sigma$, is perpendicular to the plane, and the shear stress component, $\tau$, acts along the surface of the plane in two directions. Stresses on a soil element are shown in figure 4A.2. Exactly three of the planes passing through the soil element have zero shear stress. These three principal planes are perpendicular to one another. Normal stresses acting on these planes are called principal stresses, and, from largest to smallest in magnitude, they are the major principal stress ($\sigma_1$), the intermediate principal stress ($\sigma_2$), and the minor principal stress ($\sigma_3$).
The general case of computing stresses on a plane cutting through a cube can be simplified by using a two-dimensional model. Figure 4A.3 shows an edge view of the cube cut by an inclined plane. The normal stress on the plane, \( \sigma_a \), and the shear stress in the plane, \( \tau_a \), can be computed from \( \sigma_1 \) and \( \sigma_3 \) by considering equilibrium of forces:

\[
\sigma_a = \left( \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \right) \cos 2\alpha
\]  \hspace{1cm} (4A.1)

\[
\tau_a = \left( \frac{\sigma_1 - \sigma_3}{2} \right) \sin 2\alpha
\]  \hspace{1cm} (4A.2)

Equations 4A.1 and 4A.2 describe points on a circle in a coordinate system as shown in figure 4A.4, with the x-axis representing the normal stress (\( \sigma \)) and the y-axis representing the shear stress (\( \tau \)). A graphical procedure was introduced by Mohr to solve these two equations. \( \sigma_1 \) and \( \sigma_3 \) are plotted on the coordinate system. A circle with center at \( (\sigma_1 + \sigma_3)/2 \) is drawn through \( \sigma_1 \) and \( \sigma_3 \). To find the normal and shear stresses on a plane inclined \( \alpha \) to the major stress plane, an angle \( 2\alpha \) is measured counterclockwise from the x-axis. A line is drawn at that angle from the circle center to the edge of the circle. The values of \( \sigma \) and \( \tau \) at the point of intersection of the line and the circle represent the normal stress and the shear stress, respectively.
Figure 4A.3.—Stresses on a cube cut by a plane perpendicular to the plane of \( \sigma_2 \) and making an angle of \( \alpha \) with the plane of \( \sigma_1 \) (from Sowers, 1979).*

Figure 4A.4.—Mohr’s coordinates and Mohr’s circle of stresses (from Sowers, 1979).*

Figure 4A.5 shows several common states of stress in soil. In the initial at-rest state, \( \sigma'_r \) is the overburden pressure, \( \sigma'_l = K_o \sigma'_r \) is the lateral pressure, and \( K_o \) is the coefficient of lateral earth pressure at rest. Beneath the center of a circular loaded area, the vertical stress \( (\sigma'_z) \) is the major principal stress and the radial stress \( (\sigma'_r) \) is the minor principal stress. Beneath the center of a circular excavation, the radial stress \( (\sigma'_r) \) is now the major principal stress, and the vertical stress \( (\sigma'_z) \) is the minor principal stress. Figure 4A.5(d) shows stress conditions behind a retaining structure. This is usually approximated by an assumption of plane-strain. (Plane-strain is a condition in which the intermediate principal strain \( (e_y) \) is zero.) In this case, the direction of intermediate (zero) principal strain is perpendicular to the page. The intermediate principal stress \( (\sigma'_y) \) is \( \sigma'_r \) and, in figure 4A.5(d), lies between \( \sigma'_l \) and \( \sigma'_r \) \( (\text{Wu and Sangrey, 1978}) \).

Another important feature in many stability problems is the rotation of the principal axes during loading or excavation \( (\text{figure 4A.6}) \). Prior to excavation, the state of stress is the initial at-rest state. After excavation, the principal axis rotates \( 90^\circ \) in the area of the toe with the major principal stress being in the horizontal direction. At point (b) the principal axis rotates approximately \( 45^\circ \). At point (c) no rotation takes place. The values of the principal stresses at points (a), (b), and (c) are different \( (\text{Wu and Sangrey, 1978}) \).
Effective stress is the difference between the total stress and pore-water pressure ($u$). Total stress is based on the total weight per unit area of the soil, including water in the soil structure. Effective stress results from the buoyant force of water on soil particles (Archimedes' principle). Figure 4A.7 shows an infinitesimally small element at a depth $Z$ below the ground surface and at a depth $Z_w$ below a static ground water table. It also shows a total stress and pore-water pressure perpendicular to any plane at element A in the soil profile. The total vertical stress, $\sigma_v$, acting on this element consists of the total weight above the element:

$$\sigma_v = (Z-Z_w) \gamma + Z_w \gamma_{sat} \tag{4A.3}$$

where:

$\gamma =$ soil unit weight (pcf)
$\gamma_{sat} =$ saturated soil unit weight (pcf).

In addition to the total stress there is a water pressure, $u$, acting on this element:

$$u = Z_w \gamma_w \tag{4A.4}$$

where:

$\gamma_w =$ unit weight of water (pcf).
The vertical effective stress then equals:

\[ \sigma' = \sigma_v - u \]  

(4A.5)

The following examples and figure 4A.8 illustrate the calculation of effective stress. Additional discussion can be found in sections 4E.1.1 and 4D.2.4.
(a) Vertical pressure in dry soil

Effective stress:

\[ \sigma = \sigma' = \frac{W}{A} = \gamma Z. \]

(b) Vertical pressure in saturated soil

Total stress (from eq. 4a.3):

\[ \sigma = \frac{W}{A} = Z \cdot Z_{w}\gamma + Z_w \gamma_{sat} \]

Pore pressure (from eq. 4a.4):

\[ u = \gamma_w Z_w \]

Effective stress (from eq. 4a.5):

\[ \sigma' = \sigma - u. \]

Figure 4A.8.—Calculation of effective stress (reprinted with permission of John Wiley & Sons from Fundamentals of geotechnical analysis by I.S. Dunn, L.R. Anderson, and F.W. Kiefer. Copyright © 1980 John Wiley & Sons.)
Example 1

\[ B = ul \text{(area of block)} - u2 \text{(area of block)} \]
\[ = 624 \text{ lb/ft}^2(1\text{ft}^2) - 561.6 \text{ lb/ft}^2(1\text{ft}^2) \]
\[ = 62.4 \text{ lb} \]

Buoyant weight = 100 lb - 62.4 lb = 37.6 lb

In terms of stresses:

Total stress = 9 ft (62.4 lb/ft^3) + 1 ft (100 lb/ft^3) = 661.6 lb/ft^2

Water pressure = 10 ft * 62.4 lb/ft^3 = 624 lb/ft^2

Effective stress = Total stress - water pressure
\[ = 661.6 \text{ lb/ft}^2 - 624 \text{ lb/ft}^2 = 37.6 \text{ lb/ft}^2 \]

(After Dunn et al., 1980)

Example 2

\[ \gamma_t = 120 \text{pcf} \]
\[ \gamma_w = 62.4 \text{pcf} \]

\[ z = z_w = 20 \text{ft} \]

Total stress:
\[ \sigma = \gamma_t \times z = 120 \text{pcf} (20 \text{ft}) = 2400 \text{ psf} \]

Pore pressure:
\[ u = \gamma_w \times z_w = 62.4 \text{pcf} (20 \text{ft}) = 1248 \text{ psf} \]

Effective stress:
\[ \sigma' = \sigma - u = 2400 \text{ psf} - 1248 \text{ psf} = 1152 \text{ psf} \]
4A.6 Shear Strength Parameters and Failure Criteria

4A.6.1 Angle of Internal Friction, $\phi$

The shear resistance resulting from friction can be illustrated by a free-body diagram of a stationary block on an inclined plane.

![Free-body diagram of a block on an inclined plane](image)

Resisting shear force: $S = W \sin \alpha$

Normal force: $N = W \cos \alpha$

The weight of the block is met by a resisting force of equal magnitude and opposite direction at the surface of the inclined plane. This resisting force can be separated into components parallel to the inclined plane ($S$) and normal to the inclined plane ($N$) as shown in the force triangle. If the angle $\alpha$ is increased, at some point the block will be just ready to slide. The value of $\alpha$ at the point of incipient slip is known as the friction angle, $\phi$, and $\tan \phi = S/N$ is the coefficient of friction. An expanded discussion of friction and friction angle can be found in sections 4D.2.1 and 4D.2.2. In terms of stress,

$$\tan \phi = \frac{\tau}{\sigma}$$  \hspace{1cm} (4A.6)

$$\tau = \sigma \tan \phi$$  \hspace{1cm} (4A.7)

where:

$\tau$ = resisting shear stress
$\sigma$ = normal stress.

4A.6.2 Cohesion

In equation 4A.6, the resisting shear stress is proportional to the normal stress. There are some situations in which part of the total shear resistance between particles is independent of the normal force; that is, even if the normal force is decreased to zero, there is still a measurable shear resistance (Lambe and Whitman, 1969). This shear resistance, when the normal force between two bodies is zero, is known as true cohesion. Cohesion, $c$, is in units of stress, such as psi or psf.
Coulomb (ca. 1773) presented the first hypothesis on the shear strength, $S$, of soil as a combination of cohesion and frictional resistance:

$$S = c + \sigma \tan \phi$$  \hspace{1cm} (4A.8)

or, in terms of effective stress:

$$S = c' + (\sigma - u)\tan \phi' = c' + \sigma'\tan \phi'$$  \hspace{1cm} (4A.9)

where $c'$ and $\phi'$ are effective strength parameters.

Mohr (ca. 1882) presented a theory of failure that states that while failure is essentially by shear, the critical shear stress, $\tau$, is a function of the normal stress acting on a potential surface of failure (Bowles, 1984):

$$\tau = f(\sigma').$$  \hspace{1cm} (4A.10)

One function of the normal stress is the Coulomb equation:

$$\tau = c' + \sigma' \tan \phi.$$  \hspace{1cm} (4A.11)

Today this function is commonly known as the Mohr-Coulomb failure criterion and is the most widely used method to define failure in soils. When plotted on the $\sigma$ and $\tau$ coordinates, the critical combinations of shear stress ($\tau$) and normal stress ($\sigma_n$) will form a line known as the Mohr-Coulomb failure envelope.

Figure 4A.9 illustrates the Mohr-Coulomb failure envelope. In this case, two blocks with a normal stress between them are subjected to an increasing shear stress until movement occurs along the contact plane. The stress at which movement occurs is recorded. If this test is repeated several times using different normal stresses, the resulting shear stress can be plotted as shown. Cohesion and friction angle can be determined from the plot. Additional discussion on the Mohr-Coulomb failure criterion can be found in section 4D.2.3.
4A.7 Conclusion

Stability analysis simply compares the material's shear strength available to resist failure to shear forces (stresses) that are present to cause failure. The remainder of section 4 is designed to assist in selecting the parameters used in the analysis techniques presented in sections 5 and 6.
4B. Soil Weight/Volume Relationships

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4B.1 Introduction

This section introduces the parameters used to describe soil weight and volume relationships which are used to determine essential soil stress parameters from field observations and laboratory testing and to aid in construction quality control and assurance, as illustrated in the included problems.

Once in-place soil unit weight, moisture content, and specific gravity are measured or estimated, unit weights at other moisture contents can be calculated.

4B.2 Definitions

Table 4B.1 and figure 4B.1 conceptualize and summarize the weight/volume relationships in soils. Soils are modeled as a cube divided into three parts: soil solids, water, and air. Most soils have all three phases, but saturated soils have only soil solids and water, and dry soils have only soil solids and air.
Table 4B.1.—Soil volume and weight relationships (from U.S. Department of the Navy, 1971).

<table>
<thead>
<tr>
<th>Property</th>
<th>Saturated sample ((W_s, W_w, G), are known)</th>
<th>Unsaturated sample ((W_s, W_w, G, V) are known)</th>
<th>Supplementary formulas relating measured and computed factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_s) volume of solids</td>
<td>(\frac{W_s}{G})</td>
<td>(V = (V_s - V_w))</td>
<td>(V )</td>
</tr>
<tr>
<td>(V_w) volume of water</td>
<td>(\frac{W_w}{G})</td>
<td>(V_w - V_s)</td>
<td>(SV_w)</td>
</tr>
<tr>
<td>(V_a) volume of air or gas</td>
<td>zero</td>
<td>(V = (V_s + V_w))</td>
<td>(SV_a)</td>
</tr>
<tr>
<td>(V_r) volume of voids</td>
<td>(\frac{W_r}{G_r})</td>
<td>(V = V_s - V_w)</td>
<td>(SV_r)</td>
</tr>
<tr>
<td>(V) total volume of sample</td>
<td>(V_s + V_w) measured</td>
<td>(V_s(1 - e))</td>
<td>(V_e)</td>
</tr>
<tr>
<td>n porosity</td>
<td>(\frac{V_s}{V})</td>
<td>(1 - \frac{V_s}{V})</td>
<td>(SV_s)</td>
</tr>
<tr>
<td>e void ratio</td>
<td>(\frac{V_w}{V})</td>
<td>(1 - \frac{V_w}{V})</td>
<td>(SV_w)</td>
</tr>
<tr>
<td>(W_s) weight of solids</td>
<td>measured</td>
<td>(G(V_s - V_w))</td>
<td>(SV_o)</td>
</tr>
<tr>
<td>(W_w) weight of water</td>
<td>measured</td>
<td>((1 - e))</td>
<td>(SV_w)</td>
</tr>
<tr>
<td>(W_s) weight of sample</td>
<td>(W_s)</td>
<td>(G(V_s - V_w))</td>
<td>(SV_o)</td>
</tr>
<tr>
<td>(W) total weight of sample</td>
<td>(W_s - W_w)</td>
<td>(W_s(1 - e))</td>
<td>(SV_o)</td>
</tr>
<tr>
<td>(Y_d) dry unit weight</td>
<td>(\frac{W_s}{V})</td>
<td>(\frac{W_s}{V(1 - e)})</td>
<td>(SV_o)</td>
</tr>
<tr>
<td>(Y_t) wet unit weight</td>
<td>(\frac{W_s}{V})</td>
<td>(\frac{W_s}{V(1 - e)})</td>
<td>(SV_o)</td>
</tr>
<tr>
<td>(Y_{SAT}) saturated unit</td>
<td>(\frac{W_s}{V})</td>
<td>(\frac{W_s}{V(1 - e)})</td>
<td>(SV_o)</td>
</tr>
<tr>
<td>(Y_{SUB}) submerged (buoyant)</td>
<td>(\frac{W_s}{V})</td>
<td>(\frac{W_s}{V(1 - e)})</td>
<td>(SV_o)</td>
</tr>
<tr>
<td>Moisture content</td>
<td>(\frac{W}{W_s})</td>
<td>(\frac{W}{W_s} - 1)</td>
<td>(SV_o)</td>
</tr>
<tr>
<td>Degree of saturation</td>
<td>(S)</td>
<td>(\frac{V}{V_s})</td>
<td>(SV_o)</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>(\frac{W}{V})</td>
<td>(\frac{W}{V_s})</td>
<td>(SV_o)</td>
</tr>
</tbody>
</table>

Figure 4B.1.—Diagram for soil weight/volume relationships (from U.S. Department of the Navy, 1971).
Problem 1: After compaction of a fill, a 0.22-cubic-foot sample weighing 29.8 pounds is taken. The moisture content is 12.5 percent and the specific gravity of solids is 2.70. In a laboratory compaction test of the same soil, the optimum water content is 14 percent and the maximum dry density is 125.5 pounds per cubic foot.

Determine:

- Void ratio of the fill.
- Dry density of the compacted fill.
- Degree of saturation of the compacted fill.
- Degree of compaction of the fill.
- Water content of the fill should it become saturated without change in volume after it has been compacted in the field.

\[ V_t = 0.22 \text{ ft}^3 \]
\[ W_t = 29.8 \text{ lb} \]
\[ w = 0.125 \]
\[ W'_s = \frac{W_t}{1 + w} = \frac{29.8 \text{ lb}}{1.125} = 26.5 \text{ lb} \]
\[ W_w = W_t - W'_s = (29.8 \text{ lb}) - (26.5 \text{ lb}) = 3.3 \text{ lb} \]
\[ \gamma_{\text{solid}} = \gamma_w G = (62.4 \text{ pcf})(2.70) = 168.5 \text{ pcf} \]
\[ V_s = \frac{W_s}{\gamma_{\text{solid}}} = \frac{26.5 \text{ lb}}{168.5 \text{ pcf}} = 0.157 \text{ ft}^3 \]
\[ V_w = \frac{W_w}{\gamma_w} = \frac{3.3 \text{ lb}}{62.4 \text{ pcf}} = 0.053 \text{ ft}^3 \]

\[ V_a = V_t - (V_s + V_w) = (0.22 \text{ ft}^3) - (0.157 \text{ ft}^3 + 0.053 \text{ ft}^3) \\
= 0.01 \text{ ft}^3 \]
\[ V_v = V_a + V_w = (0.01 \text{ ft}^3) + (0.053 \text{ ft}^3) = 0.063 \text{ ft}^3 \]
- Void ratio of the fill:
  \[ e = \frac{V_v}{V_s} = \frac{0.063 \text{ ft}^3}{0.157 \text{ ft}^3} = 0.40 \]

- Dry density of the compacted fill:
  \[ \gamma_d = \frac{W_s}{V_t} = \frac{26.5 \text{ lb}}{0.22 \text{ ft}^3} = 120.5 \text{ pcf} \]

- Degree of saturation of the compacted fill:
  \[ S = \frac{V_w}{V_v} = \frac{0.053 \text{ ft}^3}{0.063 \text{ ft}^3} = 84\% \]

- Degree of compaction of the fill:
  \[ \frac{\gamma_d (\text{field})}{\gamma_{lb}} = \frac{120.5 \text{ pcf}}{125.5 \text{ pcf}} = 96\% \]

- Water content of the fill after saturation:
  \[ w = \frac{V_r \gamma_w}{W_s} = \frac{(0.063 \text{ pcf})(62.4 \text{ pcf})}{26.5 \text{ lb}} = 14.8\% \]

**Problem 2:** A sample of saturated clay weighed 1,526 grams in its natural state and 1,053 grams after drying.

Determine:

- The natural moisture content.
- The void ratio of the soil in its natural state if \( G_s = 2.70 \).
- The porosity.
- Total and dry density.

\[ S = 100\% \]
\[ W_t = 1526 \text{ g} \]
\[ W_s = 1053 \text{ g} \]
\[ W_w = W_t - W_s = 1526 \text{ g} - 1053 \text{ g} = 473 \text{ g} \]
- Natural moisture content:
  \[ w = \frac{W_w}{W_s} = \frac{473}{1053} \text{ g} = 0.449 \text{ or } 44.9\% \]

- Void ratio:
  \[ e = \frac{wG}{S} = \frac{(0.449)(2.70)}{1.0} = 1.21 \]

- Porosity:
  \[ n = \frac{e}{1 + e} = \frac{1.21}{2.21} = 54.8\% \]

- Densities:
  \[ \gamma_t = \frac{(1 + w)\gamma_w}{S + \frac{1}{G}} = \frac{(1 + 0.449)(1 \text{ g/cm}^3)}{1 + \frac{1}{2.70}} = 1.77 \text{ g/cm}^3 \text{ or } 114.0 \text{ pcf} \]
  \[ \gamma_d = \frac{G\gamma_w}{1 + e} = \frac{(2.70)(1 \text{ g/cm}^3)}{2.21} = 1.22 \text{ g/cm}^3 \text{ or } 76.1 \text{ pcf} \]

**Problem 3.** For a sample of soil with \( \gamma = 110 \text{ pcf} \), \( w = 10\% \), and \( G = 2.65 \), find \( \gamma_d \), \( \gamma_{\text{sat}} \), \( \gamma_{\text{sat}} \), and \( w_{\text{sw}} \).

\[ W_t = 110 \text{ lb} \]
\[ W_w \]
\[ W_s \]

\[ \gamma = \frac{W_t}{V} = 110 \text{ pcf} \]

\[ W_t = 110 \text{ lb for } V = 1 \text{ ft}^3 \]

\[ W_s = \frac{W_t}{(1 + w)} = \frac{110 \text{ lb}}{1.10} = 100 \text{ lb} \]

\[ V_s = \frac{W_s}{G\gamma_w} = \frac{100 \text{ lb}}{(2.65)(62.4 \text{ pcf})} = 0.605 \text{ ft}^3 \]

\[ V = 1.0 \text{ ft}^3 \]
\[ V_v = V - V_s = (1.00 \text{ ft}^3) - (0.605 \text{ ft}^3) = 0.395 \text{ ft}^3 \]

\[ \gamma_d = \frac{W_s}{V} = \frac{100 \text{ lb}}{1 \text{ ft}^3} = 100 \text{ pcf} \]

\[ W_w = S \gamma_w V_v = (1)(62.4 \text{ pcf})(0.395 \text{ ft}^3) = 24.7 \text{ lb} \]

\[ \gamma_{sat} = \frac{W_s + W_w + W_d}{V} = \frac{100 \text{ lb} + 24.7 \text{ lb} + 0 \text{ lb}}{1 \text{ ft}^3} = 124.7 \text{ pcf} \]

\[ w_{sat} = \frac{W_w}{W_s} = \frac{24.7 \text{ lb}}{100 \text{ lb}} = 24.7\% \]

\[ \gamma_{sub} = \gamma_{sat} - \gamma_w = (124.7 \text{ pcf}) - (62.4 \text{ pcf}) = 62.3 \text{ pcf} \]

Alternatively:

\[ \gamma_d = \frac{\gamma}{1 + w} = \frac{110 \text{ pcf}}{1.10} = 100 \text{ pcf} \]

\[ \gamma_{sub} = \left(\frac{G - 1}{G}\right)\gamma_d = \left(\frac{2.68 - 1}{2.68}\right)(100 \text{ pcf}) = 62.3 \text{ pcf} \]

\[ \gamma_{sat} = \gamma_{sub} + \gamma_w = (62.3 \text{ pcf}) + (62.4 \text{ pcf}) = 124.7 \text{ pcf} \]

\[ w_{sat} = \frac{\gamma_w}{\gamma_d} - \frac{1}{G} = \frac{62.4 \text{ pcf}}{100 \text{ pcf}} - \frac{1}{2.68} = 24.7\% \]

**Problem 4.** The in-place moist unit weight, \( \gamma \), of a soil was measured by a nuclear gage at 125 pcf. From laboratory analysis, the dry unit weight, \( \gamma_d \), was 120 pcf and the specific gravity was 2.68. Find:

- Moisture content.
- Saturated unit weight.
- Moisture content at saturation.
- Unit weight at 10% moisture content.
- Volume of voids in 1 cubic foot of soil.

- Moisture content:

\[ w = \frac{\gamma - \gamma_d}{\gamma_d} = \frac{125 \text{ pcf} - 120 \text{ pcf}}{120 \text{ pcf}} = 4.2\% \]

- Saturated unit weight:

\[ \gamma_{sat} = \gamma_{sub} + \gamma_w = \left(\frac{G - 1}{G}\right)\gamma_d + \gamma_w \]

\[ = \left(\frac{2.68 - 1.00}{2.68}\right)(120 \text{ pcf}) + (62.4 \text{ pcf}) = 137.6 \text{ pcf} \]
- Moisture content at saturation:

\[ w_{\text{sat}} = \frac{\gamma_w}{\gamma_d} - \frac{1}{G} = \frac{62.4 \text{ pcf}}{120 \text{ pcf}} - \frac{1}{2.68} = 14.7\% \]

- Unit weight at \( w = 10\% \):

\[
\gamma_f = \frac{W_t}{V} = \frac{W_s + W_w}{V} = \frac{W_s + wW_w}{V} = \frac{W_s(1 + w)}{V} = \gamma_d(1 + w) = (120 \text{ pcf})(1 + 0.10) = 132 \text{ pcf}
\]

- Volume of voids in 1 ft³ of soil at saturation:

\[
V_v = \frac{W_w}{\gamma_w} = \frac{\gamma_{\text{sat}} - \gamma_d}{\gamma_w} = \frac{137.6 \text{ pcf} - 120 \text{ pcf}}{62.4 \text{ pcf}} = 0.282 \text{ ft}^3
\]

**Problem 5.** For a cut slope, the in-place moist unit weight is 112 pcf, moisture content is 15%, and specific gravity is 2.70. AASHTO laboratory T-99 tests show a maximum unit weight of 118 pcf and optimum moisture of 12%. For a fill to be placed at 90% of T-99 maximum (see compaction section 4B.5.3), find the expected:

- Dry unit weight for cut and fill.
- Moist unit weight if compacted at optimum moisture content.
- Saturated unit weight.
- Saturated moisture content.
- Shrinkage component of the "% cut compaction factor."

- Dry unit weight for cut and fill:

\[
\gamma_d \text{ for cut} = \frac{\gamma}{1 + w} = \frac{112 \text{ pcf}}{1.15} = 97.4 \text{ pcf}
\]

\[
\gamma_d \text{ for fill} = 0.90\gamma_{\text{max}} = (0.90)(118 \text{ pcf}) = 106.2 \text{ pcf}
\]

- Moist unit weight if compacted at optimum moisture content:

\[
\gamma \text{ for fill} = \gamma_d(1 + w) = (106.2 \text{ pcf})(1.12) = 118.9 \text{ pcf}
\]

- Saturated unit weight:

\[
\gamma_{\text{sat}} \text{ for fill} = \gamma_{\text{sub}} + \gamma_w = \left(\frac{G - 1}{G}\right)\gamma_d + \gamma_w = \frac{2.70 - 1.00}{2.70} (106.2 \text{ pcf}) + 62.4 \text{ pcf} = 129.3 \text{ pcf}
\]

- Saturated moisture content:

\[
w_{\text{sat}} \text{ for fill} = \frac{\gamma_w}{\gamma_d} - \frac{1}{G} = \frac{62.4 \text{ pcf}}{106.2 \text{ pcf}} - \frac{1}{2.70} = 21.7\%}
\]
Shrinkage component (sc) of the "% cut compaction factor" (ccf):

\[
\% \text{ ccf (sc)} = 1 - \frac{\gamma_{d_{\text{cut}}}}{\gamma_{d_{\text{fill}}}} = 1 - \frac{97.4 \text{ pcf}}{106.2 \text{ pcf}} = 8.3\%
\]

This section is a compilation of figures and tables useful for estimating the dry, moist, and saturated unit weight and relative density of soils as classified by the Unified Soil Classification System (USCS). A number of example problems are given that use several of the tables and figures, but there is no attempt at instruction in soil mechanics, and not all the terms in the tables and figures are defined.

Tables 4B.2 and 4B.3 describe soil properties pertaining to embankments and foundations and to roads and airfields, respectively. Table 4B.4 reports typical values for particle size and gradation and unit weight. Table 4B.5 relates standard penetration test (SPT) blow counts \( (N_p) \) (see section 4C.4.2.3) to relative density \( (D_r) \). Figure 4B.2 relates vertical effective stress \( (\sigma') \), relative density, and SPT. Table 4B.6 relates relative density to effective angle of internal friction \( (\phi') \) by soil type. Figure 4B.3 relates dry unit weight to moist and saturated unit weight for various moisture contents. Figure 4B.4 relates dry unit weight to effective angle of internal friction for various soil types and densities for cohesionless and non-plastic soils. Figure 4B.5 relates consistency to unconfined compressive strength \( (q_u) \), unconfined compressive strength to SPT, and relative density to unit weight by soil type for cohesive \( \phi=0 \) soils.
Table 4B.2.—Soil property characteristics pertinent to embankments and foundations (from U.S. Department of Defense, 1968).

<table>
<thead>
<tr>
<th>Major Divisions (1)</th>
<th>Letter (2)</th>
<th>Symbol (3)</th>
<th>Name (4)</th>
<th>Value for Embankments (5)</th>
<th>Permeability Cm Per Sec (6)</th>
<th>Compaction Characteristics (7)</th>
<th>Unit Dry Weight (pcf) (8)</th>
<th>Suitability Value for Foundations (9)</th>
<th>Requirements for Seepage Control (10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRAVEL</td>
<td>GW</td>
<td>Red</td>
<td>Well-graded gravel or gravel-sand mixtures, little or no fines</td>
<td>Very stable, previous shells of dike and dams</td>
<td>$k &gt; 10^{-3}$</td>
<td>Good, tractor, rubber-tired, steel-wheeled roller</td>
<td>125-135</td>
<td>Good bearing value</td>
<td>Positive cutoff</td>
</tr>
<tr>
<td>AND</td>
<td>GP</td>
<td>Yellow</td>
<td>Poorly graded gravels or gravel-sand mixtures, little or no fines</td>
<td>Reasonably stable, previous shells of dike and dams</td>
<td>$k &gt; 10^{-3}$</td>
<td>Good, tractor, rubber-tired, steel-wheeled roller</td>
<td>115-125</td>
<td>Good bearing value</td>
<td>Positive cutoff</td>
</tr>
<tr>
<td>GRAVELY</td>
<td>GM</td>
<td>Yellow</td>
<td>Silty gravels, gravel-sand-silt mixtures</td>
<td>Reasonably stable, not particularly suited to shells, but may be used for impervious cores or blankets</td>
<td>$k = 10^{-3}$ to $10^{-6}$</td>
<td>Good, with close control, rubber-tired, sheepfoot roller</td>
<td>120-135</td>
<td>Good bearing value</td>
<td>Toe trench to none</td>
</tr>
<tr>
<td>SOILS</td>
<td>GR</td>
<td>Yellow</td>
<td>Clayey gravels, gravel-sand-silt mixtures</td>
<td>Faintly stable, may be used for impervious core</td>
<td>$k = 10^{-6}$ to $10^{-8}$</td>
<td>Pair, rubber-tired, sheepfoot roller</td>
<td>115-130</td>
<td>Good bearing value</td>
<td>None</td>
</tr>
<tr>
<td>GRAINED</td>
<td>SW</td>
<td>Red</td>
<td>Well-graded sands or gravelly sands, little or no fines</td>
<td>Very stable, previous sections, slope protection required</td>
<td>$k &gt; 10^{-3}$</td>
<td>Good, tractor</td>
<td>110-130</td>
<td>Good bearing value</td>
<td>Upstream blanket and toe drainage or wells</td>
</tr>
<tr>
<td>AND</td>
<td>SP</td>
<td>Red</td>
<td>Poorly graded sands or gravelly sands, little or no fines</td>
<td>Reasonably stable, may be used in dike section with flat slopes</td>
<td>$k &gt; 10^{-3}$</td>
<td>Good, tractor</td>
<td>100-120</td>
<td>Good to poor bearing value depending on density</td>
<td>Upstream blanket and toe drainage or wells</td>
</tr>
<tr>
<td>SANDY</td>
<td>SM</td>
<td>Yellow</td>
<td>Silty sands, sand-silt mixtures</td>
<td>Faintly stable, not particularly suited to shells, but may be used for impervious cores or dike</td>
<td>$k = 10^{-3}$ to $10^{-6}$</td>
<td>Good, with close control, rubber-tired, sheepfoot roller</td>
<td>110-125</td>
<td>Good to poor bearing value depending on density</td>
<td>Upstream blanket and toe drainage or wells</td>
</tr>
<tr>
<td>SOILS</td>
<td>SC</td>
<td>Yellow</td>
<td>Clayey sands, sand-silt mixtures</td>
<td>Faintly stable, used for impervious cores for flood control structures</td>
<td>$k = 10^{-6}$ to $10^{-8}$</td>
<td>Pair, sheepfoot roller, rubber tired</td>
<td>105-125</td>
<td>Good to poor bearing value</td>
<td>None</td>
</tr>
<tr>
<td>SILTS AND CLAYS</td>
<td>ML</td>
<td>Green</td>
<td>Inorganic silts and very fine sands, rock flour, silty or clayey fine sands or clayey silts with slight plasticity</td>
<td>Poor stability, may be used for embankments with proper control</td>
<td>$k = 10^{-3}$ to $10^{-6}$</td>
<td>Good to poor, close control, essential, rubber-tired roller, sheepfoot roller</td>
<td>95-120</td>
<td>Very poor, susceptible to liquefaction</td>
<td>Toe trench to none</td>
</tr>
<tr>
<td>LL &lt; 50</td>
<td>CL</td>
<td>Green</td>
<td>Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, clayey sand</td>
<td>Stable, impervious cores and blankets</td>
<td>$k = 10^{-6}$ to $10^{-8}$</td>
<td>Pair to good, sheepfoot roller, rubber tired</td>
<td>95-120</td>
<td>Good to poor bearing</td>
<td>None</td>
</tr>
<tr>
<td>FINE</td>
<td>LL &lt; 50</td>
<td>Green</td>
<td>Organic silts and organic silt-clays of low plasticity</td>
<td>Not suitable for embankments</td>
<td>$k = 10^{-6}$ to $10^{-8}$</td>
<td>Pair to poor, sheepfoot roller</td>
<td>80-100</td>
<td>Fair to poor bearing, may have excessive settlements</td>
<td>None</td>
</tr>
<tr>
<td>GRAINED</td>
<td>MH</td>
<td>Blue</td>
<td>Inorganic silts, micaceous or diamictic coarse sandy or silty soils, elastic silts</td>
<td>Poor stability, core of hydraulic fill dam, not desirable in filled fill construction</td>
<td>$k = 10^{-6}$ to $10^{-8}$</td>
<td>Poor to very poor, sheepfoot roller</td>
<td>70-95</td>
<td>Poor bearing</td>
<td>None</td>
</tr>
<tr>
<td>SOILS</td>
<td>CH</td>
<td>Blue</td>
<td>Organic clays of high plasticity, fat clays</td>
<td>Poor stability with flat slopes, thin cores, blankets and dike sections</td>
<td>$k = 10^{-6}$ to $10^{-8}$</td>
<td>Poor to poor, sheepfoot roller</td>
<td>75-105</td>
<td>Poor bearing</td>
<td>None</td>
</tr>
<tr>
<td>LL &gt; 50</td>
<td>OH</td>
<td>Blue</td>
<td>Organic clays of medium to high plasticity, organic silts</td>
<td>Not suitable for embankments</td>
<td>$k = 10^{-6}$ to $10^{-8}$</td>
<td>Poor to very poor, sheepfoot roller</td>
<td>65-100</td>
<td>Very poor bearing</td>
<td>None</td>
</tr>
<tr>
<td>HIGHLY ORGANIC SOILS</td>
<td>PI</td>
<td>Orange</td>
<td>Peat and other highly organic soils</td>
<td>Not used for construction</td>
<td></td>
<td>Compaction not practical</td>
<td></td>
<td></td>
<td>Remove from foundations</td>
</tr>
<tr>
<td>Major Divisions (1)</td>
<td>Letter (2)</td>
<td>Symbol</td>
<td>Type</td>
<td>Color</td>
<td>Performance Value When Not Subject to Frost Action (7)</td>
<td>Performance Value When Subject to Frost Action (8)</td>
<td>Performance Value When No Subject to Frost Action (9)</td>
<td>Potential Frost Action (10)</td>
<td>Compressibility and Expansion (11)</td>
</tr>
<tr>
<td>------------------</td>
<td>-----------</td>
<td>--------</td>
<td>------</td>
<td>-------</td>
<td>-----------------------------------------------</td>
<td>-----------------------------------------------</td>
<td>-----------------------------------------------</td>
<td>---------------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>EROSIVE</td>
<td>S</td>
<td>Red</td>
<td>Well-graded gravels or gravel-sand mixtures, little or no fines</td>
<td>Good</td>
<td>None to very slight</td>
<td>Almost none</td>
<td>Excellent</td>
<td>Clay-water type, rubber-tired roller, sand-sired roller</td>
<td>125-140</td>
</tr>
<tr>
<td>GRAVELY SOILS</td>
<td>G</td>
<td>d</td>
<td>Silty gravels, gravel-sand-silt mixtures</td>
<td>Good</td>
<td>Fair to good</td>
<td>Slight to medium</td>
<td>Fair to poor</td>
<td>Rubber-tired roller, deep penetration</td>
<td>125-145</td>
</tr>
<tr>
<td>COARSE</td>
<td>S</td>
<td>Yellow</td>
<td>Clayey gravels, gravel-sand-clay mixtures</td>
<td>Good</td>
<td>Fair</td>
<td>Poor to slightly</td>
<td>Slight to medium</td>
<td>Rubber-tired roller, deep penetration</td>
<td>115-135</td>
</tr>
<tr>
<td>SANDY SOILS</td>
<td>S</td>
<td>Red</td>
<td>Well-graded sands or gravelly sands, little or no fines</td>
<td>Good</td>
<td>Fair to good</td>
<td>Poor</td>
<td>None to very slight</td>
<td>Almost none</td>
<td>Excellent</td>
</tr>
<tr>
<td>SANDY CLAYS</td>
<td>S</td>
<td>Yellow</td>
<td>Silty sands, sand-silt mixtures</td>
<td>Fair to good</td>
<td>Fair to good</td>
<td>Poor</td>
<td>Slight to high</td>
<td>Poor to practically</td>
<td>Rubber-tired roller, deep penetration</td>
</tr>
<tr>
<td>GRANULED</td>
<td>S</td>
<td>Green</td>
<td>Inorganic soils and very fine sands, silt, clay, or clayey silt or clayey silts with slight plasticity</td>
<td>Poor to fair</td>
<td>Not suitable</td>
<td>Not suitable</td>
<td>Medium to very high</td>
<td>Poor to practically</td>
<td>Rubber-tired roller, deep penetration, rubber-tired roller</td>
</tr>
<tr>
<td>FINE</td>
<td>S</td>
<td>Blue</td>
<td>Organic silts and organic silt-clays of low plasticity</td>
<td>Poor</td>
<td>Not suitable</td>
<td>Not suitable</td>
<td>Medium to high</td>
<td>Practically</td>
<td>Rubber-tired roller, deep penetration</td>
</tr>
<tr>
<td>CLAYS LL ≤ 5</td>
<td>M</td>
<td>Blue</td>
<td>Inorganic clays of low to medium plasticity, silty clays, sandy clays, or silty clays, low clays</td>
<td>Poor</td>
<td>Not suitable</td>
<td>Not suitable</td>
<td>Medium to high</td>
<td>Practically</td>
<td>Rubber-tired roller, deep penetration</td>
</tr>
<tr>
<td>ORGANIC CLAYS</td>
<td>M</td>
<td>Blue</td>
<td>Organic clays of medium to high plasticity, organic silts</td>
<td>Poor to very poor</td>
<td>Not suitable</td>
<td>Not suitable</td>
<td>Medium</td>
<td>Practically</td>
<td>Rubber-tired roller, deep penetration</td>
</tr>
</tbody>
</table>

**Table 48.3.—Soil property characteristics pertinent to roads and airfields (from U.S. Department of Defense, 1968).**
Table 4B.4.—Typical values of particle size and gradation and unit weight (adapted with permission from John Wiley & Sons from Basic Soils Engineering by B.K. Hough. Copyright© 1957 John Wiley & Sons.)

<table>
<thead>
<tr>
<th>Granular Materials</th>
<th>Particle Size and Gradation</th>
<th>In-Place Densities—Unit Weight (pcf)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D&lt;sub&gt;max&lt;/sub&gt;</td>
<td>D&lt;sub&gt;min&lt;/sub&gt;</td>
</tr>
<tr>
<td>Uniform Materials</td>
<td>a. Equal spheries (Theoretical values)—Laboratory</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>b. Standard Ottawa SAND—laboratory</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>c. Clean, uniform SAND (fine or medium) (SPI)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>d. Uniform, inorganic SILT (ML)</td>
<td>0.05</td>
</tr>
<tr>
<td>Well-graded Materials</td>
<td>a. Silty SAND (SM)</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>b. Clean, fine to coarse SAND (SW)</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>c. Micaeous SAND (SH, SM)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>d. Silty SAND and GRAVEL (SM, GP, QH, GW-QH)</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mixed Soils</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy or silty CLAY (CL, SC-CL, ML-CL)</td>
<td>2.3</td>
<td>0.001</td>
<td>0.003</td>
</tr>
<tr>
<td>Silo-graded silty CLAY with stones or rock fragment (ML-CL)</td>
<td>250</td>
<td>0.001</td>
<td>-</td>
</tr>
<tr>
<td>Well-graded GRAVEL, SAND, SILT and CLAY mixture (QC, GC-GM)</td>
<td>250</td>
<td>0.001</td>
<td>0.002</td>
</tr>
</tbody>
</table>

| Clay Soils | | | |
| CLAY (30 to 50% clay sized) (CH) | 0.05 | 5x10<sup>-6</sup> | 0.001 | - | 50 | 105 | 112 | 94 | 133 | 31 | 71 |
| Colloidal CLAY (<0.002 mm, ≥ 50%) (CH) | 0.01 | 1x10<sup>-6</sup> | - | - | 13 | 90 | 106 | 71 | 128 | 8 | 58 |

| Organic Soils | | | |
| Organic SILT (CL, QH) | - | - | - | - | 40 | - | 110 | 87 | 131 | 25 | 89 |
| Organic CLAY (30 to 50% clay sized) (OM, OL) | - | - | - | - | 30 | - | 100 | 81 | 125 | 18 | 62 |

Table 4B.5.—Relationship of standard penetration test blow counts (N<sub>S</sub>) to relative density (D<sub>r</sub>) (after U.S. Department of Agriculture, 1981). (See also Tables 4C.3, 4C.6, and Section 4C.4.2.3.)

<table>
<thead>
<tr>
<th>N&lt;sub&gt;S&lt;/sub&gt;</th>
<th>Relative Density, D&lt;sub&gt;r&lt;/sub&gt;</th>
<th>Descriptive Adjective</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>(blows per foot)</td>
<td>Descriptive Adjective</td>
<td>Percentage</td>
<td></td>
</tr>
<tr>
<td>0–4</td>
<td>Very Loose</td>
<td>0–15</td>
<td></td>
</tr>
<tr>
<td>4–10</td>
<td>Loose</td>
<td>15–35</td>
<td></td>
</tr>
<tr>
<td>10–30</td>
<td>Medium Dense</td>
<td>35–65</td>
<td></td>
</tr>
<tr>
<td>30–50</td>
<td>Dense</td>
<td>65–85</td>
<td></td>
</tr>
<tr>
<td>&gt;50</td>
<td>Very Dense</td>
<td>85–100</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4B.2.—Relationships among vertical effective stress ($\sigma_v$), relative density ($D_r$), and standard penetration test blow counts ($N$) for cohesionless soils (from U.S. Department of Agriculture, 1981).

Table 4B.6.—Relationship between relative density ($D_r$) and effective angle of internal friction ($\phi'$) by soil type (from U.S. Department of Agriculture, 1981).

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$\phi'$</th>
<th>$k_{\phi1}$</th>
<th>$k_{\phi2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_r = 0%$</td>
<td>$D_r = 100%$</td>
<td>$D_r = 0%$</td>
<td>$D_r = 100%$</td>
</tr>
<tr>
<td>GW</td>
<td>35°</td>
<td>45°</td>
<td>1.43</td>
</tr>
<tr>
<td>GP or GM or Coarse SW</td>
<td>33°</td>
<td>43°</td>
<td>1.54</td>
</tr>
<tr>
<td>Med. SW or Coarse SP or SM</td>
<td>31°</td>
<td>41°</td>
<td>1.66</td>
</tr>
<tr>
<td>Fine SW or Med. SP or SM</td>
<td>29°</td>
<td>39°</td>
<td>1.80</td>
</tr>
<tr>
<td>Fine SP or SM</td>
<td>27°</td>
<td>37°</td>
<td>1.96</td>
</tr>
<tr>
<td>ML</td>
<td>26°</td>
<td>36°</td>
<td>2.05</td>
</tr>
</tbody>
</table>

$\cot \phi = \frac{1}{\tan \phi} = k_{\phi1} - k_{\phi2} D_r$, where $k_{\phi1}$ and $k_{\phi2}$ are adjustment factors.
Figure 4B.3.—Correlation plot for estimating $\gamma_{sat}$ and $\phi'$ from $\gamma_d$ and $D_t$ for cohesionless soils (from U.S. Department of Agriculture, 1981).

Figure 4B.4.—Correlation plot for estimating $\gamma_{sat}$ and $\phi'$ from $\gamma_d$ for non-plastic soils (from Hammond et al., 1992).
Figure 4B.5.—Correlation between consistency and unconfined compressive strength ($q_u$); unconfined compressive strength and standard penetration test blow counts ($N$); and relative density ($D_r$) and unit weight by soil type for cohesive soils with $\phi = 0$ (from U.S. Department of Agriculture, 1981 with some data from Introductory soil mechanics and foundations: geotechnical engineering, fourth edition, by George F. Sowers. Copyright© 1979 by Macmillan College Publishing Company. Used with the permission of Macmillan College Publishing Company.)

**Examples**

**Example 1.** Estimate the dry, moist, and saturated unit weight for a non-plastic silt easily penetrated by driving a no. 4 rebar with a 5-pound hammer. From figure 4B.2: $D_r = 50\%$, Medium Dense ML. From figure 4B.4: $\gamma_d = 85$ pcf; using $w = 20\%$, then $\gamma = 105$ pcf, $\gamma_{sat} = 115$ pcf. From table 4B.3: $\gamma_d$ is between 90 and 130 pcf. From table 4B.4: $\gamma_d$ is between 80 and 118 pcf, $\gamma$ is between 81 and 136 pcf, and $\gamma_{sat}$ is between 115 and 137 pcf.

**Example 2.** Estimate the dry, moist, and saturated unit weight for a decomposed granite residual silty sand soil having a standard penetration test blow count of 45 blows per foot. From figure 4B.2: $D_r = 80\%$, Dense SM. From figure 4B.4: $\gamma_d = 110$ pcf; using $w = 10\%$, then $\gamma = 120$ pcf, $\gamma_{sat} = 130$ pcf. From table 4B.3: $\gamma_d$ is between 100 and 135 pcf. From table 4B.4: $\gamma_d$ is between 87 and 127 pcf.

**Example 3.** Estimate the dry, moist, and saturated unit weight for a loose fluvial coarse but poorly-graded sand. From figure 4B.2: $D_r = 25\%$, Loose SP. From figure 4B.4: $\gamma_d = 95$ pcf; using $w = 10\%$, then $\gamma = 105$ pcf, $\gamma_{sat} = 120$ pcf. From table 4B.3: $\gamma_d$ is between 105 and 135 pcf. From table 4B.4: $\gamma_d$ is between 83 and 118 pcf.
Example 4. Estimate the dry, moist, and saturated unit weight for an overconsolidated silty-sand-and-gravel glacial till. From figure 4B.2: $D_r = 90\%$, Very Dense GM. From figure 4B.4: $\gamma_d = 135 \text{ pcf}$; using $w = 5\%$, then $\gamma = 145 \text{ pcf}$, $\gamma_{sat} = 150 \text{ pcf}$. From table 4B.3: $\gamma_d$ is between 115 and 145 pcf. From table 4B.4: $\gamma_d$ is between 89 and 146 pcf.

Example 5. Estimate the dry, moist, and saturated unit weight for a well-graded gravel which can be penetrated only about a foot by a 1/2-inch rod driven with a 5-pound hammer. From figure 4B.2: $D_r = 80\%$, Dense GW. From figure 4B.4: $\gamma_d = 140 \text{ pcf}$; using $w = 5\%$, then $\gamma = 150 \text{ pcf}$, $\gamma_{sat} = 150 \text{ pcf}$. From table 4B.3: $\gamma_d$ is between 125 and 140 pcf. From table 4B.4: $\gamma_d$ is between 89 and 146 pcf.

Example 6. Estimate the dry, moist, and saturated unit weight for a plastic clay which squeezes between the fingers with fist closed. From figure 4B.5: Very Soft CL. $\gamma_{sat} = \gamma_{sub} + \gamma_w = 35 + 62.4 = 100 \text{ pcf}$, $\gamma = 95 \text{ pcf}$. Assume $w_{sat} = 40\% = (\gamma / \gamma_d) - (1/G)$.

\[
\gamma_d = \frac{\gamma_w}{w_{sat} + \frac{1}{G}} = \frac{62.4 \text{ pcf}}{0.40 + \frac{1}{2.68}} = 80 \text{ pcf}
\]

then from table 4B.3: $\gamma_d$ is between 90 and 130 pcf. From table 4B.4: $\gamma_d$ is between 60 and 135 pcf.

Example 7. Estimate the Unified Soil Classification, the consistency or relative density, and the dry, moist, and saturated unit weights for a poorly-graded sand with silt easily penetrated by a no. 4 rebar pushed by hand. Use $G = 2.68$. Estimated moist $w = 25\%$. From figure 4B.2: $D_r = 25\%$, Loose SM. From figure 4B.4: $\gamma_d = 95 \text{ pcf}$, $\gamma = 120 \text{ pcf}$, $\gamma_{sat} = 125 \text{ pcf}$. From table 4B.3: $\gamma_d = 100 \text{ pcf}$. From table 4B.4: $\gamma_d = 90 \text{ pcf}$.

Compare:

\[
\gamma_{sat} = \gamma_d(1 + w) = (95 \text{ pcf})(1.25) = 119 \text{ pcf}
\]

\[
\gamma_{sat} = \left(\frac{G - 1}{G}\right)\gamma_d + \gamma_w = \left(\frac{2.68 - 1}{2.68}\right)(95 \text{ pcf}) + 62.4 \text{ pcf} = 122 \text{ pcf}
\]

Example 8. Estimate the unified soil classification, the consistency or relative density, and the dry, moist, and saturated unit weights for a silty clay of low plasticity easily molded by finger. Estimated moist $w = 35\%$. From table 4B.3: $\gamma_d = 95 \text{ pcf}$. From table 4B.4: $\gamma_d = 70 \text{ pcf}$. From figure 4B.5: Soft CL or ML-CL.

Compare:

\[
\gamma_{sat} = \gamma_{sub} + \gamma_w = (40 \text{ pcf}) + (62.4 \text{ pcf}) = 102 \text{ pcf}
\]

\[
\gamma_{sat} = \gamma_d(1 + w) = (80 \text{ pcf})(1.35) = 108 \text{ pcf}
\]
Assume $w_{sat} = 40\%$

$$\gamma_d = \frac{\gamma_w}{w_{sat} + \frac{1}{G}} = \frac{62.4 \text{ pcf}}{0.40 + \frac{1}{2.68}} = 80 \text{ pcf}$$

Use $\gamma_d = 75 \text{ pcf}$

$$\gamma = (75 \text{ pcf})(1.35) = 100 \text{ pcf}$$

$$\gamma_{sat} = (75 \text{ pcf})(1.40) = 105 \text{ pcf}$$

**Example 9.** Estimate the unified soil classification, the consistency or relative density, and the dry, moist, and saturated unit weights for a silty sand with gravel which can be penetrated only a foot with a no. 4 rebar driven with a 5-pound hammer. Estimated moist $w = 10\%$. From figure 4B.2: $D_r = 80\%$, Dense SM. From figure 4B.4: $\gamma_d = 120 \text{ pcf}$. Using $w = 10\%$, then $\gamma = 130 \text{ pcf}$, $\gamma_{sat} = 140 \text{ pcf}$.

**Example 10.** Estimate the unified soil classification, the consistency or relative density, and the dry, moist, and saturated unit weights for a deposit of talus rock which has no soil fraction and is smaller than 18 inches in sieve-opening size. The estimated void volume is 20\% of the total volume. Estimated moist $w = 0\%$.

"Loose" talus rock

$$V_v = 0.20(V), \quad V_s = 0.80(V)$$

$$\gamma_v = G(\gamma_s) = 2.68 (62.4 \text{ pcf}) = 167.2 \text{ pcf}$$

$$W_s = V_s(\gamma_s) = 0.80 (1 \text{ ft}^3)(167.2 \text{ pcf}) = 133.8 \text{ lb}$$

$$\gamma_d = \frac{W_s}{V} = \frac{133.8 \text{ lb}}{(1.00 \text{ ft}^3)} = 135 \text{ pcf}$$

With $w = 0\%$:

$$\gamma = \gamma_d = 135 \text{ pcf}$$

$$\gamma_{sat} = \left(\frac{G - 1}{G}\right)\gamma_d + \gamma_w = \frac{1.68}{2.68} (135 \text{ pcf}) + (62.4 \text{ pcf}) \approx 145 \text{ pcf}$$

**4B.5 Compaction**

**4B.5.1 Introduction**

Compaction is the process of densifying the soil by reducing air voids and controlling the moisture content. Methods of compaction usually include the application of mechanical work energy.
Optimum compaction is that compaction at which, for the intensity of work energy applied, a maximum dry density is achieved. This compaction is associated with a moisture content called the optimum moisture content. At optimum compaction, the soil has reached relative equilibrium with optimum grain-to-grain contact and minimal air voids.

Maximum dry density can be changed by varying the intensity of work energy applied (the compactive effort). Maximum dry density increases with increasing compactive effort.

Compaction is performed to:

- Increase strength
- Decrease compressibility (settlement)
- Decrease permeability
- Improve freeze-thaw and wet-dry behavior
- Reduce erosion potential
- Control shrink and swell.

Degree of compaction is a function of:

- Dry density
- Water content
- Compactive effort (intensity of mechanical energy)
- Soil type
- Compaction equipment used.

Equipment used to compact soil includes:

- Sheepsfoot or tamping foot roller (has small feet and high unit pressure; good for compacting clayey soils);
- Rubber tire roller (good for sands to somewhat cohesive soils);
- Steel drum roller (smooth-wheel roller; good for achieving a very smooth surface; usually for coarse material);
- Vibratory roller (usually a steel drum roller; good for cohesionless soils);
- Construction equipment (scrapers, trucks, etc.; similar to rubber tire roller); and
- Hand tamping equipment (for areas the other equipment cannot access).
The moisture-density relationship is usually characterized by a moisture-density curve. Figure 4B.6 shows the general relationship.

**Figure 4B.6.—General moisture-density relationship for a soil.**

This relationship depends on the compactive effort applied. As compactive energy increases, the maximum dry density increases and the optimum water content decreases (see figure 4B.7). As can be seen, there is an infinite number of moisture-density curves, depending on the amount of mechanical energy put into the compactive effort; thus, a standard is needed to evaluate compaction. Standards for compactive effort used by the Forest Service include AASHTO T-99 and T-180. These are specifications for laboratory procedures to compact standard samples with standard compactive efforts.
Figure 4B.7.—Soil moisture-density relationship. As compactive energy increases, the maximum dry density increases and the optimum water content decreases. $S$ is soil saturation.

The family of curves for constant saturation is derived from the formulas given in table 4B.1. Note that for this soil (figure 4B.7), the optimum moisture/density point occurs at a saturation of about 85 percent. For any soil, the optimum moisture/density point will occur along a constant saturation curve; its location depends on the compactive effort used.

For practical purposes, the upper limit for percent saturation at the optimum moisture/density point is 90 percent. The 100 percent saturation line is called the zero-air-voids limit, the theoretical upper limit for compaction.

This section is based on material from chapter 20 of the U.S. Department of Agriculture Transportation Engineering Handbook (1973).

**Shrinkage**

A cubic yard of undisturbed earth will usually make less than a cubic yard of compacted embankment. This loss is called shrinkage. Shrinkage is really an apparent phenomenon due to the loss in volume through mechanical compaction. Shrinkage varies with different types of material and different locations. Sands and gravels shrink about 8 to 10 percent. Clay and loam shrink as much as 25 to 30 percent and sometimes even 50 percent.

**Subsidence**

New embankments tend to compress the underlying soils so that the original ground below the fill is lower than it was before the embankment was placed. This lowering is called subsidence. Subsidence varies from practically none, on old roadbeds or...
other stable areas, to several feet or more through swamps. Subsidence complicates
the problem of material balance.

Construction Losses

During construction, some material may be lost by erosion from rain or wind action. Also, even with careful engineering control, fills are often constructed a few inches
too wide or fill slopes are constructed a bit flatter than planned. Imbalance of cut
and fill is caused by roots, stumps, and sometimes large boulders removed from
excavation and not placed in embankments. Material is lost in transportation from
cut to fill. We can also compact material to a more dense condition than it was
before excavation. These losses are referred to as construction losses.

Compaction Factor

The sum total of shrinkage, subsidence, compaction, and construction losses is the
project shrinkage expressed as a compaction factor or earthwork adjustment factor; it
is calculated as the ratio of excavation yardage to compacted embankment yardage.
The terms compaction and shrinkage factor mean the same thing; thus, a shrinkage
of 1.15, usually referred to as 15 percent fill compaction, means that it will take 115
cubic yards of excavation from the original position to make 100 cubic yards of
compacted embankment. Stated another way, for every measured 100 cubic yards of
excavation only 87 cubic yards will be measured in the fill. Project compaction
factors vary not only with the type of material, but with the nature of the
construction. Light grading work has a higher shrinkage than heavy grading work.

Compaction factors can be defined in reference to either the fill volume or the cut
volume; but either way, the following components are included:

Compaction factor = Shrinkage (or swell) + Subsidence + Construction losses

The base equation for the compaction factor to be applied to the fill volume is (fill
compaction factor is expressed as $fcf$ and shrinkage component as $sc$):

$$\% \ fcf\ (sc) = \left( \frac{\text{Volume in Cut}}{\text{Volume in Fill}} \right) - 1 = \frac{Y_{fill}}{Y_{cut}} - 1$$ (4B.1)

The base equation (used in the Forest Service road design system) for the compaction
factor to be applied to the cut volume is (cut compaction factor is expressed as $ccf$):

$$\% \ ccf\ (sc) = 1 - \left( \frac{\text{Volume in Fill}}{\text{Volume in Cut}} \right) = 1 - \frac{Y_{cut}}{Y_{fill}}$$ (4B.2)

Swell

Solid hard rock swells when it is excavated—one cubic yard in original position
makes more than one cubic yard of embankment. On many rock projects, swell
exceeds the subsidence and construction losses and results in a project shrinkage
factor less than one. When solid rock breaks in large chunks, 1 cubic yard of rock
excavation may make 1.3 to 1.6 cubic yards of embankment, provided construction
losses due to blasting are minimal. However, the 1.6 swell is rare. Almost every
rock job has some earth or soft rock breaking into fines or materials which are
unavoidably lost because of the nature of the blasting operation.
It is not recommended to rely on any chart in selecting compaction factors. However, table 4B.7 can be used as a guide in the field to help the designer estimate compaction, shrinkage, and stripping losses for earthwork computations. Any combination can be used to determine the compaction factor. Remember, when it comes to selecting compaction factors, there is no substitute for experience, laboratory reports, and field inspection.

Table 4B.7.—Typical compaction factors for Forest Service road construction (from U.S. Department of Agriculture, 1973).

<table>
<thead>
<tr>
<th>TYPE OF EXCAVATION</th>
<th>COMPACTION FACTOR</th>
<th>TYPE OF CONSTRUCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fill</td>
<td>Below 18%</td>
<td>Through cuts</td>
</tr>
<tr>
<td>Cut</td>
<td>Below 15%</td>
<td>Previously cleared land</td>
</tr>
<tr>
<td>Fill</td>
<td>20%–25%</td>
<td>Extremely large cuts</td>
</tr>
<tr>
<td>Cut</td>
<td>17%–20%</td>
<td>Rock excavation for deep fills</td>
</tr>
<tr>
<td>Fill</td>
<td>25%–35%</td>
<td>Clay and clay-like shale soil</td>
</tr>
<tr>
<td>Cut</td>
<td>20%–26%</td>
<td>Clean common borrow</td>
</tr>
<tr>
<td>Fill</td>
<td>25%–35%</td>
<td>Medium to heavy clearing sandy soil</td>
</tr>
<tr>
<td>Cut</td>
<td>26%–33%</td>
<td>Fill-sized cobbles less than 1’ in diameter</td>
</tr>
<tr>
<td>Fill</td>
<td>Over 50%</td>
<td>Heavy clearing</td>
</tr>
<tr>
<td>Cut</td>
<td>Over 33%</td>
<td>Large timber with large roots</td>
</tr>
</tbody>
</table>

Table 4B.7 is intended only as a rough guide for the designer to estimate “ballpark factors.” The designer can estimate the shrinkage (or swell) component by a comparison between the dry densities expected in the cut and fill for a given soil type and a specified type of construction. The following examples illustrate how shrinkage can change with compactive effort.
**Shrinkage Estimate Example 1**

A new timber-purchase road is to be constructed using compaction control of the embankment. Several in-place density measurements have been made well below the topsoil and indicate a representative dry density of 105 pcf for the material to be excavated. Laboratory tests on the same material by AASHTO T-99 show a maximum dry density of 128 pcf. The material is a GM: silty gravel. Estimate the percent cut compaction factor for the shrinkage component.

Solution:

\[ \gamma_{\text{est}} \text{ (in-place dry density)} = 105 \text{ pcf} \]

Assuming only minimum compaction requirements will be achieved (90% AASHTO T-99),

\[ \gamma_{\text{fill}} = (0.90)(128 \text{ pcf}) = 115 \text{ pcf} \]

\[ \% \text{ ccf (sc)} = 1 - \frac{105 \text{ pcf}}{115 \text{ pcf}} = 9\% \text{ shrinkage} \]

Because 9 percent is a reasonable shrinkage to anticipate for this soil type, the assumption might be made that table 4B.7 would also give a reasonable total compaction factor to represent the overall type of construction. For example, if a total compaction factor of 25 percent were found to be reasonably accurate during construction of this road, the following conclusions might be drawn:

Because compaction factor = shrinkage (or swell) + subsidence + construction losses, the net effect of all components other than shrinkage of the GM soil can be considered to be the difference between the total and GM soil shrinkage, or:

\[ 25\% = 9\% + \% \text{ compaction factor (other components)} \]

\[ \% \text{ compaction factor (components other than GM shrinkage)} = 16\% \]

This is not necessarily a reasonable value to account for these other variables; only construction experience can tell. For the sake of comparison, this value will be held constant in the following examples, with variations made in shrinkage.

**Shrinkage Estimate Example 2**

Use the same conditions as in example 1, but assume 100% AASHTO T-99 compaction in embankment construction. Estimate the percent cut compaction factor for the shrinkage component.

Solution:

\[ \gamma_{\text{est}} = 105 \text{ pcf} \]

\[ \gamma_{\text{fill}} (100\% \text{ AASHTO T-99}) = 128 \text{ pcf} \]
\[
\% \text{ ccf (sc)} = 1 - \frac{105 \text{ pcf}}{128 \text{ pcf}} = 18\% \text{ shrinkage.}
\]

Using the 16\% compaction from "other components" in example 1,

\[\% \text{ ccf} = 18\% + 16\% = 34\%
\]

**Shrinkage Estimate Example 3**

Assume the same conditions as in example 1, except with no compaction control and with expected fill density the same as that of the cut. Estimate the percent cut compaction factor for the shrinkage component.

Solution:

\[
\gamma_{cut} = 105 \text{ pcf}
\]
\[
\gamma_{fill} = \gamma_{cut} = 105 \text{ pcf}
\]
\[
\% \text{ ccf (sc)} = 1 - \frac{105 \text{ pcf}}{105 \text{ pcf}} = 0\% \text{ shrinkage.}
\]

This would change the total \% cut compaction factor to:

\[\% \text{ ccf} = 0\% + 16\% = 16\%
\]

**Shrinkage Estimate Example 4**

Assume the same conditions as in example 3, except that expected fill density is 95 pcf. Estimate the percent cut compaction factor for the shrinkage component.

Solution:

\[
\gamma_{cut} = 105 \text{ pcf}
\]
\[
\gamma_{fill} = 95 \text{ pcf}
\]
\[
\% \text{ ccf (sc)} = 1 - \frac{105 \text{ pcf}}{95 \text{ pcf}} = -11\% \text{ shrinkage.}
\]
\[
= 11\% \text{ swell.}
\]

This would change the total \% cut compaction factor to:

\[\% \text{ ccf} = -11\% + 16\% = 5\%
\]
4B.5.4 Tables

Table 4B.8 lists some typical properties of compacted soils for use in preliminary analysis. For final analysis engineering property tests are necessary.

Table 4B.9 shows the relative desirability for various soil types in earth fill dams, canals, roadways, and foundations. A rating of 1 is considered the best, and 14 the least desirable.

Table 4B.10 is a summary of compaction requirements of fills for various purposes. Modify these to meet conditions and materials for specific projects.

Table 4B.11 lists commonly used compaction equipment with typical sizes and weights and guidance on use and application.
### Table 4B.8.—Typical properties of compacted soils (after U.S. Department of the Navy, 1982).

<table>
<thead>
<tr>
<th>Group Symbol</th>
<th>Soil Type</th>
<th>Range of Maximum Dry Unit Weight (pcf)</th>
<th>Range of Optimum Moisture, Percent</th>
<th>Typical Value of Compression</th>
<th>Typical Strength Characteristics</th>
<th>Range of CBR Values</th>
<th>Range of Subgrade Modulus k (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>At 20 psi</td>
<td>At 50 psi</td>
<td>Cohesion (as Compacted)(psi)</td>
<td>Cohesion (Saturated)(psi)</td>
<td>Effective Stress Envelope (Degrees)</td>
<td>Tan Ø</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Percent of Original Height</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GW</td>
<td>Well graded clean gravels, gravel-sand mixtures</td>
<td>125-135</td>
<td>11-8</td>
<td>0.3</td>
<td>0.6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GP</td>
<td>Poorly graded clean gravels, gravel-sand mix</td>
<td>115-125</td>
<td>14-11</td>
<td>0.4</td>
<td>0.9</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GM</td>
<td>Silty gravels, poorly graded gravel-sand-silt</td>
<td>120-135</td>
<td>12-8</td>
<td>0.5</td>
<td>1.1</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>GC</td>
<td>Clayey gravels, poorly graded gravel-sand-clay</td>
<td>115-130</td>
<td>14-9</td>
<td>0.7</td>
<td>1.6</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>SW</td>
<td>Well graded clean sands, gravelly sands</td>
<td>110-130</td>
<td>16-9</td>
<td>0.6</td>
<td>1.2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SP</td>
<td>Poorly graded clean sands, sand-gravel mix</td>
<td>100-120</td>
<td>21-12</td>
<td>0.8</td>
<td>1.4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SM</td>
<td>Silty sands, poorly graded sand-silt mix</td>
<td>110-125</td>
<td>16-11</td>
<td>0.8</td>
<td>1.6</td>
<td>1050</td>
<td>420</td>
</tr>
<tr>
<td>SM-SC</td>
<td>Sand-silt clay mix with slightly plastic fines</td>
<td>110-130</td>
<td>15-11</td>
<td>0.8</td>
<td>1.4</td>
<td>1050</td>
<td>300</td>
</tr>
<tr>
<td>SC</td>
<td>Clayey sands, poorly graded sand-clay mix</td>
<td>105-125</td>
<td>19-11</td>
<td>1.1</td>
<td>2.2</td>
<td>1560</td>
<td>230</td>
</tr>
<tr>
<td>ML</td>
<td>Inorganic silts and clayey silts</td>
<td>95-120</td>
<td>24-12</td>
<td>0.9</td>
<td>1.7</td>
<td>1400</td>
<td>190</td>
</tr>
<tr>
<td>ML-CL</td>
<td>Mixture of inorganic silts and clay</td>
<td>100-120</td>
<td>22-12</td>
<td>1.0</td>
<td>2.2</td>
<td>1250</td>
<td>460</td>
</tr>
<tr>
<td>CL</td>
<td>Inorganic clays of low to medium plasticity</td>
<td>95-120</td>
<td>24-12</td>
<td>1.3</td>
<td>2.5</td>
<td>1800</td>
<td>270</td>
</tr>
<tr>
<td>OL</td>
<td>Organic silts and silt-clays, low plasticity</td>
<td>80-100</td>
<td>33-21</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>MH</td>
<td>Inorganic clayey silts, plastic silts</td>
<td>70-95</td>
<td>40-24</td>
<td>2.0</td>
<td>3.8</td>
<td>1500</td>
<td>420</td>
</tr>
<tr>
<td>CH</td>
<td>Inorganic clays of high plasticity</td>
<td>75-105</td>
<td>36-19</td>
<td>2.6</td>
<td>3.9</td>
<td>2150</td>
<td>230</td>
</tr>
<tr>
<td>OH</td>
<td>Organic clays and silty clays</td>
<td>65-100</td>
<td>45-21</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

**Notes:**
- All properties are for condition of "standard proctor" maximum density, except values of CBR and CBR which are for "modified proctor" maximum.
- Typical strength characteristics are for effective strength envelopes and are obtained from USBR data.
- Compression values are for vertical loading with complete lateral confinement.
- (*) indicates that typical property is greater than the value shown.
- (...) indicates insufficient data available for an estimate.
Table 4B.9.—Relative desirability of soils as compacted fill (after U.S. Department of the Navy, 1982).

<table>
<thead>
<tr>
<th>Group Symbol</th>
<th>Soil Type</th>
<th>Rolled Earth Fill Dams</th>
<th>Canal Sections</th>
<th>Foundations</th>
<th>Roadways</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Homogeneous Embankment</td>
<td>Core</td>
<td>Shell</td>
<td>Erosion Resistance</td>
</tr>
<tr>
<td>GW</td>
<td>Well graded gravels, gravel-sand mixtures, little or no fines</td>
<td>—</td>
<td>—</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>GP</td>
<td>Poorly graded gravels, gravel-sand mixtures, little or no fines</td>
<td>—</td>
<td>—</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>GM</td>
<td>Silty gravels, poorly graded gravel-sand-silt mixtures</td>
<td>2</td>
<td>4</td>
<td>—</td>
<td>4</td>
</tr>
<tr>
<td>GC</td>
<td>Clayey gravels, poorly graded gravel-sand-clay mixtures</td>
<td>1</td>
<td>1</td>
<td>—</td>
<td>3</td>
</tr>
<tr>
<td>SW</td>
<td>Well-graded sands, gravelly sands, little or no fines</td>
<td>—</td>
<td>—</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>SP</td>
<td>Poorly graded sands, gravelly sands, little or no fines</td>
<td>—</td>
<td>—</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>SM</td>
<td>Silty sands, poorly graded sand-silt mixtures</td>
<td>4</td>
<td>5</td>
<td>—</td>
<td>8</td>
</tr>
<tr>
<td>SC</td>
<td>Clayey sands, poorly graded sand-clay mixtures</td>
<td>3</td>
<td>2</td>
<td>—</td>
<td>5</td>
</tr>
<tr>
<td>ML</td>
<td>Inorganic silts and very fine sands, rock flour, silty or clayey fine sands with slight plasticity</td>
<td>6</td>
<td>6</td>
<td>—</td>
<td>6</td>
</tr>
<tr>
<td>CL</td>
<td>Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays</td>
<td>5</td>
<td>3</td>
<td>—</td>
<td>9</td>
</tr>
<tr>
<td>OL</td>
<td>Organic silts and organic-silt-clays of low plasticity</td>
<td>8</td>
<td>8</td>
<td>—</td>
<td>7</td>
</tr>
<tr>
<td>MN</td>
<td>Inorganic silt, micaceous or diatomaceous fine sandy or silty soils, elastic silts</td>
<td>9</td>
<td>9</td>
<td>—</td>
<td>8</td>
</tr>
<tr>
<td>CH</td>
<td>Inorganic clays of High plasticity, fat clays</td>
<td>7</td>
<td>7</td>
<td>—</td>
<td>10</td>
</tr>
<tr>
<td>OH</td>
<td>Organic clays of medium-high plasticity</td>
<td>10</td>
<td>10</td>
<td>—</td>
<td>10</td>
</tr>
</tbody>
</table>

— Not appropriate for this type of use.
Table 4B.10.—Compaction requirements (after U.S. Department of the Navy, 1982).

<table>
<thead>
<tr>
<th>Fill Used for:</th>
<th>Required Density, Percent of Modified Proctor</th>
<th>Tolerable Range of Moisture About Optimum, Percent</th>
<th>Maximum Permissible Lift Thickness, Compacted In.</th>
<th>Special Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support of structure</td>
<td>95</td>
<td>-2 to +2</td>
<td>12</td>
<td>Fill should be uniform. Blending or processing of borrow may be required. For plastic clays, investigate expansion under saturation for various compaction moistures and densities at loads equal to those applied by the structure to determine condition to minimize expansion. Clays that show expansive tendencies generally should be compacted at or above optimum moisture to a density consistent with strength and incompressibility required of the fill.</td>
</tr>
<tr>
<td>Lining for canal or small reservoir</td>
<td>90</td>
<td>-2 to +2</td>
<td>6</td>
<td>For thick linings, GW-GC, GC, and SC are preferable for stability and to resist erosive forces. Single size silty sands with PI less than five generally are not suitable. Remove fragments larger than 6 inches before compaction.</td>
</tr>
<tr>
<td>Earth dam greater than 50 ft high</td>
<td>95</td>
<td>-1 to +2</td>
<td>12(+)</td>
<td>Utilize least pervious materials as central core and coarsest materials in outer shells. Core should be free of lenses, pockets, or layers of pervious material and successive lifts well bonded to each other. Amounts of oversize exceeding 1 percent of total material should be removed from the borrow prior to arrival on the embankment.</td>
</tr>
<tr>
<td>Earth dam less than 50 ft high</td>
<td>92</td>
<td>-1 to +3</td>
<td>12(+)</td>
<td>In small dams that lack elaborate zoning, materials that are the most vulnerable to cracking and piping should be compacted to 98 percent density at moisture content from optimum to 3 percent greater than optimum.</td>
</tr>
<tr>
<td>Support of pavements:</td>
<td></td>
<td></td>
<td></td>
<td>Place coarsest borrow materials at top of fill. Investigate expansion of plastic clays placed near pavement subgrade to determine compaction moisture and density that will minimize expansion and provide required soaked CBR values.</td>
</tr>
<tr>
<td>Highways...</td>
<td>See NAVFAC DM-5.</td>
<td>-2 to +2</td>
<td>8(+)</td>
<td></td>
</tr>
<tr>
<td>Airfields...</td>
<td>See NAVFAC DM-21.</td>
<td>-2 to +2</td>
<td>8(+)</td>
<td></td>
</tr>
<tr>
<td>Backfill surrounding structure</td>
<td>90</td>
<td>-2 to +2</td>
<td>8(+)</td>
<td>Where backfill is to be drained, provide pervious coarse-grained soils. For low walls, do not permit heavy rolling compaction equipment to operate closer to the wall than a distance equal to about 2/3 the unbalanced height of fill at any time. For highwalls or walls of special design, evaluate the surcharge produced by heavy compaction equipment by the methods of chapter 3 and specify safe distances back of the wall for its operations.</td>
</tr>
</tbody>
</table>
Table 4B.10.—Compaction requirements (after U.S. Department of the Navy, 1982) (continued).

<table>
<thead>
<tr>
<th>Fill Used for:</th>
<th>Required Density, Percent of Modified Proctor</th>
<th>Tolerable Range of Moisture About Optimum, Percent</th>
<th>Maximum Permissible Lift Thickness, Compacted In.</th>
<th>Special Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backfill in pipe or utility trenches</td>
<td>90</td>
<td>-2 to +2</td>
<td>8(+)</td>
<td>Material excavated from trench generally is suitable for backfill if it does not contain organic matter or refuse. If backfill is fine grained, a cradle for the pipe is formed in natural soil and backfill placed by tamping to provide the proper bedding. Where free draining sand and gravel is utilized, the trench bottom may be finished flat and the granular material placed saturated under and around the pipe and compacted by vibration.</td>
</tr>
<tr>
<td>Drainage blanket or filter</td>
<td>90</td>
<td>Thoroughly wetted</td>
<td>8</td>
<td>Ordinarily vibratory compaction equipment is utilized. Blending of materials may be required for homogeneity. Segregation must be prevented in placing and compaction. For compaction adjacent to and above drainage pipe, use hand tamping or light traveling vibrators.</td>
</tr>
<tr>
<td>Subgrade of excavation for structure</td>
<td>95</td>
<td>-2 to +2</td>
<td>-</td>
<td>For uniform bearing or to break up pockets of frost-susceptible material, scarify the upper 8 to 12 in. of the subgrade, dry or moisten as necessary, and recompact. Certain materials, such as heavily preconsolidated clays which will not benefit by compaction or saturated silts and silty fine sands that become quick during compaction, should be blanketed with a working mat of lean concrete or coarse-grained material to prevent disturbance or softening. Depending on foundation conditions revealed in exploration, a substantial thickness of loose soils may have to be removed below subgrade and recompacted or compacted in place by vibration or pile driving.</td>
</tr>
<tr>
<td>Rock fill</td>
<td>Thoroughly wetted</td>
<td>2 to 3 ft</td>
<td>-</td>
<td>For fill containing sizes no larger than 1 ft, place in layers not exceeding 24 in., thoroughly wetted and compacted by travel or heavy crawler tractors in spreading. Material with sizes up to 2 ft may be placed in 3 ft lifts. Placing should be such that the maximum size of rock increases toward the outer slopes. Rocks larger than 1 yd$^3$ in volume should be embedded on the slope.</td>
</tr>
</tbody>
</table>

Notes: Density and moisture content refer to "standard proctor" test values (ASTM D 698). Generally, a fill compacted dry of OMC will have higher strength and a lower compressibility even after saturation. Compaction of "coarse-grained, granular soil" is not sensitive to moisture content as long as bulking moisture is avoided. Where practical it should be placed saturated and compacted by vibratory methods.
Table 4B.11.—Compaction equipment and methods (after U.S. Department of the Navy, 1982).

<table>
<thead>
<tr>
<th>Equipment Type</th>
<th>Applicability</th>
<th>Requirements for Compaction of 95 to 100 Percent Standard Proctor Maximum Density</th>
<th>Possible Variations in Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Compacted Lift Thickness, in</td>
<td>Passes or Coversages</td>
</tr>
<tr>
<td>Sheepfoot Rollers</td>
<td>For fine-grained soils or dirty coarse-grained soils with more than 20 percent passing no. 200 sieve. Not suitable for clean coarse-grained soils. Particularly appropriate for compaction of impervious zone for earth dam or linings where bonding of lifts is important.</td>
<td>6</td>
<td>4 to 6 passes for fine-grained soil</td>
</tr>
<tr>
<td></td>
<td>For fine-grained soils or well graded, dirty coarse-grained soils with more than 8 percent passing the no. 200 sieve.</td>
<td>6 to 8</td>
<td>6 to 8 passes for coarse-grained soil</td>
</tr>
<tr>
<td>Rubber Tire Roller</td>
<td>For clean, coarse-grained soils with 4 to 8 percent passing the No. 200 sieve.</td>
<td>10</td>
<td>3 to 5 coverages</td>
</tr>
<tr>
<td></td>
<td>For fine-grained soils or well graded, dirty coarse-grained soils with more than 8 percent passing the no. 200 sieve.</td>
<td>6 to 8</td>
<td>4 to 6 coverages</td>
</tr>
<tr>
<td>Smooth Wheel Rollers</td>
<td>Appropriate for subgrade or base coarse compaction of well-graded sand-gravel mixtures. May be used for fine-grained soils other than in earth dams. Not suitable for clean well-graded sands or silty uniform sands.</td>
<td>8 to 12</td>
<td>4 coverages</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 to 8</td>
<td>6 coverages</td>
</tr>
</tbody>
</table>

Foot Contact Area (ft²) | Foot Contact Pressures (psi) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil Type</td>
<td></td>
</tr>
<tr>
<td>Fine-grained soil Pb&gt;30</td>
<td>250 to 500</td>
</tr>
<tr>
<td>Fine-grained soil Pb&lt;30</td>
<td>200 to 400</td>
</tr>
<tr>
<td>Coarse-grained soil</td>
<td>150 to 250</td>
</tr>
</tbody>
</table>
Table 4B.11.—Compaction equipment and methods (after U.S. Department of the Navy, 1982) (continued).

<table>
<thead>
<tr>
<th>Equipment Type</th>
<th>Applicability</th>
<th>Requirements for Compaction of 95 to 100 Percent Standard Proctor Maximum Density</th>
<th>Possible Variations in Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Compacted Lift Thickness, In.</td>
<td>Passes or Coverages</td>
</tr>
<tr>
<td>Vibrating Sheepfoot Rollers</td>
<td>For coarse-grained soils sand-gravel mixtures</td>
<td>8 to 12</td>
<td>3 to 5</td>
</tr>
<tr>
<td>Vibrating Smooth Drum Rollers</td>
<td>For coarse-grained soils sand-gravel mixtures—rock fills</td>
<td>6 to 12 (soil) to 36 (rock)</td>
<td>3 to 5</td>
</tr>
<tr>
<td>Vibrating Baseplate Compactors</td>
<td>For coarse-grained soils with less than about 12 percent passing no. 200 sieve. Best suited for materials with 4 to 8 percent passing no. 200 sieve, placed thoroughly wet.</td>
<td>8 to 10</td>
<td>3 coverages</td>
</tr>
<tr>
<td>Crawler Tractor</td>
<td>Best suited for coarse-grained soils with less than 4 to 8 percent passing no. 200 sieve, placed thoroughly wet.</td>
<td>6 to 10</td>
<td>3 to 4 coverages</td>
</tr>
<tr>
<td>Power Tamper or Rammer</td>
<td>For difficult access, trench backfill. Suitable for all inorganic soils.</td>
<td>4 to 6 in. for silt or clay, 6 in. for coarse-grained soils</td>
<td>2 coverages</td>
</tr>
</tbody>
</table>
4C. Strength and Behavior of Soil

René Renteria, Geotechnical Engineer, Intermountain Regional Office

4C.1 Shear Strength of Non-Cohesive Soils

For the purposes of this discussion, non-cohesive soil (also known as cohesionless or granular soil) refers to sands, non-plastic silts, gravels, and granular material. According to the American Society for Testing and Materials (ASTM, 1992), a cohesionless soil is defined as "a soil that when unconfined has little or no strength when air-dried and that has little or no cohesion when submerged." The general equation for shear strength given by the Mohr-Coulomb envelope is:

$$\tau' = c' + \sigma'\tan\phi'$$  (4C.1)

For non-cohesive material, the cohesion intercept is typically assumed to pass through the origin, and is thus ignored. However, in some cases cohesion is used to satisfy equilibrium for an assumed friction angle. The cohesion may be the result of apparent cohesion from capillary action or the use of a linear Mohr-Coulomb envelope instead of a curved envelope. In general, the strength of a non-cohesive material is described by the friction angle of the material ($\phi$).

4C.1.1 Angle of Repose

The angle of repose is a term used to describe the steepest stable slope of a loose granular material. This is equivalent to the angle of internal friction for the material at its loosest state (Holtz and Kovacs, 1981). This condition can be observed for the sand in an hour glass or an aggregate material being stockpiled off a conveyor belt. Further discussion can be found in section 5B.5.

4C.1.2 Behavior of Sands During Drained Shear

The shear strength of a sand is highly influenced by the initial void ratio or relative density. According to ASTM (1992), relative density is defined as:

the ratio of (1) the difference between the void ratio of a cohesionless soil in the loosest state and any given void ratio, to (2) the difference between the void ratios in the loosest and in the densest states.

The equation for relative density, $D_r$, is given as (Dunn et al., 1980):

$$D_r = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}}$$  (4C.2)

$$D_r = \left( \frac{\gamma_{d_{\text{max}}}}{\gamma_d} \right) \left( \frac{\gamma_d - \gamma_{d_{\text{min}}}}{\gamma_{d_{\text{max}}} - \gamma_{d_{\text{min}}}} \right)$$  (4C.3)

where $e$ is void ratio.
As demonstrated by equations 4C.2 and 4C.3, a relative density of 0 percent would occur when the void ratio is greatest and density is at a minimum. A relative density of 100 percent will occur at the minimum void ratio and maximum density. Because density is easier to measure than void ratio, density is more often used to calculate $D_r$.

Loose sands and dense sands behave differently during the application of load. As a load is applied to a loose sand, the soil particles move from a loose packing state to a tight packing state. This particle rearrangement produces a decrease in void ratio (and a corresponding decrease in volume) which continues with the application of load until the void ratio approaches a value $e_r$. For a dense sand, the tightly packed soil particles may shift to a slightly tighter state before particles start to over-ride each other and dilate. This increase in void ratio (and corresponding increase in volume) will continue with the application of load until the increasing void ratio approaches the value of $e_r$. Therefore, $e_r$ is defined as the critical void ratio, where the theoretical volume change would be zero. According to ASTM (1992), the critical void ratio is defined as:

$$ e_r = \text{the void ratio corresponding to the critical density—the unit weight of a saturated granular material below which it will lose strength and above which it will gain strength when subjected to rapid deformation. The critical density of a given material is dependent on many factors.} $$

Shear strength for a sand increases with the dry unit weight of the material. Although particle shape, roughness, and grain-size distribution influence shear strength, their contribution mainly provides the strength differences within a given soil type (Dunn et al., 1980). Because shear strength increases with increasing density, it can be shown from equation 4C.3 that shear strength also increases with increasing relative density. This relationship is shown in table 4C.3 and figure 4C.1.

### 4C.1.3 Behavior of Sands During Undrained Shear

**In general, the permeability of granular material is great enough to allow dissipation of changing pore-water pressures which can develop during a change in stress. However, some fine sands and silty sands may have sufficiently low permeability to behave as if undrained during application of a load. The behavior of sands during undrained shear is greatly influenced by the initial void ratio. A loose sand will tend to decrease in volume with increasing stress as the sand particles realign into a denser state. Under undrained conditions, the tendency to decrease volume results in an increase in pore-water pressure. When this increasing pressure reaches the effective stress of the soil, the soil liquefies. This phenomenon is known as liquefaction and is considered to occur under static loading conditions. Static loading includes such changes in effective stress as overburden pressure, seepage forces, and surcharge loads (Holtz and Kovacs, 1981). Mechanisms that cause changes in effective stress include streambank erosion after material is eroded, ground water exiting a newly excavated cut slope, and footstep pressure on a saturated soil deposit (commonly referred to as "quicksand").**

Dense sands behave differently than loose sands. When an increase in stress is applied to a dense sand, the volume actually increases as the particles overcome a densely packed state and dilate (roll over each other). Under undrained static loading conditions, this results in a decrease in pore-water pressure; therefore, static liquefaction is a phenomenon only of loose sands and silts. However, liquefaction
can occur in medium and dense sands under dynamic loading. This is also referred to as cyclic mobility. Dynamic loads can occur as a result of earthquakes, pile driving, or blasting. Loose sands behave as expected—few cycles are needed to create liquefaction. The response of dense sands tends to be influenced by the relative density and the initial confining pressure. Lower confining pressures require fewer cycles to failure (Holtz and Kovacs, 1981).

Methods for mitigating liquefaction potential are limited. For static loading, pore pressures can be monitored, the soil material densified or replaced with a higher density material, or ground water can be drained. For dynamic loads, the intensity and number of cycles cannot be controlled. Unlike liquefaction, cyclic mobility impacts all densities of sand. One method of mitigating cyclic mobility is to increase the confining pressure. This is often possible by placing a surcharge using non-susceptible material. Another method is to increase the soil density, because more cycles are required to liquefy a dense material than a loose material. However, predicting the potential site dynamic cycles requires a more rigorous analysis of dynamic loading. The methods of cyclic testing are not discussed here, and the reader is referred to literature on dynamic soil mechanics.

4C.1.4 Factors That Affect the Shear Strength of Granular Soils

The shear strength of sand is affected by the components of frictional resistance, $\phi$. Some factors that influence $\phi$ are given by Holtz and Kovacs (1981):

- Void ratio or relative density
- Particle shape
- Grain size distribution
- Particle surface roughness
- Water
- Intermediate principal stress
- Particle size
- Overconsolidation or prestress.

Void ratio, which is related to density, is considered the single most important variable influencing the shear strength of sands. Table 4C.1 gives a summary of influencing factors and their effect on $\phi$. 
Table 4C.1.—Summary of factors affecting friction angle (redrawn with permission of Prentice-Hall from p. 517 of An introduction to geotechnical engineering by R.D. Holtz and W.D. Kovacs. Copyright © 1981 Prentice-Hall).

<table>
<thead>
<tr>
<th>Factor</th>
<th>Effect of Increase in Factor on ( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Void ratio ( e )</td>
<td>decrease</td>
</tr>
<tr>
<td>Angularity ( A )</td>
<td>increase</td>
</tr>
<tr>
<td>Grain size distribution</td>
<td>increase</td>
</tr>
<tr>
<td>Surface roughness ( R )</td>
<td>increase</td>
</tr>
<tr>
<td>Water ( W )</td>
<td>decrease slightly</td>
</tr>
<tr>
<td>Particle size ( S )</td>
<td>no effect (with constant ( e ))</td>
</tr>
<tr>
<td>Intermediate principle stress</td>
<td>( \phi_{ps} &gt; \phi_{ix} )</td>
</tr>
<tr>
<td>Overconsolidation or prestress</td>
<td>little effect</td>
</tr>
</tbody>
</table>

The frictional strength of gravelly material is influenced by percentages of sand and gravel and by the relative density. According to Hammond et al. (1992; p. 67):

...Holtz and Ellis (1961) and Siddiqi (1984) showed that adding gravel to fine soils had no effect on friction angles until the soils contained more than about 50 to 65 percent gravel. Siddiqi explains that with less than about 50 to 65 percent gravel (depending on the specific gravity of the soil particles), the gravels merely are floating in a matrix of finer soil, and shear strength is controlled by the fine soil alone. The gravel fragments do not contribute to strength until there is a high enough percentage that the fragments are in contact with each other... Also note that many studies cited in the literature have compared friction angles of fine and gravelly soils at the same void ratio rather than at the same \( D_r \). Comparisons made on the basis of void ratio always show that the friction angles of sands are greater than those of gravels (Leslie, 1963; Marachi et al., 1969; Wu and Baladi, 1986). This is because the addition of coarse fragments decreases void ratio but also increases the limiting unit weights. Therefore, at a given void ratio, a gravel soil will behave during shear as a looser soil (lower \( D_r \)) than a sand soil will, resulting in lower friction angles for the gravel.

4C.1.5 Typical Shear Strength Values for Non-Cohesive Soils

A summary of shear strength parameters for non-cohesive soils is presented in tables 4C.2 through 4C.4 and figure 4C.1. Additional strength values can be found in the Level I Stability Analysis (LISA) manual (Hammond et al., 1992).
Table 4C.2.—Summary of suggested strength parameters for non-cohesive soils (after Hammond et al., 1992).

<table>
<thead>
<tr>
<th>Peak Strength</th>
<th>Apparent Cohesion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c'_s$</td>
<td>$\phi'_p$</td>
</tr>
<tr>
<td>0</td>
<td>From table 4C.4</td>
</tr>
<tr>
<td></td>
<td>table 4C.3</td>
</tr>
</tbody>
</table>

Table 4C.3.—Consistency versus standard penetration test blow counts (N), relative density, and friction angle for non-cohesive soils (after Peck et al., 1974).

<table>
<thead>
<tr>
<th>Consistency</th>
<th>Field Identification</th>
<th>$N$ (BPF)*</th>
<th>Relative Density (% $D_s$)</th>
<th>$\phi'$ for Sands (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Loose</td>
<td>below 4</td>
<td>below 15</td>
<td>25 to 30</td>
<td></td>
</tr>
<tr>
<td>Loose</td>
<td>Easily excavated with hand shovel</td>
<td>4 to 10</td>
<td>15 to 35</td>
<td>27 to 32</td>
</tr>
<tr>
<td>Medium</td>
<td>Difficult to excavate with hand shovel</td>
<td>10 to 30</td>
<td>35 to 65</td>
<td>30 to 35</td>
</tr>
<tr>
<td>Dense</td>
<td>Must be loosened with pick to excavate with hand shovel</td>
<td>30 to 50</td>
<td>65 to 85</td>
<td>35 to 40</td>
</tr>
</tbody>
</table>

*BPF is blows per foot, see section 4C.4.2.3
Table 4C.4.—Friction angle ($\phi'$) versus relative density ($D_r$) by soil type (U.S. Department of Agriculture, 1981).

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$\phi'$ (Degrees)</th>
<th>$D_r = 0%$</th>
<th>$D_r = 100%$</th>
<th>$k_{\phi1}$</th>
<th>$k_{\phi2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GW</td>
<td>35</td>
<td>45</td>
<td>1.43</td>
<td>0.0043</td>
<td></td>
</tr>
<tr>
<td>GP, GM, or Coarse SW</td>
<td>33</td>
<td>43</td>
<td>1.54</td>
<td>0.0047</td>
<td></td>
</tr>
<tr>
<td>Med. SW, Coarse SP, or SM</td>
<td>31</td>
<td>41</td>
<td>1.66</td>
<td>0.0051</td>
<td></td>
</tr>
<tr>
<td>Fine SW, Med. SP, or SM</td>
<td>29</td>
<td>39</td>
<td>1.80</td>
<td>0.0057</td>
<td></td>
</tr>
<tr>
<td>Fine SP or SM</td>
<td>27</td>
<td>37</td>
<td>1.96</td>
<td>0.0064</td>
<td></td>
</tr>
<tr>
<td>ML</td>
<td>26</td>
<td>36</td>
<td>2.05</td>
<td>0.0067</td>
<td></td>
</tr>
</tbody>
</table>

$\cot\phi' = k_{\phi1} \cdot k_{\phi2}D_r$; where $k_{\phi1}$ and $k_{\phi2}$ are adjustment factors.

Figure 4C.1.—Relationship between $D_r$ and $\gamma$ and $\phi'$ for non-plastic silts, sands, and gravel using $G_s = 2.68$ (from Hammond et al., 1992).
A cohesive soil is defined by ASTM (1992) as “a soil that when unconfined has considerable strength when air-dried and that has significant cohesion when submerged.” This definition includes clays as well as some non-“clay” soils (silt, sands, and gravels) with plasticity. The shear strength of cohesive soils depends on the consolidation state and stress history and the drainage conditions existing during stress loading. This section will discuss how each of these conditions influences the shear strength of cohesive soils.

4C.2.1 Consolidation State—Stress History

Consolidation theory is most often associated with settlement analysis of foundations on cohesive soils. An understanding of the soil consolidation state is required to properly define the soil strength. As the effective stress is increased on the soil, the void ratio decreases (compression). As does non-cohesive soil, cohesive soil shows an increase in strength with a decrease in void ratio (see table 4C.1); however, unlike a non-cohesive soil, which rebounds “elastically” upon removal of an added stress, a cohesive soil does not rebound to the same void ratio as before the preconsolidation stress was applied (the reader is referred to a text on consolidation theory, such as Dunn et al., 1980). Therefore, in the context of slope stability analysis, shear strength of a cohesive material at a given location is directly influenced by the consolidation stress history of the soil for that location at the time of failure.

4C.2.1.1. Normally Consolidated Soils

The normally consolidated (NC) stress state is a condition in which the soil is currently subjected to a confining stress that is equal to its maximum past stress (Dunn et al., 1980). As shown in figure 4C.2, the Mohr-Coulomb failure envelope for an NC soil passes through the origin and results in no true cohesion. When using undisturbed cohesive soil samples for laboratory strength testing (see section 4C.4.1), the confining consolidation pressure should be equal to or greater than the maximum past effective stress in order to produce the Mohr-Coulomb failure envelope for an NC soil. The in-situ effective stress of the sample is not necessarily the maximum past effective stress that the soil sample has experienced (see section 4C.2.1.2).
4C.2.1.2. Overconsolidated Soils

The overconsolidated (OC) stress state is a condition in which the soil has been subjected to an effective stress (preconsolidation) that is greater than the current effective stress (Dunn et al., 1980). Overconsolidation results from geologic conditions, such as deposition and subsequent erosion of overburden materials, glaciers, wetting and drying cycles near the ground surface, and fluctuations in pore-water pressure (Dunn et al., 1980; Hammond et al., 1992). Overconsolidation may also result from management activities, such as construction loading (Hammond et al., 1992), and possibly from modification of ground water regimes (increased water pressure resulting in decreased effective stress at depth).

For a stress below the preconsolidation stress, the soil will have a lower void ratio than an NC material at the same stress (the reader is referred to a text on consolidation theory, such as Dunn et al., 1980). The lower void ratio results in a higher shear strength than an NC soil up to the preconsolidation stress. Above the preconsolidation stress the OC soil behaves the same as the NC soil. Overconsolidation also results in true cohesion (Hammond et al., 1992). This relationship is shown in figure 4C.2.

The shear strength of OC clay soils can vary with depth. According to Hammond et al. (1992):

The strength of overconsolidated clays also is affected by weathering and fissuring, typically causing a large reduction in true $c'$ and a smaller reduction in $\phi'$. The weathering process eventually returns the clay to the normally consolidated state with its associated normally consolidated shear strength parameters. Weathering explains the common observation that overconsolidated clays are weaker near the ground surface than at depth.
Because $c'$ and $\phi'_p$ for overconsolidated clays depend on stress history, the current effective stress, and the degree of weathering, it is difficult to obtain typical values from the literature and be assured that they are appropriate for the current in situ conditions being analyzed...

Fortunately, there is a simplifying factor. Back-analyses on existing first-time failures in overconsolidated clays show that the average shear stress along the entire failure plane is much less than the peak strength of the clay as measured in the laboratory; in fact, the strength parameters corresponding to the average stress often are very close to $c'$ and $\phi'_p$ of the normally consolidated clay (Taylor and Cripps, 1987). Therefore, from a practical standpoint, it is probably not necessary to discern whether a clay is overconsolidated...

### 4C.2.2.1. Consolidated-Drained (CD)

The CD test is used to simulate most actual field conditions for effective stress analysis. The CD conditions simulate long-term consolidation and steady-state ground water. Some examples are natural slopes with steady ground water conditions, embankments with steady-state seepage, and long-term excavation or natural slopes in clay material (see figure 4C.3).

![Diagram](image)

Figure 4C.3.—Some examples of consolidated-drained (CD) analyses for clays: (a) earth dam and (b) excavation or natural slope (reprinted with permission of Charles Ladd and Prentice-Hall from p. 546 of An introduction to geotechnical engineering by R.D. Holtz and W.D. Kovacs. Copyright© 1981 Prentice-Hall. After Ladd, 1971).
Steady-state water conditions imply no buildup of undissipated pore-water pressures in the soil. Most non-cohesive soils are able to dissipate pore-water pressures created by fluctuating water levels in a short period of time (minutes to hours). Lengthier (days to weeks) steady-state water conditions are required for cohesive soils with low permeability to ensure pore-water pressures can dissipate during shear, and the resulting analysis will yield effective stress rather than total stress. It should be noted that steady-state water conditions are needed for CD conditions within the soil mass being analyzed for strength parameters. A fluctuating piezometric (as opposed to phreatic) "level" may "register" within an aquiclude over a confined aquifer but generally will not create the need to dissipate pore pressures within the aquiclude (separate water regimes).

Some typical values of shear strength for drained conditions are given in table 4C.5.

Table 4C.5.—Summary of strength parameters for saturated cohesive soils (after Hammond et al., 1992; with data added from Holtz and Kovacs, 1981).

<table>
<thead>
<tr>
<th>Consolidation State</th>
<th>Peak Strength</th>
<th>Apparent Cohesion</th>
<th>CU Parameters</th>
<th>UU Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>NC</td>
<td>0</td>
<td>From figure 4C.6</td>
<td>Depends on capillary suction. Values determined by back-analysis.</td>
<td>Undrained $\phi$ approximately 1/2 of drained $\phi'$ and cohesion approximately zero.</td>
</tr>
<tr>
<td>OC</td>
<td>Depends on stress history. Typically 100-500 psf.</td>
<td>Heavily OC 25-40°. Lightly OC 20-30°, or use NC $\phi'_c$.</td>
<td>Values of $\phi$ and $c$ may decrease slightly from CD parameters.</td>
<td>Use field conditions and estimate using table 4C.6.</td>
</tr>
</tbody>
</table>

4C.2.2.2. Consolidated-Undrained (CU)

The CU strength parameters are useful for both laboratory and field problems. The CU test with pore pressure measurements is often used to obtain the CD strength parameters. This method is much quicker than trying to run a CD test at the proper low speed for a clay soil. In the field, the CU parameters are used for total stress analysis. A total stress analysis is appropriate when a soil has been allowed to consolidate, then additional stresses are applied quickly with no drainage occurring. Some examples of this process are the typical rapid drawdown for an embankment, the clay soil foundation under a newly constructed embankment, and seismic (earthquake or blasting) forces (see figure 4C.4).
Undrained strength parameters of clay soils vary by consolidation state. Normally, consolidated clays have an undrained $\phi$ angle approximately one-half that of the drained condition and a cohesion value very close to zero. For OC and compacted clays, values of $\phi$ and $c$ under CU conditions may decrease slightly from CD parameters (Holtz and Kovacs, 1981). These recommendations are summarized in table 4C.5.

### 4C.2.2.3. Unconsolidated-Undrained (UU)

The use of UU shear strength parameters is another method of total stress analysis. The UU parameters model the situation where loading is assumed to occur quickly, with no drainage. Some examples include the completion of a cohesive embankment, the foundation for embankments and surcharges on soft cohesive material, and construction excavations (see figure 4C.5). In most cases, the critical...
condition exists immediately after the completion of the activity but before consolidation can occur. This state has the greatest increased pore pressures without the benefit of drainage.

Figure 4C.5.—Some examples of unconsolidated-undrained (UU) analyses for clays (reprinted with permission of Charles Ladd and Prentice-Hall from p. 588 of An introduction to geotechnical engineering by R.D. Holtz and W.D. Kovacs. Copyright© 1981 Prentice-Hall. After Ladd, 1971).

The UU strength parameters vary by soil consistency. An estimate of consistency can be made in the field and compared with typical values given in table 4C.6. Consistency is greatly affected by moisture, so the reader is advised to try to obtain field estimates that simulate the desired conditions. The relationship of the plasticity index (PI) to peak strength friction angle is shown in figure 4C.6 (see section 4C.3.4).
Table 4C.6.—Standard penetration test blow count ($N$) values and cohesion versus consistency for cohesive soils (after Peck et al., 1974).

<table>
<thead>
<tr>
<th>Consistency</th>
<th>Field Identification</th>
<th>$N$ (BPF)*</th>
<th>Cohesion $c$ (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Soft</td>
<td>Easily penetrated several inches by fist</td>
<td>below 2</td>
<td>0 to 250</td>
</tr>
<tr>
<td>Soft</td>
<td>Easily penetrated several inches by thumb</td>
<td>2 to 4</td>
<td>250 to 500</td>
</tr>
<tr>
<td>Firm</td>
<td>Can be penetrated several inches by thumb with moderate effort</td>
<td>4 to 8</td>
<td>500 to 1000</td>
</tr>
<tr>
<td>Stiff</td>
<td>Readily indented by thumb, but penetrated only with great effort</td>
<td>8 to 15</td>
<td>1000 to 2000</td>
</tr>
<tr>
<td>Very Stiff</td>
<td>Readily indented by thumbnail</td>
<td>15 to 30</td>
<td>2000 to 4000</td>
</tr>
<tr>
<td>Hard</td>
<td>Indented with difficulty by thumbnail</td>
<td>over 30</td>
<td>&gt; 4000</td>
</tr>
</tbody>
</table>

*BPF is blows per foot; see Section 4C.4.2.3.

Figure 4C.6.—Relationship of PI to peak strength friction angle (from an equation in Hammond et al., 1992).
4C.3. Unique Shear Strength Situations

4C.3.1 Apparent Cohesion

The following discussion has been excerpted from the LISA user’s guide (Hammond et al., 1992; p. 78 and 80).

Negative pore-water pressure develops in unsaturated soils due to capillary action (Lambe and Whitman, 1969). Negative pore-water pressure (also called capillary suction, capillary pressure, or matric suction) produces shear resistance that is called apparent cohesion ($c_{app}$).

Slope failures have been documented to occur as a result of a decrease in capillary suction, and hence apparent cohesion, without the development of positive pore-water pressure (Matsuo and Ueno, 1979). However, this is not the usual case. Slope failures usually occur below the phreatic surface where pore-water pressure is positive and apparent cohesion is zero. Even for the latter case, some $c_{app}$ may be appropriate in the analysis to account for the strength along the portion of the failure surface that passes through the unsaturated zone to the ground surface...

The magnitude of hydrostatic capillary suction is equal to the product of the height above the phreatic surface and the unit weight of water, as long as water films on the soil particles are continuous. Thus, the capillary suction at a given point in the soil profile will change as the phreatic surface fluctuates. Also, the capillary suction near the ground surface is usually greater (pore-water pressure more negative) than hydrostatic suction during dry seasons due to dessication [sic], and less (pore-water pressure more positive) than hydrostatic suction during wet seasons due to water infiltration.

Measuring and predicting the soil suction profiles with the seasons and assessing the appropriate profile to use for a particular problem is difficult. Therefore, reasonable values for $c_{app}$ to use in any [stability analysis] will likely come from back-analysis on existing failures. Cohesion determined by back-analysis would include both true and apparent cohesions.

Some recommendations for selecting apparent cohesion are presented in tables 4C.2 and 4C.5.

4C.3.2 Shear Strength of Compacted Soil Embankments

The compaction of cohesive soil as embankment produces various effects on the shear strength. In general, a compacted cohesive soil behaves much like an OC clay. The excavation and compaction of the material destroys the original soil structure. Also, the compaction of the material near the optimum moisture content will result in remolding of the material at or near the plastic limit. Because the pore pressures in an unsaturated state are difficult to measure, it is common to use a total stress analysis. Depending on the initial degree of saturation, the shear strength may exhibit a curved strength envelope. With increasing pressure, the volume of air is
reduced. At a sufficiently high pressure, the remaining air dissolves in the water. At this stress state, the soil behaves as a saturated clay, resulting in a $\phi = 0^\circ$ failure envelope. As the initial degree of saturation is increased, the overall shear strength decreases. The increase in pressure required to reach total saturation also decreases. A comparison of shear envelopes at various initial degrees of saturation is shown in figure 4C.7. It is important to note that if the embankment does become saturated, there will be a significant decrease in shear strength compared to that of unsaturated conditions. Therefore, the use of saturated shear strength parameters will be conservative for unsaturated conditions.

Because of the complex nature of unsaturated cohesive soil strength, it is recommended that strength parameters be estimated using laboratory testing that closely simulates field conditions. Selection of test methods and interpretation of the results require some knowledge of the behavior of clay materials.

An estimate of shear strength for compacted soil embankments can be made using table 4C.7 or table 4C.5 for overconsolidated (OC) soils.

\[
\begin{array}{|c|c|}
\hline
\text{Initial Degree of Saturation, } S_r & \text{Shearing Strength, } s (\text{kg/cm}^2) \\
\hline
\text{8%} & 61\% \\
\text{75%} & 75\% \\
\text{87%} & 87\% \\
\text{89%} & 89\% \\
\hline
\end{array}
\]

\[\text{Total Normal Stress, } p (\text{kg/cm}^2)\]

Figure 4C.7.—Rupture lines for undrained tests on a lean clay, in terms of total stresses, at various initial degrees of saturation (reprinted with permission of John Wiley & Sons from Foundation engineering by R.B. Peck, W.E. Hanson, and T.H. Thornburn. Copyright© 1974 John Wiley & Sons).
Table 4C.7.—Typical properties of compacted materials (reprinted from Driscoll, 1979).

<table>
<thead>
<tr>
<th>Group Symbol</th>
<th>Soil Type</th>
<th>Range of maximum dry unit weight, pcf</th>
<th>Range of optimum moisture, percent</th>
<th>Typical value of compression</th>
<th>Typical strength characteristics</th>
<th>$k$ Typical coefficient of permeability, ft/min.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Percent of original height</td>
<td>At 20 psi</td>
<td>At 50 psi</td>
<td>Cohesion (As compacted) psf</td>
<td>Cohesion (saturated), psf</td>
</tr>
<tr>
<td>GW</td>
<td>Well graded clean gravels, gravel-sand mixtures.</td>
<td>125–135</td>
<td>11–8</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GP</td>
<td>Poorly graded clean gravels, gravel-sand mix.</td>
<td>115–125</td>
<td>14–11</td>
<td>0.4</td>
<td>0.9</td>
<td>0</td>
</tr>
<tr>
<td>GM</td>
<td>Silty gravels, poorly graded gravel-sand silt.</td>
<td>120–135</td>
<td>12–6</td>
<td>0.5</td>
<td>1.1</td>
<td>...</td>
</tr>
<tr>
<td>QC</td>
<td>Clayey gravels, poorly graded gravel-sand-clay.</td>
<td>115–130</td>
<td>14–9</td>
<td>0.7</td>
<td>1.6</td>
<td>...</td>
</tr>
<tr>
<td>SW</td>
<td>Well graded clean sands, gravelly sands.</td>
<td>110–130</td>
<td>16–9</td>
<td>0.5</td>
<td>1.2</td>
<td>0</td>
</tr>
<tr>
<td>SP</td>
<td>Poorly graded clean sands, sand-gravel mix.</td>
<td>100–120</td>
<td>21–12</td>
<td>0.8</td>
<td>1.4</td>
<td>0</td>
</tr>
<tr>
<td>SM</td>
<td>Silty sands, poorly graded sand-silt mix.</td>
<td>110–125</td>
<td>16–11</td>
<td>0.8</td>
<td>1.6</td>
<td>1050</td>
</tr>
<tr>
<td>SM-SC</td>
<td>Sand-silt-clay mix with slightly plastic fines.</td>
<td>110–130</td>
<td>15–11</td>
<td>0.8</td>
<td>1.4</td>
<td>1050</td>
</tr>
<tr>
<td>SC</td>
<td>Clayey sands, poorly graded sand-clay mix.</td>
<td>105–125</td>
<td>19–11</td>
<td>1.1</td>
<td>2.2</td>
<td>1550</td>
</tr>
<tr>
<td>ML</td>
<td>Inorganic silts and clayey silts.</td>
<td>95–120</td>
<td>24–12</td>
<td>0.9</td>
<td>1.7</td>
<td>1400</td>
</tr>
<tr>
<td>ML-CL</td>
<td>Mixture of inorganic silts and clay.</td>
<td>100–120</td>
<td>22–12</td>
<td>1.0</td>
<td>2.2</td>
<td>1350</td>
</tr>
<tr>
<td>CL</td>
<td>Inorganic clays of low to medium plasticity.</td>
<td>95–120</td>
<td>24–12</td>
<td>1.3</td>
<td>2.5</td>
<td>1800</td>
</tr>
<tr>
<td>CL</td>
<td>Organic silts and silt-clays, low plasticity.</td>
<td>80–100</td>
<td>33–21</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>MH</td>
<td>Inorganic clayey silts, elastic silts.</td>
<td>70–95</td>
<td>40–24</td>
<td>2.0</td>
<td>3.8</td>
<td>1500</td>
</tr>
<tr>
<td>CH</td>
<td>Inorganic clays of high plasticity.</td>
<td>75–105</td>
<td>36–19</td>
<td>2.6</td>
<td>3.9</td>
<td>2150</td>
</tr>
<tr>
<td>CH</td>
<td>Organic clays and silty clays.</td>
<td>65–100</td>
<td>45–21</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Notes:

- All properties are for condition of "standard proctor" maximum density, except values of $k$ which are for "modified proctor" maximum density.
- Typical strength characteristics are for effective strength envelopes and are obtained from USBR data.
- Compression values are for vertical loading with complete lateral confinement.
- (> indicates that typical property is greater than the value shown.
- (..) indicates insufficient data available for an estimate.

All properties are for condition of "standard proctor" maximum density, except values of $k$ which are for "modified proctor" maximum density.

Typical strength characteristics are for effective strength envelopes and are obtained from USBR data.
Example 1

Given:

A 25-foot-high embankment was constructed using excavated cohesive material. The embankment was for a small valley stream crossing. Unknown to the maintenance crew, a beaver dam was constructed at the culvert inlet during the fall and early winter months. In February, a rain-on-snow event created a 25-year storm event, which ponded runoff behind the embankment. It was hypothesized that a week later the water escaped through the embankment, probably alongside the culvert. Slope failures were noted on the upstream and downstream sides of the embankment. The failures unjointed the culvert, which then required replacement.

An investigation of the site resulted in the following information:

<table>
<thead>
<tr>
<th>Soil Unit</th>
<th>Depth (Ft)</th>
<th>USCS</th>
<th>LL</th>
<th>PI</th>
<th>Consistency</th>
<th>Moisture</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0-20</td>
<td>MH</td>
<td>52</td>
<td>15</td>
<td>firm</td>
<td>moist</td>
</tr>
<tr>
<td>B</td>
<td>20-25</td>
<td>MH</td>
<td>54</td>
<td>16</td>
<td>soft</td>
<td>wet</td>
</tr>
</tbody>
</table>

Find:

(1) The soil strength parameters to be used to model the failure condition.

(2) The soil strength parameters to be used to model the excavation created to replace the culvert.

Solution:

(1) Assume that the week of ponded water saturated the embankment to a height of 5 feet above the bottom of the culvert. When the ponded water escaped, the soil water pressure could not equalize at the same rate. Therefore, use CU strength parameters for the saturated soil and compacted cohesive soil parameters for the moist soil. Because the values in table 4C.7 are for "standard proctor" maximum density, assume that 95 percent compaction (typical Forest Service construction specification) will be slightly lower strength parameters.

<table>
<thead>
<tr>
<th>Soil Unit</th>
<th>$c$ (psf)</th>
<th>$\phi^o$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>400</td>
<td>25</td>
<td>Table 4C.7</td>
</tr>
<tr>
<td>B</td>
<td>350</td>
<td>22</td>
<td>Table 4C.5 (OC)</td>
</tr>
</tbody>
</table>
The fast rate of excavation for the culvert replacement creates an unconsolidated-undrained condition. Therefore, use UU soil strength parameters.

<table>
<thead>
<tr>
<th>Soil Unit</th>
<th>c (psf)</th>
<th>$\phi^o$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>750</td>
<td>0</td>
<td>Table 4C.5 (OC)</td>
</tr>
<tr>
<td>B</td>
<td>350</td>
<td>0</td>
<td>Table 4C.5 (OC)</td>
</tr>
</tbody>
</table>

The determination of shear strength for rockfill embankments is a combination of laboratory, field, and empirical methods. A method to estimate the shear strength of rockfill was given by Barton and Kjaernsli (1981) and Charles and Soares (1984). An understanding of the behavior of rockfill is important to correctly apply the empirical methods of shear strength estimation.

Particle size has been well documented as a control on shear strength. Shear strength of rockfill decreases with increasing particle size. To account for this phenomenon, Barton and Kjaernsli proposed a size-dependent equivalent strength ($S$) for rockfill. Their proposed relationship is shown in figure 4C.8. The ratio of equivalent strength ($S$) to uniaxial compressive strength ($\sigma_c$) of the parent rock is given as a function of the $d_{50}$ particle size of the rockfill. The graph shows values based on triaxial and plane-strain testing. The use of plane-strain tests are advocated as more representative of practical situations. The resulting $\phi$ values are at least 2–4° higher than are those from triaxial results for the same material. The difference in strength from small-scale testing to field conditions is relatively minor, provided that the scaled material behaves as a free draining granular fill (Charles and Soares, 1984).

Figure 4C.8.—Method of estimating equivalent strength ($S$) of rockfill based on uniaxial compression strength ($\sigma_c$) and on $d_{50}$ particle size (reprinted with permission of the American Society of Civil Engineers from Shear strength of rockfill by N. Barton and B. Kjaernsli. Copyright© 1981 American Society of Civil Engineers, Geotechnical Engineering Division).
Particle shape and smoothness also affect the shear strength of rockfill. An empirical equivalent roughness can be determined based on origin, roundness, smoothness, and porosity. These relationships are presented in figure 4C.9.

Barton and Kjaernsli (1981) defined the shear strength of rockfill as:

\[ \phi' = R \log(S/\sigma') + \phi_b \]  

(4C.4)

where \( R \) is equivalent roughness, \( S \) is equivalent strength, and \( \phi_b \) is the basic angle of friction. The basic angle of friction can be estimated using simple tilt tests on sawn rock surfaces or drill core, and usually is between 25° and 35°. The basic angle of friction differs from the angle of repose in that it is a failure on a smooth rather than a rough surface. Further discussion on the basic angle of friction can be found in sections 4D.2.5.5 and 4D.3.1.6.

Another estimate of rockfill shear strength has been given by Charles and Soares (1984) as:

\[ \tau_f = A(\sigma')^b \]  

(4C.5)
Some typical values for $A$ and $b$ are given in table 4C.8.

<table>
<thead>
<tr>
<th>Rock Types</th>
<th>A</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandstone</td>
<td>18.53</td>
<td>0.67</td>
</tr>
<tr>
<td>Slate (Good Quality)</td>
<td>11.32</td>
<td>0.75</td>
</tr>
<tr>
<td>Slate (Poor Quality)</td>
<td>6.03</td>
<td>0.77</td>
</tr>
<tr>
<td>Basalt</td>
<td>7.84</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Example 2

Given:

A rockfill embankment is to be built using the material from a soil borrow source. The material is free of sand and fines and consists of subangular basalt rock fragments ranging in size from 1 to 12 inches average dimension. The $d_{50}$ size is estimated to be 4 inches (102 mm). The origin of the soil deposit is moraine. The field classification of a representative rock fragment according to the Uniform Rock Classification System (URCS) is: BBBA. From previous use of the material, the compacted rockfill density is estimated to be 135 pcf.

Find:

Plot the shear strength versus confining pressure for the proposed embankment.

Solution:

First, examine equation 4C.4 to determine what information is needed. The value of $R$ depends on rock origin, roundness, smoothness, and porosity of the compacted material. To calculate $S/\sigma_s$, one needs the $d_{50}$ particle size (in mm). And finally, the basic angle of friction is required. An estimate of porosity, $n$, can be made using a formula from Table 4B.1 and an estimate of specific gravity based on the URCS designation.

$$G = 2.7 \text{ (estimate from URCS unit weight designation and table 4D.1)}$$

$$n = 1 - \frac{W_s}{GV \gamma_w} = 1 - \frac{\gamma_d}{G \gamma_w} = 1 - \frac{135 \text{ pcf}}{(2.7)(62.4 \text{ pcf})} = 20\%$$

$$R = 8.5 \text{ (figure 4C.9)}$$
\[
\frac{S}{\sigma_c} = 0.25 \text{ (figure 4C.8 for triaxial test)}
\]

\[
\sigma_c = 12,000 \text{ psi (estimated, based on PQ designation, URCS)}
\]

\[
S = \frac{S}{\sigma_c} \cdot 3,000 \text{ psi}
\]

\[
\phi_b = 34^\circ \text{ (table 4D.4)}
\]

\[
\phi = 8.5 \log \left(\frac{3000 \text{ psi} \cdot (144 \text{ in.}^2/\text{ft}^2)}{\sigma'}\right) + 34^\circ \text{ (from equation 4C.4)}
\]

\[
\phi = 8.5 \log \frac{4.32 \times 10^5 \text{ psf}}{\sigma'} + 34^\circ \quad (4C.6)
\]

Use equation 4C.1 with \(c' = 0\) and substitute equation 4C.6 for \(\phi\). The relation of strength to effective stress is shown as the Barton and Kjaernsli line below.

\[
\tau_f = \sigma' \tan [8.5 \log \frac{4.32 \times 10^5 \text{ psf}}{\sigma'} + 34^\circ] \quad (4C.7)
\]

Next, use equation 4C.5 for comparison. The equation can be solved using the parameters from table 4C.8 for basalt.

\[
\tau_f = 7.84(\sigma')^{0.81} \quad (4C.8)
\]

The relation of strength to effective stress for equation 4C.7 is shown as the “Barton and Kjaernsli” line, and that for equation 4C.8 is shown as the “Charles and Soares” line below.

**4C.3.4 Residual Strength**

The residual strength, sometimes referred to as ultimate strength, of a soil is a result of very large strains and situations of long-term progressive failure. Residual strength is usually identified by the residual friction angle \(\phi_r\). Figure 4C.10 shows the idealized stress-strain relationships for a sand and a clay material. As strain increases, the soil strength passes a peak then decreases to a residual strength value.
For sands, the residual strength is equivalent to a loose relative density condition. The residual strength of a soil is independent of initial void ratio, confining pressure, and stress history. For clay soils, the shearing effect aligns the plate-shaped particles and breaks the adhesive bonds between clay particles. This results in a negligible cohesion intercept similar to an NC clay (Hammond et al., 1992). Therefore, for sands and clays, a residual strength analysis uses no soil cohesion ($c = 0$ psf).

![Idealized stress-strain curves for sand (a) and clay (b) soils (after Hammond et al., 1992).](image)

The residual strength of a soil can be determined using ring shear, multiple reversal direct shear, and rotational shear devices (Townsend and Gilbert, 1973). Studies have found a relationship between plasticity index (PI) and residual strength, as shown in figure 4C.11. According to Duncan and Stark (1992):

Following the recommendations of Skempton (1964, 1977, 1985), fully softened [peak] shear strengths are appropriate where no sliding has occurred previously, and residual strengths should be used where there has been sufficient shearing deformation to result in reorientation of clay particles parallel to the direction of shearing. Field shearing displacements of three feet or more are sufficient to reduce the strengths of clays to their residual values.
Figure 4C.11.—Relationship of PI to residual strength friction angle $\phi_r$, (developed with permission of the American Society of Civil Engineers from data from "Soil strengths from back analysis of slope failures" by J.M. Duncan and T.D. Stark. In: R.B. Seed and R.W. Boulanger, eds. Stability and performance of slopes and embankments-II. Copyright© 1992 American Society of Civil Engineers, Geotechnical Engineering Division).

Typical values of residual friction angle for cohesive and non-cohesive soils are shown in table 4C.9.

Table 4C.9.—Summary of residual strength parameters (after Hammond et al., 1992).

<table>
<thead>
<tr>
<th></th>
<th>Non-Cohesive Soils</th>
<th>Cohesive Soil</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c'_s$</td>
<td>$\phi_r$</td>
</tr>
<tr>
<td>$c'_s$</td>
<td>0</td>
<td>Use table 4C.4 and/or figure 4C.1 at $D_r = 0%.$</td>
</tr>
<tr>
<td>$\phi_r$ Use table 4C.4 and/or figure 4C.1 at $D_r = 0%.$</td>
<td>Same as for non-cohesive or figure 4C.11</td>
<td>Use figure 4C.11.</td>
</tr>
</tbody>
</table>

1 Clay fraction is defined as material less than 0.002 mm.
Example 3

Given:

A translational slope movement is progressing downslope 3 to 5 feet yearly. The following information from the failure zone was obtained:

<table>
<thead>
<tr>
<th>USCS</th>
<th>LL</th>
<th>PI</th>
<th>%&lt;#200</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM</td>
<td>28</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

Find:

(I) The peak strength.

(2) The residual strength.

Solution:

This soil has a PI of 10, which will provide some strength to the soil when air-dried (see subsection 4C.2). Therefore, use properties of cohesive soil.

(I) For peak strength, use table 4C.5. Assuming an NC state, use figure 4C.6. The result is \( \phi_p = 35^\circ \).

(2) For residual strength, use table 4C.9. Because only 15 percent passes the #200 sieve (0.075 mm), there will be less than 25 percent clay. From figure 4C.1, a \( D_r = 0\% \) and a GM material \( \phi_r = 28^\circ \). Using figure 4C.11, \( \phi_r = 18-25^\circ \).

4C.3.5 Anisotropy

Anisotropy refers to "having different properties in different directions" (ASTM, 1992). Anisotropy can be found in the common soil properties of permeability (water flow through soil) and shear strength. For soils exhibiting anisotropic shear strength, the value of shear strength varies as the angle between the principle planes (a property of the natural soil) and the failure surface vary.

Most soil properties reported in the literature are for homogeneous conditions. However, many natural soil deposits exhibit some anisotropic soil behavior. Because most triaxial tests are performed on isotropically consolidated samples, the resulting strength parameters tend to be at the lower end of the strength range. This results in an under-estimation of the strength and a conservative estimate for the factor of safety (Sharma, 1992). Because anisotropic soil strength varies with the angle of the failure surface, the greatest effect will be seen for circular failures rather than translational failures.

The XSTABL (1992) computer program allows the user to vary the soil strength values based on the angle of orientation for the failure surface. An example is presented in the XSTABL Technical Manual (Sharma, 1992).
4C.4 How to Measure Shear Strength

This section will examine some of the common methods for determining the shear strength of soil material. The section is divided into laboratory tests, field tests, and the use of a back-calculation analysis. Some of the test methods are explained in more detail in the cited references.

4C.4.1 Analytical Methods—Laboratory Tests

4C.4.1.1. Direct Shear Test

The direct shear test uses basic stress theory and the Mohr-Coulomb diagram to determine the shear strength of the soil. The soil specimen is held in a "shear box" which is separated horizontally in halves (see figure 4C.12). One-half of the device is fixed in position while to the other is pushed or pulled horizontally. A normal load is applied to the soil specimen. The shear stress and normal stress are determined by measuring the shear load, the horizontal and vertical deformation, and the nominal area of the soil specimen. By varying the normal load, each normal load and corresponding shear stress can be plotted on the Mohr diagram and the angle of internal friction (\(\phi\)) can be determined.

![Figure 4C.12.—Illustration of the principle components of a shear box (reprinted with permission of Oxford University Press from Hillslope materials and processes by M.J. Selby. Copyright © 1982 M.J. Selby).](image)

The advantages of a direct shear test are the low cost, simple setup, and rapid results. The disadvantages include the difficulty in controlling drainage conditions of the sample and the potential for obtaining a higher estimate of strength as a result of the forced failure surface instead of an existing plane of weakness. Therefore, a direct shear test is most appropriate for granular materials and must be used cautiously for silts and clays.
Example 4

Given:

Results from three direct shear tests on a well-graded sand are as follows:

<table>
<thead>
<tr>
<th>Test Number</th>
<th>$\sigma'$ (psi)</th>
<th>$\tau$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>6.75</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>13.49</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>20.24</td>
</tr>
</tbody>
</table>

Find:

Friction angle for the soil.

Solution:

Plot stress versus strength. The resulting line connecting the points is the Mohr-Coulomb strength envelope. The slope of the line, $34^\circ$, defines the friction angle.

4C.4.1.2. Triaxial Shear Test

The triaxial shear test is a cylindrical compression test that allows control of the drainage for the soil specimen. Unlike the direct shear device, the triaxial device allows the failure plane to occur anywhere in the soil specimen. The soil specimen is encased in a rubber membrane that separates the pressurized cell fluid from the specimen. A vertical axial load is applied through a piston. The volume change or pore-water pressure of the specimen can be measured during the test (see
For test interpretation, we assume the stresses acting on the boundary of the specimen are the principle stresses. The cell pressure represents the minor principle stress \((\sigma_3)\) and is an all-around confining pressure on the specimen. The axial load is increased until either the specimen fails on a weak plane or a bulge is noted. The axial stress provided by the piston at failure combined with the vertical stress provided by the confining pressure represent the major principle stress \((\sigma_1)\). The axial stress is referred to as the deviator stress and is equal to \(\sigma_1 - \sigma_3\). The confining pressure and the corresponding axial stress can be manipulated to provide differing Mohr circles. These circles can be plotted on a Mohr-Coulomb diagram, and the soil strength of the material determined from the strength envelope which is a line tangent to the Mohr circles (see figure 4C.14).

Figure 4C.13.—Section of triaxial shear test apparatus (reprinted with permission of John Wiley & Sons from Fundamentals of geotechnical analysis by I.S. Dunn, L.R. Anderson, and F.W. Kiefer. Copyright© 1980 John Wiley & Sons).
4C.4.1.3. Sample Drainage

Sample drainage conditions are used in shear testing to model anticipated design situations in the field. The first symbol, C or U, refers to the consolidation state occurring before shear. The second symbol, D or U, refers to the drainage state during shear. Examples of field conditions are given in section 4C.2.

Unconsolidated-Undrained (UU)

This test is characterized by placing a saturated sample under a confining pressure and applying an axial load without allowing for sample drainage. This test models the condition of rapid loading of an unconsolidated soil, such as a surcharge being placed on an embankment or foundation soil. For fully saturated soils, the lack of consolidation at any confining pressure will result in the same deviator stress, producing Mohr circles of the same diameter. The strength determined by this method is referred to as the undrained strength. The failure envelope generated by this method is a total stress envelope and is a horizontal line for fully saturated soils (see figure 4C.16). The UU soil strength is also referred to as the \( \phi = 0^\circ \) case.

Unconfined Compression Test

The unconfined compression test is a special case of UU sample drainage. The sample is neither confined nor allowed to drain. For the unconfined compression test to result in the same strength values as the UU test, the following assumptions must be satisfied (Holtz and Kovacs, 1981):

1. Specimen at 100 percent saturation—if not, compression of the air in the voids will result in a decrease in void ratio and a corresponding increase in strength.

2. The material should be cohesive, homogeneous, and intact.
(3) The material must be very fine grained.

(4) The shearing process must be rapid to ensure the total stress state of no drainage. A typical failure time range is 5 to 15 minutes. The relation of an unconfined compression test Mohr circle to that of UU total stress circles is shown in figure 4C.15.

![Diagram](image)

Figure 4C.15.—Typical strength envelope from UU test on NC clay (reprinted with permission of John Wiley & Sons from Fundamentals of geotechnical analysis by I.S. Dunn, L.R. Anderson, and F.W. Kiefer. Copyright © 1980 John Wiley & Sons).

Consolidated-Undrained (CU)

This drainage state is a result of allowing the sample to consolidate after the application of the all-around confining pressure by opening the drainage line. Upon consolidation, the drain line is closed and the axial stress is applied. Because no volume change is allowed, there is a change in pore pressure within the sample. The resulting shear strength envelope is a total stress failure envelope as shown in figure 4C.16.
Consolidated-Drained (CD)

The consolidated-drained test provides for both sample consolidation after the confining stress is applied and sample drainage during the application of the axial stress. For the sample to drain and not produce an increase in pore pressures, the axial load must be applied slowly. This test determines the drained shear strength, with the resulting shear strength envelope an effective stress failure envelope as shown in figure 4C.17. The drained shear strength parameters are typically denoted using a prime symbol (') as shown in equation 4C.1.
The UU, CU, and CD tests are sometimes referred to by the time required to run the tests. The UU test can be run quickly, and is often referred to as a quick (Q) test. The sample drainage required for the CD test can take a long time. For this reason, the test is often referred to as a slow (S) test. The CU test requires an amount of time somewhere between the UU and CD tests. The CU test is referred to as the R test, since the letter R is between the letters Q and S in the alphabet (Dunn et al., 1980).

The sample drainage method used for testing will depend on the field conditions being simulated. This topic is discussed further in section 4C.2.2.

One other property of the drainage state tests should be mentioned. For the CU shear test, water is not allowed to drain during shear. However, the resulting pore-water pressures can be measured during the test. Using these pore-water pressures, the Mohr circles plotted from the CU test can be shifted by the amount of the pore-water pressure as shown in figure 4C.16. The resulting strength envelope will be an effective stress (CD) envelope. The benefit of this procedure is that the CU test is less time-consuming than the CD test. For clays and silts, the results of the CD test can be misleading if complete drainage does not occur. Generally the fine-grained soils are tested by the CU method with pore pressure measurements to correct for the effective stress failure envelope.

A final note is given by Dunn et al. (1980):

It must be emphasized that $c$ and $\phi$ are not unique values for a given soil. Shear test results that give only values of $c$ and $\phi$ are, therefore, meaningless without a description of the type of test (drainage conditions), type of strength envelope, and the normal stress range.

---

**Figure 4C.17.**—Typical effective strength envelope from a CD triaxial shear test on an NC clay (reprinted with permission of John Wiley & Sons from Fundamentals of geotechnical analysis by I.S. Dunn, L.R. Anderson, and F.W. Kiefer. Copyright © 1980 John Wiley & Sons).
Example 5

Given:

Two samples of a saturated clay soil were consolidated and triaxially shear tested under undrained conditions with pore pressure measurements. The samples were consolidated under differing confining pressures and loaded to failure. The results are as shown:

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>$\sigma_3$ (psi)</th>
<th>$\Delta\sigma$ (at failure) (psi)</th>
<th>$\Delta u$ (at failure) (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>56</td>
<td>40</td>
<td>38</td>
</tr>
</tbody>
</table>

A third sample was tested by the unconfined compression test method and was loaded to failure at 40 psi.

Find:

1) The total stress (CU) parameters.
2) The effective stress (CD) parameters.
3) The unconfined compressive strength ($q_u$) of the soil.

Solution:

Plot the Mohr's circles for total stress and adjust by the pore pressures to find the effective stress circles. Then draw the Mohr-Coulomb strength envelopes and solve for $c$ and $\phi$. 

![Mohr's Circle Diagram](image-url)
(1) The total stress (CU) parameters are:
   \( c = 0 \) psi
   \( \phi = 16^\circ \)

(2) The effective stress (CD) parameters are:
   \( c' = 0 \) psi
   \( \phi' = 32^\circ \)

(3) The unconfined compressive strength is equal to the diameter of the Mohr circle (which is equivalent to \( 2c \)). Since the confining pressure is zero, the result is \( q_u = 40 \) psi.

4C.4.1.4. Curved Mohr-Coulomb Strength Envelope

The Mohr-Coulomb strength envelope is shown in figure 4C.18. The envelope is created by passing a line tangent to each of the stress circles at various confining pressures used during a shear test. For convenience the slightly curved path is assumed to be a straight line with a slope of \( \tan(\phi') \) and a cohesion intercept of \( c' \).

According to Hammond et al. (1992):

...several authors report large values for cohesion, up to 1,000 psf, in cohesionless sands and gravels (Holtz and Gibbs, 1956; Schroeder and Alto, 1983; Schroeder and Swanston, 1987). This, of course, is not true cohesion but may result from the way in which laboratory test results are interpreted. Figure [4C.18] illustrates that a cohesion intercept can result when a straight line Mohr's failure envelope is fit either to test data that are curved due to diminishing dilation with increasing effective stress, or to scattered test data that are due to test specimen variability or testing errors or both. The latter can result in either positive or negative intercepts. In either case, the positive \( c' \), and \( \phi' \) values reported may be inappropriate for use in stability analysis at small effective stresses (shallow soil depths or steep slopes) because shear strength will be overestimated. On the other hand, ignoring the cohesion intercept and using only the reported \( \phi' \) value to compute shear strength could underestimate the actual shear strength at all confining stresses. This problem can be alleviated by performing shear tests only at effective stresses consistent with the in situ conditions or by modeling the Mohr's failure envelope as a curve with a power function (Miller and Borgman, 1984).
A curved strength envelope can be modeled as a series of points or straight lines or as a power function. A method of estimating the shape of the strength envelope for rockfill is presented in section 4C.3.3.

4C.4.2 Empirical Methods—Field Tests

Table 4C.10 presents a summary of some of the methods used for in-situ shear strength measurements.

Figure 4C.18.—Sources of cohesion intercept from laboratory tests on cohesionless soils (from Hammond et al., 1992).
Table 4C.10.—Summary of field methods for determining shear strength (adapted from Holtz and Kovacs, 1981).

<table>
<thead>
<tr>
<th>Test</th>
<th>Use</th>
<th>Figure Number</th>
<th>Best For</th>
<th>Remarks</th>
<th>Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vane shear test (VST)</td>
<td>Lab, field</td>
<td>4C.19</td>
<td>Soft to stiff clays</td>
<td>Various sizes and configurations available for both field and lab use. Height/diameter ratio (H/D) = 2 for field vanes; H/D = 1 for lab vanes. Only lab vane sample is seen.</td>
<td>May overestimate τ; see figure 4C.20 for correction factor for very soft clays. Unreliable readings if vane encounters sand layers, varves, stones, etc., or if vane rotates too rapidly.</td>
</tr>
<tr>
<td>Dutch cone penetrometer test (CPT)</td>
<td>Field</td>
<td>4C.21</td>
<td>All soil types except very coarse granular soils</td>
<td>A 60° cone with a projected area of 10 cm² is pushed at 1 to 2 m/min. Point resistance q, and friction on the friction sleeve f, are measured either electrically or mechanically.</td>
<td>Boulders cause problems. Requires local correlation for soft clays.</td>
</tr>
<tr>
<td>Standard penetration test (SPT)</td>
<td>Field</td>
<td>4C.23</td>
<td>Granular soils</td>
<td>A standard “split-spoon” sampler is driven by a 63.5 kg hammer falling 0.76 m. The number of blows required to drive the sampler 0.3 m is called the standard penetration resistance, or blow count, N. Disturbed sample obtained.</td>
<td>Very rough correlation with τ; for cohesive soils. Boulders can cause problems. Results are sensitive to test details.</td>
</tr>
<tr>
<td>Iowa borehole shear test (BST)</td>
<td>Field</td>
<td>4C.25</td>
<td>Loessial (silty) soils</td>
<td>Device is lowered into a borehole and expanded against the side walls (σₐ). Then entire mechanism is pulled from ground surface and maximum load measured (τ). Stage test results are used to plot Mohr diagram for CD tests. Range of σₐ is from about 30 to 100 kPa.</td>
<td>Cannot be used with soils with 10% or more gravel or caving sands. Uncertain drainage conditions during shear make the test difficult to interpret. (Is it CD or CU or somewhere in between?)</td>
</tr>
<tr>
<td>Pressuremeter (PMT)</td>
<td>Field</td>
<td>4C.26</td>
<td>All soil types</td>
<td>A cylindrical probe is inserted in a drill hole (may be self-boring). Lateral pressure is applied incrementally to side of hole.</td>
<td>Requires a correlation between pₐ and τ.</td>
</tr>
<tr>
<td>Torvane (TV)</td>
<td>Lab, field</td>
<td>4C.27</td>
<td>Very soft to stiff clays</td>
<td>Hand held; calibrated spring; quick; used on tube samples or the sides of exploratory trenches, etc. Sample tested is seen.</td>
<td>Cohesive soils without pebbles, fissures, etc. Test only a small amount of soil near the surface. Only rough calibration with τ.</td>
</tr>
<tr>
<td>Pocket penetrometer (PP)</td>
<td>Lab, field</td>
<td>4C.28</td>
<td>Very soft to stiff clays</td>
<td>Same as above, except spring is calibrated in unconfined compressive strength (= 2τ).</td>
<td>Same as above.</td>
</tr>
</tbody>
</table>
4C.4.2.1. Vane Shear Test

The vane shear test (VST) uses a four-bladed vane placed into a borehole using a suitably stiff rod (see figure 4C.19). The torque required for rotation is measured and converted to a shear stress, resulting in an estimate of the undrained shear strength for the soil material. The test method is presented in ASTM D-2573. This test is appropriate for soft to medium stiff clays not containing sand lenses, gravel, or rock fragments (Dunn et al., 1980). A correction factor for very soft clays is shown in figure 4C.20.

Figure 4C.19.—(a) Principle of the vane shear test (VST); (b) Typical vane sizes; (c) Theoretical formulas for $\tau_f$ (reprinted with permission of Prentice-Hall from p. 576 of An introduction to geotechnical engineering by R.D. Holtz and W.D. Kovacs. Copyright© 1981 Prentice-Hall.)
4C.4.2.2. Dutch Cone Penetrometer

The Dutch cone penetrometer test (CPT) measures the penetration resistance to push a 60° cone at a rate of 10 to 20 mm/sec. The Begemann friction cone has a friction sleeve that allows measurement of the soil-sleeve friction. Schematics for the Dutch cone are shown in figure 4C.21. Figure 4C.22 shows a plot of cone tip resistance against depth for the Dutch cone penetrometer. The test method is presented in ASTM D-3441. Cone penetrometer data have been used to assess friction angle (Meyerhof, 1974; Dunne et al., 1980), classify soil (Sanglerat, 1972; Schmertmann, 1978; and Dunne et al., 1980), and determine the shear strength of clay soil (Gorman et al., 1975). Some advantages and disadvantages of the cone penetrometer over conventional bore procedures with standard penetration tests and tube sampling, along with some recommendations for use, are given by Dunn et al. (1980):
Figure 4C.21.—Dutch cone penetrometer (CPT): mechanical cone with friction sleeve open (top) and closed (after Vanikar, 1985).
Some advantages of using the cone penetrometer include:

1. More accurate and directly useful information about soil properties is obtained.

2. The rate of probing is fast and economical.

3. The shorter sampling interval permits more accurate identification of subsurface layers.
Some disadvantages of the cone penetrometer are:

1. No samples of subsurface material are obtained.

2. Ground water conditions cannot be readily evaluated [although electric piezometer readings are possible, but subject to interpretation].

3. The procedure is limited to soils that do not contain large rock or hard layers, which prevent penetration of the cone.

4. In some soils the interpretation of cone resistance data may be difficult.

Because no soil sample is recovered, it is recommended that the cone penetrometer be used with some supplementary information from conventional soil borings. Data from conventional borings—that is standard penetration blow count, unconfined shear strength from tube samplers, visual identification of soils, and water table data—are vital to accurate interpretation of cone penetrometer data.

4C.4.2.3. Standard Penetration Test

The standard penetration test (SPT) resistance (N) is the number of blows required to drive a standard split barrel or split-spoon sampler 12 inches into the soil. By convention, N is the sum of the number of blows for each of the last two 6-inch advances of the sampler. The sampling device is shown in figure 4C.23. The blow count is empirically correlated to unit weight and shear strength based on a 1 ton per square foot (tsf) effective overburden stress. Correction of N values for other overburdens can be made using figure 4C.24. The SPT method allows for the recovery of a disturbed sample from the barrel of the sampler for visual identification and in-situ moisture content. The procedure is described in ASTM D-1586.
Figure 4C.23.—SPT "split-spoon" sampler (reprinted with permission of Mobile Drilling Corp.)
Figure 4C.24.—Correction factor of SPT field values. $N'$ is corrected $N$ for overburden pressure at the time of the test (from an equation in Peck et al., 1974).

The SPT is used most often to estimate strength measurement in granular soils. Although it is sometimes used to measure the shear strength of cohesive materials, the correlations are much less reliable. Tables 4C.3 and 4C.6 present typical values of penetration resistance $N$ versus shear strength measure. Section 3D.5.6 describes conditions that can affect the penetration resistance results.

Example 6

Given:

SPT results were obtained at a depth of 15 feet as follows: 2-4-4.

The same soil unit was encountered from the ground surface to the test depth, and the soil was field-classified as SM using the USCS. The depth to water was measured as 5 feet. A split-spoon sample recovered above the water table was sampled using an Ely volumeter and a moisture content of 25 percent was calculated, with a resulting dry unit weight of 92 pcf.

Find:

The friction angle for the soil at the test depth.
Solution:

First, determine the N value from the field.

\[ N = 4 + 4 = 8 \text{ bpf}. \]

Next, correct the field value for overburden pressure at the time of the test.

\[
\gamma_m = 92 \text{pcf} (1 + 0.25) = 115 \text{pcf} \\
\gamma_{sat} = 120 \text{pcf (from figure 4C.1)} \\
\gamma_s = 120 \text{pcf} - 62.4 \text{pcf} = 57.6 \text{pcf} \\
p_o = 115 \text{pcf}(5 \text{ft}) + 57.6 \text{pcf}(10 \text{ft}) = 1151 \text{psf} \\
From figure 4C.24 \ N' / N = 1.2 \\
N' = 10 \text{ bpf} \\
\]

Using table 4C.3, \( D_r = 35\% \)

From figure 4C.1 for SM and \( D_r = 35\% \)

\[ \phi' = 31^\circ - 32^\circ \]

4C.4.2.4. Iowa Borehole Shear Test

The Iowa borehole shear test (BST) method, best suited for silty soils, uses a shear head device expanded against the borehole wall (confining pressure) at a given test depth. Then the entire shear head device is pulled toward the ground surface, and the force is measured to calculate the shear stress (see figure 4C.25). Stage test results can be used to plot a Mohr-Coulomb failure envelope. The uncertainty of whether drained or undrained conditions occur during shear leaves questions as to whether the test results represent CD (confined drained) or CU (confined undrained) shear strength parameters.
Figure 4C.25.—The Iowa borehole shear (BST) device, showing the pressure source and instrumentation console, the pulling device, and the expanded shear head on the sides of a borehole (from Holtz and Kovacs, 1981).

4C.4.2.5. Pressuremeter

The pressuremeter test (PMT) uses a cylindrical probe inserted into a drill hole. Lateral pressure is applied incrementally to the sides of the drill hole. The volumetric expansion and pressure over time are recorded (see figure 4C.26). An estimate of strength can be made using a correlation between limit pressure and shear stress. The method can be used for all soil types. Additional information can be found in Schmertmann (1975).
4C.4.2.6. Torvane

The Torvane (TV) is a hand-held device that provides a quick estimate of the undrained shear strength at the surface of a sample. The device uses a calibrated spring to correlate the applied torque to the shearing strength required for rotation. The device uses three different vane configurations to match the estimated strength range (figure 4C.27). The Torvane is best suited for very soft to stiff clays.
4C.4.2.7 Pocket Penetrometer

The pocket penetrometer (PP) is also a handheld calibrated spring device (see figure 4C.28). The end of the device is pushed a standard distance into the soil material, and the resistance is correlated to shear strength. The suitable soil types are the same as for the Torvane. The difference between the Torvane and the PP is that the PP results are in terms of unconfined compressive strength (equal to two times the undrained shear strength), whereas Torvane results are shear strengths.

4C.4.2.8. Williamson Drive Probe

The Williamson drive probe (WDP) is an inexpensive dynamic penetrometer developed by Douglas Williamson. The device and its uses are explained in appendix 3.6. The use of the WDP for strength determination is best accomplished using site-specific correlations with SPT (standard penetration test) or CPT (Dutch cone penetrometer test) results. The results from the WDP are reported as blows per foot based on doubling the blows recorded for a 6-inch advance of the probe. The WDP is best suited for silts and sands without gravel or rock fragments.
4C.4.2.9 Hand Tools

Hand tools—such as a pick, shovel, and even fingers and thumbs—can provide an inexpensive estimate of consistency or relative density. Some typical correlations are presented in tables 4C.3 and 4C.6.

Example 7

Given:

A soil unit described as silty sand, brown, damp, non-plastic (NPL), difficult to excavate with hand shovel.

Find:

The estimated soil strength parameters $C$, and $\phi'$.

Solution:

The material is non-plastic silty sand; therefore, use non-cohesive soil strength parameters. The consistency rates as medium (table 4C.3), so use:

$$c' = 0 \text{ psf}$$

$$\phi' = 30^\circ - 35^\circ$$

Figure 4C.28.—Hand-held pocket penetrometer (PP) which indicates unconfined compressive strength in cohesive soils (after Soiltest, Inc., Evanston, IL).
4C.4.3 Back-analysis

Calculating Strength Values

Back-analysis is an analytical tool used to solve for strength parameters by satisfying a stability equilibrium equation. As is discussed in section 5, several variables are used in most stability equations. If values can be assigned to all the variables except strength, the equation can be solved for c, φ, or both. An example of back-analysis is given in problem 1 in section 5C.

In selecting strength parameters by back-analysis, it is important to understand the methods of defining strength, such as consolidation state, drainage conditions, apparent cohesion, and residual strength. A demonstration of some of these concerns is given in example 8.

Example 8

Given:

A large re-activated translational slope movement was investigated by subsurface drilling. The failure surface was confirmed to be at the boundary of soil unit A (SU-A) and rock unit 10 (RU-10—an impermeable barrier) and parallel to the ground surface at 14°. The average depth to the failure surface was 15 feet. SU-A is described as sandy silt, red-brown, moist, above the plastic limit (APL), stiff to very stiff consistency, and lab classified by the USCS as MH. A CU triaxial shear test with pore pressure readings and lab classification resulted in the following information:

<table>
<thead>
<tr>
<th>Liquid Limit</th>
<th>PI</th>
<th>%&lt;#200</th>
<th>S.G.</th>
<th>γₙ</th>
<th>c'</th>
<th>φ'</th>
</tr>
</thead>
<tbody>
<tr>
<td>67</td>
<td>29</td>
<td>95.5</td>
<td>2.83</td>
<td>82.0 pcf</td>
<td>533 psf</td>
<td>26.8°</td>
</tr>
</tbody>
</table>

Seasonal water monitoring has confirmed a depth to water of 10 feet.

Find:

(1) The resulting factor of safety using the infinite slope equation.

(2) The resulting strength parameters using back-analysis.

(3) An explanation for the differences in strength values between those from laboratory testing and those from back-analysis.

Solution:

(1) Use the DLISA (Deterministic Level I Stability Analysis 1991) computer program. The unsaturated soil moisture can be taken as the plastic limit because the soil is APL (recall that PL is the moisture content at which the soil becomes plastic and is equal to LL–PI, or 38%). The value for φ' from the triaxial test results agrees with figure 4C.6. As is discussed in section 4C.4.1.4, the value of c' for this triaxial test is considered high and, to be conservative, could be ignored. Based on the depth and water conditions, both root strength ("root cohesion") and apparent cohesion are considered negligible. The resulting factor of safety is determined to be 1.66.
DETERMINISTIC LEVEL I STABILITY ANALYSIS

Infinite Slope Equation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil depth (ft)</td>
<td>15.00</td>
</tr>
<tr>
<td>Surface slope (deg)</td>
<td>14.00</td>
</tr>
<tr>
<td>Tree surcharge (psf)</td>
<td>0.00</td>
</tr>
<tr>
<td>Root cohesion (psf)</td>
<td>0.00</td>
</tr>
<tr>
<td>Groundwater height (ft)</td>
<td>5.00</td>
</tr>
<tr>
<td>Friction angle (deg)</td>
<td>26.80</td>
</tr>
<tr>
<td>Soil cohesion (psf)</td>
<td>0.00</td>
</tr>
<tr>
<td>Dry unit weight (pcf)</td>
<td>82.00</td>
</tr>
<tr>
<td>Moisture content (%)</td>
<td>38.00</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>2.83</td>
</tr>
<tr>
<td>Factor of safety</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Factor of safety</th>
<th>Dry unit wt. (pcf)</th>
<th>Moist unit wt. (pcf)</th>
<th>Saturated unit wt. (pcf)</th>
<th>Saturated moist. cont. (pcf)</th>
<th>Moisture content (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.66</td>
<td>82.00</td>
<td>113.16</td>
<td>115.42</td>
<td>40.76</td>
</tr>
</tbody>
</table>

(2) For back-analysis, the value for the factor of safety is set equal to 1.00. If soil cohesion is set to zero, the remaining unknown will be the friction angle. The resulting friction angle is 16.96° (use 17°).
(3) The difference in strength values between laboratory testing and back-analysis may be explained using residual strength theory. The "reactivated" description implies previous movement. Using figure 4C.11 with a PI of 29, $\phi_r = 10-20^\circ$, which would support the back-analysis value of $17^\circ$.

4C.5 Seismic Behavior

Seismic analysis becomes more important as public agencies become more aware of the occurrence of seismic events. Two reasons that dynamic analysis has been avoided are the uncertainty of assigning dynamic soil parameters, and the intensive mathematical modeling required. The use of pseudo-static analysis has been advocated as a method to check the effects of seismic loading. Although there is poor correlation between factors of safety calculated by dynamic versus pseudo-static analysis, methods using pseudo-static analysis have been shown to provide a "red flag" threshold. This method involves determining the critical seismic coefficient resulting in a factor of safety equal to one for the static critical surface. Therefore, static soil strength parameters are used. The XSTABL (1992) computer program adopted by the Forest Service uses pseudo-static analysis. The method is further described in the XSTABL Technical Manual (Sharma, 1992).

A more rigorous approach is to analyze the slope using dynamic soil mechanics. A method for determining dynamic soil properties is given by Tiedemann et al. (1984). This method is beyond the scope of this manual, and the interested reader is urged to seek this and other references on dynamic soil analysis.
4D. Strength and Behavior of Rock

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4D.1 Introduction

This section is concerned with the strength and behavior of rock as it relates to the stability of rock slopes within the forest highway prism. A review is given of some of the important concepts critical to one’s understanding of rock slope stability.

Although soils and rocks share many attributes, there are major differences between them. These differences are found both in the specimen scale and the mass scale of a project. Because of this, it is unwise to use soil mechanics when working in rock or to use rock mechanics when working in soil (Goodman, 1990). Therefore, we have one section dealing with the strength and behavior of soil and another section dealing with the strength and behavior of rock.

This is a review of current ideas and developments in rock strength and behavior; therefore, almost all of the material in this chapter was very liberally borrowed from the following excellent references: Barton (1982), Coates (1970), Golder Associates (1989), Goodman (1976, 1980, 1990), Johnson (1979), Lambe and Whitman (1969), Mitchell (1993), Piteau (1979), and Selby (1982). For more comprehensive and detailed coverage of material in this section, the reader is referred to these publications.

4D.2 Strength of Rocks

4D.2.1 Friction

There are two fundamental laws of frictional behavior. The first law is that the frictional force (shear force, $T$) between two bodies is directly proportional to the normal force, $N$, between the bodies:

$$\mu = \frac{T}{N} = \tan \phi,$$  \hspace{1cm} (4D.1)

where $\mu$ is the coefficient of friction, $\phi$ is the true intergranular friction angle, $T$ is the shear force, and $N$ is the normal force as shown in figure 4D.1. The second law is that the frictional resistance (shear resistance) between two bodies is independent of the dimensions or size of the bodies. In figure 4D.1, the value of $T$ is constant for a given value of $N$, and the size of the sliding block does not appear in the relationship. This can be shown by sliding a brick over a flat surface. The force required to pull it will be the same regardless of whether the brick lies on a face or on edge. These laws were discovered by Leonardo da Vinci (circa 1500) and restated by the French engineer Amontons in 1699. They are generally referred to as “Amontons’ laws” (Mitchell, 1993).
The basic theory of the friction process is contained in the following points:

(1) Most surfaces on a submicroscopic scale are actually rough; therefore, two objects will be in contact only at the high points, or asperities (Lambe and Whitman, 1969). The actual contact is a very small fraction of the apparent contact area (figure 4D.2).

(2) Because contact occurs at very small points, the normal stresses across these contact points will be extremely high. Even during light loading, the material will reach its yield strength (Lambe and Whitman, 1969).
The actual area of contact, $A_c$, will be:

$$A_c = \frac{N}{q_u} \quad (4D.2)$$

where $N$ is the normal load and $q_u$ is the normal stress required to cause yielding (plastic flow). Because the normal stress required to cause yielding is fixed in magnitude, an increase in total normal load between the two objects must indicate a proportional increase in the area of contact. The increase is a result of plastic flow of the asperities (Lambe and Whitman, 1969).

(3) The high contact stresses result in the two surfaces adhering chemically to each other at their points of contact (Lambe and Whitman, 1969). The adhesive strength provides shear resistance at these points; therefore, the maximum possible shear force, $T_{\text{max}}$, is

$$T_{\text{max}} = sA_c \quad (4D.3)$$

where $s$ is the shear strength of the adhered junctions and $A_c$ is the actual mineral-mineral area of contact. Combining equations 4D.2 and 4D.3 leads to the following relation:

$$T_{\text{max}} = \frac{N}{q_u} s \quad (4D.4)$$

Because $s$ and $q_u$ are material properties, $T_{\text{max}}$ is proportional to $N$. 

\[ \mu = \frac{T}{N} = \frac{\gamma}{\beta} \]

\[ \begin{align*}
N &= A q_u \\
T &= A \gamma
\end{align*} \]
In 1925, Terzaghi brought forth this hypothesis in his book on soil mechanics. However, his theories on the mechanisms of friction were overlooked for a long time. These ideas for describing frictional mechanisms were published independently in the late 1930's by various authors. Known as the Adhesion Theory of Friction, they are the foundation of almost all modern studies of friction (Lambe and Whitman, 1969).

Frictional resistance can be expressed in two ways. One way is to use the coefficient of friction, $\mu$, where $\mu = s/q$. Therefore, if $N$ is the normal force across the surface, the maximum shear force on the surface is:

$$T_{\text{max}} = N\mu. \quad (4D.5)$$

The second way is to use a friction angle $\phi$, such that:

$$\tan \phi = \mu. \quad (4D.6)$$

This is shown geometrically in figure 4D.3.

---

Figure 4D.3.—Definition of friction angle, $\phi$

The fundamental controlling factor of the strength of most rocks is the frictional resistance between mineral particles in contact (Selby, 1982). This surface friction of smooth rock and mineral surfaces is a result of micro-interlocking and adhesion. This may require rock breakage for sliding and gouging of harder minerals into a softer matrix. Various authors have conducted friction measurements on smooth rock surfaces (Goodman, 1976).
The friction properties of smooth rock surfaces vary with microroughness, normal load, weathering, and environmental conditions. Measured values vary with test apparatus and procedure (Goodman, 1976). It has been found that even relatively smooth rock surfaces (roughness = 0.001 in.) show considerable variations in friction values and that they are more responsive to changes in moisture conditions and roughness than to mineralogy. One exception is the sheet silicate minerals consisting of mica, chlorite, clay, talc, and serpentine. These minerals exhibit low friction, especially when wet. Usually the coefficient of friction \( \tan \phi \) ranges from 0.4 to 0.8 \( (\phi = 22^\circ \text{ to } 39^\circ) \). In the sheet minerals it can be as low as 0.2 \( (\phi = 12^\circ) \), and hence rocks composed mostly of such minerals may—if the platy minerals are appropriately oriented—have very low friction angles (Goodman, 1976). The presence of mica in rock does not necessarily reduce \( \mu \).

Most rock surfaces are stronger when dry than when wet. Specimens tested by various authors include granite, basalt, gneiss, sandstone, siltstone, limestone, and dolomite. The results showed higher friction after 1–3 centimeters displacement, along with secondary fracturing and formation of gouge, especially at normal pressures above 500 psi. As the shear displacement continues, wear of rock surfaces causes the rock surfaces to be coated with crushed material, affecting the frictional properties of the rock. If the material consists of dry, unweathered rock surfaces, then the value for friction angle for new material can be higher than for polished surfaces. Wear produced in a moist, weathered rock surface can produce a clay film, causing a substantial drop in the value for friction angle (Goodman, 1976).

### 4D.2.2 Friction Angle

The weight \( (N) \) of a rectangular block sitting upon a horizontal surface is resisted by an equal and opposite reaction \( (R) \), as shown in figure 4D.4. \( N \) and \( R \) together form a compressive force normal to the contact plane and the block cannot move.

If a small horizontal force \( (T) \) insufficient to move the block is applied to the block, the reaction \( R \) will no longer be normal to the contact plane. It reacts in magnitude and direction \( (\theta) \) to equal the resultant \( N \) and \( T \). The triangle of forces shows, in magnitude and direction, the relationships between \( N, T, R, \) and the angle \( \theta \).

According to the force triangle in figure 4D.4, \( \cos \theta = N/R \) and \( \sin \theta = T/R \). If the shear force \( T \) is increased until the block is just ready to slide, \( R \) will increase and so will the angle \( \theta \). At the instant when sliding begins, the frictional contact holding the block and the surface stable along the contact plane will be loosened, and \( \theta \) will have reached its largest possible value on that surface (Selby, 1982).

The maximum value of \( \theta \) is \( \phi \), known as either the angle of plane static friction or angle of shearing resistance, and \( \tan \phi = T/N \) is the coefficient of plane static friction. The resultant stress acting along a plane is broken into a normal stress, \( \sigma \), acting perpendicularly to the contact plane and a shear stress, \( \tau \), acting along the plane. If the plane is a fracture plane in a rock material, displacement along that plane due to the shearing stress will depend upon the angle of internal friction of the material, and

\[
\tan \phi = \frac{\tau}{\sigma}.
\]
The angle of friction for pure quartz is 26° to 30°, and for some clay particles it is close to 13°. These values, however, have little direct significance for determining friction angles of soils and rocks because of the great heterogeneity of these materials. The volume of voids and particle sizes have a larger control on the friction angle of soils than does mineralogy (Selby, 1982).

Water may act as an antilubricant when applied to the surfaces of minerals with large crystal structures, such as quartz and calcite, but this effect becomes smaller as mineral surfaces become rougher, and it is of little significance within soils. Water and clay particles fill in between large grains, behave like lubricants on the surfaces of platy minerals, and reduce friction angles, but this also adds apparent cohesion to a soil (Selby, 1982).
**4D.2.3 Peak and Residual Strength**

Consider a rock sample cored from a block of rock containing a discontinuity such as an absolutely planar bedding plane with no surface roughness or undulations. The bedding plane is still cemented such that a tensile stress would have to be applied to the sample on either side of the discontinuity in order to cause separation.

The sample is subjected to a constant normal stress, \( \sigma_1 \), applied across the discontinuity surface (figure 4D.5). Observed is the stress \( \tau \) required to cause a displacement \( u \).

![Figure 4D.5.—Applied stress and displacement for planar surface of failure.](image)

A typical graph of the shear stress versus shear displacement is shown in figure 4D.6. At small displacements, the shear stress increases linearly with the displacement (Golder Associates, 1989). As the shear stress overcomes the forces resisting movement, the curve becomes non-linear and reaches a peak at which the shear stress reaches its maximum value. This is the point of peak shear strength. From there the shear stress required to cause more shear displacement drops and then levels out at a constant value called the residual shear strength.
If the peak shear strength values—resulting from tests carried out at various normal stress levels—are plotted against the corresponding normal stresses, a curve similar to the one in figure 4D.7a results. This relationship meets the Mohr-Coulomb criterion of failure (section 4A.6). The curve will be nearly linear, given the accuracy of the experimental results, with a slope equal to the peak friction angle $\phi_p$ and an intercept on the shear stress axis of $c_p$, the cohesive strength of the cementing material. This cohesive part of the shear strength is independent of the normal stress; however, the frictional component increases with the increasing normal stress as shown in figure 4D.7a (Golder Associates, 1989). The peak shear strength is defined as:

$$\tau = c_p + \sigma \tan \phi_p .$$  \hfill (4D.8)

Graphing the residual shear strength against the normal stress (figure 4D.7b) results in a linear relationship governed by the equation:

$$\tau = \tan \phi_r .$$  \hfill (4D.9)
This shows that all the cohesive strength of the cementing material has been lost. The residual friction angle $\phi$ is lower than the peak friction angle $\phi_p$. Table 4D.1 shows friction and cohesion values for various types of soil and rock.

Table 4D.1.—Friction and cohesion values for various types of soil and rock (reprinted with permission from Golder Associates, 1989).

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
<th>Unit weight (Saturated/dry)</th>
<th>Friction angle degrees</th>
<th>Cohesion $lb/ft^2$</th>
<th>kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sand</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Loose sand, uniform grain size</td>
<td>118/90</td>
<td>19/14</td>
<td>28-34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dense sand, uniform grain size</td>
<td>130/109</td>
<td>21/17</td>
<td>32-40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Loose sand, mixed grain size</td>
<td>124/99</td>
<td>20/16</td>
<td>34-40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dense sand, mixed grain size</td>
<td>135/116</td>
<td>21/18</td>
<td>38-46</td>
<td></td>
</tr>
<tr>
<td><strong>Clay</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Basalt</td>
<td>140/110</td>
<td>22/17</td>
<td>40-50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chalk</td>
<td>80/62</td>
<td>13/10</td>
<td>30-40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Granite</td>
<td>125/110</td>
<td>20/17</td>
<td>45-50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Limestone</td>
<td>120/100</td>
<td>19/16</td>
<td>35-40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sand and gravel, mixed grain size</td>
<td>120/110</td>
<td>19/17</td>
<td>30-45</td>
<td></td>
</tr>
<tr>
<td><strong>Rock</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Soft bentonite</td>
<td>80/30</td>
<td>13/6</td>
<td>7-13</td>
<td>200-400</td>
</tr>
<tr>
<td></td>
<td>Very soft organic clay</td>
<td>90/40</td>
<td>14/6</td>
<td>12-16</td>
<td>200-600</td>
</tr>
<tr>
<td></td>
<td>Soft, slightly organic clay</td>
<td>100/60</td>
<td>16/10</td>
<td>22-27</td>
<td>400-1000</td>
</tr>
<tr>
<td></td>
<td>Soft glacial clay</td>
<td>110/76</td>
<td>17/12</td>
<td>27-32</td>
<td>600-1500</td>
</tr>
<tr>
<td></td>
<td>Stiff glacial clay</td>
<td>130/105</td>
<td>20/17</td>
<td>30-32</td>
<td>500-3000</td>
</tr>
<tr>
<td></td>
<td>Glacial till, mixed grain size</td>
<td>145/130</td>
<td>23/20</td>
<td>32-35</td>
<td>3000-5000</td>
</tr>
<tr>
<td></td>
<td>Hard igneous rocks —</td>
<td>**</td>
<td>25 to 30</td>
<td>35-45</td>
<td>720000-1150000</td>
</tr>
<tr>
<td></td>
<td>granite, basalt, porphyry</td>
<td></td>
<td></td>
<td></td>
<td>35000-55000</td>
</tr>
<tr>
<td></td>
<td>Metamorphic rocks —</td>
<td>160 to 180</td>
<td>25 to 28</td>
<td>30-40</td>
<td>400000-800000</td>
</tr>
<tr>
<td></td>
<td>quartzite, gneiss, slate</td>
<td></td>
<td></td>
<td></td>
<td>20000-40000</td>
</tr>
<tr>
<td></td>
<td>Hard sedimentary rocks —</td>
<td>150 to 180</td>
<td>23 to 28</td>
<td>35-45</td>
<td>200000-600000</td>
</tr>
<tr>
<td></td>
<td>limestone, dolomite, sandstone</td>
<td></td>
<td></td>
<td></td>
<td>10000-30000</td>
</tr>
<tr>
<td></td>
<td>Soft sedimentary rock —</td>
<td>110 to 150</td>
<td>17 to 23</td>
<td>25-35</td>
<td>20000-40000</td>
</tr>
<tr>
<td></td>
<td>sandstone, coal, chalk, shale</td>
<td></td>
<td></td>
<td></td>
<td>1000-2000</td>
</tr>
</tbody>
</table>

* Higher friction angles in cohesionless materials occur at low confining or normal stresses.

** For intact rock, the unit weight of the material does not vary significantly between saturated and dry states with the exception of materials such as porous sandstones.
It has been shown that in low stress situations, such as those close to the ground surface, the strength along a joint may be described by a Mohr envelope in a straight line form, and the frictional strength is independent of the area of contact and directly dependent on the normal load. It was also discovered that cohesion values are very large for clean closed joints. This may be explained by cohesive interlocking of rock grains on either side of a “hairline” joint or by showing that the Mohr envelope is curved at very low normal stresses and cohesion is therefore negligible (Selby, 1982).

There is a fundamental difference between rock and soil that is basically a question of strength (Piteau, 1979). In rock, the tendency is for the path of failure to follow pre-existing planes of weakness, but in soil, failure occurs in the soil mass itself. Rock masses' shear strength will be a result of the discontinuities (and not the intact rock strength); therefore, shear failure will tend to prefer certain planes of lower strength. In soils, the tendency will be for failure to occur within the soil mass itself, and the surfaces of failure may be curved; shear strength is accepted as being independent of direction (Piteau, 1979).

4D.2.4 Effect of Water on Rock Strength

Adding water to some rocks weakens them through a chemical deterioration of the cement or clay binder. A friable sandstone may lose up to 15 percent of its strength by saturation (Goodman, 1980). In some montmorillonite clay shales, saturation results in a large loss of strength. However, usually the effect of pore-water pressure and fissure water pressure shows the greatest influence in rock strength. If drainage is blocked during loading, water pressure increases, resulting in a consequent loss of strength. Problem 13 in section 4E illustrates this phenomenon.

Many investigators have verified that Terzaghi’s Effective Stress Law is applicable for rocks (Goodman, 1980). The effective stress \[ \sigma' = (\sigma - u) \] equals the difference between the total stress, \( \sigma \), relayed through the interparticle contacts and the stress supported by the pore water, \( u \) (Selby, 1982). For a more detailed discussion of effective stress, see sections 4A.5, 4C.2, and 4E.1.1.

The equation for effective shear strength in saturated rock is:

\[
\tau = c' + (\sigma - u)\tan \phi',
\]

which can be rewritten as:

\[
\tau = c' + \sigma' \tan \phi'
\]

where:

- \( \tau \) = shear strength at any point in the material
- \( c' \) = effective cohesion, as reduced by loss of surface tension
- \( \sigma \) = normal stress imposed by the weight of solids and water above a point
- \( u \) = pore-water pressure
- \( \phi' \) = angle of friction with respect to effective stresses
- \( \sigma' \) = effective normal stress, \( (\sigma' = \sigma - u) \)

The symbols \( c \) and \( \phi \) thus refer to total stresses and \( c' \) and \( \phi' \) to effective stresses. Effective stresses are changed by pore-water pressure and conditions of loading or
testing and are not fundamental properties of the material (Selby, 1982). See section 4C for a more detailed discussion on this subject.

Rock with low porosity, low permeability, and hence low water content will be less affected by saturation. Most rocks that are not deeply buried, however, have numerous joints, bedding planes, and other fissures. High water pressures in these openings or discontinuities can reduce effective stress enough that discontinuity water pressures are of great importance to rock strength (Goodman, 1980).

In most hard rocks the cohesive and frictional properties \((c\) and \(\phi)\) are not considerably changed by the presence of water, and, therefore, reduction in shear strength is due mostly to the reduction of normal stress across the failure surface (Golder Associates, 1989). Therefore, it is the water pressure rather than moisture content that is critical in defining the strength characteristics of hard rock. The presence of a small volume of water at high pressure, confined within the rock mass, is critical to the slope stability of hard rocks than is a large volume of water discharging from a free-draining aquifer (Golder Associates, 1989). Water pressure in joints is probably responsible for more slope failures than are all other causes combined (Johnson, 1979). Problem 13 of section 4E is an example of this phenomenon.

4D.2.5 Shear Strength of Discontinuities

4D.2.5.1. Role of Discontinuity in Slope Failure

Many authors have noted that the stability of a rock mass is more a result of the presence and nature of discontinuities within the mass than of the strength of the rock unaffected by discontinuities (Piteau, 1979).

Figure 4D.8 implies that many rock slopes are stable at steep angles and at heights reaching several hundreds of feet, yet many flat slopes fail at heights of only tens of feet. This difference is because the stability of rock slopes varies with inclination of discontinuity surfaces—such as faults, joints, and bedding planes—within the rock mass (Golder Associates, 1989). If the discontinuities are vertical or horizontal, simple sliding cannot take place, and the slope failure will involve fracture of intact blocks of rock as well as translation along some of the discontinuities (Golder Associates, 1989). However, if the rock mass contains discontinuity surfaces dipping toward the slope face at angles between 30° and 70°, simple sliding can occur, and the stability of these slopes is measurably lower than those with only horizontal and vertical discontinuities.
The influence of the inclination of a failure plane on the stability of a slope is illustrated in figure 4D.9 in which the maximum stable height—critical height—of a dry rock slope is plotted against the discontinuity angle. For this curve, it has been assumed that only one set of discontinuities is present in a very hard rock mass and that one of these discontinuities daylights at the toe of the vertical slope as shown in figure 4D.9. The critical vertical height $H$ decreases from a value exceeding 200 feet for vertical and horizontal discontinuities to about 70 feet for a discontinuity inclination of $55^\circ$. A more complete coverage of this topic is found in section 5H.
The effect of a single fracture in a rock mass is to lower the tensile strength nearly to zero in the direction perpendicular to the fracture plane (Goodman, 1980). If the joints are similar in orientation, then the result is to create pronounced anisotropy of strength, as well as of other properties in the rock mass.

In looking at the stability of a rock slope, the most important factor to consider is the geometry of the rock mass behind the slope face (Golder Associates, 1989). The geometrical relationship between the discontinuities in the rock mass and the slope and aspect of the excavated face will determine whether parts of the rock mass are free to slide or fall. The existence of discontinuities in a rock mass has a very large influence upon the stability of rock slopes. For more in-depth coverage of this topic see sections 3C.3 and 5H.

Just below geometry in importance is the shear strength of the potential failure surface, which may be a single discontinuity plane or several discontinuities involving fracture of the intact rock material. Determining reliable shear strength values is a critical part of the slope design, because relatively small changes in shear strength can result in significant changes in the safe height or angle of a slope. The ability to choose applicable shear strength values depends on two things: the availability of test data and the diligent interpretation of these data in respect to the behavior of the rock mass which makes up the slope. If failure is likely along a
single joint surface similar to the one tested, then it is possible to use these shear test results of the rock joint in designing the slope (Golder Associates, 1989).

These test results can not be used at face value in designing a slope in which complex failure, involving several joints and some intact rock failure, is predicted. In this case, the shear strength data would have to be modified to take into consideration the difference between the shearing in the test and that expected in the rock mass. Also, differences in the shear strength of rock surfaces will occur because of the influence of weathering and surface roughness, the presence of water under pressure, and the differences in scale between the surface tested and the real slope where the failure occurs (Golder Associates, 1989). The choice of appropriate shear strength values for use in a rock slope design relies upon a solid understanding both of the basic mechanics of shear failure and the influence of factors that can change the shear strength characteristics of a rock mass.

**4D.2.5.2. Sliding Due to Gravitational Loading**

(This section is taken from Golder Associates, 1989; also see sections 4E and 5A.)

A block of weight $W$ rests on a plane surface inclined at an angle $\psi$ to the horizontal (figure 4D.10). Gravity is the only force acting on the block, and therefore the weight $W$ acts vertically as shown in the free-body diagram of figure 4D.10. $W$ can be resolved into the components $W\sin \psi$ (driving force parallel to the surface) and $W\cos \psi$ (normal to the surface).

![Figure 4D.10.—Sliding of a block on an inclined plane.](image)

The normal stress, $\sigma$, is

$$\sigma = \frac{W\cos \psi}{A} \quad (4D.13)$$

where $A$ is the base area of the block.
From the definition of shear stress,

\[ \tau = c + \sigma \tan \phi \quad (4D.14) \]

Substituting the value for \( \sigma \) in equation 4D.13, the resisting shear stress, \( \tau \), is

\[ \tau = c + \frac{W \cos \psi}{A} \tan \phi. \quad (4D.15) \]

The resisting shear force, \( S \), is

\[ S = \tau A = cA + W \cos \psi \tan \phi. \quad (4D.16) \]

The block will be just on the verge of sliding or in a condition of (limiting) equilibrium when the driving force acting down the plane is exactly equal to the resisting shear force:

\[ W \sin \psi = cA + W \cos \psi \tan \phi \quad (4D.17) \]

If the cohesion is 0, the condition of limiting equilibrium shown by equation 4D.17 reduces to

\[ \psi = \phi. \quad (4D.18) \]

If water pressure is introduced between the two surfaces in contact, the resultant effect on shear strength of the two surfaces is very evident as can be seen by the demonstration experiment from shown in figure 4D.11 and example 4D.1.
Figure 4D.11.—Forces acting on a water-filled can on an inclined plane with leakage (a) and without leakage (b) from the base of the can (reprinted with permission of Golder Associates from Rock Slopes: Design, Excavation, Stabilization).

Example 1 (from Golder Associates, 1989)

An open can filled with water rests on an inclined plane as shown in figure 4D.11. The forces acting on this can are the same as those acting on the block of rock shown in figure 4D.10. Assuming no cohesion between the can and the inclined surface, the can of water will slide down the surface when $\psi_1 = \phi$.

Putting a hole in the base of the can such that water can enter the gap between the can base and inclined surface results in a water pressure or uplift force $P = uA$, where $A$ is the area of the base of the can.
The normal force $W \cos \psi_1$ is now reduced by this uplift force $P$ by the effective stress principle, and the resistance to sliding is now

$$S = (W \cos \psi_2 - P) \tan \phi.$$  \hspace{1cm} (4D.19)

If the weight per unit volume of the can plus water is $\gamma_c$ and the weight per unit volume of water is $\gamma_w$, then $W = \gamma h A$ and $P = \gamma w h w A$ where $h$ and $h_w$ are the heights as shown in figure 4D.11. From the sketch it can be shown that $h_w = h \cos \psi_1$; therefore, using $h = w / \gamma A$,

$$P = \frac{\gamma w}{\gamma} W \cos \psi_2.$$  \hspace{1cm} (4D.20)

Substituting this value for $P$ in equation 4D.19 and rearranging,

$$S = \left(1 - \frac{\gamma w}{\gamma} \right) W \cos \psi_2 \tan \phi$$  \hspace{1cm} (4D.21)

and the limiting equilibrium condition shown in equation 4D.17 becomes:

$$\tan \psi_2 = \left(1 - \frac{\gamma w}{\gamma} \right) \tan \phi.$$  \hspace{1cm} (4D.22)

If the friction angle of the interface between the can and inclined surface is $30^\circ$, the unpunctured can will slide when the plane is inclined at $\psi_1 = 30^\circ$ (from equation 4D.18). However, the punctured can will slide at a much smaller inclination because the uplift force $P$ has reduced the normal force and therefore reduced the frictional resistance to sliding. Assuming that the total weight of can and water is only slightly greater than the weight of water ($\gamma_c / \gamma = 0.9$), and keeping $\phi = 30^\circ$, equation 4D.22 shows that the punctured can will slide when the plane is inclined at $\psi_2 = 3.3^\circ$, a dramatic difference from the $30^\circ$ for the unpunctured can.

4D.2.5.3. Shearing on an Inclined Plane

Consider the case of a discontinuity surface inclined at an angle $i$ to the driving shear stress direction (figure 4D.12). The shear and normal stresses acting on the failure surface are not $\tau$ and $\sigma$ but (Golder Associates, 1989)

$$\tau_i = \tau \cos^2 i - \sigma \sin i \cos i$$  \hspace{1cm} (4D.23)

$$\sigma_i = \sigma \cos^2 i + \tau \sin i \cos i.$$  \hspace{1cm} (4D.24)
Patton's experiments on shearing regular projections.

Average dip 56-60°

Average dip 43°

Patton's observations on bedding plane traces in unstable limestone slopes.

Figure 4D.12.—Patton's experiments and observations (reprinted with permission of Golder Associates from Rock Slopes: Design, Excavation, Stabilization).
If the discontinuity surface has no cohesive strength, its shear strength is

$$\tau = \sigma \tan \phi. \quad (4D.25)$$

Substituting equations 4D.23 and 4D.24 into equation 4D.25 gives the relationship between the applied shear and normal stresses as

$$\tau = \sigma \tan(\phi + i). \quad (4D.26)$$

This equation was proven true by Patton (1966a; from Golder Associates, 1989) when he conducted a series of tests on models with regular surface projections. He measured the average value of the angle $i$ from photographs of bedding plane traces in unstable limestone outcrops. Three of the traces are drawn in figure 4D.12. It can be seen that the rougher the bedding plane trace is, the steeper the angle of the slope. Patton found that the inclination of the bedding plane trace was about equal to the sum of the average $i$ value and the basic friction angle, $\phi_b$, found from laboratory tests on planar surfaces.

A very important part of shearing on discontinuities inclined to the direction of the applied stress $\tau$ is that any shear displacement $u$ must be associated with a normal displacement $v$. This means that the overall volume of the specimen will increase (the specimen will dilate). This dilation is a major feature in the shearing behavior of actual rock surfaces (Golder Associates, 1989).

### 4D.2.5.4. Surface Roughness

The discussion in section 4D.2.5.3 was simplified because Patton (1966a; from Golder Associates, 1989) found that in order to arrive at agreement between his field observations on the dip of the unstable bedding planes and the sum of the roughness angle and the basic friction angle, it was necessary to measure only the first order roughness of the surfaces.

Figure 4D.13 shows that the first-order projections are those corresponding to the major undulations on the bedding surfaces. The small lumps and ripples on the surface have much higher $i$ values and Patton (1966a; from Golder Associates, 1989) called these second-order projections. Waviness is a measure of the first-order projections, and roughness is a measure of the second-order projections.
Barton (1973) showed that Patton’s results were associated with the normal stress acting across the bedding planes in the observed slopes. At very low normal stresses, the second-order projections come into consideration, and Barton reported a number of values of \((\phi + i)\) which were gathered at extremely low normal stresses. These values are shown in table 4D.2.

Table 4D.2.—\((\phi + i)\) values at low normal stresses (reprinted with permission of Golder Associates from Rock Slopes: Design, Excavation, Stabilization).

<table>
<thead>
<tr>
<th>Type of Surface</th>
<th>Normal Stress (\sigma)</th>
<th>((\phi + i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limestone, slightly rough bedding surfaces</td>
<td>1.57 (\text{kg/cm}^2)</td>
<td>77(^\circ)</td>
</tr>
<tr>
<td></td>
<td>2.09 (\text{kg/cm}^2)</td>
<td>73(^\circ)</td>
</tr>
<tr>
<td></td>
<td>6.00 (\text{kg/cm}^2)</td>
<td>71(^\circ)</td>
</tr>
<tr>
<td>Limestone, rough bedding surfaces</td>
<td>3.05 (\text{kg/cm}^2)</td>
<td>66(^\circ)</td>
</tr>
<tr>
<td></td>
<td>6.80 (\text{kg/cm}^2)</td>
<td>72(^\circ)</td>
</tr>
<tr>
<td>Shale, closely jointed seam in limestone</td>
<td>0.21 (\text{kg/cm}^2)</td>
<td>71(^\circ)</td>
</tr>
<tr>
<td></td>
<td>0.21 (\text{kg/cm}^2)</td>
<td>71(^\circ)</td>
</tr>
<tr>
<td>Quartzite, gneiss and amphibolite discontinuities</td>
<td>- (\text{kg/cm}^2)</td>
<td>80(^\circ)</td>
</tr>
<tr>
<td>beneath natural slopes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beneath excavated slopes</td>
<td>- (\text{kg/cm}^2)</td>
<td>75(^\circ)</td>
</tr>
<tr>
<td>Granite, rough undulating artificial tension</td>
<td>1.5 (\text{kg/cm}^2)</td>
<td>72(^\circ)</td>
</tr>
<tr>
<td>fractures</td>
<td>3.5 (\text{kg/cm}^2)</td>
<td>69(^\circ)</td>
</tr>
</tbody>
</table>
For a basic friction angle of 30°, the effective roughness angle $i$ is between 40° and 50° for very low normal stress levels. It can be assumed that almost no fracturing of the very small second order projections takes place at these low normal stresses, and therefore these steep-sided projections control the shearing process. When normal stress increases, the second-order projections are sheared off, and the first-order projections take over as the controlling factor (Golder Associates, 1989). As the normal stress increases, the first-order projections eventually will be sheared off, and finally shearing will take place through the intact rock material, which consists of the projections and results in the effective roughness angle $i$ being reduced to zero.

The nature of the mating surfaces of a discontinuity plays a very significant role in the stability of rock masses because it affects sliding resistance (Johnson, 1979). The strength along a joint relies upon the roughness of the joint surface (Goodman, 1976). Dodds (1966; from Johnson, 1979) remarked that the strength that can be mobilized along an irregular surface depends on the size of the irregularities and the inclination of the irregular surface relative to the movement direction.

Surface roughness is more important for stronger rocks than for weaker rocks (Johnson, 1979). In stronger rocks, more resistance is offered by the surface roughness before shearing takes place (Patton, 1966b; from Johnson, 1979).

Waviness has a greater effect on slope stability than does roughness (Patton, 1966b; from Johnson, 1979). The asperities that make up roughness may be sheared off, giving a smoother surface, but for two adjacent blocks to move over a wavy surface, there must be relative displacement or dilation normal to the surface for the opposing wavy sides to move parallel to the surface (Piteau, 1970; from Johnson, 1979).

Barton (1976; from Johnson, 1979) and Goodman (1976; from Johnson, 1979) explained that the increase in strength with an increase in the angle $i$ occurs only at low normal stresses at which dilation can occur easily. At higher normal stresses, the strength of the wallrock asperities controls the joint shear strength. This is because the high normal stress prevents the dilatant displacement caused by the upper block moving or riding up on the irregular surfaces (Hoek and Bray, 1974; from Johnson, 1979) as shown in figure 4D.14.
4D.2.5.5. Estimating Joint Compressive Strength and Friction Angle

It has been shown with reasonable certainty that tests on small rock specimens produce high values of strength. Size effects have been attributed to such explanations as changed stress distributions and changed machine stiffness. These arguments cannot be used to explain scale effects observed on joints. This can be shown by the tilt test (section 4D.3.1.6). Tilt angles measured during sliding tests of a large slab of jointed rock are significantly less than the tilt angles measured when the same jointed slab is cut into small samples and tilted individually or as an assembled rock mass (Barton, 1982). The tilt angles increased from 59° to 69° for individual samples and from 47° to 62° for the assembled rock mass.

It is noteworthy that rock engineers have considered the possibility that "laboratory size" samples could ever represent the characteristics of a failure surface perhaps one thousand times larger. Various authors, while investigating scale effects of rock joints in the field of rockfill, have stressed the importance of large scale tests and the importance of extra conservatism and adequate safety factors in the absence of such tests (Barton, 1982).

In a 1974 report, Hoek and Londe (from Barton, 1982) stated that when a very large structure is being designed for long term stability, the design should be based on zero cohesion and residual friction, “which can be determined in small scale laboratory tests.” This probably results in an adequate factor of safety for these major slopes (Barton, 1982). Recent work, however, has shown that the peak strength of a large
joint sample may be lower than the "residual" or "ultimate" strength, measured after a large displacement of a small sample. It is difficult to reach the true residual strength of a non-planar joint surface because dilation persists during surprisingly large displacements (Barton, 1982).

If a preliminary analysis indicates a potential slope failure, a careful characterization of individual joint sets may be in order. The appropriate input data concerning shear strength would then have to be estimated for use in a stability analysis. Methods to gather data are fairly simple and practical, yet they can produce very accurate strength-deformation data of joint behavior (Barton, 1982).

A simple method of delineating the shear behavior of rock joints was developed by Barton (1973; from Barton, 1982). It has three components: the basic or residual friction angle ($\phi_r$ or $\phi_c$), the joint roughness coefficient (JRC), and the joint wall compressive strength (JCS). A basic friction angle for flat nondilatant surfaces in fresh or weathered rock forms the limiting value. Added to this is the surface roughness component ($i$), which is a function of normal stress, JCS, and JRC (Barton, 1982). The JRC varies from 0 for smooth surfaces to about 20 for very rough surfaces (figure 4D.15). The peak drained angle of friction ($\phi'$) at any given normal effective stress ($\sigma'_n$) is

$$\phi' = \phi_r + i = \phi_r + \text{JRC} \log_{10} \frac{\text{JCS}}{\sigma'_n}.$$  \hspace{1cm} (4D.27)

---

![Examples of Roughness Profiles](image)

**Figure 4D.15.** Barton’s definition of joint roughness coefficient, JRC (reprinted with permission of Golder Associates from Rock Slopes: Design, Excavation, Stabilization).
Example 2:

Calculate $\phi'$ for $\phi_s = 25^\circ$, $JRC = 10$, $JCS = 100$ MPa, $\sigma_s' = 1$ MPa.

\[
\phi' = 25^\circ + 10 \log_{10} \left( \frac{100 \text{ MPa}}{1 \text{ MPa}} \right) = 45^\circ
\]

Examples of strength envelopes as a result of JRC values of 5, 10, and 20 are shown in figure 4D.16.

Figure 4D.16.—Method of estimating the peak shear strength of rock joints based on the JRC (20, 10, or 5) and on the JCS (100, 50, 10, or 5 MPa) (reprinted with permission of the Society for Mining, Metallurgy, and Exploration from “Shear Strength Investigations for Surface Mining” by N.R. Barion, 1982).

A method used to determine JCS and its variation with weathering involves the use of the Schmidt hammer (section 4D.3.2.2), which can be applied directly to an unprepared joint surface to get a direct estimate of the JCS (Golder Associates, 1989). The correlation between compressive strength and Schmidt hardness is given in figure 4D.17.
When no equipment is available—such as during a preliminary investigation—table 4D.3 may be used for a very rough estimate for uniaxial or joint compressive strength. Cohesive soils have been included in the table because they are important as joint filling materials.
### Table 4D.3: Approximate values for uniaxial compressive strength (or JCS) of cohesive soil and rock (reprinted with permission of Golder Associates from Rock Slopes: Design, Excavation, Stabilization).

<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Uniaxial Compressive Strength</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>lb/in²</td>
<td>kg/cm²</td>
</tr>
<tr>
<td>S1</td>
<td>VERY SOFT SOIL—easily molded with fingers, shows distinct heel marks</td>
<td>&lt;5</td>
<td>&lt;0.4</td>
</tr>
<tr>
<td>S2</td>
<td>SOFT SOIL—molds with strong pressure from fingers, shows faint heel marks</td>
<td>5–10</td>
<td>0.4–0.8</td>
</tr>
<tr>
<td>S3</td>
<td>FIRM SOIL—very difficult to mold with fingers, indented with finger nail, difficult to cut with hand spade.</td>
<td>10–20</td>
<td>0.8–1.5</td>
</tr>
<tr>
<td>S4</td>
<td>STIFF SOIL—cannot be molded with fingers, cannot be cut with hand spade, requires hand picking for excavation</td>
<td>20–80</td>
<td>1.5–6.0</td>
</tr>
<tr>
<td>S5</td>
<td>VERY STIFF SOIL—very tough, difficult to move with hand pick, pneumatic spade required for excavation.</td>
<td>80–150</td>
<td>6–10</td>
</tr>
<tr>
<td>R1</td>
<td>VERY WEAK ROCK—crumbles under sharp blows with geological pick point, can be cut with pocket knife.</td>
<td>150–3500</td>
<td>10–250</td>
</tr>
<tr>
<td>R2</td>
<td>MODERATELY WEAK ROCK—shallow cuts or scraping with pocket knife with difficulty, pick point indents deeply with firm blow.</td>
<td>3500–7500</td>
<td>250–500</td>
</tr>
<tr>
<td>R3</td>
<td>MODERATELY STRONG ROCK—knife cannot be used to scrape or peel surface, shallow indentations under firm blow from pick point.</td>
<td>7500–15000</td>
<td>500–1000</td>
</tr>
<tr>
<td>R4</td>
<td>STRONG ROCK—handheld sample breaks with one firm blow from hammer end of geological pick.</td>
<td>15000–30000</td>
<td>1000–2000</td>
</tr>
<tr>
<td>R5</td>
<td>VERY STRONG ROCK—requires many blows from geological pick to break intact sample.</td>
<td>&gt;30000</td>
<td>&gt;2000</td>
</tr>
</tbody>
</table>

The residual friction angle ($\phi_r$) of weathered joints is difficult to determine experimentally due to the large displacements required, especially if only small joint samples are available (Barton, 1982). Barton (1982) developed an empirical approach to calculate $\phi_r$:

$$\phi_r = (\phi_b - 20°) + 20(r_i/r_z) \tag{4D.29}$$

where:

$\phi_b =$ basic (minimum) friction angle of flat unweathered rock surfaces (obtained from tilt tests, figure 4D.18 and figure 4D.19).
Figure 4D.18.—Tilt tests can be used to measure $\phi_0$ (of flat surfaces) and the friction angle of joints intersecting drill core. These low stress tests are readily extrapolated to design stress levels (reprinted with permission of the Society for Mining, Metallurgy, and Exploration from "Shear Strength Investigations for Surface Mining" by N.R. Barton, 1982).
Figure 4D.19.—Tilt tests (or pull tests) can be performed relatively inexpensively on large jointed blocks of natural size. Only gravity loading is required, thereby removing the need for heavy jacking equipment. Test results are readily extrapolated to design stress levels (reprinted with permission of the Society for Mining, Metallurgy, and Exploration from "Shear Strength Investigations for Surface Mining" by N.R. Barton, 1982).

\[ r_1 = \text{Schmidt rebound on saturated, weathered joint walls, and} \]

\[ r_2 = \text{Schmidt rebound on dry unweathered rock surfaces (saw cuts, fresh fracture surfaces, etc.)} \]

**Example 3:**

Calculate \( \phi_r \) for

\[ \phi_s = 30^\circ, \quad r_1 = 30, \quad r_2 = 40 \]

Equation 4D.29 gives

\[ \phi_r = (30^\circ - 20^\circ) + (30/40) = 25^\circ. \]

If the basic friction angle (\( \phi_b \)) or residual friction angle (\( \phi_r \)) is determined by direct shear testing (section 4D.3.1.2), then the tests should be carried out over a range of normal stress levels to ensure that a linear relationship between shear strength and normal stress with zero cohesion is obtained. This is because shear strength at very low normal stress can be influenced by extremely small surface roughness of the specimen (Golder Associates, 1989).

If no test results are available, table 4D.4 can be used to obtain an estimate of the basic angle of friction.
Table 4D.4.—Approximate values for the basic friction angle $\phi_b$ for different rocks. Lower value is generally given by tests on wet rock surfaces (reprinted with permission of Golder Associates from Rock Slopes: Design, Excavation, Stabilization).

<table>
<thead>
<tr>
<th>Rock</th>
<th>$\phi$-degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amphibolite</td>
<td>32</td>
</tr>
<tr>
<td>Basalt</td>
<td>31–38</td>
</tr>
<tr>
<td>Conglomerate</td>
<td>35</td>
</tr>
<tr>
<td>Chalk</td>
<td>30</td>
</tr>
<tr>
<td>Dolomite</td>
<td>27–31</td>
</tr>
<tr>
<td>Gneiss (schistose)</td>
<td>23–29</td>
</tr>
<tr>
<td>Granite (fine grain)</td>
<td>29–35</td>
</tr>
<tr>
<td>Granite (coarse grain)</td>
<td>31–35</td>
</tr>
<tr>
<td>Limestone</td>
<td>33–40</td>
</tr>
<tr>
<td>Porphyry</td>
<td>31</td>
</tr>
<tr>
<td>Sandstone</td>
<td>25–35</td>
</tr>
<tr>
<td>Shale</td>
<td>27</td>
</tr>
<tr>
<td>Siltstone</td>
<td>27–31</td>
</tr>
<tr>
<td>Slate</td>
<td>25–30</td>
</tr>
</tbody>
</table>

The roughness angle ($i$) can be obtained from physical measurements as shown in Golder Associates (1989). The scale of the measurement should be appropriate to the scale of the problem.

JRC can be estimated by visual comparison with figure 4D.15. Two scales are given in this figure; use the scale appropriate to the problem at hand. A more reliable value for JRC can be determined by conducting a tilt test on jointed core, or on jointed blocks extracted from existing slopes, as shown in figures 4D.18 and 4D.19 (Barton, 1982; Stimpson, 1981). The value of JRC is back calculated directly from the tilt test by rearrangement of the peak strength equation (Barton, 1982):

\[
\text{JRC} = \frac{\alpha^o - \phi_i}{\log_{10} \frac{\text{JCS}}{\sigma'_{no}}}
\]

where:

- $\alpha^o =$ tilt angle when sliding occurs ($\alpha^o = \arctan \ t/\sigma'_{no} = \phi'$), in degrees
- $\sigma'_{no} =$ effective normal stress acting across joint when sliding occurs
Example 4:

Calculate the joint roughness coefficient for

\[ \alpha^0 = 75^\circ, \phi_r = 25^\circ, JCS = 100 \text{ MPa}, \sigma_{so} = 0.001 \text{ MPa}. \]

Equation 4D.30 gives \( JRC = (75 - 25)/5 = 10 \)

The values of JRC, JCS, and \( \phi_r \) are used to produce peak shear strength envelopes over the desired range of stress. Barton has shown that the value of \( \phi' \) varies inversely with the logarithm of effective normal stress. This is a fundamental result for rockjoints, rockfill, gravel, etc. (Barton, 1982; Barton and Kjaernsli, 1981).

4D.2.5.6. Size-Dependent Joint Properties

Large-scale shear tests in quartz diorite (Barton, 1982; Pratt et al., 1974; from Barton, 1982) and a large series of tests performed by Bandis et al. (1981; from Barton, 1982) have shown that larger shear displacements are required to mobilize peak shear strength as the length of joint sample is increased. The larger but less steeply inclined asperities seem to control peak strength as the length of sample is increased. The sample size does not affect the magnitude of \( \phi_r \) or \( \phi_{so} \), but it does affect the shear displacement needed to reach these values (Barton, 1982).

Both JRC and JCS are lower when the size of the joint sample is increased (Barton, 1982). A way of allowing for this double scale effect involving an approximation method for extrapolating the results of small scale laboratory shear tests to in-situ scale has been developed by Bandis et al. (1981; from Barton, 1982). The method is shown in figure 4D.20 and in example 5.

Figure 4D.20.—Approximation method for extrapolating the results of small-scale laboratory shear tests to in-situ scale (reprinted with permission of the Society for Mining, Metallurgy, and Exploration from “Shear Strength Investigations for Surface Mining” by N.R. Barton, 1982).
Example 5 (from Barton, 1982).

Lab test: \( L_o = 15 \text{ cm, JRC} = 10, JCS = 100 \text{ MPa,} \)
In-situ test: \( L_o = 90 \text{ cm, JRC} = 7, JCS = 50 \text{ MPa} \)

Using example 2 with \( \sigma_s = 1 \text{ MPa,} \) the lab value of \( \phi' \) of \( 45^\circ \) would decrease to approximately \( 37^\circ \) if measured on a 90-cm-long joint in-place sample. The potential effect of sample size on slope stability is too large to be ignored, unless joints are unusually planar, yielding a low JRC.

Planar joints have many of the characteristics of a residual surface, and sample size appears to have only a minor influence on strength (Barton, 1982).

Shear box tests or tilt tests performed on small samples of a joint may not produce reliable strength data, even after approximate correction for the scale effects of JRC and JCS. Joints can have considerable roughness on a small scale but be planar over a length of meters. It is also possible for a joint to be smooth on a scale of a drill core sample but quite undulating over a length of many meters (Barton, 1982).

Whenever possible, attempts should be made to sample the roughness of large-scale exposures of the relevant joint set, using a simple straight edge (taut wire) and offset method. Values of maximum amplitude \( (a) \) measured over a sample length \( (L) \) of up to many meters can be used to obtain a rough estimate of JRC at the proper scale, as shown in Barton (1982).

To improve reliability, we recommend that as many of the discussed methods of estimating shear strength as practical be used. Once again, the methods are:

- Tilt tests on joints sampled in drill core (estimate scale JRC and JCS from Barton, 1982).
- Tilt tests on blocks of natural size (figure 4D.19).
- Roughness profiling at different scales (from Barton, 1982).

4D.2.5.7. Shear Strength of Filled Discontinuities

A common challenge found in rock slope design is a discontinuity filled with soft material. This infilling may be detrital material or gouge from past shear movements or material deposited in open joints as a result of water moving through the rock mass. The presence of a significant thickness of soft, weak filling material can have a major influence on the stability of a rock mass (Golder Associates, 1989).

Goodman (1970; from Golder Associates, 1989) showed the importance of joint fillings in a number of tests in which constructed sawtooth joint surfaces were coated with crushed mica. The decrease in shear strength with increasing filling thickness is shown in figure 4D.21. It can be seen that once the filling thickness exceeds the amplitude of the surface projections, the strength of the joint is controlled by the strength of the filling material. A list of site-specific shear strength values for filled joints, compiled by Barton (1974; from Golder Associates, 1989) is given in table 4D.5.
If a major discontinuity has a significant thickness of filling and exists within a rock mass in which a slope is to be excavated, it is wise to assume that shear failure will occur through the filling material. Therefore, for the preliminary analysis, the influence of surface roughness should be ignored, and the shear strength of the discontinuity should be taken as that for the filling material (Golder Associates, 1989). Determination of the shear strength of this filling material should be carried out using well-established soil mechanics principles.

The effect that filled joints have on the permeability of the rock mass is of major importance. The permeability of clay gouge and similar joint filling material may be three or four orders of magnitude lower than that of the surrounding rock mass, and this can give rise to a buildup of ground water into compartments within the rock mass. When water pressure builds up behind a clay-filled discontinuity such as a fault, the overall stability of the slope can be jeopardized. The situation is made worse because the filling material has a very low shear strength, and failure of the slope may initiate along this discontinuity (Golder Associates, 1989).
Table 4D.5.—Site-specific shear strength values for filled discontinuities in rock (reprinted with permission of Golder Associates from *Rock Slopes: Design, Excavation, Stabilization*).

<table>
<thead>
<tr>
<th>Rock</th>
<th>Description</th>
<th>Peak Strength</th>
<th>Residual Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$c'$ (kg/cm²)</td>
<td>$\phi^o$</td>
</tr>
<tr>
<td>Basalt</td>
<td>Clayey basaltic breccia, wide variation from clay to basalt content.</td>
<td>2.4</td>
<td>42</td>
</tr>
<tr>
<td>Bentonite</td>
<td>Bentonite seam in chalk</td>
<td>0.15</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>Thin layers</td>
<td>0.6–1.0</td>
<td>9–13</td>
</tr>
<tr>
<td>Bentonitic shale</td>
<td>Triaxial tests</td>
<td>0–2.7</td>
<td>8.5–29</td>
</tr>
<tr>
<td>Clays</td>
<td>Over-consolidated, slips, joints and minor shears</td>
<td>0–1.8</td>
<td>12–18.5</td>
</tr>
<tr>
<td>Clay shale</td>
<td>Triaxial tests</td>
<td>0.6</td>
<td>32</td>
</tr>
<tr>
<td>Clays</td>
<td>Stratification surfaces</td>
<td>0.11–0.13</td>
<td>16</td>
</tr>
<tr>
<td>Clay shale</td>
<td>Stratification surfaces</td>
<td>0.41</td>
<td>14.5</td>
</tr>
<tr>
<td>Coal measure</td>
<td>Clay mylonite seams, 1.0 to 2.5 cm thick</td>
<td>0.14–0.3</td>
<td>15–17.5</td>
</tr>
<tr>
<td>Dolomite</td>
<td>Clay gouge (2% clay, PI = 17%)</td>
<td>0</td>
<td>26.5</td>
</tr>
<tr>
<td>Diorite, granodiorite</td>
<td>Clay filled faults</td>
<td>0–1.0</td>
<td>24–45</td>
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<td>and porphyry</td>
<td>Weakened with sandy-loam fault filling</td>
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<td>Tectonic shear zone, schistose and broken granites, distintegrated rock and gouge</td>
<td>8 cm seams of bentonite</td>
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<td>Marlaceous joints, 2 cm thick</td>
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<td>15–17.5</td>
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<td>Lignite</td>
<td>Layer between lignite and underlying clay</td>
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<tr>
<td>and siliceous schists</td>
<td>Stratification with thick clay</td>
<td>3.8</td>
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<td>33</td>
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<td>Remolded triaxial tests</td>
<td>0.42–0.9</td>
<td>36–38</td>
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<td>pyrolusite</td>
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It is critical during a site investigation program for a rock slope design that all major discontinuities filled with clay or other materials are found. If discontinuities are thought to exist, then it is important to expend extra effort to determine whether they really do. This extra effort may include drilling holes in critical locations as well as careful tracing of outcrops and their intersection with any existing excavations. It is important to determine the orientation and inclination of the discontinuities for use in the stability analyses as well as to sample the filling material for shear strength testing (see section 3C.2).

4D.2.6 Large Losses of Shear Strength Due to Displacements

The intact shear strength of rock is much larger than the “undisturbed” strength of soil having the same mineralogic composition. The residual shear strength (the strength obtained after large displacements) is approximately the same for soil as for a relatively flat joint surface in similar rock (Patton and Deere, 1971). Therefore, the loss of strength with displacements can be several orders of magnitude greater for intact rock than for soil (figure 4D.22). Figure 4D.22a shows shear strength versus displacement curves for a rock and a soil with the same mineral composition tested under the same normal stress \( \sigma_n \). The peak shear strength for intact rock is shown as inordinately larger than that for the soil, but as displacements continue the residual strength of both materials is about the same.

Figure 4D.22b is a plot showing a number of shear strength tests on a series of identical specimens of rock and a series of identical specimens of soil, with rock and soil having the same mineral composition. The maximum and residual shear strengths are plotted for each test at the applicable level of normal stress. Figure 4D.22b shows two ways that the strength loss possible in rocks is much greater than that for the majority of soils. The relatively large loss of strength per unit displacement typical of many rock discontinuities (as compared to soils) underlines the importance of focusing on small displacements in rock slope stability studies. This is one reason that failures of rock slopes many times give less advance warning than do failures in soil slopes (Patton and Deere, 1971).

The low residual strengths obtained along rock surfaces that have undergone significant displacement is one of the factors in uncemented faults and shears being important in slope stability problems (Patton and Deere, 1971). Figure 4D.23a shows a cross-section of a rock slope with an irregular joint and an uncemented fault, both with adverse orientations. Pore-water pressures are assumed to be negligible. The shear strength diagram for the irregular joint (figure 4D.23b) shows that the shearing strength at small displacements exceeds the shearing stress; therefore, the joint will remain stable. When the cut is excavated deeper, it exposes the uncemented fault. Figure 4D.23c shows that no shearing strength at any displacement along the fault will not sufficient to resist shearing stresses and a failure of the slope will occur. This shows that it is important to be able to identify faults or shear zones having low residual strength due to previous displacements.
Figure 4D.22.—Comparison of the loss of strength with displacements between soil and rock (reprinted with permission of the Society for Mining, Metallurgy, and Exploration from "Geologic Factors Controlling Slope Stability in Open Pit Mines" by F.D. Patton and D.U. Deere, 1971).

Figure 4D.23.—Significance of pre-existing displacements along faults (reprinted with permission of the Society for Mining, Metallurgy, and Exploration from "Geologic Factors Controlling Slope Stability in Open Pit Mines" by F.D. Patton and D.U. Deere, 1971).
It can be difficult to determine the shear strength of rock surfaces. Even after successful tests have been carried out, the slope designer is still faced with the problem of relating these test results to the full-scale design (Golder Associates, 1989).

The designer should consider a back-calculation analysis (back-analysis) of existing slope failures in order to determine rock slope shear strength parameters. This method has been used quite successfully in soil mechanics for many years and can be equally useful in rock slope design (Golder Associates, 1989).

The simplest type of rock slope failure on which a back-analysis can be performed is that in which strong structural control is predominant. If the plane or planes upon which sliding has occurred are clearly delineated and exposed, the dip and dip directions of the planes can be measured with a high degree of accuracy. The most important unknown in these cases is usually the water pressure distribution in the slope at the time of failure. If this distribution cannot be estimated from current data, upper and lower boundaries for ground water conditions can be found and a possible range of shear strength values can be determined by back-analysis (Golder Associates, 1989).

When doing a back-analysis of sliding on an approximately circular failure surface, the equations stating the condition of limiting equilibrium can be solved only if it is assumed that the shear strength of the failure surface is represented by a simple Mohr-Coulomb failure criterion (equation 4D.8) (Golder Associates, 1989). Even with this assumption, it still is not possible to determine the values for both the cohesive strength \( c \) and the angle of friction \( \phi \) (unless you assume \( c = 0 \)) from the back-analyses of a single slope failure. To define both material constants, one must either back-analyze several failures in the same material or determine one of the constants, usually the friction angle, by such means as direct shear testing or tilt tests.

Figure 4D.24 is a plot of cohesive strengths and friction angles obtained by back-analysis of slopes from various authors. This plot provides a good starting point for stability analysis and a check on how reasonable assumed shear strength data are (Golder Associates, 1989). The reader should add his or her own points to the plot.
4D.3 Measurement of Rock Strength

4D.3.1 Laboratory Tests

4D.3.1.1. Unconfined Compression Test

The unconfined compression test (Goodman, 1980) is frequently used to obtain a relatively accurate determination of unconfined compressive strength (figure 4D.25a). Tests should be performed on smooth NX-size or larger cores and the specimens should be cut and trimmed so that the length/diameter ratio is 2.5 to 3.0. The core specimens must have perfectly square and smooth ends.

Figure 4D.24.—Relationship between the friction angles and cohesive strengths mobilized at failure (reprinted with permission of Golder Associates from Rock Slopes: Design, Excavation, Stabilization).
At least five tests should be performed using a high capacity loading machine (Piteau, 1979). The compressive strength \( q_u \) is calculated as the ratio of peak load \( P \) to cross-sectional area \( A \):

\[
q_u = \frac{P}{A}. \tag{4D.31}
\]

Goodman (1976) stated that this test is not simple to perform properly and results can vary by a factor of more than two. Coates (1970) concluded that the uniaxial compression test is useful primarily for rock classification (for both strength and deformation properties) and for providing a rough index of drilling and grinding properties.

### 4D.3.1.2. Direct Shear Test

The recommended use of the direct shear test is for determining the properties of discontinuities at low normal pressures (Goodman, 1976). The direct shear test is not a good method for testing intact rock specimens (Goodman, 1980). The joint surface is oriented parallel to the direction of applied shear load and the two halves of the sample are fixed inside the shear box (figure 4D.26). A normal load is applied and maintained during shearing displacement.

The direct shear test can be applied for sliding of joints because normal load and shear displacement can be controlled so that surface wear is developed and residual strength measured (Goodman, 1980).
Figure 4D.26.—Shear strength testing of rock sample using a direct shear test.

4D.3.1.3. Ring Shear Test

In the ring shear test an annular (ring-shaped) sample is exposed to a constant normal stress (figure 4D.25d). The sample is confined laterally and horizontally by upper and lower confining rings. The lower half of the sample is carried on a rotating table driven by a worm gear. The upper half of the sample reacts via a torque arm against a pair of fixed proving rings that measure the shear force.

The peak shear stress (τₚ) is called the "shear strength" and is calculated by

\[ \tau_p = \frac{P}{2A} \]  

(4D.32)

where \( P \) is the peak load and \( A \) is the area across the core sample. Unlike those for the compression tests, core specimens for the ring shear test do not require perfectly square and smooth ends. As with the triaxial test, the results permit an understanding of the rate of increase in strength with the confining pressure (Goodman, 1980). The main advantages of the ring shear test are: (1) There is no change in the cross-section of the shear planes as the test proceeds, and (2) The sample can be sheared through an uninterrupted displacement of any magnitude (Selby, 1982).

4D.3.1.4. Brazilian Test (Splitting Tension)

The Brazilian test (figure 4D.25b) is best for estimating the tensile strength of rock (Goodman, 1980). Its advantage is that the rock core does not require much preparation. A rock core about as long as its diameter will split along the diameter and parallel to the cylinder axis when loaded on its side in a compression machine (Goodman, 1980). The result of such a loading is to produce a fairly uniform tensile stress over the major part of the vertical diameter (Coates, 1970).

The average tensile stress at failure is

\[ \tau_s = \frac{2PDL}{\pi} \]  

(4D.33)

where \( P \) is the external load, \( D \) is the diameter of the sample, and \( L \) is its length.
4D.3.1.5. Four-Point Flexural Test

This test places the rock sample in bending (figure 4D.25c). The flexural strength or *modulus of rupture* is the maximum tensile stress on the bottom of the rock corresponding to peak load (at the time of failure). It is calculated using conventional mechanics assuming elastic conditions. The flexural strength is found to be two to three times greater than the direct tensile strength (Goodman, 1980). For the four-point flexural test the modulus of rupture is:

\[
\tau_{mr} = \frac{16P_{\text{max}}L}{3\pi D^3}
\]

where \(P_{\text{max}}\) is the maximum load, \(L\) is the length between load reactions on the lower surface, and \(D\) is the diameter of the core.

4D.3.1.6. Triple Core Tilt Test

The triple core tilt test (figure 4D.18) can be used to find the basic friction angle \(\phi_b\) and the JRC. (See section 4D.2.5.5 for a complete explanation of this topic.)

The effective friction angle of a clean, rough joint consists of a basic friction angle \(\phi_b\) and an angle \(i\), which takes into account surface roughness (Stimpson, 1981). Residual shear tests on sandblasted, flat sawn unweathered rock surfaces seem to approximate \(\phi_b\) most closely in the lab. The triple core tilt test was found to be a good alternative to shear tests to determine \(\phi_b\) on sandblasted, sawn-cut rock surfaces (Stimpson, 1981).

A simple tilting test was found for measuring the critical angle of sliding of cylindrical core surfaces. The requirements are three pieces of core, a base that can be slowly tilted, and the ability to measure to plus or minus 0.5° the angle of tilt at the point of sliding. Two pieces of core, B and C, are placed on the horizontal base in contact with one another, and the third piece of core, A, is placed on top of B and C (figure 4D.18). Cores B and C are restrained from sliding, but core A is free to slide. The base is rotated slowly about a horizontal hinge until sliding of core A along the two line contacts with cores B and C occurs (Stimpson, 1981), and the angle of tilt \(\alpha\) is recorded.

Stimpson (1981) determined that the basic friction angle \(\phi_b\) of core A, \(\phi_A\), is

\[
\phi_A = \tan^{-1}(1.115 \tan \alpha)
\]

assuming that cores A, B, and C are homogeneous, the cylindrical rock surfaces have been prepared in a standard manner during drilling, and \(\alpha\) is the inclination of the axis of core A to the horizontal at the point of limiting equilibrium.

4D.3.2 Strength of Intact Rock

The most widely accepted measure of the strength of intact rock is the uniaxial (unconfined) compressive strength (Selby, 1982). Two field tests of rock strength—the point-load test and the Schmidt hammer test—may be correlated with unconfined compressive strength.
4D.3.2.1 Point-Load Test

This test was developed in Russia to provide a rapid strength test of irregularly shaped rock specimens in the field (Selby, 1982). The test is a result of experiments with compression of irregular pieces of rock. It was found that the shape and size effects were relatively small, and the failure was usually by induced tension (Goodman, 1980).

Specimens of either rock core ("diametral" and "axial" tests) or irregular lumps (the "irregular lump" test) are broken by applying a concentrated load using a pair of hardened steel cones, causing failure by the development of tensile cracks parallel to the axis of loading (Piteau, 1979). A point-load index ($I_p$) is obtained for use in rock strength classification. The diagram of the machine is shown in figure 4D.27.

![Diagram of the point-load testing apparatus](image)

Figure 4D.27.—Point load strength index testing apparatus (reprinted with permission of Piteau Associates from Rock Slope Reference Manual by D.R. Piteau and Associates, 1979).
Specimen dimensions required for the tests are shown in figure 4D.28. For rock that has bedded, schistose, etc., structure, tests should be carried out in both the weakest and strongest directions (Piteau, 1979). The loading should be strictly parallel or perpendicular to the weakness planes.

For each test, the failure load $P$ and distance $D$ between the conical platens should be recorded. From these the point-load strength is given by

$$I_p = \frac{P}{D^2}.$$  \hspace{1cm} (4D.36)

For classification purposes, the values of each test are corrected to a reference diameter $I_p(50)$, of 50 mm (1.97 in.) using the correction chart in figure 4D.29 (Piteau, 1979).
A representative number of tests (generally between 10 and 20) should be carried out for each rock type, and the median value chosen as the point-load index (Piteau, 1979). The median value of a set of test results may be found by deleting the highest and lowest values until only one or two remain. If two values remain, the average of these two is the required median value. If one value remains, that is the median value.

The strength anisotropy index, $I_a(50)$, is computed as the ratio of the corrected median strength indexes for tests perpendicular to planes of weakness to those for tests parallel to planes of weakness (Piteau, 1979). $I_a(50)$ assumes values close to 1.0 for isotropic rocks and higher values when the rock is anisotropic. Table 4D.6 shows a completed worksheet for point-load index tests.

Work done by Broch and Franklin (1972; from Piteau, 1979) shows a correlation between the point-load index and unconfined compression strength as:

$$
UCS = 24 \ I_a(50).
$$  \hspace{1cm} (4D.37)

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<th>Median Load, P (lb)</th>
<th>Median $i_s$ (psi)</th>
<th>Median $i_s$ (50 psi)</th>
<th>UCS (psi)</th>
<th>Run No.</th>
<th>0 Axial (in.)</th>
<th>Breaking Load, P (lb)</th>
<th>$i_s$ Axial (psi)</th>
<th>Median $i_s$ (50 psi)</th>
<th>Median UCS (Axial) (psi)</th>
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**NOTE:** Less than the required number of test runs was taken for some rocks because only a limited number of cores were available.
Point-load strength tests have been performed on many rock types—from the strongest rocks to weathered materials having the characteristics of strong soils. The irregularity of sample sizes can cause scatter in strength data, but this can be counteracted by testing a large number of samples (15–20) and by trimming irregularities off samples with a hammer. When the size correction chart (figure 4D.29) is used, error is unlikely to exceed 15 percent and will probably be much smaller (Selby, 1982).

Studies by Broch and Franklin (1972; from Selby, 1982) show that the point-load tests and unconfined compressive tests have similar scatter in data and that each is very dependent on moisture content. Sandstones may lose up to 20 to 30 percent of their strength as water content increases from a dry to saturated state, and granite may lose more than 13 percent of its strength (Selby, 1982). Therefore, it is important that tests be run at “natural” moisture content. Tests may also be run when similar samples are dried out. Running tests at different moisture contents may help establish the effect of natural water conditions on strength (Goodman, 1980).

The point-load strength test is easily carried out with equipment that can be carried in a vehicle. The test results are closely correlated with unconfined compressive strength tests; published coefficients of correlation range from 0.88 to 0.95 (Selby, 1982).

4D.3.2.2. Schmidt Hammer Test

This test was devised in 1948 by E. Schmidt for carrying out (in-situ) non-destructive tests on concrete (Selby, 1982). The test hammer measures the distance of rebound of a known mass impacting on a rock surface.

Because the elastic recovery of the rock surface depends upon the hardness of the surface, and hardness is related to mechanical strength, the distance of rebound is a relative measure of surface hardness or strength (Selby, 1982).

Three types of Schmidt hammers have value in rock slope studies. The “P” type is a pendulum hammer for testing materials of low hardness with compressive strengths of less than 70 kPa (kN/m²) (10.2 lb/in.²). The “L” type hammer is spring loaded and has a small hammer head, making it best for studying variations of hardness across a rock surface at intervals of a few millimeters (Selby, 1982). The “N” type hammer is the one used most for testing of concrete and rock. It is capable of testing rocks with compressive strength in the range of 20 MPa to about 250 MPa (2,900 lb/in.² to 36,250 lb/in.²), but is not reliable in rocks with compressive strengths less than 30 MPa (4,350 lb/in.²).

The “N” type hammer has a scale from 10 to 100. Rocks such as chalk have a rebound number (R) of 10 to 20, and, at the other end of the scale, basalts, gabbros, and quartzites have values of 55 or higher. The hammer is lightweight—only 2–3 kg (4.4–6.6 lb)—and relatively cheap. A large number of tests may be done in a short time. Weathering rinds, case hardening, individual large clasts, and rock matrices may be easily tested (Selby, 1982).

The disadvantages of the Schmidt hammer are that it is very sensitive to discontinuities in a rock; even hairline fractures may lower readings by 10 points. It is also very sensitive to water content, especially of weak rocks. To eliminate as
much variability as possible, test impact sites should be more than 60 mm (2.5 in) from any edge of a joint; surfaces should be flat and free from small pieces; and the hammer must be moved to a fresh site for each test. Twenty to 50 impacts on each sample of about 2 m$^2$ (21.5 ft$^2$) should be performed, depending on the variability of the rock. The most reliable results are obtained if the lower 20 percent of impact readings are ignored, and measurements are continued until the deviation from the mean value of the remainder does not exceed plus or minus 3 points (Selby, 1982).

Each Schmidt hammer has a slightly different rebound, so the instrument number should be recorded and all instruments should be calibrated regularly against a test anvil.

Figure 4D.17 is a correlation chart that relates the rebound number $R$ to the type “L” hammer orientation, rock unit weight, dispersion of strength, and unconfined compressive strength. Note that the hammer should always be perpendicular to the rock surface. For many instances it is not necessary to convert $R$ values to strength values as the $R$ values alone can be used as indices.

4D.3.2.3. Field Direct Shear

A portable shear machine for testing rock discontinuities in small field samples is described by Golder Associates (1989) and Piteau (1979). A drawing of the machine is shown in figure 4D.30. This machine was designed for field use, and many of the refinements present on larger machines are not included for sake of simplicity (Golder Associates, 1989).

![Diagram of portable shear machine](image-url)

Figure 4D.30.—Drawing of a portable shear machine showing the position of the specimen and the shear surface. A typical machine is 20 inches (51 cm) long and 18 inches (48 cm) high and weighs 85 lb (39 kg) (reprinted with permission of Golder Associates from Rock Slopes: Design, Excavation, Stabilization).
The specimen size for the portable shear machine is limited to about 4 by 4 inches (10 by 10 cm). This means that it is difficult to test joints with surface roughness representative of the in-situ conditions. Golder Associates (1989) recommends that this machine be used only to determine the basic friction angle $\phi$. This can be done by testing field samples and subtracting the average roughness angle $i$, measured on the specimen surface before testing, from the angle $(\phi + i)$ measured in the test.

The main objective of the use of geophysical techniques is to provide accurate data quickly and with minimal subsurface excavation. We will limit our discussion in this section to those techniques that help one discern the geometrical and mechanical properties of discontinuities in rocks.

Geophysical surveys usually produce two types of results—a determination of a physical property at different points within the rock mass, such as velocity of sound waves and density (which may be shown as an instrument reading, but which more often is calculated (Goodman, 1976)), and the distribution of measured quantities over the map (for instance, data might be interpreted to give the depth of each layer in a multilayered model).

The physical quantities measured by geophysical methods may be used directly in a design problem. The measured quantity may have an association; for example, rocks with sonic velocity less than 7,000 feet per second usually can be excavated by ripping, whereas rocks with velocities greater than 10,000 feet per second will usually require blasting (Goodman, 1976).

Generally, geophysical techniques have been used in exploration for locating major well-defined discontinuities and for giving broad ideas about less well-defined features, such as fracture sets or bedding joints (Piteau, 1979).

### 4D.3.3.1. Seismic Methods

Travel times of reflected and refracted waves can be used to measure the depth and thickness of discontinuities or other detectable surfaces and to measure rock quality or severity of fracturing (Piteau, 1979).

Knill and Price (1972; from Piteau, 1979) showed rock quality in terms of the fracture index $F$ the ratio of the in-situ velocity to velocity of the rock material. They stated that rock with a velocity ratio $(F)$ less than 0.5 would be significantly fractured. Helfrich et al. (1970; from Piteau, 1979) correlated their test results with core logging data, relating fracture frequency to seismic velocity. These relationships are shown in figures 4D.31 and 4D.32.
Cracks per meter

Figure 4D.31.—Interpretation of rock quality from drill holes, expressed in cracks per meter in relation to seismic longitudinal velocities in m/sec. The area investigated is situated in the Andes, Chile (reprinted with permission of Piteau and Associates from Rock Slope Reference Manual by D.R. Piteau and Associates, 1979. After Helfrich et al., 1970).

Figure 4D.32.—Relation between mean core lengths in centimeters and seismic velocities in m/sec (reprinted with permission of Piteau and Associates from Rock Slope Reference Manual by D.R. Piteau and Associates, 1979. After Helfrich et al., 1970).
Joints and shear zones will rarely be seen in refraction profiles but may be recognizable in reflection profiles over water (Goodman, 1976). The main problem with seismic refraction is the difficulty interpreting the data properly (Piteau, 1979). Irregularities in the rock mass, the thickness, and the degree of weathering or irregularity of geological contacts may result in significant changes in velocity not related to fractures.

4D.3.3.2. Resistivity Surveys

Most rocks are themselves nonconductive, and the electrical resistivity of a rock derives mainly from salinity in the ground water occupying pores and fractures. Therefore, rock formations will differ in resistivity because of porosity and jointing differences, and hence faults that act as contacts may be mapped by resistivity. Water may collect in faults and shears resulting in signature resistivity profiles. Interpretation of resistivity surveys is more complicated than interpretation of seismic profiles (Goodman, 1976).
4E. Ground Water Fundamentals

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4E.1 Review of Ground Water Fundamentals

In order to describe the characteristics of ground water and explain how they relate to engineering analysis and design, it is important to review certain fundamentals. Those most significant to geotechnical applications are:

- Effective stress principle
- Seepage force
- Critical hydraulic gradient
- Seepage quantity
- Flow nets.

Each of these will be defined and illustrated with sample problems. The significance of these fundamentals to geotechnical analysis will then be discussed.

4E.1.1 Effective Stress Principle

The effective stress principle (Terzaghi, 1936) defines the buoyancy effect that ground water has on a soil aquifer (see sections 4A, 4C, and 4D). It states that total stress is effective stress plus neutral stress, and total pressure is effective soil pressure plus pore-water pressure:

\[ p = p' + u_w \]  \hspace{1cm} (4E.1)

where:

\[ u_w = (H + Z)\gamma_w \]

\[ p = H\gamma_w + Z\gamma_c \]

therefore,

\[ p' = p - u_w = H\gamma_w + Z\gamma_c - (H + Z)\gamma_w = (\gamma - \gamma_w)Z = \gamma_b Z \]  \hspace{1cm} (4E.2)

where:

\[ \gamma_b = \text{buoyant unit weight.} \]
**Problem 1.** Determine the effective soil pressure at the base of slice 4 in the illustration.

\[ H_1 = 16.0 \text{ ft and } H_w = 7.5 \text{ ft} \]

**Total pressure:**

\[ p = H_1 \gamma + H_w \gamma_w \]
\[ p = (16.0 \text{ ft})(120 \text{ pcf}) + (7.5 \text{ ft})(130 \text{ pcf}) = 2895 \text{ psf} \]

**Water pressure:**

\[ u_w = H_w \gamma_w = (7.5 \text{ ft})(62.4 \text{ pcf}) = 468 \text{ psf} \]

**Effective soil pressure:**

\[ p' = p - u_w = (2895 \text{ psf}) - (468 \text{ psf}) = 2427 \text{ psf} \]

---

**4E.1.2 Seepage Force**

Seepage force results from a change in the neutral stress due to ground water flow. For the constant-head permeameter shown, we have

**Effective stress without flow:**

\[ p' = p - u_w = \gamma Z \]  \hspace{1cm} (4E.3)

**Effective stress with upward flow:**

\[ p' = \gamma Z - \Delta u_w = \gamma Z - h \gamma_w \]  \hspace{1cm} (4E.4)

**Hydraulic gradient:**

\[ i = \frac{h}{Z} \]  \hspace{1cm} (4E.5)

\[ p' = \gamma Z - i \gamma \gamma \]

The seepage pressure (friction between percolating water and soil) is \( i \gamma \gamma \gamma \) and the seepage force is \( i \gamma \gamma \gamma \).

---

**4E.1.3 Critical Hydraulic Gradient**

Seepage force acts in the direction of flow and tends to move—pipe—the soil grains in that direction. Piping will occur at hydraulic gradients greater than the critical hydraulic gradient, \( i_c \), a condition where the effective stress becomes zero:

\[ p' = \gamma Z - i_c \gamma Z = 0 \]  \hspace{1cm} (4E.6)

\[ i_c \gamma Z = \gamma Z \]  \hspace{1cm} (4E.7)

\[ i_c = \gamma Z / \gamma \]  \hspace{1cm} (4E.8)
Problem 2. For the toe of the dam with sheet pile wall illustrated, determine

a. whether the hydraulic gradient will be less than the critical hydraulic gradient without the use of a filter blanket, and

b. how thick the filter blanket will have to be adjacent to the dam if a minimum factor of safety (FOS) against piping of 2.50 is to be designed.

a. Hydraulic gradient:

\[ i = \frac{\Delta h}{d} = \frac{7.5 \text{ ft}}{10.0 \text{ ft} + 5.0 \text{ ft}} = 0. \]

Critical hydraulic gradient:

\[ i_c = \frac{\gamma_b}{\gamma_w} = \frac{47.6 \text{ pcf}}{62.4 \text{ pcf}} = 0.76 \]

Actual \( i \) is less than \( i_c \).

b. FOS against piping = 2.50

\[
\text{FOS} = \frac{\text{buoyant weight}}{\text{seepage force}} = \frac{(d\gamma_b + d_f\gamma_f)A}{(\Delta h \gamma_w d)A} = \frac{d\gamma_b + d_f\gamma_f}{\Delta h \gamma_w}
\]

\[ d_f y_f = \text{FOS} (\Delta h \gamma_w) - d\gamma_b \]

\[ d_f = \frac{2.5(7.5 \text{ ft})(62.4 \text{ pcf}) - (15.0 \text{ ft})(47.6 \text{ pcf})}{125 \text{ pcf}} = 3.7 \text{ ft} \]

Use 4 feet.
Problem 3. Estimate to what depth the illustrated trench can be excavated before sand and water can be expected to "boil" into the excavation.

Find critical hydraulic gradient, \( i_c \), and equate to \( i \):

\[
i_c = \frac{Y_b - Y_w}{Y_w} = \frac{115 \text{ pcf} - 62.4 \text{ pcf}}{62.4 \text{ pcf}} = 0.84
\]

\[
i = \frac{h}{Z}
\]

Expect piping when \( i = i_c \):

\[
i = \frac{(9 \text{ ft}) - Z}{Z} = 0.84
\]

\[
9 - Z = 0.84 Z
\]

\[
1.84 Z = 9.0
\]

\[
Z = 4.9 \text{ ft}
\]

Critical Trench Depth = (15.0 ft) - (4.9 ft) = 10.1 ft

4E.1.4 Seepage Quantity

Darcy's Law (Darcy, 1856) mathematically describes laminar ground water flow through a porous medium as

\[
Q = K_i A t
\]  
(4E.9)

where \( Q \) is discharge (seepage quantity) per unit of time, \( K \) is hydraulic conductivity, \( i = \frac{\Delta h}{\Delta L} = \frac{h}{Z} \) is hydraulic gradient (as shown for the illustrated constant-head permeameter), and \( A \) is the area normal to the direction of flow.

Darcy's law can also be expressed in terms of velocity:

\[
Q = K_i A = v A
\]  
(4E.10)

\[
v = K_i \frac{Q}{A}
\]  
(4E.11)

\[
v_s = \frac{K_i}{n_e}
\]  
(4E.12)

where \( Q \) is discharge per unit of time, \( v \) is specific discharge, \( v_s \) is linear velocity, and \( n_e \) is effective porosity (the ratio of the actual volume of the pore space through which water is seeping to the total volume).

The symbol \( Q \) is used for total discharge per unit of time, while \( q \) is sometimes used for discharge per unit time per unit width of the aquifer.
Ranges of values for hydraulic conductivity ($K$) and for specific or intrinsic permeability ($k$) for various rock and soil types are given in table 4E.1. Hydraulic conductivity is a function of both the medium (soil or rock) and the fluid (water), while intrinsic permeability is a function only of the medium. One can convert between $k$ and $K$ using the conversion factors and assumptions given in table 4E.1.

**Problem 4.**

Estimate $q$, the quantity of seepage per unit time per foot of width, of the perched ground water aquifer illustrated.

Estimate $K$ for silty sand (from table 4E.1):

$$K \approx \left(1 \times 10^{-4} \text{cm/sec}\right) \left(\frac{1 \text{ ft}}{30.5 \text{ cm}}\right) \left(\frac{60 \text{ sec}}{1 \text{ min}}\right) = 2 \times 10^{-4} \text{ ft/min}$$

$$i = \frac{\Delta h}{L} = \sin \alpha_w = 0.24$$

$A$ (area normal to flow direction for one foot of aquifer width):

$$A = h_w \cos \alpha_w (1 \text{ ft}) = (6.5 \text{ ft}) (\cos 14^\circ) (1 \text{ ft}) = 6.3 \text{ ft}^2$$

$$q = K i A \approx \left(2 \times 10^{-4} \frac{\text{ft}^3}{\text{min}}\right) (0.24) (6.3 \text{ ft}^2) = 3 \times 10^{-4} \frac{\text{ft}^3}{\text{min}}$$

$$q = \left(3 \times 10^{-4} \frac{\text{ft}^3}{\text{min}}\right) \left(\frac{7.48 \text{ gal}}{1 \text{ ft}^3}\right) = 0.002 \text{ gpm}$$
Table 4E.1.—Values of hydraulic conductivity \( (K) \) and intrinsic permeability \( (k) \) (adapted with the permission of Prentice-Hall from p. 29 of *Ground water* by R.A. Freeze and J.A. Cherry. Copyright© 1979 Prentice-Hall.)

<table>
<thead>
<tr>
<th>( k ) in ( \text{cm}^2 )</th>
<th>( k ) in ( \text{ft}^2 )</th>
<th>( K ) in ( \text{m/s} )</th>
<th>( K ) in ( \text{ft/day} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1.0764 \times 10^{-3} )</td>
<td>1.0132 ( \times 10^8 )</td>
<td>9.80 ( \times 10^2 )</td>
<td>3.22 ( \times 10^3 )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>( 1.02 \times 10^{-3} )</td>
<td>( 1.10 \times 10^{-6} )</td>
<td>( 1.04 \times 10^5 )</td>
</tr>
</tbody>
</table>

\( K = \frac{\rho \varphi}{\mu} \)

\( \rho = 1 \text{ g/cm}^3 \) (62.4 lb/ft \(^3\)) — fluid density of water

\( \varphi = 9.8 \text{ m/s}^2 \) (32.152 ft/s \(^2\)) — gravitational force

\( \mu = 1 \text{ cP} = 1 \text{ g/m-s} \) (0.72 x \( 10^{-4} \) lb/ft/s) — dynamic viscosity of water

Multiply by to get

Multiply by to get

\( k \) in \( \text{cm}^2 \)

\( K \) in \( \text{m/s} \)

\( K \) in \( \text{ft/day} \)

\( k \) in \( \text{ft}^2 \)

\( K \) in \( \text{ft/s} \)

\( K \) in \( \text{ft/day} \)

\( k \) in darcy

\( K \) in \( \text{ft/s} \)

\( K \) in \( \text{ft/day} \)

\( K \) in \( \text{ft/day} \)

\( K \) in \( \text{ft/day} \)

\( K \) in \( \text{ft/day} \)
Problem 5.

Estimate \( q \), the quantity of seepage per unit time per foot of width, of the confined ground water aquifer illustrated.

Estimate \( K \) for silty sand (from table 4E.1):

\[
K \approx \left( \frac{0.01 \text{ cm}}{\text{sec}} \right) \left( \frac{1 \text{ ft}}{30.5 \text{ cm}} \right) \left( \frac{60 \text{ sec}}{1 \text{ min}} \right) = 0.02 \text{ ft/min}
\]

\[
i = \frac{\Delta h}{L} = \frac{3.0 \text{ ft}}{25 \text{ ft} \cos 6^\circ} = 0.12
\]

\( A \) (area normal to flow direction for one foot of aquifer width):

\[
A = (2.0 \text{ ft}) (\cos 6^\circ) (1 \text{ ft}) = 2.0 \text{ ft}^2
\]

\[
q = K i A = 0.02 \text{ ft/min} \times 0.12 \times 2.0 \text{ ft}^2 = 0.005 \text{ ft}^3/\text{min}
\]

\[
q \approx 0.005 \frac{\text{ft}^3}{\text{min}} \left( \frac{7.48 \text{ gal}}{1 \text{ ft}^3} \right) = 0.04 \text{ gpm}
\]

Here we convert the measured ground distance to a horizontal distance between observation wells. In this example, the difference is insignificant (less than two inches in 25 feet), and is certainly overshadowed by the coarseness of the estimate of \( K \).

4E.1.5 Flow Nets

The movement of ground water through a porous medium can be represented graphically by two intersecting families of curves: a series of flow lines, the streamlines in the direction of seepage, intersected by a series of equipotential lines, lines having the same potential or head. Casagrande (1937) and Cedergren (1967)
gave the mathematical basis for these flow nets, and the user is referred to these sources for a complete description. The important points from these references directly affecting the analysis used in this guide are:

- The appearance of the flow net within a soil is not affected by the permeability of that soil, but deflection of the flow lines occurs as the flow passes from a soil of one permeability to another.

- Certain entrance and exit requirements of flow must be met.

- Adjacent equipotential lines have equal head loss.

- The peizometric level is the potential head or water level at any point within the soil.

- The same amount of seepage flows between adjacent pairs of flow lines.

- For isotropic soils, the flow lines and equipotential lines intersect at right angles to form areas that are basically squares.

- For anisotropic soils, the intersection of the flow lines and equipotential lines is not a right angle and becomes more acute in the direction of flow as the ratio of permeability in the flow direction increases in relation to the permeability in the equipotential direction.

- For isotropic soils, the upper flow line (phreatic surface) of a flow net for seepage into a drainage structure with an entrance slope of 180° or more from horizontal is basically parabolic in shape and can be defined mathematically with a parabolic equation. For drainage structures with entrance slopes less than 180° from horizontal, the phreatic surface can be approximated as a modification of this basic parabolic shape. Sketches for problems 6 and 7 illustrate measurement of the entrance slope angle.

**Problem 6.** For the undrained earth dam illustrated, assume the soil is homogeneous and isotropic; construct a flow net using the graphical procedures of Casagrande and Cedergren. Construct the phreatic surface using the mathematical procedure from Casagrande for comparison to the flow net solution.

a. Undrained flow net
b. Undrained mathematical-graphical

Problem 7. For the earth dam of problem 6, construct the phreatic surface for the illustrated blanket drain using flow-net and mathematical procedures.

a. Drained flow net

b. Drained mathematical
Problem 8. The procedures illustrated in problems 6 and 7 are applicable to analysis of earth dams constructed on relatively flat ground where the primary source of seepage through the dam is from the impounded reservoir. In mountainous terrain, where sloping drainage barriers with seepage parallel to those barriers are more common, flow net analysis is still applicable, but the mathematical procedures may not be quite right. In this problem, a plugged culvert is causing impoundment behind a through-fill. Seepage parallel to a drainage barrier is also present in the foundation subsoil. The location of the phreatic surface within the fill material can be estimated from flow net analysis and by a modification of Casagrande's mathematical procedure. Three relative permeability conditions are assumed in construction of the flow nets: (1) fill material impervious and subsoil pervious, (2) subsoil impervious and fill pervious, and (3) fill and subsoil both pervious with the same permeability ($k_r = k_s$).

a. Flow net for an impervious fill overlying a pervious subsoil
b. Flow net for a pervious fill overlying an impervious subsoil

![Flow net for a pervious fill overlying an impervious subsoil](image)

c. Flow net for $k_f = k_s$

![Flow net for $k_f = k_s$](image)

d. Modified mathematical-graphical

![Modified mathematical-graphical](image)
Practical Application of Solutions to Problems 6 Through 8

There are several noteworthy observations that should be made of the results of problems 6 through 8 as they relate to determining phreatic surfaces which have ground water originating from reservoir or combined reservoir/drainage-barrier-controlled sources:

- Problems 6 and 7. For an earth dam on an impervious subsoil with little or no slope, the mathematical-graphical procedures of Casagrande can be used to estimate the phreatic surface for stability analysis with reasonable accuracy as compared to the more laborious flow net construction.

- Problems 6 and 7. If the subsoil of the earth dam is not impervious, some of the ground water flow will be through the subsoil aquifer. Cedergren suggested procedures for flow-net construction for these conditions. Practically, these phreatic surfaces should be lower than for the impervious subsoil conditions. The mathematical-graphical procedure can still be used and the results will be conservative.

- Problems 8a and 8c. The effects of a sloping subsurface subsoil aquifer are illustrated in these flow nets. Some or all of the impounded water will infiltrate into the subsoil and flow under the fill through this aquifer, and the flow direction will be controlled by the slope of the impervious (or less pervious) basal barrier. At the toe, the subsoil aquifer must return to equilibrium with the drainage barrier (the upper flow line at atmospheric pressure at the ground surface), and the excess flow must exit the slope as seepage to the surface. This is the area of prime concern for piping to occur and may require protection by drainage as illustrated in problem 7.

- Problem 8a. Even though the entire ground water flow is directed through the subsoil aquifer by an impervious fill, the stability analysis must consider the artesian pore-water pressure that exists in this aquifer. This is a "confined aquifer" (further explanation to follow) where the upper flow line is not at atmospheric pressure, so an anticipated piezometric surface must be constructed from which to compute pore pressure on a potential failure surface when that surface is within the aquifer. If the fill is truly impervious, this piezometric surface should be parallel to the basal drainage barrier of the
aquifer and extend from the contact of the impounded-water surface to a location directly above the toe of the fill where the flow out of the slope can relieve the artesian pressure.

- Problem 8d. The dominating effects of the drainage barrier in controlling the direction of the flow lines is illustrated in all of the flow nets of problems 6 through 8. This suggests that the mathematical-graphical procedures of Casagrande might be modified by making them relative to the slope of the basal drainage barrier of the subsoil aquifer. This is illustrated in problem 8d.

- Problem 8e. This is a composite of all of the phreatic and piezometric surfaces constructed in problem 8. The relative permeability of the two materials must be considered in estimating the location of the anticipated phreatic surface, and Cedergren suggests procedures for flow-net construction.

Practically, for this problem regardless of the relative permeability, all of the phreatic surfaces should fall somewhere between the phreatic surface constructed for $k_f = k$, and one of the phreatic or piezometric surfaces for the extreme impervious conditions for the two materials. Also of significance is the relative thickness of the subsoil aquifer in comparison to the impounded water depth and the amount of ground water flowing in that aquifer before it is impacted by infiltrated impounded water. This also influences the location of the phreatic surface within the fill material. In this case, the thickness of the subsoil aquifer was about equal to the depth of the impounded water and the aquifer was half full above the impounded water location. A lesser aquifer depth or a greater amount of the aquifer already flowing full would result in a higher phreatic surface within the fill material. The converse would also be expected.

It is unlikely that anyone would undertake a time-consuming flow net construction for a problem such as this, which means that a simplified mathematical-graphical method is needed for the practitioner. It would appear that the modified mathematical-graphical procedure used in the solution of problem 8d could be used with judgment and supplemented by additional study:

- Use the modified mathematical-graphical procedure to predict the phreatic surface within a fill when $k_f$ is about equal to $k$, and the unsaturated depth of the subsoil aquifer before being impacted by the impounded water is about the same as the impounded water depth.

- Adjust the mathematical-graphical procedure as appropriate to account for relative differences in permeability and/or impounded depth-to-unsaturated aquifer depth ratio by adjusting the fill slope seepage intercept location, $a$. The lower limit would be $a = 0$, and the upper limit could be located by extending a line from the contact of the impounded water surface and the fill parallel to the basal drainage barrier of the subsoil aquifer to the intersection with the fill slope (similar to problem 8.a).

4E.1.5.1. Flow Nets for Natural Aquifers

To better understand the flow of ground water through natural watershed aquifers and how it differs from flow through an earth dam, the differentiation of confined versus
unconfined aquifers must be made. Figure 4E.1 shows simple flow nets for unconfined and confined aquifers.

---

**Unconfined Aquifer (Figure 4E.1a).** An unconfined aquifer is bounded at the bottom only. This lower drainage barrier is usually a material of lower permeability than the aquifer material. The slope of this drainage barrier controls the direction of flow. The phreatic surface is the upper flow line of the flow net where the water pressure is the same as atmospheric pressure.

**Confined Aquifer (Figure 4E.1b).** A confined aquifer is bounded at the top and bottom by drainage barriers (materials of lower permeability). The upper flow line is not necessarily a phreatic surface because the upper drainage barrier may be restricting the flow. This can result in "artesian" pressures within the aquifer, and the pressure at the upper flow line of the aquifer will be greater than atmospheric pressure.

**4E.1.5.2. Flow Patterns for Multiple Aquifers**

Frequently in montane watersheds, unconfined and confined aquifers exist at the same location separated by a confining layer. Depending on the water pressure differential between the two aquifers and the relative permeabilities of the aquifer and confining soils, flow patterns develop between the two aquifers through the confining soil. In figure 4E.2, only the flow lines of the flow nets are illustrated. Two important flow net construction principles are demonstrated:

1. Seepage quantities between adjacent pairs of flow lines must be constant.

2. The flow lines are deflected when the flow passes from a soil of one permeability to another according to certain entrance and exit requirements based on the relative change in permeability at the boundary.

This results in the relative difference in slope and spacing of the flow lines as shown in figure 4E.2.
4E.1.6 Analysis of Piping and Effects on Structures

Seepage force is the most significant factor to consider in the evaluation of piping potential and the effects of ground water on structures (problems 2 and 3). Note that the permeability of the soil is not a factor in seepage force analysis, that the ground water must be flowing for seepage force to occur, and that the force is related to friction between the ground water and the pores through which it is flowing.

4E.1.6.2. Slope Stability Analysis

The most important ground water factor to consider in slope stability analyses is pore-water pressure. The ground water need not flow to result in detrimental pore-water pressure. Later we will look at how hydrostatic ground water pressure affects the stability of rock slopes. If the ground water is flowing, the change in pressure in relation to the position on the slope must be taken into account in the stability analysis.

Unconfined Aquifers (Figure 4E.2a). Flow nets are a good means to define and analyze this slope-position/pore-water-pressure relationship for an unconfined aquifer. Flow nets are seldom constructed in actual practice, but the manner in which pore-water pressure is computed can still be based on flow net principles. If the phreatic surface can be defined, pore-water pressure at any location can be computed from the flow-line/equipotential-line relationship, as will be described later. Note that simple flow nets and the phreatic surface analysis that is made are based on the assumption that the porous soil or rock medium through which the ground water is flowing is homogeneous and isotropic. If that is true, the permeability of the medium need not be considered. The shape of the phreatic surface can be expected to be the same for a weathered rock as it is for a clay or a gravel under these assumptions. However, most subsurface conditions do not fit these assumptions and permeability can be a factor. There is almost always some anisotropy in the form of directional permeability differences within the unit, resulting in preferential ground water flow direction. The effect is to change the shape of the flow net. For slope
stability, the major effect on the analysis is that the equipotential lines and flow lines do not intersect at right angles, as is assumed in the calculation of pore-water pressure. If this isotropy can be defined, it can be accounted for in the analysis, as will be discussed later. Permeability is most significant in the definition of aquifer boundaries with confining layers or surfaces of significantly lower permeability forming the limits of the aquifer.

Confined Aquifers (Figure 4E.2b). Pore-water pressure within a confined aquifer is computed much differently than for unconfined aquifers. The significant difference is due to the artesian pressure within the layer which would cause the water to rise above the upper limits of the confined aquifer to some higher piezometric level. If this artesian pressure does not exist, the aquifer should be treated as an unconfined aquifer with the upper flow line as the phreatic surface. Usually, though, the artesian pressure is significant and the confined aquifer thickness is sufficiently small that the most accurate way to compute the pore-water pressure is to define the piezometric line above the aquifer and treat it hydrostatically (without correction for the flow/equipotential relationship). This is discussed more thoroughly in section 4E.4.

4E.1.6.3. Subsurface Drainage Analysis

The need for a subsurface drainage system is usually recognized in a slope stability analysis. Pore-water pressure and flow net relationships are significant factors because the subsurface drainage is intended to reduce the pore-water pressure. Algorithms for analyzing the effects of drainage on the shape of a phreatic surface during preconstruction design are discussed in section 4E.5. Permeability is more significant in drainage analysis than it is in stability analysis because the quantity of seepage must be estimated for proper installation and sizing of the drainage structure.

4E.2 Ground Water Occurrence in Soil and Rock

4E.2.1 Saturated Versus Unsaturated Flow

In order to better visualize how ground water accumulates and moves through a watershed, some simple relationships must be examined. Subsurface conditions are usually much more complex than this, but this is a good starting point. Figure 4E.3 shows a simple, but by no means universal, scheme where recharge from rain can move through a soil mass in unsaturated flow until its downward flow is impeded by a drainage barrier (the material below which is much less permeable than the material above), and saturated flow begins along the slope of the barrier. This is a simple unconfined aquifer often referred to as a “perched watertable.” The flow in the unsaturated zone would be expected to be mostly vertical and either up or down, depending on whether recharge or capillary rise is dominant at the time. At the subsurface drainage barrier, a phreatic surface can develop with a saturated zone below which ground water flow is parallel to the drainage barrier. Except during periods of recharge, flow in the unsaturated zone should be upward through capillary rise above the phreatic surface. The shape of the saturation-versus-depth curve depends at least in part on the amount of fines in the soil. Fine-grained soils can maintain capillary moisture to a greater height above the phreatic surface than can
course-grained soils. Some typical saturation-versus-depth curves for various Unified Soil Classification System (USCS) soil types are also shown in figure 4E.3.

Figure 4E.3.—Simple single-soil unconfined aquifer unsaturated and saturated flow with typical saturation-versus-depth curves.

4E.2.1.1. Ground Water Dynamics

The phreatic surface of ground water in montane watersheds is constantly fluctuating. From a simplistic view, it can be expected to increase to a higher position in the soil column during periods of recharge through precipitation and decrease to a lower position through ground water movement downslope and through capillary rise of the ground water to the ground surface where it evaporates or is transpired by vegetation. Figures 4E.4 through 4E.6 demonstrate some simplistic theoretical ground water dynamics for a silty sand (SM) soil:

- During periods of long drying, when the moisture available for capillary rise is depleted from the saturated zone, (figure 4E.4) followed by

- Intermittent recharge (figure 4E.5), with a saturated plume moving through the unsaturated zone, and gradual increase of the position of the phreatic surface in the soil column (increase in the thickness of the saturated zone), or

- High-intensity or prolonged recharge (figure 4E.6), with the entire unsaturated zone near complete saturation when the initial phreatic surface develops. In this case, ground water (the phreatic surface) would rise not gradually but rapidly toward the ground surface because of the near-saturated condition in the unsaturated zone. This would cause a sudden increase in pore-water pressure which could not only be expected to cause landslides, but to cause physical adjustments within the soil. Such phenomena as piping of the fines within the saturated zone or further consolidation of the material below the drainage barrier would likely occur under these conditions. A similar scenario could be envisioned in a confined aquifer under these extreme recharge conditions.

The saturation/depth curve (figure 4E.4) shifts from position A to B to C as the phreatic surface drops from position 1 to 2 to 3. Phreatic surface position 3 is at the
drainage barrier. Continued drying at this location destroys the phreatic surface and the saturation/depth curve shifts from position C to D to E as the saturation at the drainage barrier decreases below 100 percent.

Prior to the start of recharge, the initial position of the saturation/depth curve (figure 4E.5) is position A (the same as position E in figure 4E.4). When recharge begins, the saturation/depth curve shifts to position B due to the excess moisture at the ground surface. If the recharge is intermittent and stops, the saturation at the ground surface begins to drop as the saturation plume travels through the soil column (as shown in position C). After the plume advances to the drainage barrier, a phreatic surface can again develop at position 3, and the saturation/depth curve shifts to position D. As the phreatic surface increases from position 3 to 2 to 1 due to ground water flow through the aquifer, the saturation/depth curve moves from position D to E to F.
Prior to and at the beginning of recharge, the same saturation/depth curve relationship (figure 4E.6) exists as in figure 4E.5: position A and B are the same. In this case, recharge continues, so there is a continued excess of water at the ground surface as the plume moves down the soil column and the saturation/depth curve moves from position B to C to D to E. When the plume reaches the drainage barrier, a phreatic surface can develop; however, the phreatic surface does not necessarily develop first at the barrier, because of the high degree of saturation in the unsaturated zone, but can develop much higher in the soil column. In an extreme condition, this could be at the ground surface in position 4.

![Diagram of soil flow and saturation/depth curve](image)

**Figure 4E.6.**—Response of silty sand soil to high-intensity or prolonged recharge after a prolonged drying period.

### 4E.2.2 Compound Multi-Soil Aquifers

Not all ground water aquifers are as simple as those illustrated thus far. Frequently, there is more than one soil unit with more than one confined or unconfined aquifer. A common occurrence is an open-graded soil directly above the drainage barrier which appears to have had most of the fines piped out of it (SP or GP soils in this zone). Also, subsurface structural control (usually bedrock discontinuities) of confined aquifers within or below the drainage barrier material may be the source of ground water. Some example cross-sections are illustrated in figure 4E.7. The importance of delineating the actual subsurface conditions controlling ground water flow during the field investigation (as described in sections 3C and 3D) cannot be overemphasized.
4E.2.3 Effects of Constructed Structures

Construction of structures, such as road fills, can load an underlying soil to a degree where the subsurface flow characteristics for a confined or unconfined aquifer are affected. As illustrated in figure 4E.8a, this often will appear as a higher water level in the vicinity of the fill. If this is excessive and slope failure results, the failure may also rupture the confining layer and allow the ground water from the confined aquifer to rise into the fill material. This is illustrated in figure 4E.8b. If this occurs, the stability problem is compounded and the stabilization becomes more complex.
4E.2.4 Ground Water in Rock

Ground water affects the stability of a rock slope much as it does that of a soil slope in that it develops water pressure in the rock discontinuities. This water pressure is much like the pore-water pressure in soils in that it reduces effective stress and the frictional strength of the rock along those discontinuities. How intact and porous the rock mass is has a great deal to do with how ground water occurs and how it should be treated in the analysis. A rock mass so greatly broken up and weathered that the water moves through as in an unconfined aquifer should be analyzed similarly to a soil under those conditions. Likewise, if a porous rock unit, such as a sandstone, exists between two shale units of low permeability, it behaves as a confined aquifer in soil. The stability of a more intact rock mass that is bounded by only a few discontinuities is conventionally analyzed for two-dimensional plane failure or three-dimensional wedge failure. Ground water in these analyses is assumed to act as hydrostatic water along the discontinuities.

Figure 4E.9 illustrates variations of how hydrostatic ground water can exist and should be analyzed. This is for a plane failure, and the analytical techniques exist to allow the user all of these options under fully or partially saturated depths. Analysis
becomes more difficult for the wedge failure due to the determination of three-dimensional water pressures and the areas of the failure surfaces to which these pressures are applied. Currently, only the option for full-depth saturation for either hydrostatic with drained toe (if there is no tension crack) or failure surface pressure limited to that developed in the tension crack, if one is to be analyzed, is available to the user. These analytical techniques are covered in section 5H.

Figure 4E.9.—Variations of how hydrostatic ground water can exist—and should be analyzed—for a plane failure.

4E.3 Field Identification and Interpretation

Regardless of the type of analysis, as much as possible must be learned about the ground water conditions during the field investigation. Of primary importance is to determine:

- If ground water exists, whether it is hydrostatic or flowing;
- If it is flowing, whether it is a confined or unconfined aquifer;
- The upper and lower limits and slope of the aquifer;
• Whether there is evidence that the ground water level is periodically higher than at the time of investigation;

• The characteristics (soil type and permeability, rock discontinuities) of the aquifer;

• The characteristics of the confining surfaces;

• The proximity of the aquifer to the existing or potential failure surface; and,

• Most important, the highest phreatic surface for an unconfined aquifer or piezometric surface, or both, for a confined aquifer and, if it is an existing failure, the height of the ground water at the time of failure.

A more complete coverage of field investigations for slope stability projects is given in section 3D. The most interesting and demanding investigations are usually those in which ground water is an important factor about which to gather information. It is important that an experienced specialist makes the interpretation, mostly in the field, applying the scientific method. The investigator must form multiple hypotheses early in the investigation, gather data to verify the correct one, and be ready to form new ones as additional data are obtained. This can be done effectively only in the field during the investigation. The investigation should not be concluded until the investigator is confident that he or she has a working understanding of what is going on at the site.

Sometimes, some of the factors listed above cannot be determined during the investigation. Monitoring of the ground water level and slide movement over a period of time may be warranted. The drill holes should be cased with at least 1-inch PVC pipe if ground water monitoring by instrumentation is planned. As illustrated in figure 4E.10, the casing should be slotted to allow ground water to enter only in the anticipated aquifer zone. The annulus between the casing and the drill hole should be filled with sand in the aquifer zone and sealed with bentonite adjacent to the confining layers and at the ground surface. A minimum hole diameter of 3 inches may be required to achieve an adequate bentonite seal for a confined aquifer. Refer to section 3E for additional discussion on monitoring.
Figure 4E.10.—Drill holes should be cased, with the casing slotted to allow ground water to enter only at the anticipated aquifer zones.

Other data pertinent to ground water movement may be noted and documented during the investigation. Such evidence as a rust-stained zone might indicate a zone below the phreatic surface part of the time, with repeated wetting and drying resulting in oxidation in an aerobic environment. This is not a foolproof indicator because it can also occur in the unsaturated flow zone, but the degree of staining should be greater in a zone which is exposed to repeated wetting and drying cycles.

Better indicators usually exist for a zone which is wet nearly all the time. If the presence of ground water is fairly constant, there is usually a gray slick zone which indicates weathering in a reducing or anaerobic environment. This zone is frequently present at the confining layer and can help to determine the lower limits of an unconfined aquifer if a distinct change in soil type is not noted.

Some data, such as whether the aquifer is confined or unconfined and the limits and slope of the aquifer, can be determined only during the investigation. If an unconfined aquifer is encountered during drilling, free water should appear in the casing at all depths below the phreatic surface as long as drilling continues within the aquifer. As shown in figure 4E.10, free water in the casing should be referenced to the depth of highest confinement. If this is the bottom of the casing and the water is flowing, the elevation to which the free water rises should be less as the casing is advanced. This is because equipotential lines from further down slope are encountered as depth increases. The water in the casing will rise to the vertical height of the equipotential line intercepted at the bottom. The bottom of the
unconfined aquifer should be recognized as the contact with a material of substantially lower permeability (the existence of a drainage barrier).

If a confined aquifer is encountered, it should be below a confining layer of lower permeability, and ground water will probably enter the casing only after this confining layer is penetrated and rise to a height above the bottom of the casing. Confined aquifers in clayey soils may also exist without a change in soil type but may only be a zone of open fissures in the clay (perhaps due to prior slope failure).

Problem 9. Complete the subsurface profile using the data provided. Determine whether there are confined or unconfined aquifers present, sketch the appropriate phreatic surface or piezometric line for each, and label the aquifer soil units.
Problem 10. Construct the ground water surface or surfaces for computer analysis. The annulus between the casing and the drill hole has been effectively sealed for each observation well from the top of the perforations to the ground surface. For each soil unit, indicate the appropriate water surface to use in pore-water pressure calculations and whether the surface should be treated as a phreatic surface or a piezometric line. Keep in mind that the piezometric rise in an observation well is referenced to the point of highest effective confinement (highest perforation, seal, upper or lower confining surface, etc.).
4E.4 Ground Water in Slope Stability Analysis

4E.4.1 Developing the Model from the Field Data

Because pore-water pressure is the important ground water factor to consider in slope stability analysis, a suitable model must contain data to enable the calculation of pressure in reference to an anticipated failure surface. This can be facilitated through the construction of the phreatic surface for an unconfined aquifer and of the piezometric line for a confined aquifer. The pore-water pressure on any point on the failure surface is computed from the difference in head between that point and the water surface, $h_w$. Refer to figure 4E.11 for a comparison between phreatic and piezometric pore-water pressure calculation.

Whether a phreatic or piezometric assumption should be used depends on the subsurface conditions at the site as previously explained, on the stability analysis method, and, if a computer program is used, on the analysis method it uses. The infinite slope equation and most computer programs for its solution follow the phreatic method. Simple plane or wedge failure analyses for rock slopes usually use a hydrostatic pressure assumption, as previously described, which conforms closely to the piezometric approach.
Computer programs for Bishop’s and Janbu’s simplified methods may use either approach, but few allow the user to select the approach to be used. It is the responsibility of the program user to determine how that particular program calculates pore-water pressure. The stability analysis may lead to significantly different results if the program is not analyzing pore-water pressure by the method which best models the field conditions. The XSTABL computer program (XSTABL 1992) for these method-of-slices analyses allows the user to select either phreatic or piezometric surfaces or a combination of both in the same problem (Sharma, 1990).

In setting up the profile for an XSTABL analysis, the user defines a water surface, designates whether it is phreatic or piezometric, and designates which water surface to use for each soil unit. When a trial failure surface is within that soil unit, the program will use the designated water surface and treat it as specified in the pore-water pressure calculations.

**4E.4.2 Ground Water Effects on Slope Stability**

Saturation of a soil may have some effect on the cohesive component of the soil shear strength by destroying capillarity or “apparent cohesion” in an otherwise cohesionless soil or by the reduction of the “dry strength” of a cohesive soil. However, the greatest effect is on the frictional shear strength. This is because of the buoyant reduction of the normal force component of frictional shear strength by the pore-water pressure (the effective stress principle). Figure 4E.12 illustrates the reduction in frictional shear strength.

*Figure 4E.11.—Calculation for phreatic and piezometric pore-water pressure.*
Reduction in Frictional Strength:

With Ground Water

Slice Weight, \( W = \left( \gamma_{aw} \left( d - d_w \right) + \gamma_{aw} d_w \right) b = [120(8.3 - 5.6) + 135(5.8)]5.0 \)

\( W = 5415 \text{ lb/ft} \)

Pore Force, \( U = \gamma_w d_w \cos^2 \alpha_w L = 62.4(5.6)\cos^2(17^\circ)(5.5) = 1820 \text{ lb/ft} \)

Frictional Strength, \( \tau_f = N \tan \phi = (5415(\cos 26^\circ - 1820)\tan 35^\circ) \)

\( \tau_f = 2134 \text{ lb/ft} \)

Without Ground Water

\( W = \gamma_{aw} db = 120(8.3)5.0 = 4980 \text{ lb/ft} \)

\( U = 0 \) (Since \( d_w = 0 \))

\( \tau_f = N \tan \phi = (4980(\cos 26^\circ - 0)\tan 35^\circ) \)

\( \tau_f = 3134 \text{ lb/ft} \)

For This Slice, Ground Water Reduces Frictional Strength By:

\[
\left( \frac{3134 - 2134}{3134} \right) \times 100\% = 32\%
\]

Figure 4E.12.—Frictional shear strength of a slice is reduced by saturating the soil.
Problem 11. For the potential failure surface, determine the pore-water pressure and pore-water force on slices 4, 5, and 6. This is the same profile used in problem 4E.10, so use the results of that problem to select the appropriate water surface and type.

The XSTABL computer program uses a correction factor to correct for the slope of the phreatic surface. This correction factor is based on flow net analysis and the fact that equipotential lines intersect flow lines at right angles for flow in isotropic soils. The correction factor yields the vertical height of the equipotential line through the center of the slice if there is no curvature of that line and the soil is isotropic. The slope of the phreatic surface, $\alpha_w$, and the height of the phreatic surface above the failure surface, $h_w$, at the slice center are computed by the program. The pore-water pressure is computed as:

$$ u = \gamma_w h_w \cos^2 \alpha_w $$

(4E.11)

The ability of this analysis to model the actual conditions depends on two factors:

1. How closely to right angles the equipotential lines intersect the flow lines in a flow net (how isotropic the soil is).

2. How much curvature there is in the flow lines and equipotential lines.
4E.4.3.1. Errors from Anisotropy

In the discussion of flow nets, it was mentioned that anisotropy distorts the intersection angles of equipotential lines and flow lines due to directional permeability differences. Anisotropy may be more common in natural soil deposits than is usually accounted for because permeability along a drainage barrier might be higher than permeability down to the barrier from the ground surface. On the other hand, if this leads to piping (as much evidence seems to indicate), in effect another isotropic soil type may be created directly above the barrier which should be modelled as an independent soil unit. Much research needs to be done on natural soil deposits to further our understanding of anisotropy and piping. In practice, directional permeability differences are seldom determined. If they were, the degree of anisotropy could be accounted for in the analysis as illustrated in figure 4E.13.

![Figure 4E.13.—Accounting for degree of anisotropy of a soil.](image)

4E.4.3.2. Errors Due to Curvature of the Flow Lines and Equipotential Lines

Because the equipotential lines must intersect the flow lines at right angles in an isotropic soil, the equipotential lines curve to the same degree as the flow lines in order to form approximate squares. This can lead to some error in the phreatic surface correction factor if the curvature is great. Because most phreatic surfaces tend to be parabolic in shape, the extreme curvature is usually limited to a short distance. As the phreatic surface becomes more of a straight line in shape, as would be expected in a translational (seepage parallel to a drainage barrier) case, the correction factor becomes more accurate, as shown in figure 4E.14.
Figure 4E.14.—Accuracy of the phreatic correction factor varies with the shape of the phreatic surface.

These curvature errors may be minimized in the method-of-slices stability analysis by the manner in which the forces of all slices are summed in the overall analysis. Local effects of curvature are minimized through this process. Figure 4E.15 and table 4E.2 illustrate that point. There is an overall error of only +1.8 percent for the phreatic method (the method used in XSTABL) as compared to the flow net method (using the vertical height of the equipotential line through the center of the slice base).

Figure 4E.15.—Profile for pore-water force analysis. Results are given in table 4E.2.
Table 4E.2.—Summary of the results of the pore-water force calculation for the profile in figure 4E.15.

<table>
<thead>
<tr>
<th>Slice</th>
<th>Width (ft)</th>
<th>Phreatic Surface Slope (deg)</th>
<th>Flow Net</th>
<th>Piezometric</th>
<th>Phreatic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Height (ft)</td>
<td>Force (ppf)</td>
<td>Height (ft)</td>
</tr>
<tr>
<td>01</td>
<td>2.6</td>
<td>0</td>
<td>2.9</td>
<td>470.5</td>
<td>2.9</td>
</tr>
<tr>
<td>02</td>
<td>2.8</td>
<td>8</td>
<td>2.9</td>
<td>506.7</td>
<td>3.1</td>
</tr>
<tr>
<td>03</td>
<td>2.7</td>
<td>22</td>
<td>3.1</td>
<td>522.3</td>
<td>3.5</td>
</tr>
<tr>
<td>04</td>
<td>3.0</td>
<td>32</td>
<td>3.7</td>
<td>692.6</td>
<td>4.6</td>
</tr>
<tr>
<td>05</td>
<td>2.6</td>
<td>36</td>
<td>4.6</td>
<td>746.3</td>
<td>6.1</td>
</tr>
<tr>
<td>06</td>
<td>3.8</td>
<td>34</td>
<td>5.5</td>
<td>1304.2</td>
<td>7.5</td>
</tr>
<tr>
<td>07</td>
<td>4.7</td>
<td>31</td>
<td>6.8</td>
<td>1994.3</td>
<td>8.8</td>
</tr>
<tr>
<td>08</td>
<td>5.4</td>
<td>25</td>
<td>7.5</td>
<td>2527.2</td>
<td>9.1</td>
</tr>
<tr>
<td>09</td>
<td>5.4</td>
<td>13</td>
<td>7.0</td>
<td>2358.7</td>
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</tr>
<tr>
<td>10</td>
<td>6.9</td>
<td>6</td>
<td>3.3</td>
<td>1420.8</td>
<td>3.4</td>
</tr>
<tr>
<td>11</td>
<td>7.6</td>
<td>--</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Total Force =</td>
<td></td>
<td></td>
<td>12,543.6</td>
<td>14936.7</td>
<td>12,324.0</td>
</tr>
</tbody>
</table>

Percent Error = \(-19.1\) +1.8

*phreatic height = piezometric height \(\times\) \(\cos^\prime\) (phreatic surface slope) = \(h,\cos^\prime\alpha\).

For this example, the overall results of the phreatic method are slightly conservative (overpredicting pore-water force resulting in a lower FOS) when compared to the more correct flow net method. That is not always the case, depending on the degree and direction of curvature of the flow lines and equipotential lines. This can be visualized by comparing the results of individual slice pore-water forces, as calculated by the two methods, and comparing the discrepancies to the degree and direction of curvature of the flow net lines for those slices (compare the results for slices 2, 6, and 9).
**Problem 12.** Using XSTABL, solve for the simplified Janbu FOS for the groundwater conditions shown in the summary table for the two failure surfaces illustrated. The shear strength parameters for all soil units and the failure surfaces have been normalized to emphasize the effects of the ground water analysis.

*Summary of results for problem 4E.12.*

<table>
<thead>
<tr>
<th>Failure Surface</th>
<th>Pore Pressure Analysis Method Used</th>
<th>Water Surface Used for Soil 3</th>
<th>Simplified Janbu's FOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow</td>
<td>Phreatic</td>
<td>W1</td>
<td>1.024</td>
</tr>
<tr>
<td>Shallow</td>
<td>Piezometric</td>
<td>W2</td>
<td>0.916</td>
</tr>
<tr>
<td>Deep</td>
<td>Phreatic</td>
<td>Phreatic</td>
<td>W1</td>
</tr>
<tr>
<td>Deep</td>
<td>Phreatic</td>
<td>Phreatic</td>
<td>W2</td>
</tr>
<tr>
<td>Deep</td>
<td>Phreatic</td>
<td>Piezometric</td>
<td>W2</td>
</tr>
<tr>
<td>Deep</td>
<td>Phreatic</td>
<td>Piezometric</td>
<td>W2</td>
</tr>
<tr>
<td>Deep</td>
<td>Piezometric</td>
<td>Piezometric</td>
<td>W1</td>
</tr>
<tr>
<td>Deep</td>
<td>Piezometric</td>
<td>Piezometric</td>
<td>W2</td>
</tr>
</tbody>
</table>
4E.4.4 Ground Water in Computer Slope Stability Analysis—Rock

A computer program to do simple two-dimensional plane failure and three-dimensional wedge failure analyses is needed. The program should allow all of the ground water analysis options described in section 4E.2.4 for either partial or full saturation. An additional option to do phreatic surface analysis should also be included for analysis of ground water forces in highly fractured rock masses. A more detailed coverage of rock slope analysis is given in section 5H.
The results of the stability analysis can vary greatly depending on the method used to calculate the water pressure acting on the rock mass subjected to failure as illustrated in the plane failure analysis in problem 13.

**Problem 13.** Determine the factors of safety against failure for the illustrated rock slope with and without a tension crack as illustrated. Solve without water pressure and with the hydrostatic level at the position illustrated. Use all options described in section 4E.2.4 to calculate the water pressure acting on the rock mass and compare the results.

---

Note: Area of base for plane failure is *unit* area $A = l$ per foot of slice length (ft$^2$/ft). Also force is *unit* force per slice length (lb/ft = ppf).
- Weight with no tension crack.
  \[ w = \gamma A = \gamma \left( \frac{f}{2} \right) \]
  \[ w = (165 \text{ pcf}) \frac{(176.9 \text{ ft})(39.5 \text{ ft})}{2} = 576,473 \text{ pfp} \]

- Failure equation with no tension crack.
  \[ \text{FOS} = \frac{CA + (w \cos \theta - u) \tan \phi}{w \sin \theta} \]
  \[ = \frac{(500 \text{ psf})(176.9 \text{ ft}) + [(576,473 \text{ pfp})(\cos 40^\circ) - u] \tan 40^\circ}{(576,473 \text{ pfp})(\sin 40^\circ)} \]
  \[ \text{FOS} = \frac{88,450 + (441,604 - u) \tan 40^\circ}{370,550} \]

  (A) No tension crack and no water forces.
  \[ U = 0 \]
  \[ \text{FOS} = \frac{88,450 + (441,604) \tan 40^\circ}{370,550} = 1.24 \]

  (B) Failure condition B: No tension crack, base water force, toe drained.

\[ u = \frac{1}{2} \gamma w = \frac{1}{2} \gamma h_w \]
\[ u = \frac{1}{2} (62.4 \text{ pcf}) \left( \frac{89.4 \text{ ft}}{2} \right)(139.1 \text{ ft}) = 193,994 \text{ pfp} \]

\[ \text{FOS} = \frac{88,450 + (441,604 - 193,994) \tan 40^\circ}{370,550} = 0.80 \]
(C) No tension crack, base water force, toe drainage blocked.

\[ U = \frac{1}{2} \gamma_w l_w = \frac{1}{2} \gamma_w h_w l_w \]

\[ U = \frac{1}{2} (62.4)(89.4)(139.1) = 387,989 \text{ ppf} \]

\[ \text{FOS} = \frac{88,450 + (441,604 - 387,989) \tan 40°}{370,550} = 0.36 \]

- Geometry with tension crack at \( x_c = 30 \text{ ft} \).

\[ Y_c = h + x_c \tan \alpha = 100.0 \text{ ft} + (30.0 \text{ ft})(\tan 10°) = 105.3 \text{ ft} \]

\[ z = (x - x_c) \tan \theta - (x - x_c) \tan \alpha = (x - x_c)(\tan \theta - \tan \alpha) = 31.7 \text{ ft} \]

\[ z_w = z - (Y_c - h_w) = 31.7 \text{ ft} - (105.3 \text{ ft} - 89.4 \text{ ft}) = 15.8 \text{ ft} \]

\[ l_c = \frac{Y_c - z}{\sin \theta} = \frac{(105.3 \text{ ft}) - (31.7 \text{ ft})}{\sin 40°} = 114.5 \text{ ft} \]
\[ a = \frac{x - x_c}{\cos \alpha} = \frac{(77.8 \text{ ft}) - (30.0 \text{ ft})}{\cos 10^\circ} = 48.5 \text{ ft} \]

\[ b = a \sin (\theta - \alpha) = (48.5 \text{ ft}) \sin 30^\circ = 24.3 \text{ ft} \]

\[ w = \gamma A = \gamma \left[ \frac{1}{2} l f - \frac{1}{2} (l - l_c) b \right] \]

\[ w = (165 \text{ pcf}) \frac{1}{2} [(176.9 \text{ ft})(39.5 \text{ ft}) - (176.9 \text{ ft} - 114.5 \text{ ft})(24.3 \text{ ft})] \]

\[ = 451,376 \text{ ppf} \]

\[ \text{FOS} = \frac{CA + (w \cos \theta - U - V \sin \theta) \tan \phi}{w \sin \theta + V \cos \theta} \]

\[ = \frac{(500)(114.5) + [(451,376)(\cos 40^\circ) - U - V \sin 40^\circ] \tan 40^\circ}{(451,376)(\sin 40^\circ) + V \cos 40^\circ} \]

\[ \text{FOS} = \frac{57,250 + (345,774 - U - V \sin 40^\circ) \tan 40^\circ}{290,139 + V \cos 40^\circ} \]

(D) With tension crack and no water forces.

\[ U = 0 \text{ and } V = 0 \]

\[ \text{FOS} = \frac{57,250 + (345,774) \tan 40^\circ}{290,139} = 1.20 \]

(E) With tension crack, tension crack water force only.

On base: \( U = 0 \)

On tension crack:

\[ V = \frac{1}{2} uz_w = \frac{1}{2} \gamma_w z_w^2 = \frac{1}{2} (62.4 \text{ pcf})(15.8 \text{ ft})^2 = 7789 \text{ ppf} \]

\[ \text{FOS} = \frac{57,250 + (345,774 - 7789 \sin 40^\circ) \tan 40^\circ}{290,139 + 7789 \cos 40^\circ} = 1.16 \]
With tension crack, tension crack water, and base water at tension crack maximum.

\[ u = \gamma_c \frac{l}{2} \]

\[ u = 0 \]

\[ u = \gamma_w \frac{h}{2} \]

\[ u = 0 \]

\[ u = \gamma_h \frac{u}{2} \]

Base Water Pressure
Limited to Pressure in Tension Crack

On tension crack: \( V = 7789 \) ppf

On base:

\[ U = \frac{1}{2} \gamma_w \frac{l}{2} = \frac{1}{2} \gamma_w \frac{h}{2} \left( \frac{l}{2} - l_w \right) \]

\[ U = \frac{1}{2} (62.4 \text{ pcf})(15.8 \text{ ft})(114.5 \text{ ft}) = 56,444 \text{ ppf} \]

\[ \text{FOS} = \frac{57,250 + (345,774 - 56,444 - 7789 \sin 40^\circ) \tan 40^\circ}{290,139 + 7789 \cos 40^\circ} = 1.00 \]

With tension crack, tension crack water, and hydrostatic base water, toe drained

On tension crack: \( V = 7789 \) ppf

On base:

\[ U = \frac{1}{2} \gamma_w \frac{h}{2} \frac{l}{2} + \frac{1}{2} \gamma_w \left( \frac{h}{2} - z_w \right) \left( l_w - l_w \right) \]

\[ U = \frac{1}{2} (62.4) \left[ \frac{(89.4)(139.1)}{4} + \frac{89.4}{2} + 15.8 \left( \frac{114.5 - 139.1}{2} \right) \right] \]

\[ U = 181,845 \text{ ppf} \]

\[ \text{FOS} = \frac{57,250 + (345,774 - 181,845 - 7789 \sin 40^\circ) \tan 40^\circ}{290,139 + 7789 \cos 40^\circ} = 0.64 \]
(H) With tension crack, tension crack water, and hydrostatic base water, toe drainage blocked

\[ u = \gamma_w z \]

\[ u = \gamma_w h \]

\[ \text{Toe Drainage Blocked} \]

On tension crack: \( V = 7789 \text{ ppf} \)

On base:

\[ U = \frac{1}{2} \gamma_w (h_w + z_w) l_c \]

\[ U = \frac{1}{2} (62.4 \text{ pcf})(89.4 \text{ ft} + 15.8 \text{ ft})(114.5 \text{ ft}) \]

\[ U = 375,816 \text{ ppf} \]

\[ \text{FOS} = \frac{57,250 + (345,774 - 375,816 - 7789 \sin 40^\circ) \tan 40^\circ}{290,139 + 7789 \cos 40^\circ} \]

\[ \text{FOS} = \frac{57,250 - 29,409}{296,106} = 0.09 \]

Note: Effective stress is negative? Therefore, frictional strength is negative? More likely, plane floats when total strength becomes zero (when buoyancy overcomes cohesive strength).
**Summary of problem 4E.13 stability analyses**

<table>
<thead>
<tr>
<th>Failure Condition</th>
<th>Ground Water Height</th>
<th>FOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. No Tension Crack and No Water Forces</td>
<td>0.0</td>
<td>1.24</td>
</tr>
<tr>
<td>B. No Tension Crack, Base Water Force, Toe Drained</td>
<td>89.4</td>
<td>0.80</td>
</tr>
<tr>
<td>C. No Tension Crack, Base Water Force, Toe Drainage Blocked</td>
<td>89.4</td>
<td>0.36</td>
</tr>
<tr>
<td>D. With Tension Crack and No Water Forces</td>
<td>0.0</td>
<td>1.20</td>
</tr>
<tr>
<td>E. With Tension Crack, Tension Crack Water Force Only</td>
<td>89.4</td>
<td>1.16</td>
</tr>
<tr>
<td>F. With Tension Crack, Tension Crack Water, and Base Water at Tension Crack Maximum</td>
<td>89.4</td>
<td>1.00</td>
</tr>
<tr>
<td>G. With Tension Crack, Tension Crack Water, and Hydrostatic Base Water, Toe Drained</td>
<td>89.4</td>
<td>0.64</td>
</tr>
<tr>
<td>H. With Tension Crack, Tension Crack Water, and Hydrostatic Base Water, Toe Drainage Blocked</td>
<td>89.4</td>
<td>FOS = 0.09 Plane Floats?</td>
</tr>
</tbody>
</table>

If the effectiveness of drainage systems is to be evaluated in preconstruction by slope stability analysis, it is necessary to know how that particular drainage system affects the phreatic surface. Casagrande (1937) addressed the evaluation of drainage systems for seepage through earth dams using flow nets. The flow nets from that report (figure 4E.16) show how the angle with which the seepage enters the drain affects the shape of the phreatic surface.
Figure 4E.16.—The angle at which seepage enters a drain affects the shape of the phreatic surface (adapted from Casagrande, 1937).

Unfortunately, little has been written about drained phreatic surface shapes when the seepage is flowing along a drainage barrier of some lateral extent (an “infinite slope” seepage source). The figures in problem 14 illustrate some common drains used in the stabilization of roads and highways. Some of the principles which Casagrande used for seepage through an earth dam should also be applicable to an aquifer with an “infinite slope source” because they are derived from basic isotropic flow net construction criteria. The principles of equal head drops between equipotential lines, the manner in which flow lines intersect a seepage face, and the basic parabolic shape for a horizontal blanket drain, to name a few, should apply regardless of the seepage source. These principles and the following derivations apply to unconfined aquifers with a phreatic surface. It has not been determined whether they can be modified to estimate the effectiveness of drains in lowering the piezometric levels in a confined aquifer.

4E.5.1 Basic Parabolic Curves

For a horizontal drainage blanket (drainage slope $\beta = 180^\circ$), a basic parabolic shape for the phreatic surface should be expected. Figure 4E.17 shows flow nets constructed for seepage along drainage barriers sloping from $5^\circ$ to $40^\circ$. The head drop used between equipotential lines is that of the infinite slope source. The thickness of the aquifer and the drain depth are constant for all of these flow nets. Note how much more effective the drain is on lowering the phreatic surface on the profiles with the flatter drainage barrier slopes.
Another of Casagrande’s observations was that, for drains that intersect the phreatic surface at an angle steeper than horizontal (β less than 180°), the flow net of the basic parabolic shape is modified. Figure 4E.18 shows flow nets for the same aquifer thickness as in figure 4E.17, a drainage barrier slope of 30°, and various drainage angles steeper than horizontal (β = 45°, 60°, and 90°). The β = 45° case would be like an intercept with a 1:1 cut slope; the β = 60° like an intercept with a drain behind a structure, such as a retaining wall; and the β = 90° like an intercept with an interceptor cutoff trench drain. Note that these are all modifications of the basic parabolic shape for this location.
4E.5.3 Mathematical Analysis for Drained Phreatic Surfaces

Flow nets can provide the phreatic surface shape required for stability analysis; they are seldom used in practice but can be the basis for mathematical analysis suitable for programming into a computer subroutine. The flow nets in figures 4E.17 and 4E.18 were used by Prellwitz as the basis for curve fitting into a predictive model. The algorithms for the predictive model in figure 4E.19 can be used for estimating drained phreatic surfaces. The basic parabolic shape is estimated from the information provided by the user and modified if the drainage slope, $\beta$, is less than 180°. Program GW for the HP41 programmable calculator (Prellwitz, 1990) incorporated these algorithms and those described in section 4E.5.4, and the personal computer program XSTABL (1992) will be modified by the Intermountain Research Station to incorporate them as well.

Figure 4E.18.—Flow nets for a drainage barrier slope of 30° and various drainage angles.
Figure 4E.19.—Algorithms for a predictive model for drained phreatic surfaces.

4E.5.4
Mathematical Analysis for Parallel Drain Spacing

The estimated phreatic surfaces for a cut slope intercept and for a blanket drain, $\beta = 180^\circ$, can be used for the upper and lower limits for a phreatic surface midway between two drilled-in parallel drains (Prellwitz, 1979). The basic assumption is that the phreatic surface midway between two adjacent drains will conform to the shape of the cut slope intercept, phreatic surface $U$, if the drains are very far apart and to the shape of the blanket drain, phreatic surface $D$, if they are very close (see figure 4E.20). At optimum spacing, midway between the drains, phreatic surface $M$ is between these two and should be controlled to meet slope stability analysis requirements. Algorithms developed to conform to this approach, combined with the algorithms to estimate drained phreatic surfaces, can be used to estimate the effectiveness of parallel drains.
CALCULATED —
FROM MODIFIED GLOVER-DUMM EQUATION:
DRAWDOWN RATIO, \( \frac{h}{h_0} = 1.16 e^{-\frac{K}{h_0}} \)
WHERE:
\[ \alpha = \frac{\pi^2}{8} \left( \frac{d}{2} \sqrt{\frac{h}{h_0}} \right)^2 \]

ASSUMED:
HYDRAULIC GRADIENT, \( i = \frac{\Delta h_0 + \Delta h_b}{P + B} \)
AVERAGE FLOW DISTANCE, \( z = \frac{P + B}{2} \)
DRAWDOWN RATIO, \( \frac{h}{h_0} = \frac{y_{max} - y_{min} - \Delta h_b}{y_{max} - y_{min} - \Delta h_0} \)

Figure 4E.20.—Estimating the phreatic surface midway between two drilled-in parallel drains. Phreatic surface U occurs if the drains are very far apart; surface D if they are very close. At optimum spacing, surface M results.
Problem 14. Analyze the drained phreatic surfaces shown for a blanket drain, cutoff trench drain, drained buttress, and parallel drains (with spacings of 15 and 30 feet) using program GW.

- Blanket drain
· Cutoff trench drain

INTERCEPT

\[ h = 13 \text{ ft.} \]
\[ \beta = 0.68; \phi = 90.0 \text{ deg.} \]
\[ X_8 \quad Y_8 \quad X_8 \]
\[ 2.2 \quad 5.0 \quad -155.0 \]
\[ X_{23} \quad Y_{23} \quad X_{23} \]
\[ 4.0 \quad 3.0 \quad -26.1 \]

\[ X = 10.6 \]
\[ Y_8 \quad Y_8 \quad Y_8 \]
\[ -2.6 \quad -1.6 \quad 9.5 \]

\[ X = 8.0 \]
\[ Y_8 \quad Y_8 \quad Y_8 \]
\[ 0.0 \quad 3.0 \quad 12.1 \]

\[ X = 2.5 \]
\[ Y_8 \quad Y_8 \quad Y_8 \]
\[ 0.0 \quad 6.0 \quad 14.0 \]

\[ X = 5.0 \]
\[ Y_8 \quad Y_8 \quad Y_8 \]
\[ 1.8 \quad 6.7 \quad 14.9 \]

\[ X = 10.6 \]
\[ Y_8 \quad Y_2 \quad Y_1 \]
\[ 2.6 \quad 12.3 \quad 16.7 \]

\[ X = 28.0 \]
\[ Y_8 \quad Y_8 \quad Y_8 \]
\[ 7.3 \quad 17.7 \quad 28.4 \]

\[ X = 26.1 \]
\[ Y_8 \quad Y_8 \quad Y_1 \]
\[ 9.5 \quad 28.6 \quad 22.6 \]

\[ X = 48.0 \]
\[ Y_8 \quad Y_8 \quad Y_1 \]
\[ 14.6 \quad 25.5 \quad 27.7 \]

\[ X = 68.0 \]
\[ Y_8 \quad Y_8 \quad Y_1 \]
\[ 29.1 \quad 41.5 \quad 42.2 \]

\[ X = 128.0 \]
\[ Y_8 \quad Y_8 \quad Y_1 \]
\[ 43.7 \quad 54.4 \quad 56.6 \]

\[ X = 155.0 \]
\[ Y_8 \quad Y_8 \quad Y_1 \]
\[ 56.4 \quad 69.3 \quad 69.3 \]

\[ X = 178.0 \]
\[ Y_8 \quad Y_8 \quad Y_1 \]
\[ 61.9 \quad 75.0 \quad 75.0 \]
- Parallel drains (drain spacings of 15 and 30 feet)
4E.6 Ground Water, Precipitation, and Slide-Movement Monitoring

In section 3E, the importance of monitoring data to a subsurface investigation was discussed. On sloping watersheds, ground water height varies rapidly in response to precipitation. This will be demonstrated in hydrograph form. It is most unlikely that the critical ground water height necessary for stability analysis or drainage system design will be observed during the field investigation. The ground water in an observation well or several wells must be monitored for a reasonable length of time in order to judge what this critical ground water surface is.

4E.6.1 Instrumentation

At the very least, a simple crest gage (the "poor-boy piezometer") can be constructed to determine the maximum ground water level in a given period of time. One version consists of a length of vinyl tubing, weighted at the bottom, with a small amount of shredded cork held inside by a piece of sponge rubber. The vinyl tubing is open at the top so the water will rise up through the sponge rubber and float the shredded cork to the height of the ground water in the observation well. The static electricity between the cork and vinyl tubing tends to hold some of the cork at the highest elevation even after the ground water recedes. The method is not foolproof, however, and the cork can fall away from the vinyl tubing after it dries out. Other observations, such as water stains on the vinyl tubing, can help to substantiate the maximum water level. Crest gages can provide useful information about the maximum ground water height but do not show how the ground water height fluctuates.

Instrumentation to sense and record long-term ground water fluctuations has been under development since 1980. Prellwitz and Babbitt (1984) described the instrumentation developed by the Intermountain Research Station Engineering Research Unit. Figure 4E.21 illustrates the basic components. The ground water height above a pressure transducer is sensed periodically. After a selected recording interval, the individual sensing readings are averaged and the minimum and maximum readings determined. These three pieces of data are then stored to document how the ground water fluctuated during the recording interval.

![Figure 4E.21.—Basic components of instrumentation to sense and record long-term ground water fluctuations (reprinted from Prellwitz and Babbitt, 1984).](image-url)
Table 4E.3 shows the available recording intervals and the corresponding sensing interval for each. The maximum period to reach full storage capacity of the data storage module (DSM) is also given. Since 1984, the instrumentation has been improved and the storage capacity increased. Several variations of pressure-transducer ground water monitoring units are now available from commercial sources, but the basic principles remain the same.

Table 4E.3.—A summary of Omnidata Datapod 2K recorders' sensing intervals, recording intervals, and storage capacity.

<table>
<thead>
<tr>
<th>Recording Interval (Hours)</th>
<th>Sensing Interval (Minutes)</th>
<th>Maximum Period to Reach DSM 2K Storage Capacity (Average, Maximum, and Minimum Values Per Channel)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Single Channel Operation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Days</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>85</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>170</td>
</tr>
<tr>
<td>12</td>
<td>30</td>
<td>341</td>
</tr>
<tr>
<td>24</td>
<td>30</td>
<td>682</td>
</tr>
</tbody>
</table>

4E.6.1.1. Precipitation

Figure 4E.22 shows how the same pressure-transducer instrumentation can be used to monitor precipitation and infiltration. If the instruments are synchronized, ground water response can be correlated to precipitation.

Figure 4E.22.—The same pressure-transducer instrumentation used for sensing ground water fluctuation can be used to monitor precipitation and infiltration.
4E.6.1.2. Slide Movement

The same instrument can also be used to monitor the ground-surface movement of a landslide through a simple extensometer installation (see figure 4E.23). The pressure transducer is replaced by a 10-turn potentiometer installed in the extensometer cable assembly. This monitors the movement of the ground surface. It is most useful for relatively large movements within a short period of time to provide information as to what ground water height is critical to slide movement. It is not intended to replace the slope indicator that monitors small amounts of subsurface movement.

![Figure 4E.23.—The pressure-transducer instrument can also be used to monitor the ground-surface movement of a landslide.](image)

4E.6.2 Data Reduction

The recorded data are actually voltages between 0 and 2 volts, scaled to a corresponding ground water depth or height, an equivalent rainfall, or an amount of slide movement. The data storage module is removed from the recorder, and the voltage recordings are transferred to a data file by a reader connected to a personal computer. Once the data file has been established, the voltages can be reduced to the equivalent field units through a regression analysis in a spreadsheet program. Correlation data for the regression analysis are recorded in the field on a data sheet similar to the one in figure 4E.24. (Newer recorders operate without removing the data storage module, and the data file is created in the field with a laptop computer.)

A Lotus 1-2-3 spreadsheet macro program is available for field data reduction and processing. The printouts in figures 4E.25 through 4E.28 illustrate the Lotus procedure.
Figure 4E.24.—Sample field data sheet for ground water fluctuation instrumentation.
Actual  
Y actual  
0.00  
0.00  
16.80  
17.00  
17.60  
24.50  
read  
0.00  
0.00  
111.00  
114.00  
121.00  
168.00  
read  
0.00  
0.00  
-16.45  
-16.74  
-17.77  
-24.65  
calculated  
-0.04  
-0.04  
-16.45  
-16.74  
-17.77  
-24.65  
output  
hole a  
Regression Output:  
\( b = \text{Constant} \)  
\( \beta = 0.04281 \)  
Std Err of \( Y \) Est  
0.18966  
R Squared  
0.99972  
No. of Observations  
6.00000  
Degrees of Freedom  
4.00000  
\( \hat{Y} = a + b (\text{Reading}) \)  
\( = [0.04281 + 0.14649(\text{Reading})] x (-1) \)  
\( \text{Linear Correction Equation for Channel A Data} \)

Y actual  
0.00  
0.00  
6.90  
9.30  
9.50  
9.70  
13.50  
read  
0.00  
0.00  
70.00  
93.00  
98.00  
94.00  
132.00  
read  
0.00  
0.00  
-6.96  
-9.25  
-9.75  
-9.55  
-13.43  
calculated  
output  
hole b  
Regression Output:  
\( b = \text{Constant} \)  
\( \beta = -0.01778 \)  
Std Err of \( Y \) Est  
0.13792  
R Squared  
0.99940  
No. of Observations  
7.00000  
Degrees of Freedom  
5.00000  
\( \hat{Y} = a + b (\text{Reading}) \)  
\( = [-0.01778 + 0.09964(\text{Reading})] x (-1) \)  
\( \text{Linear Correction Equation for Channel B Data} \)

Figure 4E.25.—Regression analysis to develop the linear equation to reduce the raw data for ground water fluctuation instrumentation.
Figure 4E.26.—Spreadsheet reduction of the raw data using the linear equation or equations for ground water fluctuation instrumentation.
Figure 4E.27.—Plot of the reduced data for ground water fluctuation instrumentation.
Mt. Hood, Junction Slide
Ground water, DH 1B

Depth to Ground water, ft.

Date

02/09 03/09 04/06 05/04 06/01

Mt. Hood, Junction Slide
Equivalent Rainfall

Date

02/09 03/09 04/06 05/04 06/01

Equivalent Rainfall, ft.

Figure 4E.28.—Replot (if necessary) of specific data for correlation.
If snow is the predominant form of precipitation, it is necessary to monitor not only when and how much snow falls, but when it melts and is available for infiltration and recharge of the ground water aquifer. The hydrographs in figure 4E.29 from a site on the Clearwater National Forest (north Idaho) show the same "spikes" of infiltration and response in ground water rise for 2 years having drastically different amounts of total precipitation.

Note the magnitude of ground water rise expected from infiltration on the hydrographs in the figure—more than 5 feet of ground water rise is not uncommon for infiltration of 5 inches of equivalent rainfall.

Similar hydrographs from the Gallatin National Forest (southwest Montana) and the Nez Perce National Forest (northern Idaho), where snow is the predominant form of precipitation, are shown in figure 4E.30. In these two cases, the total infiltration (lysimeter reading) is much closer to the amount of total precipitation than for the Clearwater National Forest site.

In coastal environments where rain is the predominant form of precipitation, infiltration correlates more rapidly to precipitation events as does the corresponding ground water rise. Figure 4E.31 shows hydrographs that are composites for three sites on the Siskiyou National Forest (southwestern Oregon) where rain was the predominant form of precipitation.

Seldom is any monitoring done after a subsurface drainage system is installed to determine whether it performs according to design assumptions. The instrumentation scheme detailed here is well suited to this kind of monitoring. The hydrograph shown in figure 4E.33 is for a fill which is continuing to fail on the Siskiyou National Forest. The interceptor trench was installed without a subsurface investigation. An investigation was made to determine why the fill slope continued to fail even though the trench pipe intercepted ground water and had a substantial amount of water flowing from it. The investigation revealed a second confined aquifer below the trench elevation allowing ground water to enter the slide mass and infiltrate back to the pipe elevation.

Figure 4E.34 is a hydrograph for a similar fill failure (just a mile up the road from the site in figure 4E.33) showing similar ground water rise into the fill material. No drainage system had been installed at this site. Monitoring data together with the data gathered during the investigation will provide the needed basis for stability analysis and a subsurface drainage system design for this site.
Figure 4E.29.—Hydrographs show the same spikes of infiltration and response in ground water rise for 2 years with drastically different amounts of total precipitation. (a) Ground water depth 1984–1985, (b) rainfall 1984–1985, (c) ground water depth 1986–1987, (d) rainfall 1986–1987.
Figure 4E.30.—Hydrographs from the Gallatin and Nez Perce National Forests, with precipitation mostly as snow. Total infiltration (lysimeter recording) is close to the amount of total precipitation.
Figure 4E.31.—Composite hydrographs for three sites on the Siskiyou National Forest, October 20, 1988–January 3, 1989 where rain is the predominant form of precipitation.

Similar hydrographs for sites on the Mt. Hood National Forest (northern Oregon Cascades) (rain and snow precipitation) are shown in figure 4E.32. These hydrographs and those on the preceding pages should give you some idea of how much and why ground water can fluctuate in mountain watersheds.
Figure 4E.32.—Hydrographs for sites with rain and snow precipitation.
Figure 4E.33.—Site cross-section and hydrograph for a fill that is continuing to fail.
Figure 4E.34.—Hydrograph for a fill failure showing ground water rise in the fill material.
4F. Root Strength and Tree Surcharge

Cliff Denning, Geotechnical Engineer, Mt. Hood National Forest

4F.1 Root Strength and Tree Surcharge

In a general sense, tree roots are thought to stabilize slopes by:

- Providing a laterally reinforcing surface layer that acts as a membrane to "hold the underlying soil in place" (O'Loughlin and Ziemer, 1982);
- Anchoring an unstable soil mantle to stable subsoils or rock where the roots penetrate a potential failure surface; and
- Acting as buttress piles and/or arch abutments to support the soil uphill from the trees (Gray and Megahan, 1981).

Gray and Ohashi (1983) and O'Loughlin and Ziemer (1982) found that fibers and roots did not affect the angle of internal friction of sand; therefore, root strength can be thought of as supplemental cohesion.

Some attempts have been made to quantify the magnitude of root reinforcement by measuring the tensile strength of individual roots, by direct shear tests on soil-root masses, by pull tests on large root systems or whole trees, and by back-calculation analysis of existing failures. In addition, several researchers have used the tensile strength of individual roots in mathematical models to estimate the root resistance per unit of soil. Hammond et al. (1992) have summarized these methods and their shortcomings and state:

Additional research is necessary to increase our understanding of soil-root interaction during slope failure and to estimate defensible values for root strength to use in stability analysis. In the meantime, we suggest using root strength values reported in the literature, considering also root density and root distribution along the failure plane.

Table 4F.1 summarizes measurements of root strength per unit area of soil made by several studies. Figure 4F.1 shows a histogram created by stacking the ranges of values in each study.
Table 4F.I.—Root strength values reported in the literature (from Hammond et al., 1992).

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Soil/vegetation type</th>
<th>Root strength, $F_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burroughs and Thomas (1977)^6</td>
<td>cultivated nursery soil/1-l m alder saplings^3</td>
<td>1.6-11.5 33-240</td>
</tr>
<tr>
<td>Wu and others (1979)^6</td>
<td>Poa annua, Eto mugwort Ochads 0-5 cm depth (Bamboo)</td>
<td>10.7-12.1 224-253</td>
</tr>
<tr>
<td>Waldron and Dalekian (1981)^2</td>
<td>Type S.S. (SM)/coastal Oregon Douglas-fir</td>
<td>1.5-4.9 31-110</td>
</tr>
<tr>
<td>Waldron and others (1983)^2</td>
<td>Idaho Batholith (SM)/ Douglas-fir</td>
<td>4.2-14.0 88-293</td>
</tr>
<tr>
<td>Ziemer (1981)^2</td>
<td>SM ($\phi' = 35 - 37^\circ$)/mixed Sitka spruce &amp; hemlock</td>
<td>4.2-5.5 88-115</td>
</tr>
<tr>
<td>Ziemer (1981)^2</td>
<td>clay loam/ponderosa pine seedlings</td>
<td>4.0-10.4 104</td>
</tr>
<tr>
<td>O'Loughlin and others (1982)^6</td>
<td>coastal sands/lodgepole^3</td>
<td>0.2-17.3 4-362</td>
</tr>
<tr>
<td>Waldron and others (1983)^2</td>
<td>stony loam/beach</td>
<td>3.3 69</td>
</tr>
<tr>
<td>Rietenberg and Sovonick-Dunford (1981)^6</td>
<td>clay loam/5-year pine seedlings</td>
<td>3.7-6.4 77-134</td>
</tr>
<tr>
<td>Wu (1984)^6</td>
<td>silty clay ($\phi' = 12^\circ$) / sugar maples - head scarp</td>
<td>6.2-7.0 130-146</td>
</tr>
<tr>
<td>Wu (1984)^6</td>
<td>- slip surface</td>
<td>3.8-4.6 79-96</td>
</tr>
<tr>
<td>Wu (1984)^6</td>
<td>- average, entire slide</td>
<td>5.8 122</td>
</tr>
<tr>
<td>Wu (1984)^6</td>
<td>SM ($\phi' = 30^\circ$)/hemlock</td>
<td>5.6-12.6 117-263</td>
</tr>
<tr>
<td>Tsukamoto and Minematsu (1987)^7</td>
<td>Silice spruce</td>
<td>3.7-7.0 77-146</td>
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<tr>
<td>Tsukamoto and Minematsu (1987)^7</td>
<td>yellow cedar</td>
<td>5.4 113</td>
</tr>
<tr>
<td>Tsukamoto and Minematsu (1987)^7</td>
<td>nursery loam/Sugi</td>
<td>1.8-5.7 38-119</td>
</tr>
</tbody>
</table>

1 1 kPa = 20.9 psf
2 Direct shear tests
3 Measured over a wide range of root densities
4 Tensile strength tests on individual roots
5 Pull tests on roots
6 Referred to by Sidle and others (1985) but not reviewed by these authors
7 Isolated small trees and pulled—measuring basal shear resistance.

Figure 4F.I.—Root strength values given in 11 studies (from Hammond et al., 1992).

Root strength depends not only on the tensile strength of the individual roots, but on the pullout resistance (or skin friction) and, probably most importantly, on the morphology of the root system; that is, how many roots there are and whether they
cross the failure plane. Root tensile strength and morphology depend on species of

tree, climate, and such site factors as slope, soil, and ground water conditions.

Hammond et al. (1992) have adopted a soil-root classification scheme presented by

Tsukamoto and Kusakabe (1984). Figure 4F.2 describes the four soil-root

morphology types. Figure 4F.3 shows suggested probability distributions by

Hammond et al. for each soil-root morphology type for both densely forested and

clearcut conditions. These probability distributions were selected based on the

following observations and assumptions as discussed by Hammond et al. (1992):

- The measured values of root strength reported in the literature (table 4F.1 and

figure 4F.1) were assumed to apply to densely forested types B and C, where

roots intersect the entire failure plane. The mean and range of values are

larger for type C to account for greater tree buttressing and root penetration

along the base of the failure plane.

- The mean and range of values were reduced for types B and D based on

three-dimensional modeling of failures.

- All distributions have large standard deviations to account for the great

variability and uncertainty in reported values.

- Lognormal probability distributions were selected to reflect the tendency for

right skew in the data (figure 4F.1), thereby giving a low but positive

probability of simulating relatively high values.

Hammond et al. discussed in greater detail the rationale for selecting the suggested

probability distributions for dense timber stands. In addition, they recommended that

the user adjust the suggested distributions to account for other factors, such as roots

in saturated clay (Waldron and Dakessian, 1981), less dense timber stands, or the

user’s personal judgment and experience.

After timber harvest, root decay causes both the numbers of roots and the tensile

strength of the remaining individual roots to decrease with time (Burroughs and

Thomas, 1977). Ziemer (1981a, b) and O’Loughlin (1974) also measured a decrease

in biomass and, consequently, root strength with time after harvest using direct shear

tests. These studies indicate that the period of minimum root strength is from about

3 to 5 years until about 10 to 20 years after harvest, depending on climate, which

affects root decay and vegetation regrowth. In areas severely burned following

harvest, minimum root strength may occur even sooner—0 to 3 years. After about

10 to 20 years postharvest, root reinforcement will increase to its uncut level if

significant regrowth has occurred.

Ziemer (1981a, b) estimated that at its minimum, root reinforcement conceptually

could be 20 to 40 percent of its undisturbed value (figure 4F.4). Therefore, we

suggest using the distributions shown in figure 4F.3 to represent the time of

minimum root reinforcement after clearcut timber harvest for each soil-root

morphology type. These distributions were obtained by finding a mean and standard

deviation for a lognormal distribution which gives a mode value equal to about

30 percent of the mode for the uncut distribution.
Type A—consists of shallow soils overlying fairly competent rock that roots cannot penetrate easily. The failure plane is mostly below the root zone, except where it intersects the ground surface. Because these roots are constrained by the bedrock, root densities may be greater than those for type D allowing for greater root reinforcement.

Type B—consists of shallow soils overlying fractured or weathered rock or compact glacial till that allows some root penetration. The amount of penetration depends on the number and nature of the discontinuities in the substratum, but in general the roots are restricted somewhat by the substratum. Root reinforcement is fairly significant because roots tend to intersect the failure plane along its full length.

Type C—consists of a transition zone, that is, a nondistinct zone in which the soil shear strength and unit weight increase gradually with depth. It is assumed that the transition zone acts as a drainage barrier allowing the concentration of groundwater and the development of high pore-water pressure. As a result, the failure plane passes somewhere through the transition zone. It is assumed also that this zone is penetrated more easily by roots than is a less fractured substrate of type B. Therefore, the maximum root reinforcement is expected in type C. Examples of type C include decomposed granite over granite bedrock, and a loose ash or glacial till overlying a medium-dense compacted till over bedrock.

Type D—consists of soils and a potential failure plane both deeper than the root zone of the trees. The actual depth of the soil needed for a type D classification depends on the root morphology of the particular tree species. For example, less soil depth would be required for Sitka spruce, which has a shallow lateral root system, than for Douglas-fir, which has a deep root system. Because the bedrock does not constrain the root system, the root densities, and therefore the root strength, are assumed to be less than for those associated with type A.

Figure 4F.2.—Soil root morphology types. Suggested distributions for root cohesion values for these morphology types are shown in figure 4F.3. (Figure adapted with the permission of the East-West Center from Tsukamoto and Kusakabe, 1984.)
Figure 4F.3.—Suggested lognormal distributions describing possible ranges of Cr values for each soil-root morphology type in densely forested conditions and during the 3- to 10-year period of minimum root strength after clearcut timber harvest (from Hammond et al., 1992).
If harvesting methods other than clearcutting are used, root strength may decrease less, and the distributions in figure 4F.3 should be modified. Ziemer (1981b) discussed conceptual models for the change in relative root reinforcement following shelterwood and selection harvesting systems (figure 4F.4). A shelterwood system is described as having 70 percent of the original stand being harvested, followed by removal of the remaining trees 10 years later. For this system, Ziemer hypothesized that the root reinforcement drops to about 70 percent of its uncut value about 2 to 3 years postharvest, then rises to about 10 percent above the uncut value about 7 years after harvest as the residual trees quickly expand. About 5 years after the residual trees are harvested, root reinforcement again will drop to about 50 percent of the uncut value. The selection harvesting system is described as having 20 percent of the trees cut every 10 years. Ziemer anticipated that the root strength could decrease by about 3 percent 2 years after harvest, then increase to about 7 percent above the uncut strength due to rapid expansion of the roots of the remaining trees.

The FOS calculated by the infinite slope equation is fairly insensitive to the value of tree surcharge \( q_t \), particularly when soil depths are greater than 5 feet. Consequently, tree surcharge is often omitted from the infinite slope equation. When soil depths are less than 5 feet (and especially when less than about 2 feet), the FOS may vary slightly with tree surcharge. Simons et al. (1978) have shown that when

\[
C_s + C_r < 62.4d_w \tan \phi \cos^2 \alpha
\]

(4F.1)

where

\( C_s \) is soil cohesion in psf,
\( C_r \) is root cohesion in psf,
\(d_w\) is ground water depth in feet,  
\(\phi'\) is effective friction angle of the soil, and  
\(\alpha\) is ground surface slope,  

Tree surcharge will have a positive effect on stability. Otherwise, tree surcharge will have a negative effect.

Tree surcharge depends on the species, size, and density of the timber stand. Considering the weight to be uniformly distributed across the entire slope area is a common assumption for stability analysis (Greenway, 1987; Sidle, 1984; Wu et al., 1979). Estimates of equivalent uniform tree surcharge can be obtained from timber inventories of the volume of timber per acre and the weight per board foot of that timber. If the values given are for merchantable timber, they should be increased somewhat to account for the non-merchantable volumes. An example calculation is shown below:

\[
(3 \text{ to } 5 \text{ lb/bf})(100,000 \text{ bf/acre})(1 \text{ acre}/43560 \text{ ft}^2) = 7 \text{ to } 12 \text{ psf}
\]

Gray and Leiser (1982) discussed a slightly different method for calculating tree surcharge. They considered a Douglas-fir stand in the Cascade Range of central Oregon that contained 50,000 to 65,000 board feet of merchantable timber per acre. At 10 lb/bf, the uniform surcharge would be 12 to 15 psf. If the weight of the trees is divided by the actual basal area of the trees (300-500 ft²/acre), the stress directly under a tree would be about 1,400 psf. They then assumed that the weight of the trees is distributed over 75-square-foot circles spaced 30 feet apart in a cubic array. In this case, the 1,400 psf surface stress would produce a stress increase of 20 to 75 psf midway between trees at depths of 5 and 20 feet, respectively. They concluded that even with this more exact analysis method, tree surcharge plays an insignificant role in slope stability.

Lacking tree species and density data, estimates of tree surcharge can be taken from the literature. When doing so, care must be exercised to ascertain whether an equivalent uniform surcharge or a surcharge directly under the tree is being reported. An equivalent uniform surcharge is recommended because the stresses at depth and between trees will not be as high as the surcharge directly under the tree. Some equivalent uniform surcharge values from the literature are listed in Table 4F.2.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Species</th>
<th>(q_o) psf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greenway 1987</td>
<td>Unspecified, 30–80 meters high</td>
<td>10–40</td>
</tr>
<tr>
<td>Sidle 1984</td>
<td>Sitka spruce, Alaska</td>
<td>N(52.5, 10.4)*</td>
</tr>
<tr>
<td>Wu et al., 1979</td>
<td>Sitka spruce, 100–200 feet high</td>
<td>50</td>
</tr>
</tbody>
</table>

* Normal (Gaussian) distribution with a mean of 52.5 psf and a standard deviation of 10.4 psf.


References


References


References


SECTION 5

SLOPE STABILITY ANALYSIS

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Section 5. Slope Stability Analysis

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<td></td>
<td>References</td>
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</tbody>
</table>

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In section 4 a thorough discussion of soil and rock unit weight and shear strength was presented. Also presented were the principles of Mohr-Coulomb and effective stress. In this section, these principles will be assembled into stability analysis algorithms. These algorithms will be the core analyses for the three-level slope stability analysis system (figure 5A.1) as applicable to typical forest management activities.

**Figure 1A.1 Stability Analysis Flowchart**

```
LEVEL I
ANALYSIS

LEVEL II
ANALYSIS

LEVEL III
ANALYSIS

RESOURCE

PROJECT

SITE

LEVEL I
DATA BASE

LEVEL II
DATA BASE

LEVEL III
DATA BASE

LANDSLIDE INVENTORY

(FeEDBACK)

(FeEDBACK)
```

*Figure 5A.1—Slope stability analysis flowchart with the portion section 5 deals with shaded.*

All of the slope stability analysis techniques in the three-level system have been or are being programmed for the personal computer and are being made available to all Forest Service users. Every effort is being made to make these programs easy to use. In fact, they are so easy to use that anyone, with or without fundamental soil and rock mechanics skills, can use them to develop "answers." The danger is that by making these tools more functional for the qualified user, we increase the potential for misuse by the unqualified user. This poses a fundamental question of responsibility for the software developer. Where does the responsibility of the developer end in the application of the product? We software developers feel that it is our responsibility to develop functional software which, in the hands of the
qualified user, will provide the vehicle on which sound forest management decisions can be made in a cost-effective manner. We also feel it is our responsibility to document and instruct the qualified user on the limitations and use of the techniques.

It is the software user's responsibility to know and understand the basis for and the limitations of the software selected for analysis. If you are unfamiliar with slope stability analysis, it would be wise for you to read through the following algorithm derivations carefully and become familiar with the concepts on which the various methods are founded. None of the mathematics in the derivations goes beyond high-school-level algebra or trigonometry, so you should be able to follow it. It may take several times through before the mechanics become clear. If they do not become clear, please pursue additional training in fundamental soil and rock mechanics before using the software that employs these techniques as core algorithms. Even the experienced user will benefit from an occasional refresher in these fundamentals. If you are familiar with the derivations, you may want to skip over sections 5A.3 through 5A.12.

5A.2 Mode of Failure

It is imperative that the slope stability analysis properly model the mode of slope failure most likely to occur under the prevailing site conditions. In static soil and rock mechanical analysis, two broad groups of failure modes, translational and rotational, are analyzed based primarily on the general shape of failure surface (figure 5A.2).

Translational

Translational failures are the more planar-shaped failures with the slope of the failure surface fairly constant for the significant length of slope of interest. This is a common mode of relatively shallow failures in mountainous terrain due to the effects of ground water seepage parallel to a subsurface drainage barrier.

Rotational

Rotational failures have a circular-arc or logarithmic-spiral shaped failure surface. This is a common mode of relatively deep-seated failure for steep constructed slopes. Due to the circular shape, both a moment analysis and a force analysis can be made, as will be discussed in the derivations.

Often the shape of the failure surface or sequence of failure is such that the analysis model does not fit completely into either the translational or rotational mold, but is somewhere between. Two common examples of such failure types are block failures and progressive failures (figure 5A.2).

Block

Block failures may have a failure surface which is circular-arc in shape except for a significant planar-shaped segment adjacent to a drainage barrier, or at a contact with a material of significantly higher shear strength, or both. The shape is usually analyzed as a "block" with more than one planar surface.
Progressive failures do not occur instantaneously, but progress into different modes as a chain of cause-effect action-reaction events. Because we seldom observe slope failures as they occur, it is difficult to estimate how many failures on Forest Service lands occur in this manner, but postmortem back-calculation analyses indicate that there must be a significant number. Fill slope failures, a common type of progressive failure on Forest Service roads, begin as rotational failures with ground water entering the slide mass and progress into translational debris avalanches as the ground water builds up additional pore-water pressure due to the initial movement. This will be illustrated in the sample problems. Another example of a progressive failure is a "hummocky" slope, a series of circular-arc failures which interact into an overall translational failure of the entire slope.
All methods of slope stability analysis used in this guide conform to some degree of limiting equilibrium of static soil or rock mechanics. "Limit equilibrium" means that a relationship is established between the shear strength of the material available to resist failure and some condition of shear forces which are present to cause failure. This is commonly expressed in the form of a "factor of safety" equation:

\[
\text{Factor of Safety } = \frac{\text{Available Shear Strength}}{\text{Shearing Forces}}
\]

The available shear strength is usually defined in accordance with Mohr-Coulomb failure criteria with cohesive and frictional components:

\[
\tau = \Delta C + \Delta N' \tan \phi.
\]

Then:

\[
F = \frac{\Delta C + \Delta N' \tan \phi}{\Delta S}
\]

where:

- \(F\) = factor of safety against failure of the slope
- \(\Delta C\) = available cohesive strength of the material, ppf
- \(\Delta N' \tan \phi\) = available effective frictional strength, ppf
- \(\Delta S\) = shearing force acting on the material, ppf
- \(\tau\) = available shear strength, ppf

Frictional strength is defined in an effective stress format, which will be consistent for all analyses in the three-level system. This means that pore-water pressure must be determined and incorporated into the calculation for the effective normal force, \(\Delta N'\). The units are in a unit force basis, pounds per foot of depth in the third dimension. This is consistent with two-dimensional analysis, which will be further explained.

Equation 5A.1 can be rearranged and \(\Delta S\) defined as "mobilized shear strength," which is the portion of the available shear strength required to prevent slope failure according to certain specific limit equilibrium criteria. This will be further explained in the derivation of the various "methods of slices" algorithms. Mobilized shear strength will be used in these derivations:

\[
\Delta S = \frac{\Delta C + \Delta N' \tan \phi}{F}
\] (5A.2)

Consider a simple rectangular block of soil or rock with unit weight \(\gamma\), and dimensions \(X\), \(Y\), and \(Z\) (figure 5A.3). In the simplest case, without ground water forces acting on it, the block is placed on a plane and the angle of inclination, \(\alpha\), increased until failure occurs. At the conditions of failure, limiting equilibrium is established so that all of the available shear strength is mobilized:

\[
\Delta S = \Delta C + \Delta N' \tan \phi.
\]
In a three-dimensional analysis:

\[ \Delta W = \gamma X Y Z = (120 \text{pcf}) (5 \text{ ft}) (3 \text{ ft}) (10 \text{ ft}) = 18,000 \text{ lb} \]

\[ \Delta S = \Delta W \sin \alpha = (18,000 \text{ lb}) \sin 36.9^\circ = 10,808 \text{ lb} \]

\[ \Delta N' = \Delta W \cos \alpha = (18,000 \text{ lb}) \cos 36.9^\circ = 14,394 \text{ lb} \]

\[ \Delta C = C (\text{Area of Base}) = C X Z = (50 \text{ psf}) (5 \text{ ft}) (10 \text{ ft}) = 2,500 \text{ lb} \]

\[ \Delta N' \tan \phi = (14,394 \text{ lb}) \tan 30^\circ = 8,311 \text{ lb} \]

where:

- \( \Delta W \) is the weight of the block,
- \( \Delta S \) is shear force or mobilized shear strength,
- \( \Delta N' \) is effective normal force (with no water),
- \( \Delta C \) is available cohesive strength,
- \( \Delta N' \tan \phi \) is available frictional strength,
- \( \alpha \) is angle of inclination,
- \( \gamma \) is unit weight of the rock or soil.

At failure when limit equilibrium is established, equation 5A.1 should yield a factor of safety against failure, \( F \), of 1.00:

\[ F = \frac{\Delta C + \Delta N' \tan \phi}{\Delta S} = \frac{2500 \text{ lb} + 8311 \text{ lb}}{10,808 \text{ lb}} = 1.00 \]

Note that all of the units in the three-dimensional analysis are in pounds, which is what you would expect for forces. However, except in the rock wedge-failure analysis, a two-dimensional analysis is much more common. A fundamental "plane-strain" assumption must be made in order for a two-dimensional analysis to be applicable: the failure mass extends far enough into the third dimension so that the
end conditions are insignificant and need not be considered. In the case of this block, the edges in the "Z" direction have no constraint and the third dimension can be ignored. This assumption is never quite right but is pretty close for failures of considerable lateral extent. For failures which do have lateral constraint and "soil arching," the results of a two-dimensional analysis are conservative because these stabilizing effects are not considered. The characteristic force units in a two-dimensional analysis are in force per unit of depth into the third dimension, in this case, pounds per foot (ppf). The results of the two-dimensional analysis should be the same as for the previous three-dimensional analysis, if the assumption is valid:

$$\Delta W = \gamma X Y = (120 \text{ pcf}) (5 \text{ ft}) (3 \text{ ft}) = 1800 \text{ ppf}$$
$$\Delta S = \Delta W \sin \alpha = (1800 \text{ ppf}) (\sin 36.9^\circ) = 1081 \text{ ppf}$$
$$\Delta N' = \Delta W \cos \alpha = (1800 \text{ ppf}) (\cos 36.9^\circ) = 1439 \text{ ppf}$$
$$\Delta C = C X = (50 \text{ psf}) (5 \text{ ft}) = 250 \text{ ppf}$$
$$\Delta N' \tan \phi = (1439 \text{ ppf}) (\tan 30^\circ) = 831 \text{ ppf}$$
$$F = \frac{\Delta C + \Delta N' \tan \phi}{\Delta S} = \frac{250 \text{ ppf} + 831 \text{ ppf}}{1081 \text{ ppf}} = 1.00$$

The various method-of-slices analysis techniques differ primarily in the conditions of static limit equilibrium used in the derivation of their algorithms. Historically, the simpler methods were developed before the age of computers and more complex methods followed after. Sharma (1992) shows that beyond a certain level of accuracy, the complexity of the analysis increases to a level requiring skill beyond that of the average practitioner. This threshold level of complexity occurs when interslice forces and more than two conditions of static equilibrium are included in the analysis. Unless the user is adept at analyzing the interslice forces, the analysis can become very complex and unworkable. The methods used in level III in this guide assume that the interslice forces acting on any slice are equal and opposite, cancelling each other out of the analysis. This is not quite right, but the amount of error associated with this assumption is very small for typical forest applications. In level III analyses, a prudent practitioner using judgment will compensate for these errors by tailoring the analysis to match the existing field conditions.

There is one assumption made in each of the method-of-slices analyses that may be a more significant source of error—that the potential failure mass can be divided into "slices," and that the forces and/or moments acting on the individual slices can be summed for a total analysis of overall conditions. For this assumption to be valid, the factor of safety, $F$, must be the same on all slices, and the shearing resistance must be mobilized simultaneously along the entire failure surface. Because many failures mobilize progressively, the user is cautioned about the validity of this assumption. The best safeguard against these errors is constant field verification of the results.

Figure 5A.4 and table 5A.1 show the typical forces acting on a slice. For simplicity, only the internal forces are shown. For complete derivations (including external forces such as earthquake loading, surcharge, and external water forces) for the method-of-slices analyses used in our level III software, refer to the XSTABL.
technical manual (Sharma, 1992). A more detailed coverage of earthquake loading is given in section 5F.6.

![Diagram](image)

Figure 5A.4.—Typical forces acting on a slice.

<table>
<thead>
<tr>
<th>Force</th>
<th>Equation</th>
<th>Direction</th>
<th>Moment Arm</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slice Weight</td>
<td>$\Delta W$</td>
<td>Down</td>
<td>$R \sin \alpha$</td>
<td>Clockwise</td>
</tr>
<tr>
<td>Effective Normal</td>
<td>$\Delta N$</td>
<td></td>
<td>0</td>
<td>Through Center</td>
</tr>
<tr>
<td>Pore Water Force</td>
<td>$\Delta U_a$</td>
<td></td>
<td>0</td>
<td>Through Center</td>
</tr>
<tr>
<td>Vertical $\Delta N + \Delta U_a$</td>
<td>$(\Delta N + \Delta U_a) \cos \alpha$</td>
<td>Up</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal $\Delta N + \Delta U_a$</td>
<td>$(\Delta N + \Delta U_a) \sin \alpha$</td>
<td>Left</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mobilized Shear</td>
<td>$\Delta S$</td>
<td></td>
<td>$R$</td>
<td>Counterclockwise</td>
</tr>
<tr>
<td>Vertical $\Delta S$</td>
<td>$\Delta S \sin \alpha$</td>
<td>Up</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horizontal $\Delta S$</td>
<td>$\Delta S \cos \alpha$</td>
<td>Right</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Both horizontal and vertical force equilibrium, as well as moment equilibrium (for rotational failures), are used in the development of the commonly used method-of-slices slope stability analysis methods. However, no more than any two of these conditions are used in the derivation of a specific method. Correlations are also made to the analysis algorithms for the level I infinite slope equation and for rock slope stability analysis.
5A.6 Ordinary Method of Slices

The oldest method of slices, developed by Fellenius in 1936, satisfies only moment equilibrium (Lambe and Whitman, 1969; Sharma, 1992). This method is commonly called the "ordinary method of slices."

To satisfy moment equilibrium (figure 5A.4 and table 5A.1):

\[
\text{clockwise moments} = \text{counterclockwise moments}
\]

\[
\Delta W (R \sin\alpha) = \Delta S R.
\]

Substituting equation 5A.2 for \(\Delta S\) and rearranging, for slice \(i\) only:

\[
(\Delta W \sin\alpha) R = \left(\frac{\Delta C + \Delta N'}{\Delta W} \tan\phi\right) R.
\]

This equation is rearranged and summed for all of the slices in the failure mass as an expression for the factor of safety against failure, \(F\), along that failure surface:

\[
F = \sum_{i=1}^{n} \frac{\Delta C + \Delta N' \tan\phi}{\Delta W \sin\alpha}.
\]  

(5A.3)

Note how similar this is to equation 5A.1 and to the simple block-failure analysis, but that is only because the radius of the circle has been factored out. This is the general form of the ordinary method of slices. A form more suitable for hand-calculation consistent with effective stress analysis with pore-water pressure computed from a phreatic surface can be developed by substitution of terms from figure 5A.5.

\[
\Delta C = CL = C \frac{b}{\cos\alpha}
\]

\[
\Delta N' = \Delta N - \Delta U_a = \Delta W \cos\alpha - \Delta U_a
\]

\[
\Delta U_a = uL = u \frac{b}{\cos\alpha}
\]

\[
u = \gamma_w d_w \cos^2\alpha_w
\]

\[
\Delta W = [\gamma d + (\gamma_{sat} - \gamma)d_w] b
\]

Figure 5A.5.—Slice dimensions for hand-calculation.
The ordinary method of slices equation for hand-calculation for each slice $i$ is:

$$F = \sum_{i=1}^{n} \left[ \left( \frac{b}{\cos \alpha} \right) + \frac{[\gamma d + (\gamma_{sat} - \gamma)d_w]b \cos \phi - (\gamma_w d_w \cos^2 \alpha_w)\left( \frac{b}{\cos \alpha} \right)\tan \phi}{[\gamma d + (\gamma_{sat} - \gamma)d_w]b \sin \alpha} \right]$$

$$F = \frac{\sum_{i=1}^{n} \left( Cb + \left[ \left( \frac{\gamma d + (\gamma_{sat} - \gamma)d_w}{\cos \alpha} \right) b \cos^2 \alpha - \gamma_w d_w b \cos^2 \alpha_w \right] \frac{\tan \phi}{b \sin \alpha \cos \alpha} \right]}{\left( \gamma d + (\gamma_{sat} - \gamma)d_w \right)}$$

### 5A.7 Infinite Slope Equation

The Level I Stability Analysis (LISA) manual (Hammond et al. 1992) shows a derivation for the infinite slope equation used in the level I analysis. The correlation of equation 5A.4 to the infinite slope equation can be shown here by considering the “infinite slope with seepage parallel to the failure slope” as just a special “method of slices” case where all slices are exactly the same (in which case the slice width, $b$, can be factored out) and with seepage parallel to the failure slope, the slope of the phreatic surface, $\alpha_w$, is the same as the failure slope, $\alpha$.

Making these adjustments to equation 5A.4 results in the infinite slope equation:

$$F = \frac{C + \left[ \left( \frac{\gamma d + (\gamma_{sat} - \gamma)d_w}{\cos \alpha} \right) \frac{\cos^2 \alpha \tan \phi}{\sin \alpha \cos \alpha} \right]}{\left( \gamma d + (\gamma_{sat} - \gamma)d_w \right)}$$

Note that the units are in psf. Although this is a two-dimensional analysis, only the slope, $\alpha$, is used to define the conditions in the $X$ dimension. This is almost a one-dimensional analysis where the assumption must be made that the “end” conditions in the $X$ direction are insignificant and can be ignored in the analysis. This is never quite right and must be constantly safeguarded against in the level I analysis, as will be further discussed in section 5B.

### 5A.8 Simplified Bishop’s Method of Slices

The ordinary method of slices was generally found to be conservative when compared to actual failures (factor of safety, $F$, less than 1.00 at failure). It was not until 1955 that Bishop developed a more accurate method for analyzing rotational failures by combining vertical force equilibrium with moment equilibrium (Lambe and Whitman, 1969). In order to reduce the amount of computational time required, Bishop also reduced his more rigorous analysis to a “simplified” version by eliminating the interslice forces from the analysis. When you consider that solutions in 1955 were truly “hand” solutions, aided only by slide rule, that simplification was almost a necessity for the practitioner. Even then, a hand-calculation for one probable failure surface could easily take up to 4 hours.
For vertical force equilibrium (figure 5A.4 and table 5A.2):

upward acting forces = downward acting forces

\[(\Delta N' + \Delta U_a) \cos \alpha + \Delta S \sin \alpha = \Delta W\]
\[\Delta N' \cos \alpha = \Delta W - \Delta U_a \cos \alpha - \Delta S \sin \alpha\]
\[\Delta N' = \frac{1}{\cos \alpha} (\Delta W - \Delta U_a \cos \alpha - \Delta S \sin \alpha)\]  

(5A.6)

Substituting for \(\Delta S\) as in equation 5A.2,

\[\Delta N' = \frac{1}{\cos \alpha} \left[ \Delta W - \Delta U_a \cos \alpha - \left( \frac{\Delta C + \Delta N' \tan \phi}{F} \right) \sin \alpha \right]\]
\[F \Delta N' = \frac{1}{\cos \alpha} \left[ F(\Delta W - \Delta U_a \cos \alpha) - \Delta C \sin \alpha - \Delta N' \sin \alpha \tan \phi \right]\]
\[F \Delta N' + \frac{\Delta N' \sin \alpha \tan \phi}{\cos \alpha} = \frac{1}{\cos \alpha} \left[ F(\Delta W - \Delta U_a \cos \alpha) - \Delta C \sin \alpha \right]\]
\[\Delta N'(F + \tan \alpha \tan \phi) = \frac{F}{\cos \alpha} \left( \Delta W - \Delta U_a \cos \alpha - \frac{\Delta C \sin \alpha}{F} \right)\]
\[\Delta N' = \frac{F}{\cos \alpha(F + \tan \alpha \tan \phi)} \left( \Delta W - \Delta U_a \cos \alpha - \frac{\Delta C \sin \alpha}{F} \right)\]

Let:

\[m_\alpha = \cos \alpha \left( \frac{F + \tan \alpha \tan \phi}{F} \right) = \cos \alpha \left( 1 + \frac{\tan \alpha \tan \phi}{F} \right)\]  

(5A.7)

Then:

\[\Delta N' = \frac{1}{m_\alpha} \left( \Delta W - \Delta U_a \cos \alpha - \frac{\Delta C \sin \alpha}{F} \right)\]  

(5A.8)

For both moment and vertical force equilibrium, equation 5A.8 is substituted into equation 5A.3 and reduced to a new expression for \(F\).

For slice \(i\) forces only, equation 5A.3 becomes

\[F = \frac{\Delta C + \Delta N' \tan \phi}{\Delta W \sin \alpha}\]
\[F \Delta W \sin \alpha = \Delta C + \Delta N' \tan \phi = \Delta C + \frac{1}{m_\alpha} \left( \Delta W - \Delta U_a \cos \alpha - \frac{\Delta C \sin \alpha}{F} \right) \tan \phi\]
\[\left( F \Delta W \sin \alpha - \Delta C \right) \frac{m_\alpha}{\tan \phi} = \Delta W - \Delta U_a \cos \alpha - \frac{\Delta C \sin \alpha}{F}\]
\[\left( F \Delta W \sin \alpha - \Delta C \right) m_\alpha + \frac{\Delta C \sin \alpha \tan \phi}{F} = (\Delta W - \Delta U_a \cos \alpha) \tan \phi\]
Substituting \( m_a \) as in equation 5A.7 gives

\[
(FAW \sin \alpha - \Delta C) \cos \alpha \left( 1 + \frac{\tan \alpha \tan \phi}{F} \right) + \frac{\Delta C \sin \alpha \tan \phi}{F} = (\Delta W - \Delta U_a \cos \alpha) \tan \phi
\]

\[
(FAW \sin \alpha \cos \alpha - \Delta C \cos \alpha) \left( 1 + \frac{\tan \alpha \tan \phi}{F} \right) + \frac{\Delta C \sin \alpha \tan \phi}{F} = (\Delta W - \Delta U_a \cos \alpha) \tan \phi
\]

\[
FAW \sin \alpha \cos \alpha + \frac{FAW \sin \alpha \cos \alpha \tan \alpha \tan \phi}{F} = (\Delta W - \Delta U_a \cos \alpha) \tan \phi
\]

\[
FAW \sin \alpha \cos \alpha \left( 1 + \frac{\tan \alpha \tan \phi}{F} \right) = \Delta C \cos \alpha + (\Delta W - \Delta U_a \cos \alpha) \tan \phi.
\]

Substituting \( m_a \) as from equation 5.7 again gives

\[
FAW \sin \alpha \ m_a = \Delta C \cos \alpha + (\Delta W - \Delta U_a \cos \alpha) \tan \phi.
\]

Thus, the factor of safety equation for slice \( i \) forces only is

\[
F = \frac{\Delta C \cos \alpha + (\Delta W - \Delta U_a \cos \alpha) \tan \phi}{FAW \sin \alpha \ m_a}.
\]

Summing for all slices in a potential failure mass, the factor of safety against failure by the simplified Bishop's method is (form used is for ease of hand-calculation with pore pressure computed from a phreatic surface; see figure 5A.5):

\[
F = \sum_{i=1}^{n} \frac{C \left( \frac{b}{\cos \alpha} \right) \cos \alpha + \left\{ \gamma_d + (\gamma_{sat} - \gamma) d_w \right\} b - \gamma_w d_w \cos^2 \alpha \left( \frac{b}{\cos \alpha} \right) \cos \alpha \left( 1 + \frac{\tan \alpha \tan \phi}{F} \right) \tan \phi}{\left[ \gamma_d + (\gamma_{sat} - \gamma) d_w \right] b \sin \alpha m_a}
\]

\[
F = \sum_{i=1}^{n} \left[ \frac{Cb + \left\{ \gamma_d + (\gamma_{sat} - \gamma) d_w \right\} b \tan \phi}{\left[ \gamma_d + (\gamma_{sat} - \gamma) d_w \right] b \sin \alpha m_a} \right]
\]

where, as before,

\[
m_a = \cos \alpha \left( 1 + \frac{\tan \alpha \tan \phi}{F} \right).
\]

Note that the equation for the factor of safety against failure, \( F \), also contains \( F \) in the expression for \( m_a \). Therefore, the solution is not as direct as the ordinary method of slices solution. In practice, an assumed value of \( F \) is used to calculate \( m_a \) and the equation is solved for the calculated value of \( F \). The solution is then recalculated using the new value of \( F \) until the two values of \( F \) are the same. This can be very time consuming in hand-calculations but converges rapidly in computer analysis. In section 5E, a procedure to speed up the reiteration process for hand-calculation is presented. This procedure uses the more direct solution by the ordinary method of slices as a conservative first approximation to \( F \).
5A.9 Janbu's Simplified Method

In 1956, Janbu, Bjerrum, and Kjaernsli published a method of slices applicable to failures of any general shape (Lambe and Whitman, 1969). As does Bishop’s method, the simplified version of this procedure ignores interslice forces. The procedure differs from the simplified Bishop’s method by combining horizontal force equilibria, rather than moment equilibria, with vertical force equilibria. By eliminating moment equilibria, the analysis could be made applicable to other than rotational failures.

For horizontal force equilibrium (figure 5A.4 and table 5A.2):

\[(\Delta N' + \Delta U_a)\sin\alpha = \Delta S \cos\alpha\]

For both horizontal and vertical force equilibria, substitute equation 5A.6 for \(\Delta N'\):

\[
\left(\frac{\Delta W - \Delta U_a \cos\alpha - \Delta S \sin\alpha}{\cos\alpha} + \Delta U_a\right) \sin\alpha = \Delta S \cos\alpha
\]

\[
\Delta W \left(\frac{\sin\alpha}{\cos\alpha}\right) = \Delta S \left(\frac{\sin^2\alpha}{\cos\alpha}\right) = \Delta S \cos\alpha
\]

\[
\Delta W \tan\alpha = \Delta S \left(\frac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha}\right)
\]

\[
\Delta W \tan\alpha = \frac{\Delta S}{\cos\alpha}.
\]

Using equation 5A.2,

\[
\Delta W \tan\alpha = \frac{\Delta C + \Delta N' \tan\phi}{F \cos\alpha}
\]

\[
\Delta W F \cos\alpha \tan\alpha = \Delta C + \Delta N' \tan\phi.
\]

Substitute equation 5A.8 for \(\Delta N'\):

\[
\Delta W F \cos\alpha \tan\alpha = \Delta C + \frac{1}{m_s} \left(\Delta W - \Delta U_a \cos\alpha - \frac{\Delta C \sin\alpha}{F}\right) \tan\phi
\]

\[
(\Delta W F \cos\alpha \tan\alpha - \Delta C) m_s = \left(\Delta W - \Delta U_a \cos\alpha - \frac{\Delta C \sin\alpha}{F}\right) \tan\phi
\]

\[
(\Delta W F \cos\alpha \tan\alpha - \Delta C) m_s + \frac{\Delta C \sin\alpha \tan\phi}{F} = (\Delta W - \Delta U_a \cos\alpha)\]
Substitute equation 5A.7 for $m_o$:

\[
\Delta W F \cos \alpha \tan \alpha - \Delta C \cos \alpha \left(1 + \frac{\tan \alpha \tan \phi}{F}\right) + \frac{\Delta C \sin \alpha \tan \phi}{F} = (\Delta W - \Delta U_a \cos \alpha) \tan \phi
\]

\[
\Delta W F \cos^2 \alpha \tan \alpha - \Delta C \cos \alpha + \frac{\Delta W F \cos^2 \alpha \tan^2 \alpha \tan \phi}{F} = (\Delta W - \Delta U_a \cos \alpha) \tan \phi
\]

\[
\Delta W F \cos^2 \alpha \tan \alpha \left(1 + \frac{\tan \alpha \tan \phi}{F}\right) = \Delta C \cos \alpha + (\Delta W - \Delta U_a \cos \alpha) \tan \phi
\]

\[
F = \frac{\Delta C \cos \alpha + (\Delta W - \Delta U_a \cos \alpha) \tan \phi}{\Delta W \cos^2 \alpha \tan \alpha \left(1 + \frac{\tan \alpha \tan \phi}{F}\right)}
\]

But:

\[
\frac{1}{\cos^2 \alpha} = \sec^2 \alpha = 1 + \tan^2 \alpha.
\]

For slice $i$ forces only:

\[
F = \frac{[\Delta C \cos \alpha + (\Delta W - \Delta U_a \cos \alpha) \tan \phi][1 + \tan^2 \alpha]}{\Delta W \tan \alpha \left(1 + \frac{\tan \alpha \tan \phi}{F}\right)}
\]

Summing for all slices in a potential failure mass:

\[
F = \sum_{i=1}^{n} \frac{\{Cb + [(\gamma d + (\gamma_{sat} - \gamma) d_w) b - \gamma_{sat} d_w \cos^2 \alpha_w \tan \phi] + \tan^2 \alpha\}}{[\gamma d + (\gamma_{sat} - \gamma) d_w] b \tan \alpha \left(1 + \frac{\tan \alpha \tan \phi}{F}\right)}
\]

Janbu recognized some difference in the results of the simplified version (without the interslice forces) and his generalized version (with interslice forces). To account for the difference, he developed a correction factor, $f_o$, based on 40 solutions. Figure 5A.6 is a plot of $f_o$, showing how the correction factor varies with the shape of the failure surface and the shear strength of the material.

An equation for $f_o$, developed from the graph in figure 5A.6, is programmed into XSTABL (Sharma, 1992):

\[
f_o = 1.0 + k \left[\frac{d}{L} - 1.4 \left(\frac{d}{L}\right)^2\right]
\]

(5A.10)

where $k$ varies according to the soil type: for C-only soils, $k = 0.69$,

for C and $\phi$-soils, $k = 0.50$, and

for $\phi$-only soils, $k = 0.31$. 
Hydrostatic Water Force in Tension Crack

5A.10
Hydrostatic Water Force in Tension Crack

5A.10.1 Ordinary Methods of Slices Modified for Tension Crack Water Forces

\begin{equation}
F = \sum_{i=1}^{n} \frac{Cb + [\gamma d + (\gamma_{sat} - \gamma) d_w] b \cos^3 \alpha - \gamma \sigma d_w b \cos^2 \alpha_{w}}{[\gamma d + (\gamma_{sat} - \gamma) d_w] b \sin \alpha + \frac{1}{2} \gamma \sigma Z_w^2 (\frac{d}{R})} \tan \phi.
\end{equation}
5A.10.2 Bishop's Simplified Method of Slices Modified for Tension Crack Water Forces

Including the additional driving force to equation 5A.11 in a force format gives

\[
F = \sum_{i=1}^{n} \left( Cb + \left[ \gamma d + (\gamma_{sat} - \gamma) d_w - \gamma_w d_w \cos^2 \alpha_w \right] b \tan \phi \right)
\]

\[
\left\{ \gamma d + (\gamma_{sat} - \gamma) d_w \right\} b \sin \alpha + \frac{1}{2} \gamma_w Z_w^2 \left( \frac{c}{\gamma_v} \right) m_a
\]

5A.10.3 Janbu's Simplified Method of Slices Modified for Tension Crack Water Forces

Including the additional driving force to equation 5A.9 in a moment format gives

\[
F = \sum_{i=1}^{n} \left( f_z \left( Cb + \left[ \gamma d + (\gamma_{sat} - \gamma) d_w - \gamma_w d_w \cos^2 \alpha_w \right] b \tan \phi \right) \left[ 1 + \tan^2 \alpha \left( \frac{\cos^2 \alpha}{m_a} \right) \right] \right)
\]

\[
\left\{ \gamma d + (\gamma_{sat} - \gamma) d_w \right\} b \tan \alpha + \frac{1}{2} \gamma_w Z_w^2
\]

Figure 5A.7.—Hydrostatic water force in tension crack (reprinted with permission of Golder Associates from Rock Slopes: Design, Excavation, Stabilization).

5A.11 Rock Plane-Failure Analysis

Rock plane-failure analysis is very similar to the block failure analysis discussed in section 5A.4. The simple equation for \( F \) is similar to equation 5A.1 (figure 5A.8):

\[
F = \frac{CL + N' \tan \phi}{W \sin \theta}
\]
where:

\[ N' = W \cos \theta - uL, \text{ so} \]

\[ F = \frac{CL + (W \cos \theta - uL) \tan \phi}{W \sin \theta} \]  \hspace{1cm} (5A.12)

Section 4E contains a detailed description of ground water in rock slopes and the computational procedure for calculating the ground water pressure, \( u \), for various potential drainage conditions. Section 5H.2 and sample problems 5H.1 and 5H.2 contain an in-depth discussion of rock plane-failure analysis.

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**Figure 5A.8.—Rock slope geometry for simple plane- and wedge-failure analyses (adapted with permission of Golder Associates from Rock Slopes: Design, Excavation, Stabilization).**

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### 5.A.12 Rock Wedge-Failure Analysis

Simple rock wedge-failure analysis is also simplistic in principle, but becomes very complicated in application. A three-dimensional analysis similar to the three-dimensional block failure in section 5A.4 is required. The complication comes in determining:

- The mobilized shear strength on two potential failure surfaces,
- The geometry and weight of the failure wedge,
- The wedge contact area on each failure surface,
- The ground water force acting on each failure surface, and
- The failure surface area on which the ground water force is acting.

The general form of the wedge-failure analysis is:

$$F = \frac{C_A A_A + C_B A_B + N_A' \tan \phi_A + N_B' \tan \phi_B}{W \sin \Psi_i}$$  \hspace{1cm} (5A.13)

Section 5H.3 and sample problem 5H.3 contain an in-depth discussion of rock wedge-failure analysis.

5A.13 Deterministic or Probabilistic?

The above equations can be used in either a deterministic or a probabilistic analysis. In a deterministic analysis, a single value is used for each variable in the solution. This type of analysis is applicable when the user can measure, back-calculate, or otherwise select single parametric values applicable to the conditions of the analysis. In a back-calculation analysis, the value of one of the parameters in the equation about which there is some uncertainty is evaluated while the values of all of the other parameters are set. The results of the analysis must be compared to field observations of stability or instability of the slope being analyzed, and the parametric value in question is adjusted until the user is satisfied that the equation and the combination of parametric values is consistent with these field observations. These field conditions are usually extreme, and the prudent practitioner will use many observations and combinations of physically possible parametric values to test and adjust a single parametric value to gain confidence that the uncertainty in the parametric value determination and in the selection of the failure model are properly accounted for in the analysis. This tailors the analysis to "relative" failure conditions and is particularly functional for analysis levels II and III. Sample problem 5C.1 and section 5F demonstrate the use of back-calculation analysis to evaluate single parametric values for deterministic analysis. Deterministic analysis is also very applicable in a "sensitivity" evaluation. As in back-calculation analysis, the sensitivity of the slope stability analysis to the anticipated variability of parametric values is tested by varying one value at a time through its entire anticipated range while the other parametric values are held constant. This will be demonstrated in section 5D.

When variability is anticipated in most of the parametric values in a slope stability analysis, the interaction of that variability can best be modelled with a probabilistic analysis. Both natural variability of the values and the uncertainty of the user in determining these values can be handled in a probabilistic analysis. Therefore, this type of analysis is most applicable to level I and II analyses where the spatial scope of the analysis is large and the data base is less than ideal. For levels I and II, the planned form of probabilistic analysis is the use of the Monte Carlo simulation. Simplistically, the Monte Carlo simulation allows the user to define parametric values according to a wide range of possible spatial distributions and then solves the appropriate deterministic equation using possible combinations of single values selected from these population distributions. Through this simulation, in the stability analysis setting the probability of landslide occurrence is the ratio of the number of
solutions which result in a factor of safety less than or equal to one to the total number of solutions made in the simulation. This will be further described in section 5D.

A probabilistic version of analysis is also needed for level III. The decision analysis required to compare stabilization alternatives and select a stabilization measure could be aided by analyzing the inherent “probability of failure” of the possible alternatives. In a decision analysis, this “probability of failure” could be combined with the “consequences of failure” for each alternative and evaluated in a risk analysis format. Due to the complexity of the analyses used in level III, the Monte Carlo simulation technique may not be as applicable at this level as it is for levels I and II. This analysis complexity factor is offset somewhat by the quality of data available for level III analyses which reduces the amount of uncertainty. Section 5E describes a few options available for a probabilistic level III analysis.

Regardless of whether a deterministic or a probabilistic analysis is used, the factor of safety equation used becomes a relative comparison of the assumed shear strength to the assumed shearing forces acting in an assumed failure model and all are calculated from a certain combination of assumed parametric values. This is meant not to belittle the accuracy of the method, but to emphasis the importance of verification through parametric value and failure model evaluation by field observations.

5A.14 What Is Stable?

It is important to consider some fundamental differences in how deterministic and probabilistic analyses are applied by experienced users. The basic differences lie in how the experienced user evaluates parametric values for the analysis, how uncertainty and variability are handled in that evaluation, and how one perceives when a slope can be considered to become unstable. In section 5B.4, it is demonstrated that the experienced deterministic analyzer judges whether a slope can be expected to be stable on whether the factor of safety is greater than or equal to one, and back-calculation analysis is used to evaluate parametric values with that as the standard. However, in section 5B.5, it is shown that if parametric values are somewhat normally distributed, a slope with a mean factor of safety of one still has a probability of failure of about 0.50 (about a 50-50 chance of becoming unstable). To the experienced probabilistic analyzer, this is hardly a stable slope. Why the conflict on using a factor of safety of one as the dividing line between stability and instability? When does a slope become unstable? The basic difference in how these two experienced users view the dividing line between stability and instability stems from the manner in which each arrives at the parametric values to use in the analysis and when the degree of stability enters the analysis. For the example from section 5B.6, if the experienced deterministic analyzer used back-analysis based on a factor of safety of one as the dividing line between stability and instability, it is unlikely that each of the single parametric values selected for deterministic analysis would be identical to the mean values of the ranges that the experienced probabilistic analyzer used. They should be close and both should be physically possible, but the deterministic analyzer would typically use more conservative values for those parameters about which there is some uncertainty in value. If properly applied and the two concepts are kept separate, neither is more correct than the other. How successful they are in providing a stability analysis that realistically addresses the problem at hand will depend more on the individual’s experience and ability to read and define the field conditions than on the type of analysis used.
5A.14.1 The Deterministic Concept

The deterministic analyzer must select a single value to realistically represent a parameter in the analysis. The uncertainty in determination of the parametric value and in the equation to model the actual field and failure conditions can be accounted for either by applying a high factor of safety (much greater than one) to the results of the analysis or by evaluation of the parametric value through back-calculation analysis using a factor of safety of one as the stability standard. Through back-calculation analysis, this individual will develop parametric values that best define the observed slope stability conditions. In this manner, the values of which the user is most uncertain will normally tend toward the conservative side of the anticipated range rather than toward the mean values. Those values which are the result of back-calculation, even though they are determined to be physically possible, must be isolated from the measured values and must not be entered into a data base because they are relative values related to the assumptions used in model selection and to all of the other values used in the evaluation.

To the deterministic analyzer who uses back-calculation analysis to evaluate parameters, a factor of safety of one is realistically the dividing line between stability and instability because all possible variations in parametric values should have been considered in the evaluation and selection of the single values. To the deterministic analyzer who is using this form of analysis in the evaluation of stabilization alternatives the discussion of “What is stable?” must also extend into “How stable should it be?” The degree of stability enters the picture in the evaluation of stabilization alternatives through comparison of the stabilized to the unstabilized conditions. The unstabilized conditions may have been used with back-calculation analysis to evaluate parametric values. In that case, this degree of stability is in part relative to the degree of conservatism used in the selection of the individual parametric values. It must be viewed as an increase in stability over the unstabilized slope, and the experienced user must balance the increase in stability against construction costs, maintenance costs, and the consequences of slope failure. That is why in section 6 there is a range of factors of safety being used for the design of stabilization alternatives. The discussion on the selection of a factor of safety for the design of stabilization alternatives is continued in section 6A. Section 6A also contains a discussion of selecting a “stable” factor of safety in a litigation or review capacity.

5A.14.2 The Probabilistic Concept

The probabilistic analyzer can select a range of values for each parameter and let the analysis determine a degree of stability. Field observations are equally important to the probabilistic analyzer, but the comparison to observed slope stability conditions is made on a degree basis and not on a single factor of safety. Back-calculation analysis is just as likely to result in the adjustment of the range of possible values and not necessarily the mean value of a parameter about which there is uncertainty. However, the possible range of that value must still be physically possible, and adjustments through back-calculation must be made accordingly. To this individual, a factor of safety of one is viewed only as a means of comparison of the results of individual trial solutions using only one possible combination of parametric values to establish the degree of stability. It is that degree of stability that is the standard of comparison to observed field conditions.
Regardless of whether a deterministic or probabilistic analysis is used, the ability of the selected equation to accurately model the exact failure conditions anticipated in the field must always be considered. Does the infinite slope equation accurately account for such near-surface conditions as root strength? Can interslice forces be eliminated without introducing significant error? Is a two-dimensional analysis valid under these conditions? Is the failure surface going to be translational or rotational? These types of questions concerning the validity of the analysis selected to model a specific set of field conditions must be answered, and the model and/or parametric values used in the analysis must be tailored to the specific problem.
5B. Soil Slopes—Level I
Analysis—Natural Slopes

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5B.1 Introduction

A level I slope stability analysis is used to support decisions in the resource allocation phase of land management and to provide an initial stability analysis of natural slopes prior to level II and III stability analyses of constructed slopes. For resource allocation, a probabilistic analysis is most appropriate because it allows the user to account for uncertainty and natural variability in evaluating relative landslide hazards of landforms for use in forest planning, timber sale environmental assessment, transportation planning, and cumulative effects assessment. In addition, the concept of probability of slope failure has more meaning to decision makers than does the factor of safety concept.

For analysis of constructed slopes, the prudent designer will begin a level II or III analysis with a level I analysis of the natural slope on which the constructed slope is (or will be) superimposed to determine why the natural slope exhibits the degree of stability that it does. Once this basic link is made, the design of constructed slopes is likely to be more successful. Level I analyses performed prior to level II or III analyses may be either probabilistic or deterministic, but the user must be aware of the differences in the definition of stable which might exist between the two methods, as described in section 5A.14. Most of the subsurface conditions which are analyzed in level I also apply to level II. Level II analysis and the link to level I will be discussed in section 5C.

5B.2 The Infinite Slope Equation

A stability analysis on natural slopes is most easily accomplished with the infinite slope equation. The LISA manual (Hammond et al., 1992) provides a derivation of that equation. Figure 5B.1 illustrates and defines the terminology and variables used in that derivation. The infinite slope equation for the factor of safety against failure, $F$, of the entire soil mantle (with the failure surface at the soil/rock contact) is:

$$F = \frac{C_r + C_r + [q_o + \gamma_d + (\gamma_{sat} - \gamma) d_w] \cos^2 \alpha \tan \phi}{[q_o + \gamma_d + (\gamma_{sat} - \gamma) d_w] \sin \alpha \cos \alpha}.$$

(5B.1)

Except for the inclusion of terms for tree root strength, $C_r$, and tree surcharge, $q_o$, equation 5B.1 is identical to equation 5A.5, which was derived in section 5A as a special case of the ordinary method of slices. The equation is an expression for factor of safety against failure as the ratio of resisting stresses to driving forces.
\( \alpha = \) slope of the ground surface, phreatic surface, and failure surface (soil/rock contact), degrees

\( d = \) total soil depth to soil/rock contact, ft

\( d_p = \) height of phreatic surface above soil/rock contact, ft

\( q_o = \) tree surcharge, psf

\( C_r = \) tree root strength expressed as cohesion, psf

\( C_s = \) effective soil cohesion, psf

\( \phi = \) effective soil angle of internal friction, degrees

\( \gamma = \) moist soil unit weight, pcf

\( \gamma_{sw} = \) saturated soil unit weight, pcf

\( \gamma_w = \) unit weight of water, pcf

*Figure 5B.1.—Terminology and variables used in the infinite slope equation.*
A factor of safety greater than one thus implies a stable slope. Refer to section 5A.14 for a discussion of how this may be interpreted differently in a deterministic analysis than in a probabilistic analysis. The equation has inherent inaccuracies stemming largely from the infinite slope assumptions which do not describe the actual conditions found in nature. These assumptions are:

- The failure plane, ground water (phreatic) surface, and ground surface are parallel to one another.
- The failure plane is of infinite extent.
- Only one soil type is considered.
- A two-dimensional analysis is applicable.

If properly used, however, the inaccuracies can be compensated for through back-calculation and minor parametric value adjustment, and the equation becomes a powerful analysis tool. For additional discussion on these inaccuracies, refer to chapter 3 of the LISA manual (Hammond et al., 1992).

### 5B.3 Level I
#### Computer Programs

Both deterministic and probabilistic personal computer programs are available for level I analysis. The level II programs may be used for level I analysis along road corridors, as well.

---

#### 5B.3.1 Deterministic

An HP-41 programmable-calculator program, SSIS, was developed for the infinite slope equation. The SSIS manual (Prellwitz, 1988) gives additional background and correlation between the level I and level II analyses. The equations from SSIS have been programmed for the personal computer as a deterministic program DLISA (Deterministic Level I Stability Analysis ver. 1.02 1991). This program is particularly useful for level I sensitivity analysis (described in section 5D).

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#### 5B.3.2 Probabilistic

LISA (Level I Stability Analysis ver 2.0 1991) is the probabilistic personal computer program for level I stability analysis. The user enters probability distributions rather than single values for each of the variables in the infinite slope equation. Monte Carlo simulation is used to estimate the probability of slope failure.

---

#### 5B.3.3 Road Corridor Analysis

Although LISA’s primary use is for assessing the relative landslide hazard of landforms for resource allocation, it is being integrated with the probabilistic level II personal computer program, Stability Analysis for Road Access (SARA), currently under development. Used with SARA, LISA is also a tool for level II analysis to analyze the stability of natural slopes along road corridors. Background fundamentals and the use of DLISA and LISA are thoroughly described in the LISA manual (Hammond et al., 1992).
Figure 5B.2 is an illustration of site conditions used for the comparison of the computation of equation 5B.1 to the results of DLISA shown in figure 5B.3.

Factor of safety for the site conditions in figure 5B.2 according to equation 5B.1 is

\[ F = \frac{C_r + C_s + \left[ q_o + \gamma d + (\gamma_{sat} - \gamma_w - \gamma)d_w \right]\cos^2\alpha \tan\phi}{\left[ q_o + \gamma d + (\gamma_{sat} - \gamma)d_w \right] \sin\alpha \cos\alpha} \]

\[ F = \frac{40 + [30 + 120 \times 10.0 + (130 - 62.4 - 120) \times 5.0] \cos^2(26.6^\circ) \tan 32^\circ}{[30 + 120 \times 10.0 + (130 - 120) \times 5.0] \cos 26.6^\circ \sin 26.6^\circ} \]

\[ F = 1.02, \]

which is what DLISA gives as shown in figure 5B.3.
The following deterministic level I sample problems will illustrate the application of
the infinite slope equation to the analysis of natural slopes and to the back-analysis
for reasonable parametric values. These values may be for use in either deterministic
or probabilistic analyses; however, the user is cautioned that the singular values for
deterministic analysis may not be the same as the mean values used in probabilistic
analysis, as discussed in section 5A.13. The site conditions illustrated in figure 5B.4
will be used for both level I and level II (section 5C) deterministic sample problems
to establish a correlation between the stability of natural slopes and constructed
slopes superimposed on those natural slopes.

The interpretation of the results in terms of “What is stable?” is consistent with the
deterministic concept described in section 5A.13: a factor of safety of one is the
dividing line between stability and instability. To illustrate, note that in these level I
and II deterministic sample problems that a factor of safety of one is used in
conjunction with field observations of stability to judge the limiting values for such
parameters in question as soil shear strength, surface slope, and depth of ground
water. This factors the uncertainty in parametric value determination into the back-
calculation analysis. If these predictions of stability, based on the level I and II
stability analyses, do not match the field observations, this back-analysis will result
in adjustment of the singular deterministic value for the parameter in question. The
significance of variations in surface slope, soil depth, and ground water depth to the
stability of a slope will be illustrated in these problems. The strength parameters for
these level I deterministic problems were developed from back-analysis in sample
problem 5C.1.
Figure 5B.4.—Site conditions for level I and II deterministic sample problems.
In these deterministic sample problems, root strength and effective soil cohesion are replaced by one cohesion value, C. This is the logical way to handle these two, because a combined value is the result of the back-analysis, and it is unnecessary to separate them for the purpose of these problems. That is not always the case, particularly when the management impacts of tree removal are part of the analysis. The user must be aware of this in the interpretation of the results. Also, it must be noted that the value of C can vary between the level I and level II analyses due to the differences in the shapes of the failure surfaces being analyzed.

Tree surcharge, \( q_o \), is assumed to be zero in the following problems. The value is generally small and usually has little effect on the analysis. For example, in figure 5B.2, \( q_o = 30 \) psf was used, and \( F = 1.02 \) was calculated. These same site conditions were used for problem 5C.11 with \( q_o = 0 \) psf, and \( F = 1.02 \) was still the result of the analysis of the natural slope conditions.

For simplicity, if \( C = C_r + C_r \) and \( q_o = 0 \), equation 5B.1 can be reduced further to:

\[
F = \frac{C + [\gamma d + (\gamma_{sat} - \gamma) d_w] \cos^2 \alpha \tan \phi}{[\gamma d + (\gamma_{sat} - \gamma) d_w] \sin \alpha \cos \alpha}.
\]  

(5B.2)

If \( d_w = 0 \), this equation can be further reduced to:

\[
F = \frac{C + \gamma d \cos^2 \alpha \tan \phi}{\gamma d \sin \alpha \cos \alpha}.
\]  

(5B.3)

**Problem 1.**

If ground water is not considered to be a problem (no phreatic surface develops) at location A, is 50 percent (26.6 degrees) the maximum stable natural slope?

\[
F = \frac{C + \gamma d \cos^2 \alpha \tan \phi}{\gamma d \sin \alpha \cos \alpha}
\]

\[
F = \frac{(110 \text{ psf}) + (120 \text{ pcf}) (15 \text{ ft}) \cos^2 26.6^\circ \tan 30^\circ}{(120 \text{ pcf}) (15 \text{ ft}) \sin 26.6^\circ \cos 26.6^\circ}
\]

\[
F = 1.31.
\]

\( F \) is greater than 1.00; therefore, 50 percent is not the maximum stable slope at Location A.
Problem 2.

At location B in figure 5B.4, is 70 percent the maximum stable natural slope if ground water is not considered to be a factor?

From equation 5B.3,

\[
F = \frac{C + \gamma d \cos^2 \alpha \tan \phi}{\gamma d \sin \alpha \cos \alpha}
\]

\[
F = \frac{(110 \text{ psf}) + (120 \text{pcf}) (10.0 \text{ ft}) \cos^2 35.0^\circ \tan 30^\circ}{(120 \text{pcf}) (10.0 \text{ ft}) \sin 35^\circ \cos 35^\circ}
\]

\[
F = 1.02
\]

\(F\) is greater than 1.00; therefore, 70 percent is not the maximum stable slope at Location B, but it is closer to failure than is the 50 percent slope at Location A.

Problem 3.

Would you expect the natural slope at location B in figure 5B.4 to be stable if the soil depth were 15 feet (ground water still not a factor)?

Using equation 5B.3,

\[
F = \frac{C + \gamma d \cos^2 \alpha \tan \phi}{\gamma d \sin \alpha \cos \alpha}
\]

\[
F = \frac{(110 \text{ psf}) + (120 \text{pcf}) (15.0 \text{ ft}) \cos^2 35.0^\circ \tan 30^\circ}{(120 \text{pcf}) (15.0 \text{ ft}) \sin 35^\circ \cos 35^\circ}
\]

\[
F = 0.95
\]

\(F\) is less than 1.00; therefore, the slope at Location B would be unstable, if the soil depth were 15 ft.
Problem 4.

Is the ground water phreatic surface at the critical level for the stability of the slope at location D? (Solve for the critical level by setting \( F = 1.00 \) and solving for \( d_w \). Compare \( d_w \) to 14 ft.)

Using equation 5B.2,

\[
F = \frac{C + [\gamma d + (\gamma_{sat} - \gamma_w - \gamma)d_w] \cos^2 \alpha \tan \phi}{[\gamma d + (\gamma_{sat} - \gamma)d_w] \sin \alpha \cos \alpha}
\]

\[
1.00 = \frac{50 \text{ psf} + [(120 \text{ pcf})(20.0 \text{ ft}) + (130 - 62.4 - 120)d_w] \cos^2 20^\circ \tan 30^\circ}{[(120 \text{ psf})(20.0 \text{ ft}) + (130 - 120)d_w] \sin 20^\circ \cos 20^\circ}
\]

\[
d_w = 16.8 \text{ ft}
\]

\( d_w = 14.0' \) is less than the critical \( d_w \), so the slope is stable.

Problem 5.

If, in the springtime, ground water is expected to concentrate at location A, how high could a phreatic surface rise in the soil column before the slope becomes unstable? (Solve for the “critical” level in a manner similar to problem 4.)

Using equation 5B.2,

\[
F = \frac{C + [\gamma d + (\gamma_{sat} - \gamma_w - \gamma)d_w] \cos^2 \alpha \tan \phi}{[\gamma d + (\gamma_{sat} - \gamma)d_w] \sin \alpha \cos \alpha}
\]

\[
1.00 = \frac{50 + 1223.55 - 26.71d_w}{771.35 + 3.21d_w}
\]

\[
771.35 + 3.21d_w = 1273.55 - 26.71d_w
\]

\[
29.92d_w = 502.20
\]

\[
\text{Critical } d_w = 16.8 \text{ ft}
\]

\[
d_w = 16.0' \text{ is less than the critical } d_w, \text{ so the slope is stable.}
\]

5B.6 What Is “Angle of Repose”?

“Angle of repose” can be thought of as the maximum slope at which a cohesionless soil would be stable in its loosest state. Sometimes, however, those unfamiliar with slope stability analysis will refer to the “angle of internal friction,” \( \phi \), as the angle of repose of the soil. From the results of the previous problems, it should be clear that all factors in the infinite slope equation must be properly accounted for in the
determination of the repose angle for a soil and that the value of this angle depends on the specific set of conditions. There is only one set of conditions under which the angle of repose is equal to the angle of internal friction (as illustrated in problem 6). If the talus slope illustrated is truly at its angle of repose and if this is also the angle of internal friction, then soil cohesion, root strength, and ground water depth must all be zero and the deposit must be in its loosest state. Some (but not all) talus materials exist in nature at their maximum stable slope because of the nature of their origin. However, that is not always true due to the other factors that can control that slope angle. Rock talus deposits can be particularly troublesome if they have an angle of internal friction of 40° or less and a stable road is to be constructed across them (as will be illustrated in problem 5C.4).

**Problem 6.**

If the illustrated talus slope is at its angle of repose (maximum stable slope), what is the minimum value of the angle of internal friction, $\phi$? In this problem, groundwater is not considered to be a factor (no phreatic surface is expected to develop in this case). What if it is?

Since $d_w=0$, use the equation for $F$ in problem 1:

$$\alpha = \tan^{-1}\left(\frac{80\%}{100\%}\right) = 38.7^\circ$$

$$\tan \alpha = \frac{\gamma d \cos \alpha \tan \phi}{\gamma d \sin \alpha \cos \alpha}$$

Since $C = 0$:

$$F = \frac{\gamma d \cos \alpha \tan \phi}{\gamma d \sin \alpha \cos \alpha} = \frac{\cos \alpha \tan \phi}{\sin \alpha}$$

$$F = \frac{\tan \phi}{\tan \alpha}$$

When $F = 1.00$:

$$\phi_{\text{min}} = \alpha = 38.7^\circ$$

Note that when $C$ and $d_w$ are both zero, soil depth, $d$, is no longer a factor in the analysis. If $C$ is zero but $d_w$ is not, then the relationship between $\alpha$ and $\phi$ becomes:

$$F = \frac{C + [\gamma d + (\gamma_{\text{sat}} - \gamma_w - \gamma)d_w] \cos^2 \alpha \tan \phi]}{[\gamma d + (\gamma_{\text{sat}} - \gamma)d_w] \sin \alpha \cos \alpha}$$

Since $C = 0$:

$$F = \left(\frac{\gamma d + (\gamma_{\text{sat}} - \gamma_w - \gamma)d_w}{\gamma d + (\gamma_{\text{sat}} - \gamma)d_w}\right) \tan \phi \tan \alpha$$

However, at the extreme case, when $d_w = d$, then soil depth, $d$, is again no longer a factor:

$$F = \left(\frac{\gamma_{\text{sat}} - \gamma_w}{\gamma_{\text{sat}}}\right) \tan \phi \tan \alpha.$$
## Level One Stability Analysis
### LISA Version 2.00

---

**ID:** Figure 5B.3 Comparison.

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<th>Prellwitz</th>
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<tr>
<td>Probability of failure</td>
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</table>

**INPUT DATA**
---

**NATURAL DATA**
---

| Soil depth (ft) | Normal Mean: 10.00 Std.: 1.50 |
| Ground slope (%) | Normal Mean: 50.00 Std.: 5.00 |
| Tree surcharge (psf) | Normal Mean: 20.00 Std.: 5.00 |
| Root cohesion (psf) | Normal Mean: 20.00 Std.: 5.00 |
| Friction angle (deg) | Normal Mean: 32.00 Std.: 5.00 |
| Soil cohesion (psf) | Normal Mean: 20.00 Std.: 5.00 |
| Dry unit weight (pcf) | Normal Mean: 109.00 Std.: 5.00 |
| Moisture content (%) | Normal Mean: 10.00 Std.: 2.00 |

**DESCRIPTIVE STATISTICS OF SIMULATED VALUES -- NATURAL SLOPE**
---

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<thead>
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<th>MAXIMUM</th>
<th>MEAN</th>
<th>S.D.</th>
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<tbody>
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<tr>
<td>Ground slope (%)</td>
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<td>65.45</td>
<td>49.95</td>
</tr>
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<td>Tree surcharge (psf)</td>
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<td>45.91</td>
<td>29.96</td>
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<td>Groundwater ratio (Dw/D)</td>
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**Histogram of natural slope factor of safety**
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<tr>
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<td>124</td>
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<tr>
<td>0.81 - 0.95</td>
<td>217</td>
</tr>
<tr>
<td>0.95 - 1.10</td>
<td>250</td>
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<tr>
<td>1.10 - 1.24</td>
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<td>1.24 - 1.39</td>
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<td>1.96 - 2.11</td>
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</tbody>
</table>

**Histogram Statistics**
---

| Number of iterations | 1000 | Sample minimum | 0.52 |
| Sample mean | 1.05 | Sample maximum | 2.11 |
| Sample median | 1.03 |
| Sample standard deviation | 0.24 |
| P( FS <= 1 ) | 0.450 |

---

*Figure 5B.5.—Probabilistic comparison using LISA for the slope of figure 5B.3.*
Section 5A.13 discussed the differences between selection of single parametric values for deterministic analysis and a range of likely values for probabilistic analysis. It was noted that the single values used in deterministic analysis were more likely to be on the conservative end of the range, rather than the mean values from a probabilistic analysis for the same slope. In this section, that point is further illustrated through an example correlation between DLISA and LISA. Figure 5B.5 is the printout of the results of LISA for the DLISA example from figure 5B.3. The deterministic values from figure 5B.3 were used as mean values for the LISA run. Even though not all of the distributions are normal, the distributions about the mean are fairly symmetrically distributed. The distribution of factors of safety, $F$, resulting from the analysis is also roughly equally distributed about the mean factor of safety. The mean factor of safety from LISA is shown to be very near the deterministic factor of safety (about 1.00) from DLISA and the probability of slope failure is estimated at about 0.5. From this example one can conclude that in probabilistic analysis, if parametric values are more or less symmetrically distributed about the mean, a factor of safety of one can be interpreted as having about a 50-50 chance of being stable or unstable. In deterministic analysis that is far from the dividing line between stability and instability, and single parametric values would be selected more conservatively through back-calculation analysis.
5C. Soil Slopes—Level II
Analysis—Constructed Slopes

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5C.1 Introduction

Level II stability analysis is the type that will support decisions made during the project planning phase of land management. The infinite slope equation used in level I is also useful as a level II analysis to support those decisions involving natural slopes. For instance, the selection of timber-harvest techniques may be supported if the impact of tree removal on the stability of natural slopes can be modelled and analyzed with the infinite slope equation. Usually, the greater land management impact on slope stability results from road construction. This section will describe procedures useful in the preliminary analysis of constructed road cut and fill slopes. Figure 5C.1 illustrates the continuity and interaction of subsurface conditions between the two types of analysis. This figure also defines the geometric terms used in both methods.

![Figure 5C.1. Relationship between the geometric terms used in level I and level II analyses.](image)

The analysis of constructed slopes in level II consists of superimposing the anticipated road template on the natural slope conditions and estimating the stability of these slopes with stability number solutions. Stability number solutions are only
empirical correlations to the more complicated level III analysis. This provides a rapid assessment of whether a planned road cut or fill slope can be expected to be stable under the anticipated site conditions (see section 5A.14 for a discussion of “What is stable?” and how this interpretation can vary from deterministic to probabilistic analyses). This makes it very functional when used in conjunction with digital terrain models or computer-aided road design methods. Although the anticipated stability is predicted, the analysis tells the designer nothing about the shape of the potential failure surface such as the level III analysis does. There is also some loss in accuracy compared to the level III analysis. Researchers at the Intermountain Research Station are attempting to develop level II algorithms which will predict to within ±10 percent of the results of the simplified Bishop’s level III method.

5C.2 Fundamentals

There are two fundamental forms of stability number analyses: those that predict the critical height of a constructed slope under the anticipated site conditions, and those that estimate the factor of safety, $F_s$, of the level III analysis for the slope. Stability number solutions from Chen and Giger (1971) and Cousins (1978) have been selected by the Intermountain Research Station as the core analyses for these two approaches, respectively. The Cousins method is very similar in procedure and results to the methods presented by Duncan and Buchignani (1975) which were patterned after Janbu (1968). None of these methods was found to yield consistent results within 10 percent of the results of the simplified Bishop method of slices used in level III because they do not adequately account for the subsurface conditions. To improve the accuracy, correction factors are being developed by the Intermountain Research Station. These correction factors use dimensionless parameters to account for the subsurface conditions illustrated in figure 5C.2. Prellwitz (1988) documents the original algorithms for these correction factors developed for the HP41 programmable calculator program, SSCHFS (1988). Algorithms are now being developed to further improve the accuracy and extend the range of applicable shear strength parameters. These improved algorithms will be programmed into a deterministic program, DSARA. They will also be coupled with a Monte Carlo simulation technique for a probabilistic version, SARA. These programs will interact and share data files with their level I counterpart, LISA. The technical manual and user's guide for SARA will document all of the algorithms and suggestions for use in level II analysis much the same as the LISA manual (Hammond et al., 1992) has done for level I.

SARA and DSARA will allow the user either to define the constructed slope geometry or to select a standard road template and let the computer solve for the slope geometry. Three standard road templates are available: a self-balance road section, a full-bench road section, and a through-fill road section. Figure 5C.3 illustrates these three sections. In the future, the “user-defined” option may be coupled to computer-aided road design computer programs. The road template option superimposes the defined template onto the anticipated site conditions and will function well in conjunction with a digital terrain model. The self-balance template also includes an analysis for compaction factor which is required to “balance” the cut volume with the fill volume. A discussion of compaction factor was included in section 4B. Construction losses and subsidence are modeled as “stripping depth”, $d_s$, and shrinkage is analyzed in relation to the anticipated unit weights of the cut and fill material. Problems 11 through 13 in section 5C.9 demonstrate the use of the template options that will be available with SARA.
5C.3 Critical Height Analysis

Using critical height analysis allows the designer to estimate the maximum or critical height, $H_\text{c}$, to which a given cut or fill slope might be stable under the anticipated site conditions. It is useful for back-calculation analysis to estimate the minimum shear-strength parameters for a soil (as demonstrated in problem 1). It is also quite useful in determining or testing road design limits and the effects of changes in alignment. This is demonstrated in problems 2 through 7.

Figure 5C.4 is a plot of Chen and Giger's (1971) stability number solution. Note that the stability number, $N$, approaches infinity as the constructed slope, $\beta$, approaches the angle of internal friction, $\phi$, of the soil. To account for this, Chen and Giger limited their analysis to $\beta$ greater than $\phi + 5^\circ$. Also note that when cohesion, $C$, is zero, $H_\text{c}$ can be either zero or infinite depending on the value of $N$. This will be further illustrated in problem 4.

Figure 5C.5 is a plot of the original correction factor, $S$, developed for Chen and Giger's solution. This figure will be used in the solutions of the sample problems. The algorithms now under development will improve on these original ones, and the answers may not be the same. When in question, the level III analysis should be used to determine the accuracy of the solution.
I. Self-Balance Template

![Diagram of Self-Balance Template]

User Specifications:
- $d$, $L$, $E_x$, $E_y$, $E_z$, $a$, $x_0$, $h_0$, $k_x$, $k_y$, $k_z$

b. Full-Bench Template

![Diagram of Full-Bench Template]

User Specifications:
- $d$, $L$, $E_x$, $E_y$, $E_z$, $a$, $x_0$, $h_0$

c. Through-Fill Template

![Diagram of Through-Fill Template]

User Specifications:
- $d$, $E_x$, $E_y$, $E_z$, $a$, $x_0$, $h_0$

Figure 5C.3. —Road template options for level II analysis.
Figure 5C.4.—Plot of Chen and Giger's stability number solution (adapted with permission of the American Society of Civil Engineers from "Limit analysis of stability of slopes" in Proceedings of the American Society of Civil Engineers, Journal of the Soil Mechanics and Foundations Division, 97 (SM1) by W.F. Chen and M.W. Giger, 1971).
APPROXIMATE CHART SOLUTION FOR CORRECTION FACTOR S

This chart was developed for $\phi = 30$ degrees and $\beta = 1:1$. Checks for $\phi$ ranging from 25 to 40 degrees and $\beta = \phi + 5$ degrees resulted in a mean $FS = 0.82$ with std. dev. = 0.10. Checks for the same range of $\phi$ and $\beta = 65$ degrees resulted in a mean $FS = 1.19$ with std. dev. = 0.11.

$H_0 = SNCG
\gamma$

Figure 5C.5.—Original correction factor, S, for the Chen and Giger solution.
The critical height of the slope is

\[ H_c = SN(C/\gamma) \]  

(5C.1)

where:

- \( H_c \) = Critical or maximum stable height of the constructed slope, ft
- \( S \) = Correction factor based on the parameters of figure 5C.2
- \( N \) = Chen and Giger’s stability number based on \( \alpha \), \( \beta \), and \( \phi \)
- \( \alpha \) = Natural slope, degrees
- \( \beta \) = Constructed slope, degrees
- \( \phi \) = Effective angle of internal friction of the soil, degrees
- \( C \) = Effective cohesive strength of the soil, psf
- \( \gamma \) = Moist or saturated unit weight of the soil, pcf.

### 5C.4 Estimated Factor of Safety

The estimated factor of safety analysis for level II has an advantage over the critical height analysis in that it provides a direct correlation to the anticipated results of the level III analysis for rotational failures (specifically, the simplified Bishop’s method of slices). Because it is on a factor of safety basis, it also provides the best continuity between the level I and level III analyses.

Section 5C.9 provides a summary correlation between the estimated factor of safety analysis and the critical height analysis for sample problems 11, 12, and 13. A comparison is also made for these problems to the results of three level III personal computer programs, DTIS*BISHOP (1969; Bailey and Christian, 1969), STABL3FW (1986; Siegel, 1984, modified to include phreatic surface analysis and Janbu’s \( f_o \) correction factor), and XSTABL (1992; Sharma, 1991), using the simplified Bishop’s option (see table 5C.1). The original correction factor algorithms developed for the HP41 programmable calculator (Prellwitz, 1988) were used in the solution of these problems, and the user is reminded that improved algorithms are under development for use with SARA.

Figure 5C.6 is a plot of Cousins’ (1978) stability number charts. Algorithms for computer analysis were developed by Pratt et al. (1984) from these data. Prellwitz (1988) added an original correction factor, \( k \), to account for the dimensionless parameters of figure 5C.2. The estimated factor of safety expected from the simplified Bishop’s method-of-slices analysis by this modified method is:

\[ \text{Estimated } F = \frac{kN_f(C/\gamma)}{h} \]  

(5C.2)

where:

- \( k \) = Correction factor based on the parameters of figure 5C.2
- \( N_f \) = Cousins’ stability number based on \( \beta \), \( \lambda \), and \( r_u \)
- \( \beta \) = Constructed slope, degrees
- \( \lambda \) = \((\gamma h \tan \phi)/C\)
- \( r_u \) = Pore-pressure ratio = \( \gamma_w h_u/\gamma h \)
- \( h \) = Height of the constructed slope, ft
- \( h_u \) = Height from the toe of the slope to the ground water surface, ft
- \( \gamma_w \) = Unit weight of water (62.4 pcf)
\[ \gamma = \text{Moist or saturated unit weight of the soil, pcf} \]
\[ \phi = \text{Effective angle of internal friction of the soil, degrees} \]
\[ C = \text{Effective cohesive strength of the soil, psf} \]

Figure 5C.6. —Cousins’ stability number charts. Limitation of the Cousins’ data and Pratt’s algorithms are: (1) \( \lambda \) must be greater than zero and (2) \( \beta \) must be less than 65°. (a) \( r_s = 0 \) (b) \( r_s = 0.25 \) (c) \( r_s = 0.50 \).

5C.5 Critical Height Sample Problems

The following seven problems demonstrate the use of deterministic level II analyses. These solutions are limited to the critical height analysis because it is the simpler of the two methods to demonstrate. Section 5C.9 compares the results of the two methods using the original HP41 algorithms and a comparison to other methods of stability analysis.

Figure 5B.4 shows four sites on a road location which have the same soil type. Three of these sites were used with the section 5B problems to illustrate the application of the infinite slope equation to the analysis of the stability of natural
slopes at these sites. They will be used with problems 1 through 3 and 5 through 7 to illustrate the application of level II analysis to the design of cut slopes.

Problem 1 is a back-calculation analysis for a cohesion value to use in road design. This value must include all but the frictional strength of the soil. In addition to the effective cohesion of the soil, additional strength from root strength, $C_r$, and apparent cohesion, $C_a$, are solved for in this type of back-analysis. These can be significant, however, because small values of "cohesion" often enable the construction of short, steep constructed slopes on low-volume roads. This back-calculation analysis, using a factor of safety of one as the dividing line between stability and instability, is consistent with the discussion of deterministic analysis in section 5A.13. The shear strength parameters of the soil are not measured; they are only assumed and must be considered as "relative" to the conditions under which they were evaluated. The user must keep that in mind and continue to monitor the accuracy of the stability predictions made with analyses using these parameters when the slopes are actually constructed. Also, because they are not measured, they should not be entered into a data base for that soil.

Problems 2 and 3 evaluate the stability of several ratios of cut slope available for design. There is no ground water influence on the stability of these constructed slopes. Note the effects of the steepness of the natural slope on the results. Problem 4 is similar but is unique in that the material is a truly cohesionless talus material. Construction difficulties in talus material are not uncommon if the slope of the talus is already at or near the angle of internal friction of the material due to the extremely low or absent cohesive strength. Refer to section 5B.5 for additional discussion of the relationship between the angle of internal friction and the angle of repose.

Problems 5 and 6 demonstrate the influence of ground water on the stability of the cut slope. Note how minor changes in the road alignment might stabilize a cut slope constructed where ground water is present. Problem 7 demonstrates that a cut slope is likely to become unstable long before a natural slope during seasonally high ground water. Subsurface drainage systems to prevent cut slope failures at locations such as this will be covered in section 6D.

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**Problem 1.**

Use the soils data and site conditions given in figure 5B.4 to estimate soil parametric values for use in the design of the road. Note that this type of back-analysis requires the assumption that the observed constructed slopes are at the maximum stable conditions. In practice, many observations must be made on which to base these calculations to reduce the inherent conservatism of this approach.

**Soils Data**

- **Unified Class:** SM - Silty Sand
- **Plasticity Index:** Nonplastic to 10 max.
- **In-place Density:** $\gamma = 120$ to $125$ pcf
- $\gamma_p = 107$ to $110$ pcf
- **Specific Gravity:** 2.7
From Fig. 4B.4, for $\gamma = 120$ pcf, $\gamma_b = 107$ pcf, and SM soil class, find:

\[
\phi' = 30^\circ \quad \text{(Most conservative value)}
\]

\[
\gamma_{sat} = 130 \text{ pcf}
\]

\[
(\gamma_b = \gamma_{sat} - \gamma_w = 130 \text{ pcf} - 62.4 \text{ pcf} = 67.7 \text{ pcf})
\]

From Fig. 5C.4 for Location A conditions ($h = 12$ ft, $\alpha = 26.6^\circ$, $\beta = 1:1$) and $\phi = 30^\circ$ as determined above, find:

\[
N = H_c \left( \frac{\gamma}{C} \right) = 13.2
\]

set $h$ to the critical height $H_c = 12$ ft and solve for $C$:

\[
C = \frac{h\gamma}{N} = \frac{(12 \text{ ft}) (120 \text{ pcf})}{13.2} = 109.1 \text{ psf}
\]

Use $C = 110$ psf above the phreatic surface

For Location C conditions ($h_w = h = 14.2$ ft., $\alpha = 15^\circ$, $\beta = 1:1$) and $\phi' = 30^\circ$, $\gamma_{sat} = 130$ pcf as determined above, find:

\[
N = H_c \left( \frac{\gamma}{C} \right) = 34.5 \quad \text{Fig. 5C.4.}
\]

For $\frac{\alpha_w}{\phi} = 15^\circ \frac{30^\circ}{0.5} = 0.5$ or $\alpha_w = \frac{\phi}{2}$, $\frac{h_w}{h} = 1.0$, $\frac{h_R}{h} < -0.1$

find: $S = 0.31 \quad \text{Fig. 5C.5.}$

\[
H_c = SN \left( \frac{C}{\gamma} \right) \text{ or } C = \frac{H_c \gamma}{SN}
\]

Set $h$ to the critical height $H_c = 4.2$ ft and solve for $C$:

\[
C = \frac{h\gamma}{SN} = \frac{(4.2 \text{ ft}) (130 \text{ pcf})}{(0.31) (34.5)} = 51.1 \text{ psf}
\]

Use $C = 50$ psf below the phreatic surface
Problem 2.

At location A of figure 5B.4, the design calls for an additional 10 feet of road width to be added by extending the section into the cut. Using the parametric values from problem 1, evaluate the stability of 0.50:1, 0.75:1, 1.00:1, and 1.25:1 cut slopes with this 10 feet of widening. Note that because the existing 0.50:1 cut slope at this location was used to estimate the value of "cohesion" to use in cut slope design, a 0.50:1 cut must be at its maximum height already at this location and cannot be constructed any higher. A cut slope flatter than 0.50:1 at this location is dictated by this assumption.

\[
v_z = \sqrt{\frac{3}{2}} \frac{Z}{Z_0} = \frac{3}{2} \frac{30}{30} = 1.5
\]

\[
v_{z0} = \sqrt{\frac{3}{2}} = \sqrt{\frac{3}{2}} \frac{30}{30} = 1.73
\]

\[
\phi = 30^\circ, \beta = 120 \text{ psi}, C = 110 \text{ psi}
\]

\[
C' = \frac{110}{120} = 0.92
\]

**SOLUTION:**

<table>
<thead>
<tr>
<th>Cut Slope, ( s ) (( F_z, S.C. ))</th>
<th>Critical Height ( H_c = N(C'/Y) )</th>
<th>Actual Height ( h ) (By Scale)</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1/2:1 )</td>
<td>13.2</td>
<td>12.1</td>
<td>Unstable</td>
</tr>
<tr>
<td>( 1/4:1 )</td>
<td>20.0</td>
<td>18.3</td>
<td>Unstable</td>
</tr>
<tr>
<td>( 1:1 )</td>
<td>33.5</td>
<td>30.7</td>
<td>Stable</td>
</tr>
<tr>
<td>( 1/4:1 )</td>
<td>66.0</td>
<td>60.5</td>
<td>Stable</td>
</tr>
</tbody>
</table>

Use \( 1:1 \) Cut Slope.
Problem 3.

At location B of figure 5B.4, would the cut slope ratio selected for location A be stable for a full-bench cut to provide a 15-foot-wide subgrade width with no ditch? Compare the results to the results of problem 2 and note the effects that steep natural slopes have on the design of a stable cut slope.

<table>
<thead>
<tr>
<th>Cut Slope, B</th>
<th>Critical Height $h_c = \frac{N(C/F)}{k}$</th>
<th>Actual Height (By Scale)</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>3:1</td>
<td>12.7</td>
<td>16.5</td>
<td>Unstable</td>
</tr>
<tr>
<td>4:1</td>
<td>17.7</td>
<td>22.8</td>
<td>Unstable</td>
</tr>
<tr>
<td>1:1</td>
<td>29.9</td>
<td>34.5</td>
<td>Unstable</td>
</tr>
<tr>
<td>1:4</td>
<td>62.5</td>
<td>82.5</td>
<td>Unstable</td>
</tr>
<tr>
<td>1:4.1</td>
<td>33.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

No stable full-bench cut can be made without removing all of the soil down to bedrock ($B = 1/4.1$).
Problem 4.

To further illustrate the effects of even small values of cohesion on the design of stable cut slopes, consider the design of a road through a clean talus rock material with the parametric values as shown in the following sketch and select a cut slope with the minimum stable cut height. Frequently, stable cut slopes of 1.00:1 or steeper are not possible in talus material. Compare also to the analysis of section 5B.5 for a correlation to the stability of a natural talus slope.

SOLUTION:
(a) Similar to Prob. 5B.5, recognize that when \( c = 0 \), the maximum stable slope is \( \phi = 40^\circ \) regardless of density or depth. Select a cut slope slightly flatter than \( 40^\circ \) which is \( 38^\circ \). Graphically, for \( \theta = 38^\circ \) and \( \phi = 40^\circ \).

(b) Or use Stability Number Chart, Fig. 5C.1

<table>
<thead>
<tr>
<th>Cut Slope (( b:1 )</th>
<th>(Fig. 5C.4)</th>
<th>Critical Height ( H = h (\tan \phi) )</th>
<th>Actual Height ( h ) (in. calc)</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1/2:1 )</td>
<td>2.2</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>Unstable</td>
</tr>
<tr>
<td>( 3/4:1 )</td>
<td>4.7</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>Unstable</td>
</tr>
<tr>
<td>( 1:1 )</td>
<td>16.5</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>Unstable</td>
</tr>
<tr>
<td>( 1/4:1 )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>Stable</td>
</tr>
</tbody>
</table>

* NOTE: That mathematically \( \infty \) is indeterminate and can either equal 0 or \( \infty \). Test simply by using very small values of cohesion.
Problem 5.

Will the design cut planned for location D as shown in the sketch be stable?

\[ \text{SOLUTION:} \]

\text{Cross-section Data:} \hspace{1cm} \text{Parameters Below Phreatic Surface:}

\[ h = 11 \text{ ft} \quad a = 20^\circ \quad \gamma_{sat} = 130 \text{pcf} \]
\[ h_w = 1 \text{ ft} \quad 4' = 30^\circ \]
\[ h_r = -21 \text{ ft} \quad a_v = a_r = 20^\circ \quad C' = 50 \text{ pcf} \]

\text{Stability Number, } N:

\[ \text{From Fig. 5C.4, for } \beta = 11^\circ, \gamma = 20^\circ, \phi = 30^\circ \]
\[ \text{Find: } N = 3.40 \]

\text{Correction Factor, } S:

\[ \text{From Fig. 5C.5, for } \frac{h_h}{h_m} = \frac{10}{110} = 0.09, \frac{h_h}{h_m} = -\frac{210}{110} = -1.9 \]
\[ \text{From Fig. 5C.5, for } \frac{h_h}{h_m} = -\frac{210}{110} = -1.9 \]
\[ \text{Find: } S = 0.74 \]

\text{Critical Cut Height, } H_c:\n\[ H_c = SN(\phi/\gamma) = 0.74 \times 33.0 \times (50/130) = 9.7 \text{ ft.} \]

\text{Design Height, } h_d, \text{ exceeds Critical Height, } H_c:
\[ \text{Expect it to be UNSTABLE.} \]
Problem 6.

If the design for location D is modified by shifting the alignment to the left, resulting in a slight fill and a reduced cut height, would the cut slope be stable under these ground water conditions?

\[ h_i = 9.6 \text{ ft} \]
\[ h_o = 0 \text{ ft} \]
\[ b_r = -22 \text{ ft} \]
\[ a_r = a_r = 20^\circ \]
\[ C' = 50 \text{ psi} \]

**SOLUTION (Similar to Prob. 5C.5):**

**Cross-section Data:**
- \( h_i = 9.6 \text{ ft} \)
- \( h_o = 0 \text{ ft} \)
- \( b_r = -22 \text{ ft} \)
- \( a_r = a_r = 20^\circ \)

**Parameters Below Phreatic Surface:**
- \( \gamma_{sat} = 120 \text{pcf} \)
- \( \gamma' = 30^\circ \)
- \( C' = 50 \text{ psi} \)

**Stability Number, \( N \):**
\[ N = 34.0 \quad \text{(Same as Prob. 5C.5)} \]

**Correction Factor, \( S \):**

From Fig. 5C.5, for \( \frac{h_w}{\gamma} = 9.5^\circ \), \( h_i = 9.6 \text{ ft} \), \( h_i' = 0.00 \), \( h_o = 0 \) and \( h_o'' = 0.00 \).

\[ S = 0.87 \quad \text{(Same as Prob. 5C.5)} \]

**Critical Cut Height, \( h_c \):**
\[ h_c = 5N(\gamma') = 0.87 \times 34.0 \times (50/30) = 144 \text{ ft} \]

Design Height, \( h_i \), is less than Critical Height, \( h_c \):
Expect it to be **SLOPE**.
Problem 7.

Which would you expect to become unstable first under seasonally high ground water at location A: the natural slope or the 1.00:1 cut slope designed in problem 5C.2? (Refer to the solution of problem 5B.5 for the critical ground water depth for location A and determine the critical height of cut slope under those ground water conditions.)

Solution:
(Plot Critical $d_w$ from Problem 5B.5 on sketch, determine $h_w$ and solve as in problems 5 and 6.)

Cross-section Data:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>28 ft</td>
</tr>
<tr>
<td>$a$</td>
<td>26.6°</td>
</tr>
<tr>
<td>$b_a$</td>
<td>10 ft</td>
</tr>
<tr>
<td>$h_R$</td>
<td>-2 ft</td>
</tr>
</tbody>
</table>

Parameters below Phreatic Surface:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{sat}$</td>
<td>180 psf</td>
</tr>
<tr>
<td>$v'$</td>
<td>30°</td>
</tr>
<tr>
<td>$q_w$</td>
<td>26.6°</td>
</tr>
<tr>
<td>$c'$</td>
<td>50 psf</td>
</tr>
</tbody>
</table>

Stability Number, $N$:

$$N = 33.5 \quad (Same \ as \ Prob \ 5C.2)$$

Correction Factor, $S$:

From Fig. 5C.5, for $h_w/a = 16.0/28.0 = 0.57$, $h_w/a = -5.80 = -0.07$,

$$S = 0.60$$

Critical Cut Height, $h_c$:

$$h_c = SN(c'/a) = 0.60 \times 33.5 \times (50/30) = 27 \text{ ft}$$

Design Height, $h_d$, is much Greater than Critical Height, $h_c$: Expect Cut to become Unstable before Groundwater reaches $27'$. 
The level II stability number methods used for fill slope analysis assume that the failure surface will exit at the toe of the fill slope. However, that is not always the case. Figure 5C.7 shows four possible modes of fill failure. A debris avalanche with a translational failure surface at the rock line is just as likely a mode of fill slope failure under high ground water conditions on steep slopes. This will be demonstrated further in section 5D in a discussion of progressive rotational-to-translational failures of fill slopes. To alert the user of the potential for this type of failure under extreme ground water conditions, a "red flag" option is available in SSCHFS and is being programmed into DSARA.

Possible Failure Modes:

1) Failure surface exits at toe of failure.
2) Failure surface tangent to rock line.
3) Fill sliding in organic debris.
4) Debris avalanche with translational failure surface at rock line.

Figure 5C.7.—Four modes of fill slope failure.

This red flag option is in the form of an exaggerated ground water surface extending into the fill material. The ground water phreatic surface can be forced up into the fill material, and this phenomenon has been measured and monitored on occasion (section 4E.6.1). Unfortunately, there is little measured data on which to base reliable predicting analysis. There may be several causes for this phenomenon—such as subsidence of the subsoil or organic surficial material, if it were not stripped before fill placement; infiltration of snowmelt or rain into the fill material developing a perched ground water surface within the fill; infiltration from inadequate ditch drainage, which is most likely to occur in a through-fill section without organic surficial material stripping; and a plugged culvert in a through-fill section (sample problem 4E.8). The location of the exaggerated ground water surface in this red flag option is an arbitrary approach developed by Prellwitz (1988) based on the relationship between: (1) the ratio of the slope of the phreatic surface to the angle of internal friction of the subsoil, $\alpha_s/\phi$, and (2) the ratio of the height of the fill to the depth of the subsoil, $h/d$, and the unexaggerated depth of ground water in the subsoil to the depth of the subsoil, $d/\bar{d}$. 
Figure 5C.8 shows the location of the exaggerated ground water surface (labeled $h_e$) in relationship to the other ground water options (drained and intercept) available in SSCHFS and DSARA.

![Diagram of ground water options]

1) Exaggerated
2) Intercept
3) Drained

Figure 5C.8.—Ground water options available for programmed fill slope analysis.

5C.7 The Center-of-Gravity Infinite Slope (C.G.I.S.) Approximation

In section 5A.6, it was demonstrated how the infinite slope equation relates to the fundamentals of the method-of-slices methods used in level III. This relationship suggests that the infinite slope equation can be used to estimate the factor of safety against failure, $F$, anticipated for a level III analysis. The following procedure (the C.G.I.S. approximation), documented by Prellwitz (1975), uses the infinite slope equation with the conditions that exist at the center of gravity of a potential failure mass (refer to figure 5C.9):

Step 1. Replace the upper boundaries of the failure mass with one chord line and locate the center of gravity of the mass. For a circular arc failure surface, the following equation can be used to determine the distance from the arc origin along the arc angle bisector to the center of gravity, $R_{cg}$:

\[ R_{cg} = \frac{2R \sin^3(\Delta/2)}{0.02618 \Delta - 1.5 \sin \Delta} \]  \hspace{1cm} (5C.3)

Where:

- $R_{cg}$ = Distance from the arc origin to the center of gravity
- $R$ = Radius of the arc
- $\Delta$ = Central arc angle, degrees.

Step 2. Graphically measure $\theta$, $d$, and $d_w$ on the vertical section through the center of gravity using the "effective" chord line used to replace the upper failure mass boundaries in step 1.

Step 3. Solve the infinite slope equation.
Before the age of programmable calculators and personal computers, this technique was most useful as a substitute for the time-consuming method-of-slices methods. It will be used here to demonstrate the correlation to the level III analysis. One of the drawbacks of the level II analysis is that it tells the designer nothing about the mode of failure. The next three sample problems demonstrate the analysis of potential failure surfaces to locate the critical or most likely failure surface. Sample problems 8 and 9 are for 1.00:1 and 2.00:1 cut slopes at location D of figure 5B.4. In each case, there is a "critical" arc location where arcs on either side have higher factors of safety. Notice that "flattening the slope" from 1.00:1 to 2.00:1 in an attempt to stabilize it increased the factor of safety only slightly, but increased the size of the failure mass which might be mobilized. This is not uncommon when seasonal ground water is present. Subsurface drainage systems (as described in section 6D) would be more successful than flattening the slope. Problem 10 is a through-fill at the same location and demonstrates that the potential of a rotational failure progressing into a translational debris avalanche is great for a fill located on a steep slope with ground water in the subsoil.

In using the C.G.I.S. approximation and making comparisons to the results of level III analyses, the user must remember that the infinite slope equation is a special case of the ordinary method of slices (as was shown in the derivation in section 5A.6) and that the results of the ordinary method of slices analysis are usually conservative compared to the simplified Bishop's or Janbu's results. The user must then expect conservatism (up to 15 percent) in the results of the C.G.I.S. approximation (as will be demonstrated in section 5C.9 for these sample problems). However, this conservatism is a useful attribute for an approximate analysis which does not have the "search for critical failure surface" capabilities that computer programs do.
Problem 8.

Using the C.G.I.S. approximation, estimate where the critical failure surface might be for a full-bench 1.00:1 cut slope at location D of figure 5B.4 as shown in the sketch. Use the parameters from problem 1.

<table>
<thead>
<tr>
<th>ARC</th>
<th>d</th>
<th>d_w</th>
<th>θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.3</td>
<td>0.0</td>
<td>43°</td>
</tr>
<tr>
<td>2</td>
<td>3.7</td>
<td>0.7</td>
<td>38.5°</td>
</tr>
<tr>
<td>3</td>
<td>4.4</td>
<td>1.3</td>
<td>29°</td>
</tr>
<tr>
<td>4</td>
<td>14.0</td>
<td>8.0</td>
<td>20°</td>
</tr>
</tbody>
</table>
**ARC No. 1**

$$F.S. = \frac{C' + \frac{\phi_0}{\pi} C \cos \theta \tan \phi'}{\gamma d \cos \theta \sin \theta}$$

$$= \frac{110 + 120 \times 3.3 \times \cos^2 (43^\circ) \times \tan 30^\circ}{120 \times 3.3 \times \cos 43^\circ \times \sin 43^\circ}$$

$$F.S. = \frac{110 + 122.29}{197.52} = 0.56$$

*Using $C' = 50$ psf:*

$$F.S. = \frac{50 + 122.29}{197.52} = 0.87$$

**ARC No. 2**

$$F.S. = \frac{C' + \frac{(\phi_d + (\phi_f - \phi) \sigma)}{\sigma d \cos \theta \sin \theta}}{\frac{(\phi_d + (\phi_f - \phi) \sigma)}{\sigma d \cos \theta \sin \theta}}$$

$$= \frac{50 + (120 \times 6.1 + (130 - 120) \times 0.7}){10 \times 6.1 + (130 - 120) \times 0.7}) \times \cos 35.5^\circ \times \sin 35.5^\circ}$$

$$= \frac{50 + 266.07}{349.87} = 0.90$$

*Using $C' = 110$ psf:*

$$F.S. = \frac{110 + 266.07}{349.87} = 0.90$$

**ARC No. 3**

$$F.S. = \frac{50 + (120 \times 9.4 + (130 - 120) \times 3.3}){120 \times 9.4 + (130 - 120) \times 3.3}) \times \cos 28^\circ \times \sin 28^\circ}$$

$$F.S. = \frac{50 + 29.88}{481.26} = 1.00$$

**ARC No. 4**

$$F.S. = \frac{50 + (120 \times 14.0 + (130 - 120) \times 8.0}){120 \times 14.0 + (130 - 120) \times 8.0}) \times \cos 20^\circ \times \sin 20^\circ}$$

$$F.S. = \frac{50 + 642.77}{565.65} = 1.22$$

"Critical Failure Arc is near Arc No. 2."
Problem 9.

As predicted, the 1.00:1 cut failed the first year after construction as a result of seasonally high ground water. If the slope is reconstructed by "flattening" to 2.00:1, would that slope be stable with the same seasonally high ground water? Compare the shape and location of the "critical" failure surfaces and the amount of material mobilized for the two cut slope ratios.
ARC No. 2

$$F.S. = \frac{C' + \frac{1}{2} d_w \cos^2 \Theta \tan \phi'}{V_{oc} \frac{1}{2} d_w \cos \Theta \sin \Theta}$$

$$F.S. = \frac{50 + 6 \times 10 \times \cos^2 (26^o) \times \tan 30^o}{130 \times 10 \times \cos 26^o \times \sin 26^o}$$

$$F.S. = \frac{50 + 31.53}{51.22} = 1.59$$

ARC No. 3

$$F.S. = \frac{50 + 47.6 \times 3.4 \times \cos^2 (24.5^o) \times \tan 30^o}{130 \times 3.4 \times \cos 24.5^o \times \sin 24.5^o}$$

$$F.S. = \frac{50 + 109.88}{166.77} = 0.96$$

ARC No. 4

$$F.S. = \frac{C' + \left[7d + (\frac{1}{2} - x) \right] \cos^2 \Theta \tan \phi'}{\left[7d + (\frac{1}{2} - x) \right] \cos \Theta \sin \Theta}$$

$$F.S. = \frac{50 + \left[120 \times 1.3 + (67.6-120) \times 0.7 \times \cos^2 (23\degree) \times \tan 30\degree\right]}{\left[120 \times 1.3 + (130-120) \times 0.7 \times \cos 23\degree \times \sin 23\degree\right]}$$

$$F.S. = \frac{50 + 458.29}{514.49} = 0.98$$

ARC No. 5

$$F.S. = \frac{50 + \left[120 \times 19.6 + (67.6-120) \times 13.9 \times \cos^2 (20.5\degree) \times \tan 30\degree\right]}{\left[120 \times 19.6 + (130-120) \times 13.9 \times \cos 20.5\degree \times \sin 20.5\degree\right]}$$

$$F.S. = \frac{50 + 822.44}{817.12} = 1.07$$

"Critical" Failure Arc is probably between Arc No. 3 and Arc No. 4.
Problem 10.

Rather than the full-bench cut of problem 8 or 9, if a through-fill were constructed at location C, where would the location of the possible "critical" failure surface be? Note the "critical" location and the potential of this failure progressing into a translational debris avalanche. Based on these solutions, which would you expect to be most likely to be stable at this location under seasonally high ground water: a full-bench cut, a self-balance section, or a through-fill section?
Arc No. 1
\[
F.S. = \frac{C' + \frac{f'd}{2} \cos^2 \theta \tan \phi}{vd \cos \theta \sin \theta}
\]
\[
= \frac{110 + 120 \times 9.8 \times \cos^2(28^\circ) \times \tan 30^\circ}{120 \times 9.8 \times \cos 28^\circ \times \sin 28^\circ}
\]
\[
F.S. = \frac{110 + 529.32}{187.47} = 1.31
\]

Arc No. 2
\[
F.S. = \frac{g' + \frac{f'd}{2} d_x \cos^2 \theta \tan \phi'}{\left(f'd + (f_x - f) d_x \cos \theta \sin \theta \right)}
\]
\[
= \frac{50 + \frac{120 \times 15.6 + (67.6 - 120)^2 \times 7.8 \times \cos^2(23.5^\circ) \times \tan 30^\circ}{\left(120 \times 15.6 + (130 - 120) \times 7.8 \times \cos 23.5^\circ \times \sin 23.5^\circ
\]
\[
F.S. = \frac{50 + 710.50}{713.07} = 1.07
\]

Arc No. 3
\[
F.S. = \frac{50 + \frac{120 \times 17.8 + (67.6 - 120)^2 \times 11.7 \times \cos^2(22.5^\circ) \times \tan 30^\circ}{\left(120 \times 17.8 + (130 - 120) \times 11.7 \times \cos 22.5^\circ \times \sin 22.5^\circ
\]
\[
F.S. = \frac{50 + 750.47}{794.56} = 1.00
\]

Arc No. 4
\[
F.S. = \frac{50 + \frac{120 \times 19.7 + (67.6 - 120)^2 \times 13.2 \times \cos^2(22^\circ) \times \tan 30^\circ}{\left(120 \times 19.7 + (130 - 120) \times 13.2 \times \cos 22^\circ \times \sin 22^\circ
\]
\[
F.S. = \frac{50 + 830.02}{866.93} = 1.02
\]

"Critical" Failure Arc is probably between Arc 3 and Arc 4.
Because both the full-bench cut and through-fill have potential for failure, the least likely section to have a failure would be the self-balance section, which would minimize both the cut and fill height. However, the design selection must also consider such other factors as (1) significance to the road alignment, (2) significance to surface drainage, and (3) consequences of failure (which type of failure has the least impact on the watershed and is easiest to mitigate).

In this section, options to generate the road template sections shown in figure 5C.3 will be demonstrated. These problems will also be used for comparison of the results of the two level II options to the results of level III analysis and to the results of the C.G.I.S. approximation. The summary of these analyses (table 5C.1) shows the results of three computer programs that use the simplified Bishop's option: DTIS*BISHOP (1969), STABL3FW (1984), and XSTABL (1992).

Problem 11.

For the self-balance road template in the sketch:

a. Solve with the self-balance option of DSARA (with HP41 algorithms),

b. Solve with the self-balance option of SSCHFS,

c. Check with XSTABL (simplified Bishop option), and

d. Solve using the C.G.I.S. approximation option of SSIS.
a. **DSARA printout**

**NATURAL SITE DATA**

- ID: Station 134+50
- Soil depth: 19.00'
- Surface slope: 15.27°
- Friction angle: 32.00°
- Cohesion (CvCr): 46.00 psi
- Dry unit weight: 119.99 lbs
- Moisture content: 10.00%
- Specific gravity: 2.65
- Groundwater table: 0.50'
- Factor of Safety: 1.0

**SELF-BALANCED ROAD SECTION DATA**

- Road width: 16.00'
- Cut slope: 45.00°
- Ditch slope: 1.00°
- Cut depth: 1.00'
- Ditch depth: 1.00'
- Cut edge: 19.43'
- Fill slope: 14.84°
- Fill edge: 16.15'
- Total cut height: 14.71'
- Cut water height: 4.71'
- Cut rock height: 5.20'
- Fill water height: 16.30'
- Fill rock height: 16.30'
- Cut area: 93.92
- Fill area: 44.77

**MATERIAL DATA**

- Cut cohesion: 40.00 psi
- Cut dry unit weight: 119.00 lbs
- Cut moist content: 10.00%
- Rock dry unit wt: 115.00 lbs
- Rock moisture content: 10.00%
- Friction angle: 5.00

**CRITICAL WEIGHT ANALYSIS**

- Critical Height: 15.14 UNSTABLE
- Cut/lf height: 14.71 16.56
- Rock height: 1.29 -16.38 Data set 10
- Water Height: 4.71 -1.79 Phr. 5 deg
- Stability number: 42.74 14.94 (1.85)
- Seepage correction: 0.46 0.51 (1.33)

**ESTIMATED FACTOR OF SAFETY ANALYSIS**

- Estimated F.S.: 0.93 UNSTABLE
- (Phre): 13.73
- Stability F: 0.93

**ROAD DATA**

- Cut width: 1.00'
- Ditch: 1.00'
- Cut slope: 45.00°
- Ditch: 1.00°
- Road width: 16.00'

**SELF-BALANCE SECTION**

- Cut width: 16.00'
- Ditch: 1.00'
- Road width: 16.00'

**GROUNDWATER TABLE**

- Cut height: 14.71'
- Fill height: 16.30'
- Cut water height: 4.71'
- Fill water height: 16.30'
- Cut rock height: 5.20'
- Fill rock height: 16.30'
- Cut area: 93.92
- Fill area: 44.77

**CRITICAL WEIGHT ANALYSIS**

- Critical Height: 15.14 STABLE
- Cut/lf height: 14.71 16.56
- Rock height: 1.29 -16.38 Data set 10
- Water Height: 4.71 -1.79 Phr. 5 deg
- Stability number: 42.74 14.94 (1.85)
- Seepage correction: 0.46 0.51 (1.33)

**ESTIMATED FACTOR OF SAFETY ANALYSIS**

- Estimated F.S.: 0.93 STABLE
- (Phre): 13.73
- Stability F: 0.93

**GROUNDWATER TABLE**

- Cut height: 14.71'
- Fill height: 16.30'
- Cut water height: 4.71'
- Fill water height: 16.30'
- Cut rock height: 5.20'
- Fill rock height: 16.30'
- Cut area: 93.92
- Fill area: 44.77

b. **SSCHFS printouts**

**STA. 134+50**

**MTLS. DATA**

- I.S. = MTL - SOIL
- USCP=S-1.5,4.7,5.5
- Drv. den.: 20.0pcf
- Sat. den.: 16.0pcf
- Cut = ROCK
- Fill = SOIL
- Cut/lf: 1.00'
- Road width: 16.00'

**SITE DATA**

- d = 3.0'
- 1.1 = 1.1
- 5 = 5
- 1.1 = 1.1
- 1.1 = 1.1
- 1.1 = 1.1

**STABLE**

**c. XSTABL printouts**

**STA. 134+50, 1:1 CUT, WET, INTERCEPT**

3 most critical surfaces, MINIMUM BSHMP FOS = 0.949

**STA. 134+50, 1.5:1 FILL, WET, EXAG.**

3 most critical surfaces, MINIMUM BSHMP FOS = 0.933
d. C.G.I.S. approximation with SSIS

1.00:1 CUT SLOPE

<table>
<thead>
<tr>
<th>R</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>34°</td>
<td>26°</td>
<td>20°</td>
</tr>
<tr>
<td>B</td>
<td>27°</td>
<td>30°</td>
<td>45°</td>
</tr>
<tr>
<td>C</td>
<td>39°</td>
<td>33°</td>
<td>30°</td>
</tr>
<tr>
<td>D</td>
<td>5°</td>
<td>7°</td>
<td>8°</td>
</tr>
<tr>
<td>E</td>
<td>6°</td>
<td>3°</td>
<td>0°</td>
</tr>
</tbody>
</table>

Example:

For R = 50°, Graphically find:

\[ A = 54.5° + 80.0° - 45° = 88.2° \]

Then:

\[ R_y = \frac{2A}{A^2 - 3B^2} \]

Plot \( R_y \) on center bounce from arc center to find the center of gravity of arc (CG). Draw vertical line through CG, and measure: \( d_y = 38.0°, d_x = 80.0°, d_o = 94° \)

Safe Infinite Slope Equation:

\[ FS = \frac{0.75 \times 36.65 \times 2.0}{32.0 \times 38.0} \]

\[ FS = 0.87 \]

I.B.: 134+50 C.G.I.S.

C.G.I.S.

R = 38.80°

delta 56.50°

Rev 27.91°

TRIAL 1

MATERIALS DATA

usg:SP-50 r.s.f.: 2.63
dry den.: 190pcf
sat. den.: 130pcf

Corr.: 88.0 psi

Site Data:

tree 8.0 psi
cor: 6.0 psi
dr: 5.5 ft.
dr: 2.4 ft.

al anh 78.1° 32.0°

FS: unknown

FS: 0.87

I.B.: 134+50 C.G.40

C.G.I.S.

R = 48.80°

delta 61.60°

Rev 36.71°

TRIAL 1

MATERIALS DATA

usg:SP-50 r.s.f.: 2.63
dry den.: 190pcf
sat. den.: 130pcf

Corr.: 88.0 psi

Site Data:

tree 8.0 psi
cor: 6.0 psi
dr: 7.8 ft.
dr: 3.6 ft.

al anh 66.21° 33.5°

FS: unknown

FS: 0.90

I.B.: 134+50 C.G.50

C.G.I.S.

R = 58.80°

delta 60.50°

Rev 45.91°

TRIAL 1

MATERIALS DATA

usg:SP-50 r.s.f.: 2.63
dry den.: 190pcf
sat. den.: 130pcf

Corr.: 88.0 psi

Site Data:

tree 8.0 psi
cor: 6.0 psi
dr: 6.8 ft.
dr: 3.8 ft.

al anh 69.13° 31.6°

FS: unknown

FS: 0.92
**1.50:1 FILL SLOPE**

**TRIAL 1**
**MATERIALS DATA**
- use: 3SP-5H
- asr. = 2.43
dry. den. = 115pcf
- sat. den. = 130pcf

**SITE DATA**
- tree age = 8.0 yrs
- Cx = 46.8 psf
- de = 3.2 ft.
- d = 6.0 ft.
- manned
- slope = 57.7% 28.6 deg.

**TRIAL 1**
**MATERIALS DATA**
- use: 3SP-5H
- asr. = 2.43
dry. den. = 115pcf
- sat. den. = 130pcf

**SITE DATA**
- tree age = 8.0 yrs
- Cx = 46.8 psf
- de = 6.0 ft.
- d = 9.3 ft.
- manned
- slope = 51.0% 27.0 deg.

---

**C.C.I.S.**
- R = 73.81 ft.
delta = 38.56 ft.

**C.C.I.S.**
- R = 158.04 ft.
delta = 31.04 ft.

**C.C.I.S.**
- R = 225.04 ft.
delta = 33.64 ft.

---

**STATION 134+50**
Problem 12.

For the full-bench road template in the sketch:

a. Solve with the full-bench option of DSARA (with HP41 algorithms),

b. Solve with the full-bench option of SSCHFS,

c. Check with XSTABL (simplified Bishop's option), and

d. Solve using the C.G.I.S. approximation option of SSIS.
a. DSARA printout for the 1.00:1 cut in the sketch.

- NATURAL SITE DATA
  - Soil depth: 11.00 ft
  - Surface slope: 32.92°

- CUT SECTION DATA
  - Cut slope: 22.80°
  - Area of cut: 1.00:1
  - Water table: 1.00 ft
  - Moisture content: 0.00
  - Stability: 5.00
  - Factor of safety: 1.00

- CRITICAL STABILITY ANALYSIS
  - Critical height: 14.60
  - Cut/fill ratio: 33.33
  - Stabilization factor: 1.00
  - Estimated factor of safety: 1.00

b. SSCHFS printouts

- STA. 145.60
  - MTLS. DATA
    - 1.5 x 50 cut @ 50 ft
  - SITE DATA
    - 12.05 @ 40 ft
  - ROAD DATA
    - road width: 16.01 ft

- FALL-BOUCH SECTION
  - Cut: 112.3 sl
  - Fill: 112.3 sl
  - Unstable

- CST
  - CST: 11.4 21.4 8.9
  - Unstable

- STABLE
  - h h h

- UNSTABLE
  - lambda = 1.1
  - Final factor = 0.9

- STABLE
  - h h h
  - lambda = 0.9
c. XSTABL printout for the 1.00:1 cut (other cuts summarized in table 5C.1).
d. C.G.I.S. approximation with SSIS
Problem 13.

For the through-fill road template in the sketch:

a. Solve with the through-fill option of DSARA (with HP41 algorithms),

b. Check with the stability analysis of SSCHFS,

c. Check with XSTABL (simplified Bishop's option), and

d. Solve using the C.G.I.S. approximation option of SSIS.

a. DSARA printout for the 1.30:1 fill in the sketch

---

**NATIONAL HYDRAULIC**

**NATIONAL HYDRAULIC**

**NATIONAL HYDRAULIC**

---

**THROUGH-FILL ROAD SECTION DATA**

---

**THROUGH-FILL ROAD SECTION DATA**

---

**MATERIAL DATA**

---

**CRITICAL HEIGHT ANALYSIS**

---

**OPTIMIZED FACTOR OF SAFETY ANALYSIS**

---

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b. SSCHFS printouts

c. XSTABL printout for the 1.30:1 fill (other fills summarized in table 5C.1)
d. C.G.I.S. approximation with SSIS

1.00:1 AND 1.30:1 FILL SLOPES

<table>
<thead>
<tr>
<th>Fill</th>
<th>1.00:1</th>
<th>1.30:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>150.0'</td>
<td>300.0'</td>
</tr>
<tr>
<td>A</td>
<td>240.0'</td>
<td>300.0'</td>
</tr>
<tr>
<td>Hg</td>
<td>147.7'</td>
<td>147.7'</td>
</tr>
<tr>
<td>Hg2</td>
<td>370.0'</td>
<td>370.0'</td>
</tr>
<tr>
<td>A</td>
<td>50.0'</td>
<td>89.0'</td>
</tr>
<tr>
<td>dw</td>
<td>3.5'</td>
<td>14.7'</td>
</tr>
<tr>
<td>dw1</td>
<td>0.0'</td>
<td>0.7'</td>
</tr>
<tr>
<td>dw2</td>
<td>0.0'</td>
<td>0.0'</td>
</tr>
</tbody>
</table>

STATION 144+30
(1:1/1.3:1 FILL SLOPES)

TRIAL 1
MATERIALS DATA
uscs=SP-50, s.s. = 2.65
den. = 115pcf
tot. den. = 130pcf, ur = 12%
sat. den. = 130pcf, ur = 12%
Cv = 75.8 osf
phi = 38.0 deg.
SITE DATA
tree cut 8.0 osf
Cv = 75.0 osf
d = 8.5 ft.
durinnum
alpha = 37.0 deg.
FSnumnum

TRIAL 1
MATERIALS DATA
uscs=SP-50, s.s. = 2.65
den. = 115pcf
tot. den. = 130pcf, ur = 12%
sat. den. = 130pcf, ur = 12%
Cv = 75.8 osf
phi = 38.0 deg.
SITE DATA
tree cut 8.0 osf
Cv = 75.0 osf
d = 8.5 ft.
durinnum
alpha = 37.0 deg.
FSnumnum

TRIAL 1
MATERIALS DATA
uscs=SP-50, s.s. = 2.65
den. = 115pcf
tot. den. = 130pcf, ur = 12%
sat. den. = 130pcf, ur = 12%
Cv = 75.8 osf
phi = 38.0 deg.
SITE DATA
tree cut 8.0 osf
Cv = 75.0 osf
d = 8.5 ft.
durinnum
alpha = 37.0 deg.
FSnumnum

C.G.I.S.
R = 150.4ft.
delta = 29.5deg.
Rem = 147.7ft.

C.G.I.S.
R = 150.4ft.
delta = 29.5deg.
Rem = 147.7ft.

C.G.I.S.
R = 150.4ft.
delta = 29.5deg.
Rem = 147.7ft.

C.G.I.S.
R = 150.4ft.
delta = 29.5deg.
Rem = 147.7ft.

C.G.I.S.
R = 300.0ft.
delta = 31.0deg.
Rem = 292.4ft.
1:50:1 FILL SLOPE

<table>
<thead>
<tr>
<th>Fill</th>
<th>1/2:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>155.0' 30.0'</td>
</tr>
<tr>
<td>Δ</td>
<td>47.0' 33.0'</td>
</tr>
<tr>
<td>R_h</td>
<td>146' 308.8'</td>
</tr>
<tr>
<td>d_h</td>
<td>30.0' 29.5'</td>
</tr>
<tr>
<td>d</td>
<td>14.5' 15.0'</td>
</tr>
<tr>
<td>d_w</td>
<td>14.0' 15.0'</td>
</tr>
<tr>
<td>d_w</td>
<td>7.8' 7.9'</td>
</tr>
<tr>
<td>d_w</td>
<td>4.8' 4.9'</td>
</tr>
<tr>
<td>d_w</td>
<td>0.0' 0.0'</td>
</tr>
</tbody>
</table>

STATION 164+30
(1/2:1 FILL SLOPE)

TRIAL 1
MATERIALS DATA
uscs=SP-5H, r.s. = 2.65
dry den. = 115pcf
sat. den. = 93pcf
Crs = 75.0 psf
shin = 34.0 deg.
SITE DATA
tree ex = 2.0 p/s
Crs = 8.0 psf
d = 15.0 ft.
dem = 5.0 ft.

ALPHA = 57.2' x 20.0 deg.
FS = unknown

d_w = 15.0 ft., d_w = 14.9'
FS = 1.25

TRIAL 1
MATERIALS DATA
uscs=SP-5H, r.s. = 2.65
dry den. = 115pcf
sat. den. = 93pcf
Crs = 75.0 psf
shin = 34.0 deg.
SITE DATA
tree ex = 2.0 p/s
Crs = 8.0 psf
d = 15.0 ft.
dem = 5.0 ft.

ALPHA = 56.6' x 29.3 deg.
FS = unknown

d_w = 15.0 ft., d_w = 14.8'
FS = 1.25
Table 5C.1—Comparison of the results of solutions of sample problems 11 through 13.

<table>
<thead>
<tr>
<th>PROBLEM ID.:</th>
<th>LEVEL III ANALYSES</th>
<th>LEVEL II ANALYSES</th>
<th>LEVEL I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Station, Slope, Gndwrtr. Surf.</td>
<td><strong>DTIS+B</strong></td>
<td><strong>STABLJ</strong></td>
<td><strong>STABL</strong></td>
</tr>
<tr>
<td></td>
<td>Mtr</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>134+50</td>
<td>1:1 Cut</td>
<td>1</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>1.5:1 Fill</td>
<td>1</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1.16</td>
</tr>
<tr>
<td>145+60</td>
<td>0.5:1 Cut</td>
<td>1</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>0.75:1 Cut</td>
<td>1</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>1:1 Cut</td>
<td>1</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>1.25:1 Cut</td>
<td>1</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1.15</td>
</tr>
<tr>
<td>164+30</td>
<td>1:1 Fill</td>
<td>1</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.85</td>
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<tr>
<td></td>
<td></td>
<td>3</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>1.3:1 Fill</td>
<td>1</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>1.5:1 Fill</td>
<td>1</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>1.23</td>
</tr>
</tbody>
</table>

**Mean Percent Error** (Using DTIS+BISHOP F as Standard) = ±2 Standard Deviation = ±6 ±10 ±7

*From Linear Regression Correlation of "CH" to DTIS+BISHOP:
Est. F = 0.55 + 0.48 Hc/h (R² = 0.83). Est. F = 1.04 for Hc/h = 1.00.*
5D. Soil Slopes—Transition from Deterministic to Probabilistic Analyses

Rodney W. Prellwitz, Geotechnical Engineer, Intermountain Research Station

5D.1 Introduction

Thus far, the sample problems in section 5 have illustrated deterministic analysis where single parametric values have been used in the solution. It is important to recognize that in nature everything varies to some degree, and in addition to this natural variability, there is uncertainty—the inability of the analyzer to measure values precisely for certain parameters. Sections 5A.12 and 5A.13 discuss how experienced deterministic and probabilistic analyzers treat variability and uncertainty differently. The deterministic analyzer must account for both of these factors in the selection of representative single parametric values. Values about which there is some uncertainty are likely to tend toward the conservative side of a possible range of values rather than toward the mean value of the range in the deterministic analysis. The probabilistic analyzer can use a range of values for a particular parameter selected according to whatever areal distribution is a reasonable match to what actually exists in nature, and the uncertainty in the determination must be taken into account in the selection of that representative range. In probabilistic analyses, such as the Monte Carlo simulation used in the computer programs LISA (Level I Stability Analysis ver. 2.0 1991) and SARA (Stability Analysis for Road Access), deterministic equations are used with many representative combinations of parametric values selected according to the probabilistic distributions defined by the user. Sample sizes of 500 to 1,000 or more are common in probabilistic analysis to reduce the amount of variation in results due to individual samples. One of the results reported is the "probability of failure"—the ratio of the number of solutions in a sampling indicating failure (factor of safety, $F$, less than or equal to one) to the total number of samples.

5D.2 Levels I and II Sensitivity Analyses

As a transition from single-value deterministic analysis to multi-value probabilistic analysis, this section will deal with sensitivity analyses for the level I and II deterministic algorithms presented in sections 5B and 5C. This differs somewhat from the probabilistic techniques of LISA and SARA in that only the range (minimum and maximum) and median (midpoint of the range) of the possible values are used for the parameters, not the probabilistic areal distributions. What is tested in a sensitivity analysis is how sensitive the analysis algorithms are to changes over the anticipated range of a value if all of the other parametric values remain at the median value. This will be illustrated with sample problems using level I and II algorithms and the parametric values for map unit 45 (table 5D.1). The mapping unit boundaries are defined by geomorphic feature, geologic origin, parent rock type, soil type, and parametric value criteria.
Table 5D.1. Parametric values for map unit 45

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parametric Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Range</td>
</tr>
<tr>
<td>$\gamma_d$ (pcf)</td>
<td>90 to 110</td>
</tr>
<tr>
<td>$w$ (%)</td>
<td>6 to 18</td>
</tr>
<tr>
<td>$C_v$ (psf)</td>
<td>0 to 60</td>
</tr>
<tr>
<td>$\phi$ (degrees)</td>
<td>28 to 34</td>
</tr>
<tr>
<td>$q_o$ (psf)</td>
<td>0 to 80</td>
</tr>
<tr>
<td>$C_r$ (psf)</td>
<td>20 to 80</td>
</tr>
<tr>
<td>$d$ (ft)</td>
<td>6 to 18</td>
</tr>
<tr>
<td>$d_o/d$ (ratio)</td>
<td>0.00 to 1.00</td>
</tr>
<tr>
<td>$\alpha$ (%)</td>
<td>30 to 60</td>
</tr>
</tbody>
</table>

The sensitivity analysis is a determination of the percent change in the results of the analysis (in the case of level I and II algorithms, the percent change in the factor of safety, $F$) in relation to the corresponding percent change in one of the input variables while all of the other variables remain constant at their median value. In the analysis, it is important to consider only the amount of variability that a parameter is expected to have (the anticipated range). This is because, even if the analysis is very sensitive to change in a specific variable, if that variable is nearly constant in nature, other variables may be more significant at a particular location. For example, in all of the following sample problems, the sensitivity graphs show that the analyses are very sensitive to the angle of internal friction of the soil, $\phi$, but for only a short range of possible change in output. For this type of soil, the practitioner skilled in measuring or predicting $\phi$ would place a greater effort on the determination of more significant parametric values.

Results of sensitivity analyses are often plotted as percent change in the overall variable of interest (here, factor of safety, $F$) against percent change in one of the input variables, $X$. These are calculated as follows:

$$\Delta X = \frac{x_i - \overline{x}}{\overline{x}} \times 100 \%$$

(5D.1)

$$\Delta F = \frac{F[x_i] - F[\overline{x}]}{F[\overline{x}]} \times 100 \%$$

(5D.2)

where

$\Delta X$ = percentage change in parameter $X$

$x_i$ = value for parameter $X$
5D.3 Level I and II Sensitivity Sample Problems

Figure 5D.1 illustrates the median conditions used in all of the sample problems in this section.

a. Natural Slope

b. Self-Balance Template

c. Full-Bench Cut Template

d. Through-Fill Template

Figure 5D.1.—Median conditions used in the sensitivity analyses.

\[ \bar{x} = \text{median value for parameter } X \]
\[ \Delta F = \text{corresponding percentage change in factor of safety} \]
\[ F[x_i] = \text{factor of safety using } x_i \]
\[ F[\bar{x}] = \text{factor of safety using } \bar{x} \]
Problem 1.

Make a sensitivity analysis of the natural slope conditions of map unit 45 using DLISA. In DLISA, the user may specify a range of values for one variable and solve for another as a function of the first while holding all the others constant. This option is available for factor of safety as a function of all of the variables and for certain other combinations, such as soil friction angle as a function of soil cohesion or soil depth as a function of ground water height. This is a powerful feature for back-calculation analysis as well as for sensitivity analysis.

DLISA Analysis of the Median Conditions

<table>
<thead>
<tr>
<th>Trial</th>
<th>Soil depth (ft)</th>
<th>12.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface slope (%)</td>
<td>45.00</td>
<td></td>
</tr>
<tr>
<td>Rock cohesion (psi)</td>
<td>40.00</td>
<td></td>
</tr>
<tr>
<td>Groundwater height (ft)</td>
<td>6.00</td>
<td></td>
</tr>
<tr>
<td>Friction angle (deg)</td>
<td>31.00</td>
<td></td>
</tr>
<tr>
<td>Soil cohesion (psi)</td>
<td>20.00</td>
<td></td>
</tr>
<tr>
<td>Dry unit weight (pcf)</td>
<td>100.00</td>
<td></td>
</tr>
<tr>
<td>Moisture content (%)</td>
<td>12.00</td>
<td></td>
</tr>
</tbody>
</table>

Determistic Level 1 Stability Analysis

Infinite Slope Equation

SOLVE FOR Factor of safety

Dry unit wt. (pcf) 100.00 12.00 174.66 24.66 12.00

Sample Output: F as a function of C

Plot of the sensitivity analysis using results from DLISA:
Note that in defining the sensitivity of the analysis to changes in soil depth, \( d \), it is important to define whether the analysis is being made with constant \( d \), or constant \( d/d \) ratio. The sensitivity of the analysis to the soil dry density, \( \gamma_d \), to moisture content, \( w \), or to surcharge, \( q_o \), was insignificant and not plotted. In contrast, in sample problems 2 through 5, it is shown that the level II analyses are sensitive to the anticipated changes in \( \gamma_d \) and \( w \). Surcharge, \( q_o \), is not considered in the level II analyses (\( q_o = 0 \)).

**Problem 2.**

Make a sensitivity analysis of the natural slope conditions of map unit 45 using DSARA. Because DSARA does a level I (natural slope) analysis of the natural slope conditions prior to a level II analysis of the constructed slope(s), it can also be used for a sensitivity analysis for level I. It is not as convenient for this as DLISA is, however, since it does not allow the incremental analysis. Also, DSARA does not consider surcharge (\( q_o \)).

For map unit 45 conditions, comparison of the analyses with the median conditions using DLISA and DSARA shows little difference—\( F = 1.14 \) with \( q_o = 40 \) psf and \( F = 1.13 \) without \( q_o \).
Problem 3.

Make a sensitivity analysis for a self-balance road template superimposed on map unit 45 site conditions using conditions shown in figure 5D.1, cut slopes ranging from 0.50:1 to 1.25:1, and fill slopes ranging from 1.00:1 to 1.50:1.

*Level II sensitivity analysis, map unit 45, self-balance template*

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>SENSITIVITY (% CHANGE IN F)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NATURAL</td>
</tr>
<tr>
<td></td>
<td>F, ΔF, %</td>
</tr>
<tr>
<td>ft.</td>
<td>rto.</td>
</tr>
<tr>
<td>20.0</td>
<td>28.0</td>
</tr>
<tr>
<td>1.02</td>
<td>-0.7</td>
</tr>
<tr>
<td>0.10</td>
<td>-2.7</td>
</tr>
<tr>
<td>1.25</td>
<td>10.6</td>
</tr>
<tr>
<td>1.19</td>
<td>5.3</td>
</tr>
<tr>
<td>0.81</td>
<td>-28.3</td>
</tr>
<tr>
<td>1.49</td>
<td>31.9</td>
</tr>
<tr>
<td>1.08</td>
<td>-4.4</td>
</tr>
<tr>
<td>1.02</td>
<td>-9.7</td>
</tr>
<tr>
<td>1.08</td>
<td>-4.4</td>
</tr>
<tr>
<td>1.19</td>
<td>5.3</td>
</tr>
<tr>
<td>0.97</td>
<td>-14.2</td>
</tr>
<tr>
<td>0.81</td>
<td>-28.3</td>
</tr>
<tr>
<td>1.25</td>
<td>10.6</td>
</tr>
<tr>
<td>1.19</td>
<td>5.3</td>
</tr>
<tr>
<td>0.97</td>
<td>-14.2</td>
</tr>
<tr>
<td>0.81</td>
<td>-28.3</td>
</tr>
<tr>
<td>1.25</td>
<td>10.6</td>
</tr>
</tbody>
</table>

The plot is shown in figure 5D.2.
Figure 5D.2.—Level II sensitivity analysis of map unit 45 using DSARA from problems 2, 3, 4, and 5.
Problem 4.

Make a sensitivity analysis for a full-bench cut superimposed on the natural site conditions of map unit 45 using the conditions shown on figure 5D.1 and cut slopes ranging from 0.50:1 to 1.25:1.

*Level II sensitivity analysis, map unit 45, full-bench cut template*

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>SENSITIVITY (% CHANGE IN E)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NATURAL</td>
</tr>
<tr>
<td></td>
<td>AF. %</td>
</tr>
<tr>
<td>12.0</td>
<td>1.13</td>
</tr>
<tr>
<td>6.0</td>
<td>1.28</td>
</tr>
<tr>
<td>9.0</td>
<td>1.18</td>
</tr>
<tr>
<td>15.0</td>
<td>1.10</td>
</tr>
<tr>
<td>18.0</td>
<td>1.08</td>
</tr>
<tr>
<td>0.00</td>
<td>1.49</td>
</tr>
<tr>
<td>0.25</td>
<td>1.31</td>
</tr>
<tr>
<td>0.75</td>
<td>0.97</td>
</tr>
<tr>
<td>1.00</td>
<td>0.81</td>
</tr>
<tr>
<td>20.0</td>
<td>1.02</td>
</tr>
<tr>
<td>50.0</td>
<td>1.08</td>
</tr>
<tr>
<td>110.</td>
<td>1.19</td>
</tr>
<tr>
<td>140.</td>
<td>1.25</td>
</tr>
<tr>
<td>28.0</td>
<td>1.02</td>
</tr>
<tr>
<td>30.0</td>
<td>1.10</td>
</tr>
<tr>
<td>32.0</td>
<td>1.17</td>
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<td>34.0</td>
<td>1.25</td>
</tr>
<tr>
<td>90.0</td>
<td>1.12</td>
</tr>
<tr>
<td>110.</td>
<td>1.15</td>
</tr>
<tr>
<td>140.</td>
<td>1.13</td>
</tr>
<tr>
<td>6.0</td>
<td>1.14</td>
</tr>
<tr>
<td>18.0</td>
<td>1.13</td>
</tr>
<tr>
<td>30.0</td>
<td>1.35</td>
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<tr>
<td>37.5</td>
<td>1.25</td>
</tr>
<tr>
<td>52.5</td>
<td>0.98</td>
</tr>
<tr>
<td>60.0</td>
<td>0.87</td>
</tr>
<tr>
<td>0.50:1</td>
<td>0.70</td>
</tr>
<tr>
<td>0.75:1</td>
<td>0.79</td>
</tr>
<tr>
<td>1.00:1</td>
<td>0.88</td>
</tr>
<tr>
<td>1.25:1</td>
<td>0.96</td>
</tr>
</tbody>
</table>

The sensitivity plot is given in figure 5D.2.
Problem 5.

Make a sensitivity analysis for a through-fill road section superimposed on the natural slope conditions of map unit 45 using the conditions shown on figure 5D.1 and fill slopes ranging from 1.00:1 to 1.50:1.

Level II sensitivity analysis, map unit 45, through-fill template

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>SENSITIVITY (% CHANGE IN F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ft. rto.</td>
<td>12.0 0.50 80.0 31.0 100 12 45.0</td>
</tr>
<tr>
<td>$d_0$/d</td>
<td>6.0</td>
</tr>
<tr>
<td>9.0</td>
<td>1.18 4.4 1.03 2.0</td>
</tr>
<tr>
<td>15.0</td>
<td>1.10 -2.7 1.02 1.0</td>
</tr>
<tr>
<td>18.0</td>
<td>1.08 -4.4 1.02 1.0</td>
</tr>
<tr>
<td>0.00</td>
<td>1.49 31.9 1.02 1.0</td>
</tr>
<tr>
<td>0.25</td>
<td>1.31 15.9 1.02 1.0</td>
</tr>
<tr>
<td>0.75</td>
<td>0.97 -14.2 0.86 -14.9</td>
</tr>
<tr>
<td>1.00</td>
<td>0.81 -28.3 0.72 -28.7</td>
</tr>
<tr>
<td>20.0</td>
<td>1.02 -9.7 0.80 -20.8</td>
</tr>
<tr>
<td>50.0</td>
<td>1.06 -4.4 0.91 -9.9</td>
</tr>
<tr>
<td>110.</td>
<td>1.19 5.3 1.10 8.9</td>
</tr>
<tr>
<td>140.</td>
<td>1.25 10.6 1.18 16.8</td>
</tr>
<tr>
<td>28.0</td>
<td>1.02 -9.7 0.91 -9.9</td>
</tr>
<tr>
<td>30.0</td>
<td>1.10 -2.7 1.02 1.0</td>
</tr>
<tr>
<td>32.0</td>
<td>1.17 3.5 1.04 3.0</td>
</tr>
<tr>
<td>34.0</td>
<td>1.25 10.6 1.12 10.9</td>
</tr>
<tr>
<td>60.</td>
<td>1.12 -0.9 1.04 3.0</td>
</tr>
<tr>
<td>110.</td>
<td>1.15 1.8 0.99 2.0</td>
</tr>
<tr>
<td>6</td>
<td>1.13 0.0 1.03 2.0</td>
</tr>
<tr>
<td>18.</td>
<td>1.14 0.9 1.00 -1.0</td>
</tr>
<tr>
<td>30.0</td>
<td>1.68 48.7 1.29 27.7</td>
</tr>
<tr>
<td>37.5</td>
<td>1.35 19.5 1.13 11.9</td>
</tr>
<tr>
<td>52.5</td>
<td>0.98 -13.3 0.73 -27.7</td>
</tr>
<tr>
<td>60.0</td>
<td>0.87 -23.0 0.53 -47.5</td>
</tr>
<tr>
<td>1.00:1</td>
<td>0.96 -5.0</td>
</tr>
<tr>
<td>1.30:1</td>
<td>1.07 5.9</td>
</tr>
<tr>
<td>1.50:1</td>
<td>1.12 10.9</td>
</tr>
</tbody>
</table>

The plot of the sensitivity analysis is given in figure 5D.2.
Comparison of the level I sensitivity analysis results of problem 1 to the level II results in figure 5D.2 shows some interesting features. It is clear that some of the variables (such as \( \phi, C, \) and \( \alpha \)) affect each of the slopes in somewhat the same manner but with differences in degree. Other variables (such as \( d \) and \( d' \)) affect each of the slopes in drastically different ways. Variables which affect the weight (such as soil dry unit weight, \( \gamma_d \), soil moisture, \( w \), and tree surcharge, \( q_o \)) may have some effect on the stability analysis, but can largely be ignored in the range that they normally occur. Some important, perhaps obvious, conclusions should be stated:

- Each set of natural slope conditions is unique in the way it is likely to affect stability conditions of not only the natural slopes, but of constructed slopes superimposed on them. (Sensitivity plots are likely to differ depending on the site conditions.)

- It is quite likely that a constructed slope can be more sensitive to changes in natural conditions than is the natural slope on which it is superimposed, and the constructed slope may become unstable first.

- Not all constructed slopes react the same way to changes in the natural slope conditions.

- A cut or fill slope in a self-balance section is less sensitive to natural slope changes than a cut or fill constructed to the same ratio in a full-bench or through-fill section, respectively, due to the reduced cut or fill height required for a given width of road.

- For a given width of road, the stability of a cut or fill slope is relatively insensitive to slope ratio within a reasonable range because as the slope steepens, the corresponding required slope height decreases, which helps balance the stability. This is especially true for the self-balance section. This would not be true if unreasonably steep (such as vertical) constructed slopes were used. Such other considerations as ease in revegetation, roadway obstruction and safety must also be considered in selection of slope ratio.

- Strength parameters are frequently easier variables for the experienced practitioner to evaluate for stability analyses than are subsurface variables such as soil depth and ground water depth.

Some of these observations will be pursued in greater depth in the next section, which will use the same map unit 45 conditions in a comparison of the probabilistic characteristics of level I and level II analyses.

[Editor’s note: At press time, the probabilistic level II program was not completed, so a comparison is not yet available.]
5E. Soil Slopes—Levels I and II
Probabilistic Analyses

Rodney W. Prellwitz, Geotechnical Engineer, Intermountain Research Station

In this section we intended to illustrate the use of probabilistic level I and II analyses. However, at the time of printing, the algorithms for the probabilistic level II program, SARA, were not completed; therefore, a comparative illustration of probabilistic level I and II analyses was not possible. Much of the pertinent information on probabilistic analysis at these levels is or will be covered in the LISA and SARA manuals. Intended to be covered here was a supplement to those references to demonstrate the application of these programs. When defining the areal distribution of parametric values for probabilistic analysis, it is important that the distribution is relative to the area under consideration. For example, in section 5D the sensitivity sample problems were developed around map unit 45, which had some “broad brush” ranges for the parametric values. In this section, the intent was to define probabilistic distributions for parametric values as they are anticipated to vary over the entire map unit 45 and for subunits within the overall unit. The subunit distributions are expected to fall within the overall range but to be more representative of specific subunit characteristics. In this manner, the stability of draws, ridges, and other unique geomorphic features used in zonation of the area can be considered.

Probabilistic analyses for the natural slopes using LISA were intended to be run for both the overall unit and for the subunits. This can be visualized as conducting a probabilistic analysis for the natural slopes first with a level I data base and then with more refined data from a level II reconnaissance. It was anticipated that the probability of landslide occurrence in the natural slopes would be greater for some subunits than for others, and the overall probability of landslide occurrence on the entire map unit 45 would be lower than for some of the more sensitive subunits. This technique is very useful for correlation to landslide inventories and for doing a level II analysis to evaluate management impacts on natural slope stability of specific locations within the unit.

A probabilistic analysis of constructed slopes using SARA was then intended. All three of the standard road templates were to be analyzed for the overall unit characteristics. For the subunits, only the most likely templates were to be analyzed—through-fill sections in the draws, full-bench sections at steeply plunging ridges, and self-balance sections for average slopes between the ridges and draws.

Finally, a comparison of the probability of landslide occurrence on natural slopes and of the most likely constructed slopes was intended. It was anticipated that the results would better illustrate some of the observations made of the comparison of the sensitivity analyses in section 5D.4.
5F. Soil Slopes—Level III
Stability Analysis

Rodney W. Prellwitz, Geotechnical Engineer, Intermountain Research Station

5F.1 Introduction

A level III stability analysis is used for analyzing the stability of a specific site and to support the selection of a stabilization measure for that site. Various stabilization alternatives can be contrasted by this method by comparing the relative increase in stability over the unstabilized case. This comparison, in conjunction with the relative costs and risks of the alternatives, is the basis for the decision of what, if anything, to do to stabilize the site. This technique will be covered thoroughly in section 6I. The method of analysis is by method of slices. These analysis methods are more complicated than those used in levels I and II because more detail must be considered for the proper evaluation of the stabilization measures. Section 5A includes the derivation and comparison of the ordinary (Fellenius) method of slices, the simplified Bishop's method of slices, and the simplified Janbu’s method of slices. In this section, the emphasis will be on manual computations by these three methods. The ordinary method of slices is used only to provide an initial factor of safety estimate for the other two methods. XSTABL (1992) is recommended for general use at this level. This program uses the simplified Bishop’s and Janbu’s methods. The sample problem hand-calculated in this subsection is correlated to the solution using this program. The user is referred to the excellent user’s guide (Sharma, 1990) and companion technical manual (Sharma, 1992) for XSTABL for detailed background information.

A version of XSTABL that will allow the user to do detailed interslice-force analysis using either Spencer’s method of slices (Spencer, 1967) or Janbu’s rigorous methods of slices (Janbu, 1973) is under development. This program will use an XSTABL data file of slice forces for a critical failure surface created through initial search by the simplified Bishop’s or simplified Janbu’s method.

5F.2 Reiteration for Hand-Calculation

Both the simplified Bishop’s and Janbu’s methods of slices require an iterative solution (i.e., start with an estimated factor of safety value, use it to calculate an approximate factor of safety, use the approximate factor of safety to calculate a new factor of safety, and so on until two successive calculated factors of safety are the same). The analysis converges rapidly and takes very little time on a computer, but it can be quite time consuming by hand-calculation for several trials. The number of trials required by either method is reduced in the manual computation procedure used in this section using an analysis patterned after the algorithm illustrated by Lambe and Whitman (1969) which assumes a linear relationship between the calculated and assumed values for only two trials and interpolates to estimate where the two values
are equal. This can be done graphically as illustrated in figure 5F.1 or by the following equation:

\[ Y_3 = \frac{(X_2 - X_1)Y_1 - (Y_2 - Y_1)X_1}{(X_2 - X_1) - (Y_2 - Y_1)} \]  

(5F.1)

where:

- \( Y_3 \) = \( F \) where calculated \( F \) = assumed \( F \)
- \( X_1, Y_1 \) = minimum assumed \( F \) and resulting calculated \( F \)
- \( X_2, Y_2 \) = maximum assumed \( F \) and resulting calculated \( F \)

Because the results of the ordinary method of slices (OMS) usually is conservative by 10 percent or so compared to the results of either of the other two methods, the method used in this section uses:

\[ X_1 = \text{Minimum assumed } F = \text{OMS } F \text{ (rounded to nearest 0.10)} \]

\[ X_2 = \text{Maximum assumed } F = X_1 + 0.20 \]

The results of this analysis are usually sufficiently accurate to yield a calculated \( F \) approximately equal to the assumed \( F \) rounded to the nearest 0.01. To check, a third trial may be used with the assumed \( F \) equal to \( F \) as estimated from the interpolation. Accuracy within 0.01 is sufficient for hand-calculation and usually matches or exceeds the accuracy of the parametric values.
Figures 5F.2 through 5F.4 are spreadsheet forms for hand-calculation by the three methods of slices. The algorithms are consistent with equations 5A.4, 5A.9, and 5A.11 as modified to include hydrostatic water force in a tension crack. They have been rearranged slightly for ease in hand-calculation by all three methods. For simplicity, at most three soils (two if the weight for one is to be calculated using either a moist or saturated unit weight) and 20 slices can be analyzed using these forms. Pore pressure is computed using a "phreatic" water surface as the spreadsheet computations are directed on the form. If a "piezometric" water surface is to be used, either the $\cos^2 \alpha_w$ term must be eliminated or $\alpha_w$ must be set to zero. Janbu’s $f_c$ coefficient is computed from equation 5A.10. The simplified Bishop’s and/or Janbu’s iteration to find calculated $F = \text{assumed } F$ is done graphically in accordance with figure 5F.1.
### Slope Stability - Manual Computation of Method(s) of Slices Analyses

<table>
<thead>
<tr>
<th>Slice No.</th>
<th>( \alpha ) (Degrees)</th>
<th>( \alpha_W ) (Degrees)</th>
<th>( d_W ) (ft)</th>
<th>( b ) (ft)</th>
<th>Pore Pressure</th>
<th>Slice Weight</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Tension Crack w/ Hydrostatic Pressure

1. \( Z_W = \_ \) ft.
2. \( \frac{1}{2} \alpha W \cdot z_W^2 = 3.1 \times \frac{w^2}{E} = 3.1 \times \frac{2}{E} = \_ \) \( p_{of} \)
3. \( \sigma = \_ \) \( p_{of} \)
4. \( \frac{1}{2} \alpha W \cdot z_W^2 (\sigma) = \_ \) \( p_{of} \)
5. \( H = \_ \) ft.

---

Figure SF.2.—Spreadsheet 1 of 3 for methods of slices hand calculations.
Figure 5F.3.—Sheet 2 of 3 for methods of slices hand calculations.
Figure 5F.4.—Sheet 3 of 3 for methods of slices hand calculations.
The critical circle as defined by XSTABL for the 1.5:1 fill slope of problem 5C.11.C will be used as a sample problem to demonstrate hand-calculation and to provide a correlation to the output file of XSTABL. The maximum segment length was increased to 10 feet to reduce the number of slices necessary for hand-calculation, and XSTABL was rerun for this critical circle. Figure 5F.5 is the input file and figure 5F.6 is the plot of the results using the simplified Bishop's method. Figure 5F.7 is a plot of the results using the simplified Janbu's method. Figure 5F.8 is a portion of the output file for the simplified Bishop's analysis which summarizes the individual slice data, the modified Bishop factor of safety, and the resisting and driving moments. Also at the bottom of figure 5F.8 is the portion of the output file for the simplified Janbu analysis which summarizes the corrected Janbu factor of safety, Janbu's $f_r$ factor, and the resisting shear strength and total driving shear force. The following will demonstrate how these data compare to similar data from hand-analysis. Figure 5F.9 is a plot of the slice geometry as defined on figure 5F.8 for use in scaling dimensions and slopes for hand-calculation. Figures 5F.10 through 5F.12 are the spreadsheet hand-calculations for this problem.

<table>
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<th>ft</th>
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</tr>
<tr>
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</tr>
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<td>SURB2</td>
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*Figure 5F.5.—XSTABL input file for the critical circle of problem 5C.11 using the simplified Bishop's method of slices.*
Figure 5F.6.—XSTABL plot for the critical circle of problem 5C.11 using the simplified Bishop's method of slices.

Figure 5F.7.—XSTABL plot of the critical circle of problem 5C.11 using the simplified Janbu's method of slices.
### Summary of a Portion of the Output File for the Simplified Bishop's Analysis

<table>
<thead>
<tr>
<th>Slice</th>
<th>x-base (ft)</th>
<th>y-base (ft)</th>
<th>height (ft)</th>
<th>width (ft)</th>
<th>alpha</th>
<th>beta</th>
<th>weight (lb)</th>
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<td>23.88</td>
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<td>4.760</td>
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<td>1927.9</td>
</tr>
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</table>

### Summary of a Portion of the Output File for the Simplified Janbu's Analysis

For the single specified surface,  
Modified BISHOP factor of safety = .961  
Resisting Moment = 966.95E+03 ft-lb  
Driving Moment = 100.63E+04 ft-lb

### Figure 5F.8—Portions of the XSTABL output files for the critical circle of problem 5C.11.
Figure 5F.9.—Slice geometry for the critical circle of problem 5C.11 as used to measure dimensions and slopes for hand-calculation.
Figure SF.10.—Spreadsheet 1 of hand-calculations for the critical circle of problem 5C.11.
Figure 5F.11.—Spreadsheet 2 of hand-calculations for the critical circle of problem 5C.11.
Figure SF.12.—Spreadsheet 3 of hand-calculations for the critical circle of problem SC.11.
The effective normal pressure acting on the base of the slice, sigma ($\sigma'$), reported by XSTABL (see figure 5F.8) was not defined in section 5A and does not appear on the hand-calculation spreadsheet. Sharma (1992) defined $\sigma'$ as:

$$
\sigma' = \frac{\Delta N'}{L} = \frac{b \Delta N'}{\cos \alpha} \tag{5F.2}
$$

$$
\Delta N' = \frac{1}{m_\alpha} \left( \Delta W - \Delta U_a \cos \alpha - \frac{\Delta C \sin \alpha}{F} \right) \tag{5F.3}
$$

$$
m_\alpha = \cos \alpha \left( 1 + \frac{\tan \alpha \tan \phi}{F} \right) \tag{5F.4}
$$

where $\Delta N'$ is the effective normal force (refer to the illustration and section 5A.8).

5F.5 Comparison of Results of Hand-Calculation to XSTABL

Table 5F.1 shows a comparison of the individual slice data from the hand-calculation Bishop analysis to the data as summarized on figure 5F.8 for the XSTABL analyses, demonstrating the utility of a computer program like XSTABL that allows the user this indepth comparison and a better understanding of the program operation.
Table 5F.1.—Comparison of hand-calculated analysis results to those of XSTABL for the individual slice data of figure 5F.8.

<table>
<thead>
<tr>
<th>Slice, Analysis</th>
<th>$b$ (ft.)</th>
<th>$\alpha$ (deg.)</th>
<th>$L$ (ft.)</th>
<th>$M_a$ * (ppf)</th>
<th>$\Delta W$ (UL ppf)</th>
<th>$\Delta C$ (CL ppf)</th>
<th>$\Delta N^*$ (UL ppf)</th>
<th>$\sigma^*$ (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Hand XSTABL</td>
<td>5.2</td>
<td>4.0</td>
<td>5.2</td>
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<tr>
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<td>1.7</td>
<td>1.115</td>
<td>693.0</td>
<td>68</td>
<td>47</td>
<td>322.0</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>53.83</td>
<td></td>
<td></td>
<td>684.6</td>
<td></td>
<td></td>
<td>318.3</td>
</tr>
<tr>
<td>8. Hand XSTABL</td>
<td>1.2</td>
<td>53.8</td>
<td>2.0</td>
<td>1.115</td>
<td>604.0</td>
<td>81</td>
<td>481</td>
<td>237.0</td>
</tr>
<tr>
<td></td>
<td>1.18</td>
<td>53.83</td>
<td></td>
<td></td>
<td>587.5</td>
<td></td>
<td></td>
<td>233.4</td>
</tr>
<tr>
<td>9. Hand XSTABL</td>
<td>2.2</td>
<td>53.8</td>
<td>3.7</td>
<td>1.157</td>
<td>429.0</td>
<td>153</td>
<td>0</td>
<td>70.0</td>
</tr>
<tr>
<td></td>
<td>2.24</td>
<td>53.83</td>
<td></td>
<td></td>
<td>445.4</td>
<td></td>
<td></td>
<td>71.8</td>
</tr>
</tbody>
</table>

* $M_a$ is given in equation 5F.4

$\Delta N^*$ is given in equation 5F.3

$\sigma^*$ is given in equation 5F.2
A similar table could have been developed for the individual slice data from the simplified Janbu's analysis; however, only factor "sigma" (\(\sigma'\)) is different between the XSTABL output files for individual slice data for the simplified Bishop's and Janbu's analyses. Sigma differs between the two analyses because \(m_\text{w}\) and \(\Delta N'\) used in the calculation are a function of \(F\) which differs between the two methods.

A comparison (hand-calculation results to those of XSTABL) of the factors of safety and the resisting and driving forces and/or moments for the two analysis methods shows:

**Simplified Bishop's Method of Slices**

**Factor of Safety**

By hand-calculation, \(F = 0.96\)

From XSTABL, \(F = 0.961\)

**Resisting Moment**

By hand, (figure 5F.12, interpolated for \(F = 0.96\))

- resisting force = 16,760 ppf
- circle radius = 57.59 ft
- resisting moment = 16,760 \times 57.59 = 965,000 fppf

From XSTABL (figure 5F.8), resisting moment = 966,950 fppf

**Driving Moment**

By hand, driving force (figure 5F.12) = 17,515 ppf

- driving moment = 17,515 \times 57.59 = 1,009,000 fppf

From XSTABL (figure 5F.8), driving moment = 1,006,300 fppf

**Simplified Janbu's Method of Slices**

**Factor of Safety**

By hand (figures 5F.11 and 5F.12), \(f_0 = 1.054\) and corrected \(F = 0.95\)

From XSTABL (figure 5F.8), \(f_0 = 1.054\) and corrected \(F = 0.946\)

**Driving Forces**

By hand (figure 5F.12 for uncorrected \(F = 0.90\)),

- driving forces = 19,868 ppf

From XSTABL (figure 5F.8), driving forces = 19,856 ppf
Resisting Forces

By hand (figure 5.12), resisting forces = 22,179 ppf

From XSTABL (figure 5F.8), resisting forces = 22,108 ppf

5F.6 Seismic Loading

Sharma (1992) gives an explanation and example of how seismic loading is incorporated into the XSTABL program by applying additional loads to the centroid of a slice in vertical and/or horizontal directions. These applied loads are a function of the slice weight (ΔW) and the seismic coefficients, $k_v$ and $k_h$:

Horizontal load: $\Delta H = k_h\Delta W$

Vertical load: $\Delta V = k_v\Delta W$

The Corps of Engineers (U.S. Department of the Navy, 1982) further defines the horizontal seismic force as the mass (ΔW/g) involved times the horizontal acceleration ($a_h$):

Horizontal load: $\Delta H = \Delta W/g \times a_h = k_h\Delta W$

The horizontal seismic coefficient is then equal to the horizontal acceleration divided by the acceleration due to gravity ($g = 32$ ft/sec²):

$k_h = a_h/g$

Figure 5F.13 is a general seismic zone map for the United States with the corresponding seismic coefficient per zone.

5F.7 Probabilistic Level III

A probabilistic level III analysis is needed to provide the "probability of landslide occurrence" necessary for a practical risk assessment. Currently, there are analysis techniques available which use method of slices analysis with normal distribution for soil shear strength parameters as the only probabilistic variables. Our experience with level I probabilistic analysis is that soil shear strength parameters in typical forest watersheds are the least variable of parameters in the analysis and are usually the easiest to determine. Parameters such as ground water depth and soil depth are much more variable and need to be given probabilistic distributions as well. The Monte Carlo simulation used for levels I and II probabilistic analysis was a good technique for allowing the user the latitude to allow variations in more than just the soil shear strength parameters. However, Monte Carlo simulation may be too cumbersome and time-consuming for level III with search routines, and such. If the failure surface is known or predetermined in deterministic analysis, Monte Carlo simulation may still be applicable to level III. Much research will be required to perfect these techniques.
Figure 5F.13.—Seismic zone map of the contiguous United States and Puerto Rico (adapted from U.S. Department of the Navy, 1982).
5G. Soil Slopes—Sample Problem Including All Three Levels of Analysis

Rodney W. Prellwitz, Geotechnical Engineer, Intermountain Research Station

5G.1 Sample Problem Background

This is a hypothetical field problem to illustrate typical stability analysis of conditions common to colluvial soils of granitic parent rock type. The decomposed granitic rock underlying the colluvium can be considered as residual soil because its shear strength and slope failure modes are more like soil than rock. However, the residual decomposed granitic parent material is usually much less permeable than the overlying colluvium. The unit weight of the colluvium also usually is much lower than that of the residual material. This frequently leads to unconfined aquifer development within the colluvium with the colluvium/residual contact acting as a drainage barrier. Natural slope and typical forest road slope failures are usually within the colluvium because the lower shear strength and the extent of the aquifer is within that soil unit. In this case, the colluvium/residual contact can be considered the lower limit of slope failure in the stability analysis.

5G.2 Typical Site Conditions

Natural ground surface slopes in colluvium/decomposed granite terrain can vary from 0 to 70 percent or more, but typical steep slopes are in the 55 to 70 percent range. Soil depths vary from less than 1 foot to over 20 feet but usually range from 4 to 12 feet. Steeper slopes usually have the shallower soils and less ground water concentration. This is consistent with the development of colluvial deposits. Ground water can occur on all slopes but can be expected to concentrate in topographic lows or concave-shaped troughs where the soils are usually deeper and the slopes are usually not as steep as the adjacent convex-shaped ridges or intermediate slopes. This is also consistent with the development of colluvial soil deposits which, like ground water, concentrate in these locations. Ground water in these shallow aquifers can be expected to respond rapidly to precipitation and to fluctuate rapidly with the seasons.

5G.3 Typical Soil Properties

Both the colluvial and residual soils are usually classified as SM (silty sand) by the Unified Soil Classification System. The sand fraction is usually poorly graded and can vary from fine to medium in predominant size. This is a function of the individual grain size of the parent rock material. Because the parent rock type is composed mostly of quartz and feldspar minerals, the typical specific gravity of the soil is controlled by these mineral types and is in the range of 2.6 to 2.7. The colluvium in a field sample will appear homogeneous and more oxidized compared to the “salt-and-pepper” texture of the residual material that has not undergone transportation.
The colluvium is loose because it has not undergone any over-consolidation during and after deposition. Typical standard penetration tests conducted above the phreatic surface and at a depth of 5 to 10 feet usually result in blow counts in the range of 4 to 20 blows per foot (bpf) with an average of about 8 to 10 bpf. Typical AASHTO T-99 laboratory test results are in the range of 105 to 115 pcf for the optimum dry density at optimum moisture content.

The loose nature of the colluvium also makes it relatively permeable. The base of the colluvium may be more permeable because it is often more open-graded and may even grade into an SP (poorly graded sand) at the colluvial/residual contact. This may be due partly to segregation during deposition, but is probably also due to piping of the fines adjacent to the drainage barrier. If the drainage barrier is saturated constantly, the reducing anaerobic environment can lead to the development of more plastic materials directly at the barrier. This appears as a gray, often slimy, zone at this location. This zone is usually not very extensive but should be identified during investigation because the shear strength of this material is less than that of the SM soil and must be taken into account in the analysis.

The soil particles are angular for both the colluvium and residual soil. This is a characteristic most useful in distinguishing a colluvial deposit from other types of deposits, such as fluvial or glacial, which may have entirely different shear strength characteristics. The significance of particle shape to slope stability is that the shear strength is generally higher for soils with angular particle shape, even in a looser state. If index properties and charts are used for typical SM soils in the range of their in-situ relative densities to estimate the effective angle of internal friction ($\phi'$) of the soil, the results will usually be less by a few degrees from the result of a direct shear or triaxial test conducted at the same unit weight. You can usually use these published index property correlations to estimate a value for $\phi'$ knowing that it is slightly on the conservative side of the probable range, and adjust it upward according to field experience and back-analysis.

Except for the anaerobic material previously described, the fines in the colluvium are usually nonplastic or only slightly plastic. What this means to slope stability is that soil cohesion ($C_s$) can be expected to be very low to nonexistent. For the analysis of slopes at most forest locations, however, a value for combined cohesive strength ($C$) is still appropriate to account for such cohesion-like strengths as root strength ($C_r$) and apparent cohesion ($C_a$) from capillary tension. The values of soil cohesion from laboratory tests do not account for these factors but still are usually excessively high. This is due to the nature of the typical laboratory test and to the linear interpolation of the Mohr-Coulomb failure envelope used to arrive at a value for $C_s$. Therefore, it is better to arrive at a safe value for $C$ from back-calculation analysis of known field conditions. Short steep cut slopes on spur roads or undercut by streambanks are good locations to use for back-analysis, especially if these have slumped back to a stable slope and ground water did not appear to be a contributing factor to the failure.
A new road is to be located for the Stumpfarm timber sale. The task is to use the information provided to evaluate the suitability of a standard road design template at the proposed location and to do a preliminary evaluation of the potential for failure of constructed slopes, the need for stabilization, and the feasibility of construction of a stabilization measure for a specific critical site (station 178+50) on the location. The task is divided into five separate problems:

- **Problem 1. Develop shear strength parametric values using index properties and charts from section 4 with back-analysis by level II methods.**

- **Problem 2. Evaluate the stability of a typical full-bench cut slope for the location using level II methods.**

- **Problem 3. Evaluate critical ground water depths at station 178+50 using level I methods.**

- **Problem 4. Evaluate an unstabilized self-balance road template constructed at station 178+50 using level II methods.**

- **Problem 5. Evaluate the probable mode of failure of constructed slopes at station 178+50 and the feasibility of stabilizing these slopes. Analyze using level III methods.**

### Problem 1. Develop Shear Strength Parameters.

Develop typical shear strength parameters for the colluvial soil at the Stumpfarm timber sale road location. The soil is of granitic parent rock type as described in section 5G.3. The grain size of the sand fraction is predominantly medium grained. The fines are nonplastic. Spur road cut slopes (originally cut at 0.50:1 or steeper) and streambanks have been observed to slump back to slopes in the range of about 55° (at a height of about 8 feet) to about 60° (at a height of about 14 feet). The natural slopes at the locations where the failure observations were made were in the range of 55 to 70 percent, and ground water did not appear to be a factor in the failures.

a. Estimate relative density, \( D_r \).

Using SPT Data:

\[
N = 8 \text{ to } 10 \quad @ \quad d = 5 \text{ to } 10 \text{ ft.} \quad (\text{Use } N=9 \text{ & } d=7.5 \text{ ft.})
\]

Moist unit weight? At least = 100 pcf

Effective overburden pressure with no watertable,

\[
\sigma_v' = (7.5 \text{ ft}) (100 \text{ pcf}) = 800 \text{ psf or } 0.40 \text{ tsf}
\]

Using \( N = 9 \):

\[
D_r = 30\% \text{ from table } 4B.5 \\
D_r = 30\% \text{ from figure } 4B.2
\]

Use \( D_r = 30\% \) (Loose)
b. Estimate γ₀, γ, γₛₐₜ, and φ' (using figure 4B.4)

From figure 4B.4 for coarse SM @ Dₜ = 30%, find:

γ₀ = 94 pcf, γ = 107 pcf @ w = 15%, γₛₐₜ = 121 pcf, φ' = 32°

Round unit weights:
γ₀ = 95 pcf, γ = 110 pcf, γₛₐₜ = 120 pcf

For angular colluvium, Increase φ' to 33°

c. Estimate C.

Using steep slope failure data with Fig. 5C.4 and back-calculating for C (φ' = 33°, γ = 110 pcf, and α = φ):

<table>
<thead>
<tr>
<th>β</th>
<th>N</th>
<th>Hₛ</th>
<th>C = Hₛγ</th>
</tr>
</thead>
<tbody>
<tr>
<td>55°</td>
<td>22.5</td>
<td>18 ft.</td>
<td>88 pcf</td>
</tr>
<tr>
<td>60°</td>
<td>17.2</td>
<td>14 ft.</td>
<td>90 pcf</td>
</tr>
</tbody>
</table>

Use C = 90 pcf.
**Problem 2. Evaluation of the Stability of Full-Bench Cut Slope**

a. Evaluate whether a 3/4:1 cut slope can be expected to be stable for a 16-foot subgrade width (no ditch) in a full-bench template for a natural slope of 60 percent and soil depths to 10 feet if ground water is not a factor.

The 1:1 cut slope would be more likely to be stable at deeper soil depth locations but probably would not be required in most cases. Recommend the 3/4:1 cut; if a few slumps occur in deeper soils, they should be shallow and ravel back to a stable slope.

b. Would this 3/4:1 full-bench cut be stable at locations where the natural slope is 70 percent if soil depths range from 5 to 10 feet and there are no ground water problems anticipated? Would a 1:1 cut slope be stable under these conditions? Which cut slope ratio would you recommend for full-bench design?
Figure 5G.1.—Stability number determination for level II analyses of problems 1 and 2 using figure 5C.4 (adapted with permission of the American Society of Civil Engineers from "Limit Analysis of Stability of Slopes" in Proceedings of the American Society of Civil Engineers, Journal of the Soil Mechanics and Foundation Division, 97 (SM1) by W.F. Chen and M.W. Giger, 1970).

The road location crosses a draw at station 178+50 where ground water is expected to concentrate. A self-balance road template is expected to be used at this location with a ditch and 16-foot subgrade width. Figure 5G.2 is a cross-section of this site with the proposed self-balance design template. Using level I analysis, estimate the critical height of the phreatic surface for the natural slope conditions.

For natural slope and soil depth conditions as shown on figure 5G.2:

<table>
<thead>
<tr>
<th>From X:</th>
<th>To X:</th>
<th>Average Natural Slope, $\alpha$ (%)</th>
<th>Average Soil Depth, $d$ (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>40.0</td>
<td>55</td>
<td>8</td>
</tr>
<tr>
<td>40.0</td>
<td>67.0</td>
<td>52</td>
<td>9</td>
</tr>
<tr>
<td>67.0</td>
<td>120.0</td>
<td>45</td>
<td>10</td>
</tr>
<tr>
<td>120.0</td>
<td>160.0</td>
<td>35</td>
<td>11</td>
</tr>
</tbody>
</table>

Use the level I HP41 program SSIS or PC program DLISA to find the critical ground water depth at each break in slope by setting $F = 1.00$ and solving for $d_w$:

Using SSIS:

**TRIAL 1**

<table>
<thead>
<tr>
<th>MATERIALS DATA</th>
<th>SITE DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>usc=SR s=0.1 t=2.65</td>
<td>tree x= 0.0 msl</td>
</tr>
<tr>
<td>atm den. = 106pcf, w = 15%</td>
<td>C= 98.8 msl</td>
</tr>
<tr>
<td>sat. den. = 122pcf, w = 29%</td>
<td>C= 98.8 msl</td>
</tr>
<tr>
<td>C= 98.8 msl</td>
<td>d= 9.8 ft.</td>
</tr>
<tr>
<td>alpha= 55.8 ft = 28.8 deg.</td>
<td>d= unknown</td>
</tr>
<tr>
<td>Find: Critical $d_w$ = 5.2 ft.</td>
<td>F5= 1.00</td>
</tr>
</tbody>
</table>

**TRIAL 2**

<table>
<thead>
<tr>
<th>MATERIALS DATA</th>
<th>SITE DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>usc=SR s=0.1 t=2.65</td>
<td>tree x= 0.0 msl</td>
</tr>
<tr>
<td>atm den. = 106pcf, w = 15%</td>
<td>C= 98.8 msl</td>
</tr>
<tr>
<td>sat. den. = 122pcf, w = 29%</td>
<td>C= 98.8 msl</td>
</tr>
<tr>
<td>C= 98.8 msl</td>
<td>d= 9.8 ft.</td>
</tr>
<tr>
<td>d= unknown</td>
<td>alpha = 52.1 ft = 27.5 deg.</td>
</tr>
<tr>
<td>F5= 1.00</td>
<td>d= 6.2 ft.</td>
</tr>
</tbody>
</table>

**TRIAL 3**

<table>
<thead>
<tr>
<th>MATERIALS DATA</th>
<th>SITE DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>usc=SR s=0.1 t=2.65</td>
<td>tree x= 0.0 msl</td>
</tr>
<tr>
<td>atm den. = 106pcf, w = 15%</td>
<td>C= 98.8 msl</td>
</tr>
<tr>
<td>sat. den. = 122pcf, w = 29%</td>
<td>C= 98.8 msl</td>
</tr>
<tr>
<td>C= 98.8 msl</td>
<td>d= 9.8 ft.</td>
</tr>
<tr>
<td>d= unknown</td>
<td>alpha = 45.4 ft = 24.2 deg.</td>
</tr>
<tr>
<td>F5= 1.00</td>
<td>d= 3.6 ft.</td>
</tr>
</tbody>
</table>

**TRIAL 4**

<table>
<thead>
<tr>
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<th>SITE DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>usc=SR s=0.1 t=2.65</td>
<td>tree x= 0.0 msl</td>
</tr>
<tr>
<td>atm den. = 106pcf, w = 15%</td>
<td>C= 98.8 msl</td>
</tr>
<tr>
<td>sat. den. = 122pcf, w = 29%</td>
<td>C= 98.8 msl</td>
</tr>
<tr>
<td>C= 98.8 msl</td>
<td>d= 9.8 ft.</td>
</tr>
<tr>
<td>d= unknown</td>
<td>alpha = 25.8 ft = 19.3 deg.</td>
</tr>
<tr>
<td>F5= 1.00</td>
<td>d= 12.5 ft.</td>
</tr>
</tbody>
</table>
Plot the critical ground water profile as the undrained phreatic surface on figure 5G.2:

<table>
<thead>
<tr>
<th>At X:</th>
<th>Critical $d_w$</th>
<th>Drainage Barrier $Y$</th>
<th>Undr. Phreatic Surface $Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>5.2</td>
<td>6.0</td>
<td>11.2</td>
</tr>
<tr>
<td>40.0</td>
<td>$(5.2 + 6.2)/2 = 5.7$</td>
<td>28.0</td>
<td>33.7</td>
</tr>
<tr>
<td>67.0</td>
<td>$(6.2 + 8.6)/2 = 7.4$</td>
<td>42.0</td>
<td>49.4</td>
</tr>
<tr>
<td>120.0</td>
<td>8.6</td>
<td>66.0</td>
<td>74.6</td>
</tr>
<tr>
<td>160.0</td>
<td>$d_w = d = 12.0$</td>
<td>80.0</td>
<td>92.0</td>
</tr>
</tbody>
</table>

Figure 5G.2.—Cross-section at station 178+50 showing data for and solutions from problems 2 and 3.

**Problem 4. Level II Analysis of Self-Balance Road Template at Station 178+50.**

Using either the HP41 program SSCHFS (1988) or the PC program DSARA, determine the dimensions for a self-balance road template with a 16-foot subgrade width, 1-foot-deep ditch on a 3.00:1 slope, 0.75:1 cut slope, and 1.30:1 fill slope. Using level II analysis, estimate the stability of the cut and fill slopes if the phreatic surface approaches the critical conditions as estimated in problem 3 and the stabilizing effect that drainage might have on that stability.
Conclusion: Both cut and fill slopes can be expected to be unstable without drainage, but can be stabilized even under extreme ground water conditions with proper drainage.
Problem 5. Level III Analysis of Self-Balance Road Template at Station 178+50.

Using XSTABL, verify the level II stability estimations for the self-balance cut and fill slopes. Determine whether the failures are likely to be rotational or translational. Determine the effects on the stability of the fill slope of not removing the organic surface material (duff) before fill placement.

a. Construct a drained phreatic surface at cut-slope intercept. Refer to section 4E.5 for the analysis of drained phreatic surfaces. Using the HP program GW with the intercept conditions of the cut slope with the undrained phreatic surface (X and Y measured from the origin at the toe of the cut slope):

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3.0</td>
<td>4.0</td>
</tr>
<tr>
<td>8.0</td>
<td>8.0</td>
</tr>
<tr>
<td>18.0</td>
<td>13.6</td>
</tr>
<tr>
<td>58.0</td>
<td>33.3</td>
</tr>
<tr>
<td>160.0</td>
<td>92.0</td>
</tr>
</tbody>
</table>

Plot as the cut slope intercept phreatic surface on figure 5G.2 and use for the level III undrained analysis of the cut slope.

b. Construct of phreatic surfaces for the fill-slope analysis. The phreatic surfaces for the following fill slope stability analyses were estimated to conform to the fill $h_w$ as predicted by DSARA.
c. XSTABL analysis. Figures 5G.3 through 5G.11 are the input files and plots for the analysis using XSTABL of the cut and fill slopes at station 178+50.

Figure 5G.3.—XSTABL input file and plot for the undrained circular failure analysis of the cut slope at station 178+50.
Figure 5G.4.—XSTABL input file and plot for the undrained translational failure analysis of the cut slope at station 178+50.
Figure 5G.5.—XSTABL input file and plot for the drained circular failure analysis of the cut slope at station 178+50.
Figure 5G.6.—XSTABL input file and plot of the drained translational failure analysis of the cut slope at station 178+50.

### PROFILO

<table>
<thead>
<tr>
<th>PROFIL</th>
<th>CUT SLOPE, DRAINED</th>
<th>FILE: STUMPFDT R-20-92 6-97 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>STUMPFARM ROAD</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>24.4</td>
<td>14.0</td>
<td>24.4</td>
</tr>
<tr>
<td>24.4</td>
<td>27.7</td>
<td>43.0</td>
</tr>
<tr>
<td>41.0</td>
<td>42.0</td>
<td>42.0</td>
</tr>
<tr>
<td>41.0</td>
<td>55.0</td>
<td>42.0</td>
</tr>
<tr>
<td>59.0</td>
<td>42.0</td>
<td>42.0</td>
</tr>
<tr>
<td>59.0</td>
<td>71.4</td>
<td>51.0</td>
</tr>
<tr>
<td>71.4</td>
<td>55.0</td>
<td>91.0</td>
</tr>
<tr>
<td>91.0</td>
<td>63.0</td>
<td>125.0</td>
</tr>
<tr>
<td>120.0</td>
<td>76.0</td>
<td>160.0</td>
</tr>
<tr>
<td>24.4</td>
<td>27.7</td>
<td>51.0</td>
</tr>
</tbody>
</table>

### SOIL

<table>
<thead>
<tr>
<th>SOIL</th>
<th>110.0</th>
<th>120.0</th>
<th>80.0</th>
<th>33.0</th>
<th>0.000</th>
<th>.0</th>
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<tbody>
<tr>
<td>WATER</td>
<td>62.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>11.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>33.7</td>
<td></td>
<td></td>
<td></td>
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<td>67.0</td>
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### LIMITS

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<td>67.0</td>
</tr>
<tr>
<td>70.0</td>
<td>43.4</td>
<td>120.0</td>
</tr>
</tbody>
</table>

3 most critical surfaces, MINIMUM JANBU POS = 1.219
Figure 5G.7.—XSTABL input file and plot of the undrained translational failure analysis of the fill slope at station 178+50 with the duff layer unremoved.
Figure 5G.8.—XSTABL input file and plot of the undrained shallow circular failure analysis of the fill slope at station 178+50.
Figure 5G.9.—XSTABL input file and plot of the undrained deep circular failure analysis of the fill slope at station 178+50.
<table>
<thead>
<tr>
<th>X-AXIS (feet)</th>
<th>Y-AXIS (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>40</td>
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<td>60</td>
<td>60</td>
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<td>80</td>
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<td>140</td>
<td>140</td>
</tr>
<tr>
<td>160</td>
<td>160</td>
</tr>
</tbody>
</table>

Figure 5G.10.—XSTABL input file and plot for the drained shallow circular failure analysis of the fill slope at station 178+50.
Figure 5G.11.—XSTABL input file and plot for the drained deep circular failure analysis of the fill slope at station 178+50.
d. Summarize level III analyses. The following is a summary of the XSTABL analyses as plotted on figures 5G.3 through 5G.11:

<table>
<thead>
<tr>
<th>Slope</th>
<th>Drainage</th>
<th>Failure Mode</th>
<th>Method of Analysis</th>
<th>Factor of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut</td>
<td>Undrained</td>
<td>Circular</td>
<td>Simplified Bishop’s</td>
<td>0.935</td>
</tr>
<tr>
<td>Cut</td>
<td>Undrained</td>
<td>Translational</td>
<td>Simplified Janbu’s</td>
<td>0.961</td>
</tr>
<tr>
<td>Cut</td>
<td>Drained</td>
<td>Circular</td>
<td>Simplified Bishop’s</td>
<td>1.203</td>
</tr>
<tr>
<td>Cut</td>
<td>Drained</td>
<td>Translational</td>
<td>Simplified Janbu’s</td>
<td>1.219</td>
</tr>
<tr>
<td>Fill</td>
<td>Drained</td>
<td>Translational</td>
<td>Simplified Janbu’s</td>
<td>0.917</td>
</tr>
<tr>
<td>Fill</td>
<td>Undrained</td>
<td>Shallow Circular</td>
<td>Simplified Bishop’s</td>
<td>1.114</td>
</tr>
<tr>
<td>Fill</td>
<td>Undrained</td>
<td>Deep Circular</td>
<td>Simplified Bishop’s</td>
<td>1.021</td>
</tr>
<tr>
<td>Fill</td>
<td>Drained</td>
<td>Shallow Circular</td>
<td>Simplified Bishop’s</td>
<td>1.410</td>
</tr>
<tr>
<td>Fill</td>
<td>Drained</td>
<td>Deep Circular</td>
<td>Simplified Bishop’s</td>
<td>1.170</td>
</tr>
</tbody>
</table>

e. Conclusions from level III analysis. As predicted in level II and shown on figure 5G.3, a rotational cut slope failure is quite likely under critical high ground water conditions. Compare figures 5G.3 and 5G.4: even though the rotational failure is most likely, there still is potential for a translational failure. If the 0.75:1 cut slope were constructed without drainage protection, it would fail as illustrated in figure 5G.3, and the rotational slide mass would probably buttress the slope above and prevent the progression into a larger translational failure. If this were not recognized as a potential problem, removal of the rotational failure mass and reconstruction of the cut slope to a flatter slope could cause a much larger translational failure (see problem 5C.8 and 5C.9). The cut slope could be stabilized against rotational or translational failure by construction of an interceptor trench approximately 12 feet deep somewhere between X = 100 and 130 feet as illustrated on figures 5G.5 and 5G.6. (The drained phreatic surface illustrated at X = 130 feet is estimated to be the response to an interceptor trench constructed to the drainage barrier at that location.)

The importance of removal of the organic duff layer and keying the fill into the subsoil is illustrated on figure 5G.7. This is a common form of failure of sidecast fills. If the duff layer is removed prior to fill placement, the fill is shown on figures 5G.8 and 5G.9 to be marginally stable even under undrained critical ground water conditions. This was predicted by the level II analysis. However, if failure did occur, it is likely to be a deep rotational failure with a failure surface tangent to the drainage barrier as illustrated on figure 5G.9. A failure surface of this type does have the potential for progression into a translational debris avalanche. Proper drainage to prevent the failure of the cut slope should also prevent failure of the fill slope as illustrated on figures 5G.10 and 5G.11, provided the trench drain water is collected and discharged below the fill location. (Section 6D contains an in-depth coverage of stabilization with subsurface drainage systems.)
5H. Rock Slopes—Fundamentals and Sample Problems

Michael T. Long, Engineering Geologist, Willamette National Forest

5H.1 Introduction

The reader is referred to Golder Associates (1989) for detailed coverage of rock slope analysis. In this section, sample problems to illustrate rock plane-, wedge-, and toppling-failure analyses will be illustrated.

In section 5A.11, simple rock slope plane-failure analysis was introduced. In this section, rock plane-failure analysis will be expanded to include external forces from surcharge, seismic acceleration, and artificial support for stabilization. Sample problems will be used to illustrate rock plane-failure analysis. All of the computational procedures for hydrostatic ground water pressures (as described in section 4E) will be illustrated in these problems.

The fundamentals of rock wedge-failure analysis were briefly described in section 5A.12. In this section, a hand-calculation for a simple rock wedge failure (without tension crack) will be made to illustrate the complexity of this three-dimensional analysis. The application of the use of stereo nets to the analysis of rock wedge-failure potential will also be illustrated.

The ROCKPACK personal computer program (Rockslope Stability Computerized Analysis Package 1988) can be used to analyze rock plane and wedge failures (Watts and Associates, 1986). Not all of the hydrostatic ground water force equations used in the following sample problems have been programmed into ROCKPACK, but several of the problems are solved both by the equations and by ROCKPACK.

5H.2 Rock Plane-Failure Sample Problems

Figure 5H.1 illustrates the rock plane-failure analysis equation when external forces from surcharge, seismic acceleration, and artificial support are included in the analysis. Figures 5H.2 and 5H.3 show the equations used to calculate the variables and water pressures for a plane-failure analysis when there is no tension crack. Figures 5H.4, 5H.5, and 5H.6 show the equations used when there is a tension crack. The equations used to calculate hydrostatic water pressures are consistent with those discussed in section 4E.
Figure 5H.1.—Equation for rock plane-failure analysis, including surcharge, seismic acceleration, and artificial support external forces.
Figure 5H.2.—Equations for calculation of variables for rock plane-failure analysis with no tension crack.
Figure 5H.3.—Equations for calculation of hydrostatic ground water pressure for rock plane-failure analysis without a tension crack.
Figure 5H.4.—Equations for calculations of variables for rock plane-failure analysis with a tension crack.
Figure 5H.5.—Equations for calculation of hydrostatic ground water pressures for rock plane-failure analysis with a tension crack: (a) Pressure in tension crack only, and (b) Pressure in tension crack and base pressure limited to crack pressure.
Figure 5H.6.—Equations for calculation of hydrostatic ground water pressure for rock plane-failure analysis with a tension crack: (a) With toe drainage, and (b) With toe drainage blocked.
Problem 1. Plane-Failure Analysis for Road Location.

During a proposed road location investigation, it was determined by drive probes and outcrop mapping that a full-bench cut slope would intersect dense basalt with a brecciated and secondary mineralized altered basalt contact as illustrated. A cross-section of the anticipated subsurface conditions along the dip line of the basalt follows.

![Cross-section diagram of subsurface conditions](image)

Given:

- Turning radius = 100 ft
- True dip/dip direction = 40/270
- Bearing of alignment
  - tangent to dip vector = North
- Friction angle of basal contact = 35°
- Density of rock (γ) = 160 pcf
- Cohesion of soil (C) = 500 psf
a. Using Markland's test (section 3C.2) on an equatorial equal area stereo net, determine whether a plane failure is kinematically possible for the conditions of this problem.

The dip vector falls within the critical zone ± 20° of dip direction of slope, so failure is possible.

b. Determine the approximate length of the failure zone along this road segment.

Turning radius = 100 ft.

\[ c = 2\pi r \]

\[ c = (2)(3.1416)(100 \text{ ft}) \]

\[ c = 628 \text{ ft.} \]

Failure length = (628 ft) \( \left( \frac{40'}{360'} \right) \approx 70 \text{ ft.} \)

It is common practice in engineering hand calculations to convert pounds to kips (kilo-pounds) in order to simplify the calculations. The following problems are in units of kips, unless noted.
c. Using the equations from figures 5H.1, 5H.2, and 5H.3, compute the factor of safety for the proposed cut slope (assume no tension crack, dry conditions, and no seismic acceleration or surcharge).

\[ FOS = \frac{CA + [(W + S)(\cos \theta - g \sin \theta) - U - V \sin \theta + T \cos \delta] \tan \phi}{(W + S)(\sin \theta + g \cos \theta) + V \cos \theta - T \sin \delta} \]

\[ X = \frac{h}{\tan \theta - \tan \alpha} \left(1 - \frac{\tan \theta}{\tan \beta}\right) = \frac{100 \text{ ft}}{\tan 40' - \tan 20'} \left(1 - \frac{\tan 40'}{\tan 76'}\right) \approx 166.4 \text{ ft} \]

\[ Y = \left(X + \frac{h}{\tan \beta}\right) \tan \theta = \left(166.4 \text{ ft} + \frac{100 \text{ ft}}{\tan 76'}\right) \tan 40' \approx 160.5 \text{ ft} \]

\[ e = \frac{h}{\sin \beta} = 103.1 \text{ ft} \]

\[ f = e \sin(\beta - \theta) = (103.1 \text{ ft}) \sin(76' - 40') = 60.60 \text{ ft} \]

\[ I = \frac{Y}{\sin \theta} = \frac{160.5 \text{ ft}}{\sin 40'} = 249.70 \text{ ft} \]

\[ W = \frac{Ih}{2} = \frac{(249.7 \text{ ft})(1 \text{ ft})(60.6 \text{ ft})(160 \text{ lb/ft}^3)}{2} = 1,210,546 \text{ lb} = 1210.5 \text{ kips} \]

\[ FOS = \frac{CA + W \cos \theta \tan \phi}{W \sin \theta} \]

\[ = \frac{(.5 \text{ kips/ft}^2)(249.7 \text{ ft})(1 \text{ ft}) + (1210.5 \text{ kips}) \cos 40' \tan 35'}{(1210.5 \text{ kips}) \sin 40'} \]

\[ = \frac{774}{778} = 0.99. \]

The ROCKPACK computer program gives the following:

ROCKPACK
ROCKslope Stability Computerized Analysis PACKage
(c) C.F. WATTS, 1988

Select one of the following:

A. AFTRDAT (Data storage)
B. DISDAT (Data display and rectangular plot)
C. EQNET (Equal-area stereo net plot)
D. MARKLND (Stereo net test for plane failure)
E. GRTCRCRL (Stereo net test for wedge failure)
F. RECTCON (Contours rectangular orientation plot)
G. SNETCON (Projects orientation contours to stereo net)
H. PLANE (Wedge failure safety factor and artificial support)
I. RAPWEDG (Wedge failure safety factor)
J. CMPWEDG (Comprehensive wedge analysis & artificial support)
K. TOPPLE (NOT yet available)
d. If the slope is to be stabilized by artificial support, determine the increase in resisting forces necessary to increase the factor of safety to 1.50.

\[ \uparrow F_{RP} = F_D(\text{FOS}) - F_R = (778 \text{ kips})(1.5) - (774 \text{ kips}) = 393 \text{ kips} \]

where:

\[ \uparrow F_{RP} = \text{Increased passive resisting force,} \]

\[ F_D = \text{Driving force,} \]

\[ F_R = \text{Resisting force,} \]

\[ \text{FOS} = \text{Design factor of safety.} \]

This would be correct for passive support such as a buttress.

e. If the artificial support is to be achieved through tensioned rock bolts, determine the optimum bolt angle from perpendicular to the failure plane as illustrated.
The optimum angle from horizontal ($\delta_H$) for rock bolt installation to achieve the maximum increase in resisting forces ($F_{rT}$) is reached when:

\[
\tan (\theta + \delta_H) = \tan \phi \\
\delta_{H_{opt}} = \phi - \theta.
\]
To convert $\delta_H$ to the bolt angle measured from the perpendicular to the failure surface ($\delta_p$),

$$\delta_{p, opt} = 90^\circ - (\theta + \delta_H) = 90^\circ - \phi$$

$$\delta_H = 90^\circ + (\delta_p - \theta).$$

Determine the optimum bolt angle from the perpendicular to the failure plane:

$$\phi = 35^\circ$$

$$\theta = 40^\circ$$

$$\delta_H = 35^\circ - 40^\circ = -5^\circ$$

$$\delta_{p, opt} = 90^\circ - 35^\circ = 55^\circ$$

f. Determine the number of rock bolts with a working capacity of 35 kips that will be necessary to increase the factor of safety to 1.50 (see problem 5H.1.d, 393 kips of additional passive force was calculated).

For *active* support, such as tensioned rock bolts or cables, forces are resolved for both driving, $F_D$, and resisting, $F_R$. If, for example, the 393 kips of additional resisting force needed in the *passive* state were solved by determining the number of 35-kips working-load tensioned bolts with $\delta_p = 55^\circ$

$$\frac{393 \text{ kips}}{35 \text{ kips/bolt}} \approx 11 \text{ bolts per foot or 385 kips/ft.}$$

Inserting this value for $T$ into the factor of safety equation in (c), we get

$$\text{FOS} = \frac{CA + (W \cos \theta + T \cos \delta_p) \tan \phi}{W \sin \theta - T \sin \delta_p}$$

$$= \frac{124.9 + (927.3 + 385 \cos 55^\circ) \tan 35^\circ}{778.1 - 385 \sin 55^\circ} = 2.0$$

which results in an overdesigned support system (FOS = 2.0, not 1.5 as desired).

To determine the increase in resisting force necessary in the *active* state to reach an FOS of 1.50, solve for $T = F_{Ra}$ (increased active resisting force), where from (c) again,

$$\text{FOS} = \frac{CA + (W \cos \theta \tan \phi + T \cos \delta \tan \phi)}{W \sin \theta - T \sin \delta}$$

$$= \frac{774 + T \cos 55^\circ \tan 35^\circ}{778 - T \sin 55^\circ} = 1.50.$$
Solve for $T$ where $T = \uparrow F_{RA}^*\dagger$

$$(1.50)(778 - T \sin 55^\circ) = T \cos 55^\circ \tan 35^\circ + 774$$

$1167 \text{ kips} - 1.50T \sin 55^\circ = T \cos 55^\circ \tan 35^\circ + 774$

$393 \text{ kips} - 1.50T \sin 55^\circ = T \cos 55^\circ \tan 35^\circ$

$$\frac{393 \text{ kips}}{T} = \cos 55^\circ \tan 35^\circ + 1.50 \sin 55^\circ$$

$$T = \frac{393 \text{ kips}}{\cos 55^\circ \tan 35^\circ + 1.50 \sin 55^\circ} = 241 \text{ kips.}$$

$$\frac{241 \text{ kips}}{35 \text{ kips/bolt}} \approx 7 \text{ bolts per foot.}$$

$$\uparrow F_{RA} = \frac{\uparrow F_{RP}}{\cos \delta_p \tan \phi + (FOS) \sin \delta_p}$$

$$\uparrow F_{RA} = \frac{F_{D}(FOS) - \uparrow F_R}{\cos \delta_p \tan \phi + (FOS) \sin \delta_p}.$$

Check: 7 bolts at 35 kips at $\delta_p = 55^\circ = 241 \text{ kips/ft active force applied.}$

* Increased active resisting force

From (c):

$$\text{FOS} = \frac{(774 \text{ kips}) + (241 \text{ kips}) \cos 55^\circ \tan 35^\circ}{(778 \text{ kips}) - (241 \text{ kips}) \sin 55^\circ} = \frac{870.8 \text{ kips}}{580.6 \text{ kips}} = 1.5.$$ 

ROCKPACK reports the same:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Height</td>
<td>100 feet</td>
</tr>
<tr>
<td>Inclination of slope face</td>
<td>76 degrees</td>
</tr>
<tr>
<td>Inclination of upper slope face</td>
<td>20 degrees</td>
</tr>
<tr>
<td>Inclination of potential failure plane</td>
<td>20 degrees</td>
</tr>
<tr>
<td>Horizontal acceleration due to blasting or earthquake</td>
<td>0 g</td>
</tr>
<tr>
<td>Bolt tension</td>
<td>$X (== \text{resolve})$</td>
</tr>
<tr>
<td>Bolt angle</td>
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</tr>
<tr>
<td>Cohesive strength of failure surface</td>
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</tr>
<tr>
<td>Friction angle of failure surface</td>
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<tr>
<td>Density of water</td>
<td>62.4 pcf</td>
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<tr>
<td>Amount of the discontinuity saturated with water</td>
<td>0%</td>
</tr>
<tr>
<td>Additional vertical surcharge</td>
<td>0 psf</td>
</tr>
</tbody>
</table>
g. Assume that during winter conditions, water can migrate along the basal contact with free drainage at the toe. Determine the factor of safety using the equations of figures 5H.2 and 5H.3.

Given \( h_w = 70 \text{ ft} \) and \( T = 241 \text{ kips} \) (for ROCKPACK, enter \( h_w \) as a decimal percentage of the height \( Y \), or \( h_w/Y = 70/160.5 = 44\% \)).

\[
I_w = \frac{h_w}{\sin \theta} = \frac{70 \text{ ft}}{\sin 40^\circ} = 108.9 \text{ ft}
\]

\[
U = \frac{1}{4} \gamma_w h_w I_w = \frac{1}{4} (62.4 \text{ lb/ft}^3)(70 \text{ ft})(108.9 \text{ ft}) = 119 \text{ kips}
\]

\[
\text{FOS} = \frac{(500)(249.7)(1) \cos 40^\circ - 119 + 241 \cos 55^\circ \tan 35^\circ}{1210.5 \sin 40^\circ - 241 \sin 55^\circ}
\]

\[
= \frac{(124.9 \text{ kips}) + (946.5 \text{ kips}) \tan 35^\circ}{580.7 \text{ kips}} = \frac{787.6 \text{ kips}}{580.7 \text{ kips}} = 1.36
\]
The ROCKPACK computer program gives:

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>100 feet</td>
</tr>
<tr>
<td>Inclination of slope face</td>
<td>76 degrees</td>
</tr>
<tr>
<td>Inclination of upper slope face</td>
<td>20 degrees</td>
</tr>
<tr>
<td>Inclination of potential failure plane</td>
<td>40 degrees</td>
</tr>
<tr>
<td>Horizontal acceleration due to blasting or earthquake</td>
<td>0 g</td>
</tr>
<tr>
<td>Starting rock bolt angle</td>
<td>55 degrees</td>
</tr>
<tr>
<td>Ending rock bolt angle</td>
<td>55 degrees</td>
</tr>
<tr>
<td>Increment step for rock bolt angles</td>
<td>10 degrees</td>
</tr>
<tr>
<td>Starting bolt tension</td>
<td>241000 pounds</td>
</tr>
<tr>
<td>Ending bolt tension</td>
<td>241000 pounds</td>
</tr>
<tr>
<td>Tension increment step</td>
<td>10000 pounds</td>
</tr>
<tr>
<td>Cohesive strength of failure surface</td>
<td>500 psf</td>
</tr>
<tr>
<td>Friction angle of failure surface</td>
<td>35 degrees</td>
</tr>
<tr>
<td>Density of rock</td>
<td>160 pcf</td>
</tr>
<tr>
<td>Density of water</td>
<td>62.4 pcf</td>
</tr>
<tr>
<td>Amount of the discontinuity saturated with water</td>
<td>0.44 decimal %</td>
</tr>
<tr>
<td>Horizontal distance from crest to failure plane</td>
<td>166.4365 feet</td>
</tr>
<tr>
<td>Contact area</td>
<td>249.8149 feet</td>
</tr>
<tr>
<td>Weight of slice</td>
<td>1210662 pounds</td>
</tr>
<tr>
<td>Water force normal to failure plane</td>
<td>121153 pounds</td>
</tr>
<tr>
<td>Factor of safety</td>
<td>1.3538</td>
</tr>
</tbody>
</table>

h. If freezing occurs at the toe and drainage is blocked, what would the factor of safety be?

\[
U = \frac{Y_w h_w l_w}{2} = \frac{(62.4 \text{ lb/ft}^3)(70 \text{ ft})(108.9 \text{ ft})(1 \text{ ft})}{2} = 237.8 \text{ kips}
\]

\[
FOS = \frac{(124.9 \text{ kips}) + (927.3 \text{ kips} - 237.8 \text{ kips} + 138.2 \text{ kips}) \tan 35^\circ}{580.7 \text{ kips}} = \frac{704.5 \text{ kips}}{580.7 \text{ kips}} = 1.21.
\]

i. Assume a magnitude 7.0 earthquake load with a peak acceleration of 0.15 g and compute the factor of safety.

\[
FOS = \frac{124.9 + [1210.5 (\cos 40^\circ - 0.15 \sin 40^\circ) - 237.8 + 138.2] \tan 35^\circ}{1210.5 (\sin 40^\circ + 0.15 \cos 40^\circ) - 197.4} = \frac{622.7}{719.8} = 0.87.
\]
j. If a tension crack develops 40 feet from the top of the cut (as illustrated under (e)), calculate the factor of safety for dry slope conditions using the equations from figures 5H.4 and 5H.2.

\[ Y_c = h_w + X_c \tan \alpha = (100 \text{ ft}) + (40 \text{ ft}) \tan 20' = 114.6 \text{ ft} \]

\[ Z = (X - X_0)(\tan \theta - \tan \alpha) = (166.4 \text{ ft}-40 \text{ ft})(\tan 40' - \tan 20') = 60.1 \text{ ft} \]

\[ Z_w = Z - (Y_c - h_w) = (60.1 \text{ ft}) - (114.6 \text{ ft} - 100 \text{ ft}) = 45.5 \text{ ft} \]

\[ I_w = \frac{h_w}{\sin \theta} = \frac{100 \text{ ft}}{\sin 40'} = 155.6 \text{ ft} \]

\[ I_c = \frac{Y_c - Z}{\sin \theta} = \frac{114.6 \text{ ft} - 60.1 \text{ ft}}{\sin 40'} = 84.8 \text{ ft} \]

\[ a = \frac{(X - X_0)}{\cos \alpha} = \frac{166.4 \text{ ft} - 40 \text{ ft}}{\cos 20'} = 134.5 \text{ ft} \]

\[ b = a \sin(\theta - \alpha) = (134.5 \text{ ft}) \sin(40' - 20') = 46.0 \text{ ft} \]

\[ W = \gamma A = \gamma \left[ \frac{1}{2} I_{0} - \frac{1}{2} (I_{0}) b \right] \]

\[ W = (160 \text{ lb/ft}^3) \left[ \frac{1}{2} (249.7 \text{ ft})(60.6 \text{ ft}) - \frac{1}{2} (249.7 \text{ ft} - 84.8 \text{ ft})(46 \text{ ft}) \right] \text{ (1 ft)} \]

\[ = 603.7 \text{ kips} \]

\[ \text{FOS} = \frac{CA + W \cos \theta \tan \phi}{W \sin \theta} \]

\[ = \frac{(0.5 \text{ kips/ft}^3)(84.8 \text{ ft})(1 \text{ ft}) + (603.7 \text{ kips}) \cos 40' \tan 35'}{(603.7 \text{ kips}) \sin 40'} \]

\[ = \frac{366.2}{388.1} = 0.94. \]

ROCKPACK's results are:

| ===) PLANE | (Wedge failure safety factor and artificial support) |
| Height | 100 feet |
| Inclination of slope face | 76 degrees |
| Inclination of upper slope face | 20 degrees |
| Inclination of potential failure plane | 40 degrees |
| Horizontal acceleration due to blasting or earthquake | 0 g |
| Starting rock bolt angle | 0 |
| Ending rock bolt angle | 0 |
| Increment step for rock bolt angles | 0 |
| Starting bolt tension | 0 |
| Ending bolt tension | 0 |
| Tension increment step | 0 |
k. If the same artificial support from tensioned rock bolts were added to the conditions of (j), as was calculated in (f) as required for a factor of safety of 1.50 for the slope without a tension crack, what would be the increase in factor of safety? Add 241 kips of artificial support at $\delta = 55^\circ$.

$$FOS = \frac{(0.5 \text{ kips/ft}^3)(84.8 \text{ ft})(1 \text{ ft}) + (462.5 \text{ kips} + (241 \text{ kips} \cos 55^\circ)) \tan 35^\circ}{388.1 \text{ kips} - (241 \text{ kips} \sin 55^\circ)}$$

$$= \frac{463.0 \text{ kips}}{190.7 \text{ kips}} = 2.43.$$  

ROCKPACK's results are:

<table>
<thead>
<tr>
<th>PLANE (Wedge failure safety factor and artificial support)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
</tr>
<tr>
<td>Inclination of slope face</td>
</tr>
<tr>
<td>Inclination of upper slope face</td>
</tr>
<tr>
<td>Inclination of potential failure plane</td>
</tr>
<tr>
<td>Horizontal acceleration due to blasting or earthquake</td>
</tr>
<tr>
<td>Starting rock bolt angle</td>
</tr>
<tr>
<td>Ending rock bolt angle</td>
</tr>
<tr>
<td>Increment step for rock bolt angles</td>
</tr>
<tr>
<td>Starting bolt tension</td>
</tr>
<tr>
<td>Ending bolt tension</td>
</tr>
<tr>
<td>Tension increment step</td>
</tr>
<tr>
<td>Cohesive strength of failure surface</td>
</tr>
<tr>
<td>Friction angle of failure surface</td>
</tr>
<tr>
<td>Density of rock</td>
</tr>
<tr>
<td>Density of water</td>
</tr>
<tr>
<td>Amount of the discontinuity saturated with water</td>
</tr>
<tr>
<td>Horizontal distance from tension crack to crest</td>
</tr>
<tr>
<td>Relative height of water in tension crack</td>
</tr>
<tr>
<td>Tension crack depth</td>
</tr>
<tr>
<td>Weight of slice</td>
</tr>
<tr>
<td>Contact area</td>
</tr>
<tr>
<td>Bolt angle</td>
</tr>
<tr>
<td>Tension</td>
</tr>
</tbody>
</table>

*FOS Factor of safety | 2.4323
1. Using the supported slope conditions of (k), what would the factor of safety be under the various potential hydrostatic ground water conditions from figures 5H.5 and 5H.6?

**Tension crack, water force only**

\[ V = \frac{1}{2} \gamma_w Z_w^2 = \frac{1}{2} (62.4 \text{ kips})(45.5 \text{ ft})^2(1 \text{ ft}) = 64.6 \text{ kips} \]

\[ \text{FOS} = \frac{42.4 + [462.5 - 64.6 \sin 40^\circ + 138.2]}{388.1 + 64.6 \cos 40^\circ - 197.4} \tan 35^\circ \]

\[ = \frac{434.0}{240.2} = 2.13. \]

**Tension crack and base water (hydraulically connected), base water force at tension crack maximum**

\[ V = 64.6 \text{ kips} \]

\[ U = \frac{1}{2} \gamma_w Z_w^2 l_c = \frac{1}{2} (62.4 \text{ lb/ft}^3)(45.5 \text{ ft})(84.8 \text{ ft})(1 \text{ ft}) = 120.4 \text{ kips} \]

\[ \text{FOS} = \frac{42.4 + [462.5 - 120.4 - 41.5 + 138.2]}{388.1 + 49.5 - 197.4} \tan 35^\circ \]

\[ = \frac{349.7}{240.2} = 1.46. \]

ROCKPACK gives the same results:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>100 feet</td>
</tr>
<tr>
<td>Inclination of slope face</td>
<td>76 degrees</td>
</tr>
<tr>
<td>Inclination of upper slope face</td>
<td>20 degrees</td>
</tr>
<tr>
<td>Inclination of potential failure plane</td>
<td>40 degrees</td>
</tr>
<tr>
<td>Horizontal acceleration due to blasting or earthquake</td>
<td>0 g</td>
</tr>
<tr>
<td>Starting rock bolt angle</td>
<td>55 degrees</td>
</tr>
<tr>
<td>Ending rock bolt angle</td>
<td>55 degrees</td>
</tr>
<tr>
<td>Increment step for rock bolt angles</td>
<td>0 degrees</td>
</tr>
<tr>
<td>Starting bolt tension</td>
<td>241000 pounds</td>
</tr>
<tr>
<td>Ending bolt tension</td>
<td>241000 pounds</td>
</tr>
<tr>
<td>Tension increment step</td>
<td>0 pounds</td>
</tr>
<tr>
<td>Cohesive strength of failure surface</td>
<td>500 psf</td>
</tr>
<tr>
<td>Friction angle of failure surface</td>
<td>35 degrees</td>
</tr>
<tr>
<td>Density of rock</td>
<td>160 pcf</td>
</tr>
<tr>
<td>Density of water</td>
<td>62.4 pcf</td>
</tr>
<tr>
<td>Amount of the discontinuity saturated with water</td>
<td>1 decimal %</td>
</tr>
<tr>
<td>Horizontal distance from tension crack to crest</td>
<td>40 feet</td>
</tr>
<tr>
<td>Relative height of water in tension crack</td>
<td>0.757 decimal %</td>
</tr>
<tr>
<td>Tension crack depth</td>
<td>60.0737 feet</td>
</tr>
<tr>
<td>Depth of water in crack</td>
<td>45.4758 feet</td>
</tr>
<tr>
<td>Weight of slice</td>
<td>603021 pounds</td>
</tr>
<tr>
<td>Contact area</td>
<td>84.7638 feet</td>
</tr>
<tr>
<td>Water force normal to failure plane</td>
<td>1020266.6 pounds</td>
</tr>
<tr>
<td>Horizontal water force on tension crack</td>
<td>64523.11 pounds</td>
</tr>
<tr>
<td>Bolt angle</td>
<td>55 degrees</td>
</tr>
<tr>
<td>Tension</td>
<td>241000 pounds</td>
</tr>
<tr>
<td><strong>FOS</strong></td>
<td>Factor of safety</td>
</tr>
</tbody>
</table>
Tension crack water and hydrostatic base water force (tension crack and base not hydraulically connected—toe drained)

\[
U = \frac{1}{2} \gamma_w \left( \frac{h_w}{4} \right) + \frac{1}{2} \gamma_w \left( \frac{h_w}{2} + Z_w \right) \left( l_c - \frac{l_w}{2} \right) \\
= \frac{1}{2} (62.4) \left[ \left( \frac{100 \text{ ft}}{4} \right) \left( \frac{155.6 \text{ ft}}{4} \right) \right] + \frac{1}{2} \left( \frac{155.6 \text{ ft}}{2} \right) \left( \frac{100 \text{ ft}}{2} + 45.5 \text{ ft} \right) \left( 84.8 \text{ ft} - \frac{155.6 \text{ ft}}{2} \right) \\
= 173.4 \text{ kips.}
\]

\[
FOS = \frac{42.4 + (462.5 - 173.4 - 41.5 + 138.2) \tan 35^\circ}{240.2} = \frac{312.5}{240.2} = 1.30.
\]

Tension crack water and hydrostatic base water force (tension crack and base not hydraulically connected—toe drainage blocked)

\[
U = \frac{1}{2} \gamma_w (h_w + Z_w) l_c = \frac{1}{2} (62.4 \text{ lb/ft}^3)(100 \text{ ft} - 45.5 \text{ ft})(84.8 \text{ ft})(1 \text{ ft}) = 385.0 \text{ kips}
\]

\[
FOS = \frac{42.4 + (462.5 - 385.0 - 41.5 + 138.2) \tan 35^\circ}{240.2} = \frac{164.4}{240.2} = 0.68.
\]

**Problem 2. Plane-failure analysis for bridge abutment.**

During a bridge foundation investigation, it was determined that the rock beneath one of the abutment spread footings had a potential for slope failure under certain conditions. Among the retrofit design recommendations were grouting the extension crack and installing rock bolts with shotcrete on the face (as shown). Evaluate these two stabilization techniques for this site.
a. Calculate the present factor of safety for the unsupported conditions illustrated assuming free drainage at the toe and a hydraulically connected tension crack and base.
\[ X = \frac{h}{\tan \theta - \tan \alpha} \left( 1 - \tan \theta \right) \left( 1 - \tan \frac{30^\circ}{\tan 60^\circ} \right) = 46.2 \text{ ft} \]

\[ Y = \left( X + \frac{h}{\tan \beta} \right) \tan \theta = \left( 46.2 \text{ ft} + \frac{40 \text{ ft}}{\tan 60^\circ} \right) \tan 30^\circ = 40.0 \text{ ft} \]

\[ Y_c = h + X_c \tan \alpha = 40.0 \text{ ft} \]

\[ Z = (X - X_c)(\tan \theta - \tan \alpha) = (46.2 \text{ ft} - 18 \text{ ft}) \tan 30^\circ = 16.3 \text{ ft} \]

\[ Z_w = Z - (Y c - h_w) = 16.3 \text{ ft} - (40 \text{ ft} - 37 \text{ ft}) = 13.3 \text{ ft} \]

\[ e = \frac{h}{\sin \beta} \approx \frac{40 \text{ ft}}{\sin 30^\circ} = 46.2 \text{ ft} \]

\[ f = e \sin (\beta - \theta) = (46.2 \text{ ft}) \sin(60^\circ - 30^\circ) = 23.1 \text{ ft} \]

\[ l = \frac{Y}{\sin \theta} = \frac{40 \text{ ft}}{\sin 30^\circ} = 80 \text{ ft} \]

\[ l_c = \frac{Y_c - Z}{\sin \theta} = \frac{40 \text{ ft} - 16.3 \text{ ft}}{\sin 30^\circ} = 47.4 \text{ ft} \]

\[ a = \frac{X - X_c}{\cos \alpha} = \frac{46.2 \text{ ft} - 18 \text{ ft}}{1} = 28.2 \text{ ft} \]

\[ b = a \sin(\theta - \alpha) = (28.2 \text{ ft}) \sin 30^\circ = 14.1 \text{ ft} \]

\[ W = \gamma \left[ \frac{1}{2}lf - \frac{1}{2}(l - l_e)b \right] \]

\[ = (165 \text{ lb/ft}^3) \left[ \frac{1}{2}(80 \text{ ft})(23.1 \text{ ft}) - \frac{1}{2}(80 \text{ ft} - 47.4 \text{ ft})(14.1 \text{ ft}) \right] (1 \text{ ft}) \]

\[ = 114.5 \text{ kips} \]

\[ V = \frac{1}{2} \gamma_w Z_w^2 = \frac{1}{2}(62.4 \text{ lb/ft}^3)(13.3 \text{ ft})^2(1 \text{ ft}) = 5.5 \text{ kips} \]

\[ U = \frac{1}{2} \gamma_w Z_w l_c = \frac{1}{2}(62.4 \text{ lb/ft}^3)(13.3 \text{ ft})(47.4 \text{ ft})(1 \text{ ft}) = 19.7 \text{ kips} \]
\[
FOS = \frac{CA + [(W + S) \cos \theta - U - V \sin \theta] \tan \phi}{(W + S) \sin \theta + V \cos \theta} = \frac{(1 \text{ kip/ft}^2)(47.4 \text{ ft})(1 \text{ ft}) + [(114.5 + 10.2) \cos 30^\circ - 19.7 - 5.5 \sin 30^\circ] \tan 35^\circ}{(114.5 + 10.2) \sin 30^\circ + 5.5 \cos 30^\circ} = \frac{107.3}{67.2} = 1.60.
\]

ROCKPACK gives:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>40 feet</td>
</tr>
<tr>
<td>Inclination of slope face</td>
<td>60 degrees</td>
</tr>
<tr>
<td>Inclination of upper slope face</td>
<td>0 degrees</td>
</tr>
<tr>
<td>Inclination of potential failure plane</td>
<td>30 degrees</td>
</tr>
<tr>
<td>Horizontal acceleration due to blasting or earthquake</td>
<td>0 g</td>
</tr>
<tr>
<td>Starting rock bolt angle</td>
<td>0</td>
</tr>
<tr>
<td>Ending rock bolt angle</td>
<td>0</td>
</tr>
<tr>
<td>Increment step for rock bolt angles</td>
<td>0</td>
</tr>
<tr>
<td>Starting bolt tension</td>
<td>0</td>
</tr>
<tr>
<td>Ending bolt tension</td>
<td>0</td>
</tr>
<tr>
<td>Tension increment step</td>
<td>0</td>
</tr>
<tr>
<td>Cohesive strength of failure surface</td>
<td>1000 psf</td>
</tr>
<tr>
<td>Friction angle of failure surface</td>
<td>35 degrees</td>
</tr>
<tr>
<td>Density of rock</td>
<td>160 pcf</td>
</tr>
<tr>
<td>Density of water</td>
<td>62.4 pcf</td>
</tr>
<tr>
<td>Amount of the discontinuity saturated with water</td>
<td>1 decimal %</td>
</tr>
<tr>
<td><em>S</em> Additional vertical surcharge</td>
<td>10200 psf</td>
</tr>
<tr>
<td>* Horizontal distance from tension crack to crest</td>
<td>18 feet</td>
</tr>
<tr>
<td>* Relative height of water in tension crack</td>
<td>0.82 decimal %</td>
</tr>
<tr>
<td><em>Z</em> Tension crack depth</td>
<td>16.2744 feet</td>
</tr>
<tr>
<td><em>Zw</em> Depth of water in crack</td>
<td>13.3450 feet</td>
</tr>
<tr>
<td>Weight of slice</td>
<td>111102.3 pounds</td>
</tr>
<tr>
<td><em>le</em> Contact area</td>
<td>47.4513</td>
</tr>
<tr>
<td><em>U</em> Water force normal to failure plane</td>
<td>19756.97 pounds</td>
</tr>
<tr>
<td><em>V</em> Horizontal water force on tension crack</td>
<td>5556.357 pounds</td>
</tr>
<tr>
<td>Bolt angle</td>
<td>0 degrees</td>
</tr>
<tr>
<td>Tension</td>
<td>0 pounds</td>
</tr>
<tr>
<td>* FOS* Factor of safety</td>
<td>1.6075</td>
</tr>
</tbody>
</table>

b. If the tension crack is sealed with grout but seepage continues to occur along the basal contact and is blocked due to ice damming during the winter, what would the factor of safety be?
\[ V = 0 \]

\[ U = \frac{1}{2} \gamma w (h_w + Z_w) l_c = \frac{1}{2} (62.4 \text{ lb/ft}^3)(37.0 \text{ ft} + 13.3 \text{ ft})(47.7 \text{ ft})(1 \text{ ft}) = 74.4 \text{ kips} \]

\[ \text{FOS} = \frac{CA + [(W + S) \cos \theta - U] \tan \phi}{(W + S) \sin \theta} \]

\[ = \frac{(1 \text{ kips/ft}^3)(47.4 \text{ ft})(1 \text{ ft}) + [(124.7 \text{ kips}) \cos 30^\circ - 74.4 \text{ kips}] \tan 35^\circ}{(124.7 \text{ kips}) \sin 30^\circ} \]

\[ = \frac{70.9 \text{ kips}}{62.4 \text{ kips}} = 1.14. \]

c. To the conditions of (b) add a horizontal seismic acceleration of 0.15g and determine the factor of safety.

\[ \text{FOS} = \frac{CA + [(W + S)(\cos \theta - g \sin \theta) - U] \tan \phi}{(W + S)(\sin \theta + g \cos \theta)} \]

\[ = \frac{47.4 + [124.7 \text{ cos } 30^\circ - 0.15 \text{ sin } 30^\circ] - 74.4 \tan 35^\circ}{124.7 (\sin 30^\circ + 0.15 \cos 30^\circ)} \]

\[ = \frac{64.3}{78.5} = 0.82. \]

d. Determine the artificial support necessary to bring the present factor of safety up to 2.50 (assuming the tension crack is grouted and there is no tension crack hydrostatic water force and the toe is free draining).

\[ \text{FOS} = \frac{CA + [(W + S) \cos \theta - U] \tan \phi}{(W + S) \sin \theta} \]

\[ = \frac{47.4 + [124.7 \text{ cos } 30^\circ - 19.7] \tan 35^\circ}{124.7 \sin 30^\circ} \]

\[ = \frac{109.2}{62.4} = 1.75 \]

\[ \uparrow F_{RP} = F_D(\text{FOS}) - F_R = (62.4 \text{ kips}) 2.50 - (109.2 \text{ kips}) \]

\[ = 46.8 \text{ kips} \]

\[ \delta_{R,\text{opt}} = 90^\circ - \phi = 55^\circ \]

In this case, construction constraints allow for \( \delta_r = 30^\circ \) only, as shown in the problem illustration.
\[ F_{RA} = \frac{F_{RP}}{\cos \delta \tan \phi + (FOS) \sin \delta} \]

\[ = \frac{46.8 \text{ kips}}{\cos 30^\circ \tan 35^\circ + 2.50 \sin 30^\circ} = \frac{46.8}{1.9} = 24.6 \text{ kips.} \]

Check:

\[ = \frac{109.2 + 24.6 \cos 30^\circ \tan 35^\circ}{62.4 - 24.6 \sin 30^\circ} = \frac{124.1}{50.1} \approx 2.50. \]
Figure 5H.7 illustrates a simple rock wedge failure with no tension crack or external forces and the four planar surfaces in the analysis.

Figure 5H.7.—Simple rock-wedge failure geometry (reprinted with permission of Golder Associates, 1988).
The variables used in the simple rock wedge-failure analysis as illustrated in figure 5H.7 are (Golder Associates, 1989):

\[ \text{4} \] = Planar surfaces

\[ H_w \] = Height of wedge
\[ H \] = Height of upper slope above intersection daylight point
\[ \gamma \] = Unit weight of rock
\[ \phi \] = Angle of internal friction
\[ C \] = Cohesion
\[ \beta \] = Dip or plunge angle
\[ \alpha \] = Dip direction or trend

\[ x, y, z \] = Coordinate axes with origin at 0
\[ \hat{a} \] = Unit vector in the direction of the normal to plane 1 with components \([a_x, a_y, a_z]\)
\[ \hat{b} \] = Unit vector in the direction of the normal to plane 2 with components \([b_x, b_y, b_z]\)
\[ \hat{f} \] = Unit vector in the direction of the normal to plane 4 with components \([f_x, f_y, f_z]\)
\[ \hat{g} \] = Vector of the intersection of planes 1 and 4 with components \([g_x, g_y, g_z]\)
\[ \hat{i} \] = Vector of the intersection of planes 1 and 2 with components \([i_x, i_y, i_z]\)

\[ i = -i_z \]

\[ q \] = component of \( \hat{g} \) in the direction of \( \hat{b} \)
\[ r \] = component of \( \hat{a} \) in the direction of \( \hat{b} \)

\[ K = |\hat{i}|^2 = i_x^2 + i_y^2 + i_z^2 \]
\[ l = W/A_2 \]
\[ p = A_1/A_2 \]

Assuming contact on both planes:

\[ n_1 = N_1/A \]
\[ n_2 = N_2/A \]
\[ |l_1|/\sqrt{k} = S A_2 \]
Contact on plane 1 only:

\[ m_1 = N_1/A_2 \]
Denominator of FOS = \( S_1/A_2 \)

Contact on plane 2 only:

\[ m_2 = N_2/A_2 \]
Denominator of FOS = \( S_2/A_2 \)

Water pressure with discontinuities filled and free draining at toe

\[ U_1 = U_2 = \frac{\gamma_w H_w}{6} \]
\[ n = -1 \text{ if the slope face overhangs the slope toe.} \]
\[ n = 1 \text{ if the slope face does not overhang the slope toe.} \]

The sequence of the calculations for the factor of safety in simple wedge-failure analysis is as follows (from Golder Associates, 1989):

1. \((x, y, z) = \{ \sin \beta, \sin (\alpha_1 - \alpha_2), \sin \beta_1 \cdot \cos (\alpha_1 - \alpha_2), \cos \beta_1 \} \)
2. \((f_x, f_y, f_z) = \{ \sin \beta_4 \cdot \sin (\alpha_4 - \alpha_2), \sin \beta_4 \cdot \cos (\alpha_4 - \alpha_2), \cos \beta_4 \} \)
3. \(b_y = \sin \beta_2 \)
4. \(b_z = \cos \beta_2 \)
5. \(i = a_x b_y \)
6. \(g_z = f_x b_y - f_y a_x \)
7. \(q = b_y(f_x a_x - f_x a_z) + b_z g_z \)
8. If \( nq/i > 0 \), or if \( n(f_z - \frac{q}{i}) \tan \beta_3 > \sqrt{1 - f_z^2} \) and
\[ \alpha_3 = \alpha_4 \pm (1 - n) \frac{\pi}{2}, \text{ no wedge is formed.} \]
9. \( r = a_y b_y + a_z b_z \)
10. \( K = 1 - r^2 \)
11. \( l = \frac{(\gamma H_q)}{3 g_z} \)
12. \( p = \frac{-b_y f_x}{g_z} \)
13. \( n_1 = \left\{ \left( \frac{l}{K} \right) (a_z - rb_z) - pu_1 \right\} \cdot \frac{p}{|p|} \)
14. \( n_2 = \left( \frac{1}{K} \right) (b_z - ra_z) - u_2 \)
15. \( m_1 = (la_z - nu_2 - pu_1) \cdot \frac{p}{|p|} \)
16. \( m_2 = lb_z - rpu_1 - u_2 \)
17A. If \( n_1 > 0 \) and \( n_2 > 0 \), there is contact on both planes, and

\[
FOS = (n_1 \cdot \tan \phi_1 + n_2 \cdot \tan \phi_2 + |p| C_1 + C_2) \frac{\sqrt{K}}{|l|}
\]

17B. If \( n_1 < 0 \) and \( m_1 > 0 \), there is contact on plane 1 only, and

\[
FOS = \frac{m_1 \cdot \tan \phi_1 + |p| C_1}{\sqrt{l^2(1 - a_z^2) + Ku_2^2 + 2(ra_z - b_z)\mu_2}}
\]

17C. If \( n_1 < 0 \) and \( m_2 > 0 \), there is contact on plane 2 only, and

\[
FOS = \frac{m_2 \cdot \tan \phi_2 + C_2}{\sqrt{l^2b_y^2 + Kp^2u_1^2 + 2(rb_z - a_z)\mu_1}}
\]

17D. If \( m_1 < 0 \) and \( m_2 < 0 \), contact is lost on both planes and the wedge "floats" as a result of water pressure acting on planes 1 and 2. In this case, the factor of safety falls to zero.

---

**Problem 3. Simple Wedge Failure**

a. Calculate the factor of safety for a wedge with the following properties (from Golder Associates, 1989):

<table>
<thead>
<tr>
<th>Plane</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dip(( \beta ))</td>
<td>32°</td>
<td>42°</td>
<td>15°</td>
<td>75°</td>
</tr>
<tr>
<td>Dip Dir (( \alpha ))</td>
<td>268°</td>
<td>178°</td>
<td>240°</td>
<td>240°</td>
</tr>
<tr>
<td>Strike</td>
<td>N2W</td>
<td>N88E</td>
<td>N60E</td>
<td>N60E</td>
</tr>
<tr>
<td>( \phi )</td>
<td>35°</td>
<td>28°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C )</td>
<td>100 psf</td>
<td>200 psf</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. \( a_x = (\sin 32°) \sin (268° - 178°) = 0.5299 \)

2. \( a_y = (\sin 32°) \cos (268° - 178°) = 0 \)

3. \( a_z = \cos 32° = 0.8480 \)

2. \( f_x = (\sin 75°) \sin (240° - 178°) = 0.8529 \)

3. \( f_y = (\sin 75°) \cos (240° - 178°) = 0.4535 \)

4. \( f_z = \cos 75° = 0.2588 \)

3. \( b_y = \sin 42° = 0.6691 \)
4. \( b_z = \cos 42^\circ = 0.7431 \)

5. \( i = (0.5299)(0.6691) = 0.3546 \)

6. \( g_z = (0.8529)(0) - (0.4535)(0.5299) = -0.2403 \)

7. \( q = 0.6691[(0.2588)(0.5299) - (0.8529)(0.8480)] + (0.7431)(-0.2403) \\
   = -0.5707 \)

8. \( \frac{np}{i} = \frac{(1)(-0.5707)}{0.3546} < 0 \quad \text{ok} \)

9. \( n\left(f_z - \frac{g}{i}\right)\tan 15^\circ = 0.5006 < \sqrt{1 - f_z^2} = 0.9659 \quad \text{ok} \)

   \( \therefore \) wedge is formed

10. \( r = (0)(0.6691) + (0.8480)(0.7431) = 0.6301 \)

11. \( K = 1 - (0.6301)^2 = 0.6030 \)

12. \( l = \frac{(165)(70)(-0.5707)}{3(-0.2403)} = 9.14355 \)

13. \( n_1 = \left[\frac{9.14355}{0.6030}, \frac{(0.8480 - (0.6301)(0.7431) - 2.3748(0))}{0.6300}\right] = 5.75866 \)

14. \( n_2 = \frac{9.14355}{0.6030}(0.7431 - ((0.6301)(0.8480)) - 0 = 3.16575 \)

15. \( m_1 = (9.14355)(0.8480)(1) = 7.75373 \)

16. \( m_2 = (9.14355)(0.7431) = 6.79457 \)

\( FOS = \sqrt{((5.75866)(\tan 35^\circ) + (3.16575)(\tan 28^\circ) + (2.3748)(100) + 200)\sqrt{0.6030}} \)

\( = \frac{4777.99}{3242.30} \approx 1.47 \)
ROCKPACK gives the same results:

**ROCKPACK**

ROCKslope Stability Computerized Analysis PACKage  
(c) C.F.WATTS, 1986

---

**RAPWEDG** (Wedge solution for rapid computation, after Hoek & Bray, 1981)

* Dry slope  
  Slope is not undercut

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Unit weight rock in wedge</td>
<td>165 pcf</td>
</tr>
<tr>
<td>Height of crest above intersection</td>
<td>70 feet</td>
</tr>
<tr>
<td>Plane 1 dip</td>
<td>32 degrees</td>
</tr>
<tr>
<td>Plane 1 dip direction</td>
<td>268 degrees</td>
</tr>
<tr>
<td>Plane 2 dip</td>
<td>42 degrees</td>
</tr>
<tr>
<td>Plane 2 dip direction</td>
<td>178 degrees</td>
</tr>
<tr>
<td>Plane 3 dip</td>
<td>15 degrees</td>
</tr>
<tr>
<td>Plane 3 dip direction</td>
<td>240 degrees</td>
</tr>
<tr>
<td>Plane 4 dip</td>
<td>75 degrees</td>
</tr>
<tr>
<td>Plane 4 dip direction</td>
<td>240 degrees</td>
</tr>
<tr>
<td>Plane 1 cohesion</td>
<td>100 psf</td>
</tr>
<tr>
<td>Plane 1 friction angle</td>
<td>35 degrees</td>
</tr>
<tr>
<td>Plane 2 cohesion</td>
<td>200 psf</td>
</tr>
<tr>
<td>Plane 2 friction angle</td>
<td>28 degrees</td>
</tr>
</tbody>
</table>

There is contact on both planes

\[
Ax = 0.52992 \quad Ay = 1.0318E-7 \quad Az = 0.84805 \quad I = 0.35459
\]

\[
Fx = 0.85286 \quad Fy = 0.45347 \quad Fz = 0.25882 \quad K = 0.60282
\]

\[
By = 0.66913 \quad Bz = 0.74315 \quad L = 9144.46
\]

\[
Gz = 0.24030
\]

\[
P = 2.37480 \quad Q = 0.57077 \quad R = 0.63022
\]

\[
n1 = 5759.88 \quad n2 = 3165.66 \quad m1 = 7754.94 \quad m2 = 6795.66
\]

*FOS* Factor of safety against sliding  1.47353

b. For the wedge in (a), calculate the factor of safety with the discontinuities fully saturated and free draining at the toe intersection of planes 1 and 2.

In order to calculate the water force, $H_w$ must be determined.
\[ u_1 = u_2 = \gamma_w \cdot \frac{H_w}{6} \]

\( \theta \) = Plunge angle of plane 1/2 intersection

\( \alpha_3 \) = Dip angle of plane 3

\( \alpha_4 \) = Dip angle of plane 4

\( H_w = H + \Delta Y \)

\( \Delta Y = \tan \alpha_3 (X) \)

\[ X = \frac{H}{\tan \theta - \tan \alpha_3} \left( 1 - \frac{\tan \theta}{\tan \alpha_4} \right) \]

\( \theta \) may be determined by either of the following two methods.

### Cotangent Method

1. Plot the strike lines from planes 1 and 2 radiating from a common point.

2. Calculate \( \cot 42^\circ = \frac{\cos 42^\circ}{\sin 42^\circ} = 1.1 \)

   Measure 1.1" from the origin, up the strike line of plane 1 and make a reference mark.

3. Calculate \( \cot 32^\circ = 1.6 \) and draw a perpendicular from the strike line of plane 1 out 1.6". This is the point of intersection of the two cotangents.

4. Draw a line from the origin to the point of cotangent intersection. This is the trend of the line of intersection of planes 1 and 2. Measure the trend and add 180° in this case, because the planes dip to the southern hemisphere.

5. The plunge of the line of intersection (\( \theta \)) may now be determined using the dip angle of either plane (\( \alpha \)) and the angle between the strike of the plane and the trend of the intersection (\( \delta \)).
\[ \tan \theta = \tan \alpha \sin \delta \]
\[ \theta = \tan^{-1}(\tan \alpha \sin \delta) \]
\[ = \tan^{-1}(\tan 32^\circ \sin 55^\circ) \]
\[ = 27.3^\circ \]

or
\[ \theta = \tan^{-1}(\tan \alpha_2 \sin \delta_2) \]
\[ = \tan^{-1}(\tan 42^\circ \sin 35^\circ) \]
\[ = 27.3^\circ \]

**Stereo Net Method**

The trend and plunge can both be plotted directly.

\[ X = \frac{70 \text{ ft}}{\tan 27^\circ - \tan 15^\circ} \left(1 - \frac{\tan 27^\circ}{\tan 75^\circ}\right) = 250.2 \text{ ft} \]

\[ \Delta Y = (\tan 15^\circ)(250.2 \text{ ft}) = 67 \text{ ft} \]

\[ H_w = 70 \text{ ft} + 67 \text{ ft} = 137 \text{ ft} \]

\[ u_1 = u_2 = \frac{(62.4 \text{ pcf})(137 \text{ ft})}{6} = 1,424.8 \text{ psf} \]
This can be checked using ROCKPACK:

\[ n_1 = \left( \frac{9143.55}{.6030} \right) \left( .8480 - (.6301)(.7431) - 2.3748(1424.8) \right) \]  
\[ n_1 = 2,374.70 \]

\[ n_2 = \frac{9143.55}{.6030} \left( .7431 - (.6301)(.8480) - 1424.8 \right) = 1740.95 \]

\[ m_1 = \left( (9143.55)(.8480) - (.6301)(1424.8) - (2.3748)(1424.8) \right) = 3,472.35 \]

\[ m_2 = (9143.55)(.7431) - (.6301)(2.3748)(1424.8) = 3,237.76 \]

\[ FOS = \frac{[(2374.70)(\tan 35^\circ) + (1740.95)(\tan 28^\circ) + (2.3748)(100) + 200]}{(9143.55)(.3546)} \]

\[ = \frac{2349.74}{3242.30} \]

\[ = 0.72 \]

This can be checked using ROCKPACK:

<table>
<thead>
<tr>
<th>RAPWEDG (Wedge solution for rapid computation, after Hock &amp; Bray, 1981)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cracks completely filled (free-draining)</td>
</tr>
<tr>
<td>Slope is not undercut</td>
</tr>
<tr>
<td>Unit weight rock in wedge</td>
</tr>
<tr>
<td>Height of crest above intersection</td>
</tr>
<tr>
<td>Plane 1 dip</td>
</tr>
<tr>
<td>Plane 1 dip direction</td>
</tr>
<tr>
<td>Plane 2 dip</td>
</tr>
<tr>
<td>Plane 2 dip direction</td>
</tr>
<tr>
<td>Plane 3 dip</td>
</tr>
<tr>
<td>Plane 3 dip direction</td>
</tr>
<tr>
<td>Plane 4 dip</td>
</tr>
<tr>
<td>Plane 4 dip direction</td>
</tr>
<tr>
<td>Plane 1 cohesion</td>
</tr>
<tr>
<td>Plane 1 friction angle</td>
</tr>
<tr>
<td>Plane 2 cohesion</td>
</tr>
<tr>
<td>Plane 2 friction angle</td>
</tr>
<tr>
<td>Overall vertical height of wedge</td>
</tr>
<tr>
<td>* There is contact on both planes</td>
</tr>
<tr>
<td>* Water pressure on both planes</td>
</tr>
<tr>
<td>Ax = 0.52992</td>
</tr>
<tr>
<td>Fx = 0.85286</td>
</tr>
<tr>
<td>By = 0.66913</td>
</tr>
<tr>
<td>Gz = 0.24030</td>
</tr>
<tr>
<td>P = 2.37480</td>
</tr>
<tr>
<td>m1 = 5759.88</td>
</tr>
<tr>
<td><em>FOS</em> Factor of safety against sliding</td>
</tr>
</tbody>
</table>
Figure 5H.8 illustrates the conditions under which toppling failure is kinematically possible and under which it occurs.

IF \( \frac{Y}{\Delta X} > \cot \alpha \)
THEN TOPPLING OCCURS

EQUAL AREA STEREO NET PROJECTION

POLES PLOTTED IN THIS AREA ARE KINEMATICALLY POSSIBLE TO FAIL

Figure 5H.8.—Toppling failure analysis.
**Problem 4.** *Will the illustrated proposed cut be stable to the right of the interbed slope break?*

![Proposed cut diagram]

To the right of the interbed slope break

\[
\alpha = 5^\circ
\]

\[
\cot \alpha = \frac{1}{\tan \alpha} = 11.4
\]

\[
\frac{y}{\Delta x} = \frac{50}{10} < \cot 5^\circ \therefore \text{stable.}
\]

**Problem 5.** *Will the blocks of rock to the left of the interbed slope break be stable if the rocks to the right are removed?*

To the left of the interbed slope break \(\alpha\) increases to \(15^\circ\). Therefore,

\[
\alpha = 15^\circ
\]

\[
\cot 15^\circ = 3.7
\]

\[
\frac{y}{\Delta x} = \frac{50}{10} > \cot 15^\circ \therefore \text{unstable due to toppling.}
\]


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