Previous stochastic models in harvest scheduling seldom address explicit spatial management concerns under the influence of natural disturbances. We employ multistage stochastic programming models to explore the challenges and advantages of building spatial optimization models that account for the influences of random stand-replacing fires. Our exploratory test models simultaneously consider timber harvest and mature forest core area objectives. Random fire samples are built into the model, creating a sample average approximation (SAA) formulation of our stochastic programming problem. Each model run reports first-period harvesting decisions along with recourse decisions for subsequent time periods reflecting the influence of stochastic fires. In each test, we solve 30 independent, identically distributed (i.i.d.) replicate models and calculate the persistence of period one solutions. Harvest decisions with the highest persistence are selected as the solution for each stand in a given test case. We explore various sample sizes in our SAA models. Monte Carlo simulations of these solutions are then run by fixing first-period solutions and solving new i.i.d. replicates. Multiple comparison tests identify the best first-period solution. Results indicate that integrating the occurrence of stand-replacing fire into forest harvest scheduling models can improve the quality of long-term spatially explicit forest plans.

Keywords: sample average approximation, mixed integer programming, harvest scheduling, spatial optimization

Natural disturbances such as fire, wind, insects, and diseases interact with forest management activities across space and time. These disturbances can influence timber flow, economic return, forest age-class distribution, and forest spatial structure. A number of research studies have highlighted the importance of accounting for the influence of such stochastic disturbances in forest planning models (e.g., MacLean 1990). In many forests, fire is a major disturbance factor influencing timber supplies. Interest in planning harvest schedules that account for the risk of fire blossomed in the 1980s and has continued through today, as outlined below.

Stochastic programming models to develop harvest schedules that consider fire risk were introduced as early as 1980. Stochastic programming models are formulated presupposing a sequence of decisions and observations wherein a first stage contains decisions that must be taken “here and now” considering the available data. These data are presumed to include conditional probabilities for random events that could affect the outcomes of those decisions. Subsequent stages contain “wait and see” recourse decisions that can be used to take corrective measures or adjustment plans as random events preceding the second stage (and possibly later decision stages) are revealed (Birge and Louveaux 1997). Martell (1980) developed an aspatial stochastic programming model to optimize the harvest schedule for a single stand of trees with a given fire risk. His work demonstrated the value of fire management activities, finding that these activities often boosted timber production enough to more than pay for themselves. Gassmann (1989) developed an aspatial stochastic programming model with seven time periods to evaluate strata-based optimal forest wide harvesting in the presence of fire. He found in some cases that he could use one-point discretizations for the last three planning periods (aggregating periods four to seven into a single decision stage), thus reducing the model to four stages, without affecting the quality of first-period solutions. In other cases, all seven planning periods had to be modeled as individual decision stages. Gassmann suggested using penalty terms in the objective function instead of restricting timber flow through constraints because he found that timber flow constraints are difficult to enforce in a stochastic programming framework. His model suggested that...
uncertainty leads to smaller optimal harvest sizes. Similarly, Boychuck and Martell (1996) developed an aspatial multistage stochastic programming model with 10 discrete planning periods, the first four modeled as decision stages. They examined the impacts of timber flow limitations and fire severity and found that timber flow limitations in a stochastic environment force a much smaller optimal harvest. Boychuck and Martell suggest the use of buffer stock to allow a forest to produce a nondeclining timber supply, as they also found timber flow constraints difficult to enforce.

In other stochastic modeling work, Routledge (1980) incorporated the probability of a catastrophe and the expected salvage proportion in a stand into the classic Faustmann formulation for identifying optimal stand rotation, demonstrating the importance of including stochastic information in harvest scheduling models. More recent aspatial stochastic models have been used to examine balancing timber production with other goals. One such model was developed by Spring and Kennedy (2005), who used a stochastic dynamic programming (SDP) model to study the tradeoffs between producing timber and protecting endangered species under the threat of random fires. While their study did include effects of random fires, it did not analyze how the risk of fire would change trade-off decisions. Ferreira et al. (2011) developed an SDP model that assumed management of each stand would be implemented at the end of the stage, after fires occur. They found that higher wildfire risk tends to decrease rotation lengths.

Zhou and Buongiorno (2006) incorporated Markov chain models describing stand transitions of mixed loblolly pine-hardwood forests under the influence of natural disturbances into a stochastic optimization model to study the tradeoffs between landscape diversity and timber objectives. They found that both large-scale and small-scale disturbances can be beneficial to diversity, but achieving higher landscape diversity has a high timber production cost. Markov chain models have also been incorporated into a forest level goal programming simulation/optimization model using simulated annealing, which was used to search for the optimal harvest schedule for a forest subject to random wildfires (Campbell and Dewhurst 2007). When the objective was to return the landscape to a historical condition, Campbell and Dewhurst found that it could be difficult to meet timber objectives.

Because stochastic optimization modeling can be computationally expensive and slow, researchers have experimented with other methods to examine the impact of fire on timber harvests, although in many cases recourse decision opportunities are not modeled. Van Wagner (1983) used simulation to examine the weaknesses of forest level harvest schedules that fail to take fire into account. He analyzed the effect of fire on timber yield and found that the optimal yield is decreased by fire risk. The use of these decreased annual harvests led to an annual yield that was much less sensitive to fire events. Reed and Errico (1986) formulated an aspatial forest level linear programming harvest scheduling model with flow constraints to maximize the return from harvests under the influence of mean value fire disturbance rates. While this was initially formulated as a stochastic problem, the authors decided to solve only the mean value problem. The authors suggest that in large enough forests the mean value problem is a satisfactory simplification. Their results were similar to Van Wagner’s (1983). Reed and Errico (1989) later enhanced their model by accommodating the possibility of partial salvage, multiple timber types, accessibility constraints, and variable recovery costs. Armstrong (2004) developed an aspatial stochastic simulation model to test deterministic annual allowable cut (AAC) solutions in Alberta, Canada. After determining an AAC using a linear programming model, Armstrong evaluated the results using Monte Carlo simulations by assuming the proportion of area burned in each period is random. The study compared the effects of fire on AAC under different harvest schedules in different types of forest. More recently, a deterministic aspatial model was built by Pasalodos-Tato et al. (2010) to evaluate the interaction between fire and timber management in stands of Maritime Pine (Pinus pinaster). Both the Armstrong (2004) and Pasalodos-Tato et al. (2010) results are consistent with previous work, finding that increased fire risk leads to a decreased optimal harvest.

In some recent work, researchers have investigated the potential effects that harvesting has on fire spread rates for future fires. Kono-shima et al. (2008) tested a two stage spatially explicit stochastic dynamic programming model in a regularly shaped landscape with seven hexagonal stands. Random fire events were captured by probabilities of ignition and different weather scenarios. Under the assumption that harvesting a stand produces higher fire spread rates across the stand, rotation lengths are not necessarily shortened; given the higher spread rates a fire in a younger stand tended to threaten more stands than a fire in an older stand. In contrast, Acuna et al. (2011) built a spatial model that assumes fire does not spread through harvested stands. This model integrated fire ignition, fire spread, fire suppression, and fire protection values in a combined simulation and spatial timber model to develop harvest schedules that purposefully impact landscape flammability. The harvest scheduling component in the model is an extension of Reed and Errico’s 1986 model. Acuna et al. (2010) found that they could increase harvest volumes when looking at harvesting as a fire mitigation strategy. Because they assumed fire does not spread through harvested areas, they could reduce the risk to high value stands by harvesting stands nearby.

As Boychuck and Martell (1996) summarized, the effects of fire disturbance on timber supplies over time appear to vary considerably depending on a number of factors. In multistand or multistrata harvest scheduling models with forest-level constraints such as non- declining flow, a frequent outcome is that “attempting implementation of mean value problem solutions in a stochastic system leads to infeasibility with high probability” (e.g., see Pickens and Dress 1988, Hof et al. 1995). They observed, however, that mean value problem solutions generally provided fair approximations to stochastic programming problem first-period solutions. In systems where periodic replanning occurs, they suggest that mean value solutions may even be good approximations where allowances are made for fire risk by harvesting less than the solution indicates (i.e., by retaining a timber supply buffer stock). Boychuck and Martell (ibid) indicate that more complex stochastic programming methods “would be justified in areas with a tight timber supply, lacking sufficient overmature areas, having high and highly variable fire losses, and where harvest quantity declines are particularly unwanted.”

Many forest planning problems include objectives besides allowable cut or financial returns from timber harvests. These nontimber objectives often are spatially explicit and many require spatial optimization methods to account for landscape patterns and arrangement effects (e.g., see Hof and Bevers 1998, 2002, Murray 2007). Forest planning problems with harvest area (“adjacency”) constraints (e.g., Murray 1999, Weintraub and Murray 2007, McDill et al. 2002, Goycoolea et al. 2005), forest patch considerations (Tóth and McDill 2008) or habitat requirements for species that
dwell in the interior ("core area") of mature forests (e.g., Öhman and Eriksson 1998, 2002, Öhman 2000, Hoganson et al. 2005, Wei and Hoganson 2007, Zhang et al. 2011) are typical examples. We note that, so far, few studies on the subject of forest planning under fire risk have been conducted with spatially explicit models. We anticipate that mean value approaches to modeling fire disturbance may be untenable in many spatially explicit stand- or cut block-based landscape-wide harvest scheduling problems, motivating this research.

In this paper, we explore the use of a spatial multistage stochastic programming model to select optimal harvest schedules for timber and core area joint production under the influence of wildfires. Because core area production is affected by the arrangement of stand conditions over a landscape and the number of possible arrangements is large for even a small forest, we incorporate random stand-replacing fire events into our model using a sample average approximation (SAA) formulation (Kleywegt et al. 2001, Bevers 2007). This model is similar in many respects to the model III harvest scheduling formulation (Gunn 1991), which models forest growth and harvesting by balancing the area of forest entering and leaving each state (age class) in each stage (planning period). Boychuck and Martell (ibid.) developed their stochastic programming formulation from a model III framework by replacing mean values for fire disturbance rates with discrete two-point probability distributions of fire disturbance. In this study, we employ random stand-replacing fires that maintain stand boundaries to address not only fire effects on allowable cut but the additional spatial concern of retaining mature forest core area over time. We built and solved our models on a 64-bit workstation equipped with two quad-core processors and 32 GB of memory using IBM’s ILOG-CPLEX Parallel v.12.2 solver. The models proved difficult to solve in many cases. Consequently, we conducted our experiments on simplified artificial forest problems to study model performance and the effects of fires on timber and core area production. We also resorted to using relatively small sample sizes and solving multiple independent, identically distributed (i.i.d.) model replications, followed by statistical selection methods to evaluate candidate solutions, as suggested for some problems by Kleywegt et al. (2001) and described below.

Methods

We first introduce a deterministic even-aged harvest scheduling formulation that is constructed to maintain the integrity of stand boundaries. We step through the development of our model in subsequent sections.

A Deterministic Timber Harvesting Formulation

We consider the stand as the smallest management unit so that decisions need to be made for an entire stand. For example, if clearcutting is the only management prescription to be applied, this model will assume we can either clearcut a whole stand or not cut it at all. Binary variables are used to track management decisions for each stand in each period. This model coordinates stand level decisions across time to maximize forest level timber-based economic returns.

\[
\text{Max } \sum_{i} \sum_{j} \sum_{k} \sum_{t} v_{ijk} X_{ijk} - q \sum_{t \in \{2, T\}} Q_{t} \quad (1.1)
\]

\[
\text{St. } \sum_{j} X_{ijk1} = 1 \quad \forall i \quad (1.2)
\]

\[
\sum_{j} \sum_{k} g_{ij} X_{ij(t-1)} - \sum_{j} X_{ijk} = 0 \quad \forall i, j, t, \in \{2, T\} \quad (1.3)
\]

\[
Q_{t} \geq \sum_{i} \sum_{j} \sum_{k} c_{ijk} X_{ij(t-1)} - \sum_{j} \sum_{k} g_{ijk} X_{ijk} \quad \forall t \in \{2, T\} \quad (1.4)
\]

\[
X_{ijk} \text{ is binary } \forall i, j, k, t \quad (1.5)
\]

The objective function maximizes the total discounted return of managing a forest for \(T\) periods minus a penalty on any declines in timber production between the \((T-1)\) pairs of consecutive periods. Penalties or constraints on declines in timber production commonly are used to secure a sustainable flow of timber supply to local timber markets and to prevent overharvesting in earlier time periods when discount rates have less effect (Nautiyal and Pearse 1967). We employed this penalty instead of a nondeclining yield constraint (similar to Gassmann 1989), using a positive coefficient \(q\) as the penalty for each unit of timber decline between each pair of consecutive periods. Constraint 1.2 requires that exactly one stand management activity, either clearcut or no cut, is selected for each stand at period
Table 1. Three-period example illustrating the sampling notation used in our sample-based stochastic programming model.

<table>
<thead>
<tr>
<th>Period 1 sample set $N_1$</th>
<th>Period 2 sample set $N_2$</th>
<th>Period 3 sample set $N_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $_i$</td>
<td>(1,1)$_i$, (1,1)$_j$</td>
<td>(1,1)$_k$</td>
</tr>
<tr>
<td>(2) $_h$</td>
<td>(2,3)$_h$, (2,3)$_j$</td>
<td>(2,3)$_k$</td>
</tr>
<tr>
<td>(2) $_j$</td>
<td>(2,4)$_j$, (2,4)$_k$</td>
<td>(2,4)$_k$</td>
</tr>
</tbody>
</table>
| We assume random fires can occur in each of the three periods. Suppose two random sample sets are drawn for a stand in period one and two more samples are drawn for each resulting state in subsequent periods. In this example, we would have two sample forest states at the end of period one, four at the end of period two, and eight at the end of period three for this one forest stand. $N_i$ denotes the set of randomly generated stand sample states during period $i$. We index each random state in the set $N_i$ in period $i$ by $(n)_i$. To identify each random state in the set $N_i$, we use $(n)_i$ to index each random state in the set $N_i$ in period $i$. This SAA formulation is given by

\[
\text{Max} \sum_{n \in \{1, 2, \ldots, 8\}} 1_{N_i} \sum_{n} \sum_{j} \sum_{k} \sum_{h} D_{ij} h(n)_i = 0 \quad \forall i, j, t \in \{1, 2, \ldots, 8\}, (n)_i \in N_i^{(t)} (2.2)
\]

\[
Q_{ij}(n)_i = \sum_{j} \sum_{k} \sum_{h} c_{ijk} X_{ijk}(n)_i - \sum_{j} \sum_{k} \sum_{h} c_{ijk} X_{ijk}(n)_i = 0 \quad \forall i, j, t \in \{1, 2, \ldots, 8\}, (n)_i \in N_i^{(t)} (2.5)
\]

where $X_{ijk}(n)_i$ are binary variables. $H_{ij}$ is the set of fire disturbances under which a stand $i$ transits from age class $j$ to $k$. $h$ is the index of the disturbances in the set $H_{ij}$. For the simplified cases in this study, each set can hold two disturbance types: “stand-replacing fire” and “no-fire.” Other types of fire disturbances such as stand-damaging fire could also be modeled. $D_{ij} h(n)_i$ is a binary variable used to track the actual occurrence of fire disturbance type $h$ in stand $i$ for sample paths where it is in age class $j$ in time period $t$, based on the random sample $(n)_i$. $D_{ij} h(n)_i = 1$ indicates that fire disturbance type $h$ would occur in sample $(n)_i$, as explained below.

Adding Random Fires With a Fixed Sequence Between Harvesting and Fires

In the example formulation below, we first build random samples of stand-replacing fires into our model based on the assumption that fires always occur after management activities within each planning period. This assumption will be relaxed in the section that follows. Another simplification in this work is that we do not explicitly model fire spread from one stand to another.

In the formulation below, $N$ denotes a constant number of i.i.d. landscape-wide samples of stand-replacing fires generated as outcomes for each existing stand sample state in each time period (see Table 1). The beginning of each time period is a decision stage in our model. Either a clearcut or a no cut decision is made for each stand sample state at the start of the period. Logical “branches” in the constraints model implementation of the decision for that stand sample state in that stage followed by $N$ stand-replacing fire sample outcomes (either burned or not burned). Thus, decisions after the first period are recourse decisions, unique to the state of the stand at that point in time in that sample path, and nonanticipativity is imposed implicitly by presenting each decision with multiple random outcomes (Birge and Louveaux 1997).

We use $\bar{N}_i$ to denote the set of randomly generated stand sample states during period $i$. $\bar{N}_1$ includes only the initial stand state. Using the example in Table 1, if $N$ is set to two, $\bar{N}_1$ would include two sampled states in period one for each stand. Branching from each period one state creates two new states for each stand at period two; therefore, $\bar{N}_2$ would include four sample states. $\bar{N}_3$ would include eight states. By tracing from each of these eight sampled states back to the current stand state, we can identify a sample forest stand succession pathway across three periods (Table 1). We use $(n)_i$ to index each random state in the set $\bar{N}_i$ in period $i$. This SAA formulation is given below
stand in a planning period to build Constraints 2.3 and 2.4. In this example, we assume a stand could be in one of three age classes at the start of each period: age class one, two, or three. Harvesting and no cut are the only two available management options for each stand. For this exploratory study, we assume the probability of fire in each stand depends on its age class. See Reed (1994) for a method of estimating the historic probability of stand-replacing fires. An example (denoted as sample A) was built based on random draws to reflect the possible fire occurrence in this stand. Constraints 2.3 and 2.4 can then be designed to reflect the fire occurrence as a function of stand condition (age class) in sample A. For simplicity we omit the subscripts of the stand and the sample index (sample A) from each decision variable. We use this example to demonstrate how to set up the transitions between $X_i$ and $D_i$. Without harvesting, this stand will stay in the same age class in the same period (reflected by Equations SA.1 to SA.3). Harvesting would reset the stand age to zero (Equation SA.4). Equations SA.1 to SA.4 enumerate all possible age class transitions for this stand within the same planning period. After randomly drawing fires in this sample, we observe that some of these transition paths may not be possible. The example below depicts a case where a fire will occur if and only if the stand is in age class two or three and not harvested in this time period. We set the corresponding $D_i$s to zero through Equations SA.5 to SA.8 to block other paths.

\[
X_{age\_1, no\_cut} = D_{age\_1, no\_fire} + D_{age\_1, fire} \quad (SA.1)
\]

\[
X_{age\_2, no\_cut} = D_{age\_2, no\_fire} + D_{age\_2, fire} \quad (SA.2)
\]

\[
X_{age\_3, no\_cut} = D_{age\_3, no\_fire} + D_{age\_3, fire} \quad (SA.3)
\]

\[
X_{age\_1, cat} + X_{age\_2, cat} + X_{age\_3, cat} = D_{age\_0, no\_fire} + D_{age\_0, fire} \quad (SA.4)
\]

\[
D_{age\_0, fire} = 0 \quad (SA.5)
\]

\[
D_{age\_1, fire} = 0 \quad (SA.6)
\]

\[
D_{age\_2, no\_fire} = 0 \quad (SA.7)
\]

\[
D_{age\_3, no\_fire} = 0 \quad (SA.8)
\]

For a different sample, e.g., sample B in Figure 1, the same set of $X$ variables will be connected to another set of $D$ variables that reflect fire sample B. Constraint 2.5 advances each stand into an older age class. Constraint 2.6 tracks the declining timber flow in each sample between every pair of consecutive periods. In Constraint 2.6, $(n)_{t-1}$ is used to index the direct parent sample of $(n)$ as we described for Constraint 2.4. We use many samples to represent random fires and each sample has its own discounted return and associated timber flow. The new objective function (2.1) again has two components: the discounted return from managing the forest and the penalty on average timber production declining from any two consecutive periods. Discounted returns and timber flow penalties from each sample are weighted by one over the appropriate sample size $N^{-1}$ or $N^t$, creating a SAA model that maximizes the resulting estimate of expected net return.

**Modeling Random Sequences Between Management and Fire**

Stand-replacing fires in reality can happen before or after management activities in each period. For example, harvesting may liquidate the timber value before it can be destroyed by a stand-replacing fire; however, a stand-replacing fire might also make the harvesting impractical if the fire happens first. The assumption that harvesting always precludes fire may overestimate the allowable cut. In this study, we tested a more general formulation that assumes the sequence between harvesting and fire within a time period is also random. Following this assumption, we built an enhanced SAA formulation

\[
\begin{align*}
Max \quad & \sum_{i \in [1, T]} \frac{1}{N} \sum_{(n)} \sum_{j} \sum_{k} W_{ijk(n)} Q_{ijk(n)} \\
& - q \sum_{i \in [2, T]} \frac{1}{N} \sum_{(n)} Q_{ijk(n)},
\end{align*}
\]

subject to

\[
\sum_{i} X_{ijk(n+1)} = 1 \quad \forall i \quad (3.2)
\]

\[
W_{ijk(n)} = 0 \quad \forall i, j, k = 1, t \in [1, T], (n) \in \mathcal{N}_{i,j,k} \quad (3.3)
\]

\[
D_{ijk(n)} = 0 \quad \forall i, j', h, t \in [1, T], (n) \in \mathcal{N}_{i,j',k} \quad (3.4)
\]

\[
X_{ijk(n-1)} - W_{ijk(n)} = 0 \quad \forall i, j, k, t \in [1, T], (n) \in \mathcal{N}_{i,j,k} \quad (3.5)
\]
Table 2. Management decisions and fire occurrence can interact within a stand during each planning period. Randomly drawn samples model these interactions. In each sample, \( W_{ijk(n)} \) tracks whether a clearcut could be implemented as a consequence of the within-period timing of fire occurrence. This table lists six possible scenarios of fire occurrence and forest stand management (mgmt) in a given time period. The action associated with the appropriate value of \( W_{ijk(n)} \) is shown for each scenario.

<table>
<thead>
<tr>
<th>Sampled fire occurrence</th>
<th>Sampled sequence between mgmt and fire</th>
<th>Implemented mgmt ( W_{ijk(n)} )</th>
<th>Actual fire occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>no cut</td>
<td>irrelevant’</td>
<td>no cut</td>
</tr>
<tr>
<td>No</td>
<td>clearcut</td>
<td>irrelevant’</td>
<td>cut</td>
</tr>
<tr>
<td>Yes</td>
<td>no cut</td>
<td>irrelevant’</td>
<td>cut</td>
</tr>
<tr>
<td>Yes</td>
<td>clearcut</td>
<td>mgmt, fire</td>
<td>cut</td>
</tr>
<tr>
<td>Yes</td>
<td>no cut</td>
<td>irrelevant’</td>
<td>no cut</td>
</tr>
<tr>
<td>Yes</td>
<td>clearcut</td>
<td>mgmt, fire</td>
<td>no cut</td>
</tr>
</tbody>
</table>

*The sequence between fire and management does not matter in a given period if there is no fire, or if clearcut is not selected to manage this stand.

\[
\sum_{i} \sum_{kj} K_{ij} W_{ijk(n)} - \sum_{h} H_{ij} D_{ij(h(n))} = 0 \\
\forall i, j, t \in \{1, T\}, (n), j \notin N_{ij(t)} \tag{3.6}
\]

\[
\sum_{i} \sum_{kj} H_{ij} D_{ij(h(n))} - \sum_{i} X_{ijk(n)} = 0 \\
\forall i, j, t \in \{0, T - 1\}, (n), j \notin N_{ij(t)} \tag{3.7}
\]

\[
Q(n) = \sum_{i} \sum_{j} \sum_{k} c_{ijk} W_{ijk(n-1)} - \sum_{i} \sum_{j} \sum_{k} c_{ijk} W_{ijk(n)} \\
\forall (n), j \notin N_{ij(t)} , t \in \{2, T\} \tag{3.8}
\]

\( X_{ijk(n)}, W_{ijk(n)}, D_{ij(h(n))} \) are binary variables.

Modeling Forest Core Area

Preserving interior mature forest some distance from edge effects (core area) has been one of the important spatial forest management concerns in landscape forest planning (Zipper 1993, Bassent and Jordan 1995, Fischer and Church 2003). Mature forest core is the area of mature forest protected by a buffer area from edge effects of surrounding habitats. Core area has been integrated into forest planning through dynamic programming (Hoganson et al. 2005), mixed integer programming (Wei and Hoganson 2007), and heuristic models (Ohman and Eriksson 1998, 2002, Ohman 2000). Core area can be modeled by tracking the states of many predefined influence zones across time (Hoganson et al. 2005). While core area has been modeled in this fashion before, it has not previously been modeled in the presence of wildfire to our knowledge.

The concept of influence zones can be used to account for how forest management could preserve mature forest core area across a planning horizon. Influence zones are delineated through a separate geographic information system (GIS) process (Hoganson et al. 2005). Each influence zone can be considered as a portion of the forest where a unique set of stands influence whether the area in the zone will produce core area of mature forest in a given time period. An example forest of four stands (A, B, C, and D; Figure 2A) (Wei and Hoganson 2006) illustrates this concept and the associated modeling method. By buffering outward from the boundaries of each stand for a predefined distance, a set of influence zones (A, B, C, AB, BC, BD, CD, and BCD) can be identified (Figure 2B). Using influence zone AB as an example, the ages of both stand A and B must satisfy the age requirement of mature forest for this influence zone to be classified as mature forest core area. It is clear that implementing the concept of influence zone requires the model to maintain the stand boundaries. The probability of each zone becoming core area depends on the joint probability of all stands forming the zone to meet the age requirement of mature forest. Modeling joint
probability directly is challenging due to potential nonlinearity. This makes the sampling-based approach more appealing.

The influence zone concept has been modeled in a deterministic context, in which harvesting decisions determine whether each influence zone would become core area at the end of each period. We substitute Equations 3.5 and 3.6, Equation 4.2 can be used to replace Equation 4.1 in the stochastic model.

Fire occurrence is simulated in each randomly generated sample. The SAA model tracks the amount of core area produced in each sample at the end of each planning period. For this study, we chose to use average minimum core area as our performance measure. To address this, a Max(Min(β)) approach (e.g., Bevers 2007) is used. This is accomplished as follows

(1) For each sampled forest succession pathway, calculate the minimum core area produced from period one to T.
(2) Build a bookkeeping constraint to average the minima core area calculated in step 1 across all sampled succession pathways.
(3) Value the average minimum core area calculated from step 2 in the objective function.

Based on the above discussion, we built an integrated model to incorporate the impact of fire into a spatially explicit harvest-scheduling model with core area concerns. By including core area estimation, our SAA stochastic programming model addresses a much
more complex landscape spatial optimization problem. This new model includes Constraints 3.2 to 3.7 to model fire occurrence and stand age class succession. Constraint 3.8 is included to track timber production declines between consecutive periods. Constraint 4.2 is included to track whether each influence zone produces forest core area at the end of each period given fire disturbances and management actions. Objective function (5.1) below is used to maximize the total weighted average return from timber, the average minimum core area along all sampled forest succession pathways, and a penalty for average timber production declines between any two consecutive periods across all samples. Bookkeeping Constraints 5.2, 5.3, and 5.4 below are used to support the Max(Min()) model format. By using spatially explicit constraints to track core area preservation, this model maximizes the estimated expected total joint benefits from timber harvesting and core area conservation in a forest over a given planning horizon.

\[
\text{Max} \sum_{a(1,2)} \sum_{(n)} \sum_{i} \sum_{j} v_{ijk} w_{ijk}(n) - q \sum_{a(2,3)} \sum_{(n)} q_{a} + q' V_{a} \tag{5.1}
\]

subject to

\[
V_{(a)} = \sum_{i \in \alpha} o_{i} v_{(a)} \quad \forall t \in \{1, T\}, \alpha \in \mathcal{N}
\tag{5.2}
\]

\[
V_{(a)} \leq V_{(n)} \quad \forall (n), t \in \{1, T\}, (n) \in \mathcal{P}(n)
\tag{5.3}
\]

\[
V_{a} = \frac{1}{N} \sum_{(a)} V_{(n)}
\tag{5.4}
\]

The area of influence zone \(z\) is denoted as \(o_{z}\). Constraint 5.2 calculates \(V_{(a)}\), which is the total core area produced in each random sample at the end of planning period \(t\). \(V_{(n)}\) is the minimum total core area preserved in the forest along each sampled forest succession pathway. The term \(P_{(n)}\) denotes the set of samples along the succession pathway ending with \((n)\). Using the example in Table 1, \(P_{(5)}\) represents a sample set \(\{(2), (3), (5)\}\). Equation 5.4 calculates \(V_{a}\) that denotes the average \(V_{(n)}\) across all sampled pathways.

**Selecting the First-Period Solution**

A primary purpose for using a multistage stochastic harvest scheduling model is to identify first-period harvests that perform well over a range of plausible future conditions (Hoganson and Rose 1987). Harvesting decisions for the first period need to be implemented immediately. For periods one decisions may also have longer-term impacts on future timber production and spatial forest structure. The quality of the first-period decisions can be used to evaluate overall model performance.

In our SAA model, randomly generated fires are used to estimate the expected consequences of stand-replacing fires and recourse management actions. Replicate models built on different independent i.i.d. random fire samples sometimes suggest different period one harvest schedules. Larger sample sizes can be used to better inform the model about future fires up to a point. However, more samples also increase model complexity and make the model more difficult to solve. An alternative approach is to build multiple SAA models using different sets of i.i.d. fire samples (see Kleywegt et al. 2001). Solutions from the \(R\) different models can be used to estimate the “persistence” (Bertsimas et al. 2007) of first-period harvest deci-

**Test Cases and Results**

We built a computer-simulated forest with eleven stands as the test site (Figure 3). We divided a 120-year planning horizon into three 40-year planning periods. At the beginning of each period, a stand is in either age-class one (\(\leq 39\) years), two (\(40–79\) years), or three (\(80+\)). At the beginning of the planning horizon, stands 1, 4, 7, and 10 are assumed in age class one; stands 2, 5, 8, and 11 are in age class two; and stands 3, 6, and 9 are in age class three. Stands older than 40 years (in either age class two or three) are considered
mature forest for core area calculations. Areas of mature forest 50 m away from the boundary of a young forest (<40 years old) or the boundary of the forest are counted as interior forest habitat, or core area. Using this assumption about 8,300 influence zones are identified to model core area.

Two management alternatives are assumed available for each stand in each planning period: no cut or clearcut. Clearcuts reset the stand age back to zero in the period of harvest. We assume clearcutting a stand in age class one generates no financial returns. Stand-replacing fire sets the stand age back to zero in the period of harvest. We assume clearcutting of all stands at age class two and three should be harvested during the first period to maximize expected returns (Table 3). The benefits of using larger sample sizes are also reflected by the decreased standard deviations of the average minimum core area, the timber productivity of each period, and the objective function value across the 30 tested SAA model runs (Table 4).

We assumed stands in age classes two or three are mature forest and can contribute to forest core area when they occur away from edges. However, older forest could be more susceptible to fires. Under this assumption, higher fire probability increases the cost of maintaining core area. Test results show, when core area price is set at $500/acre under the high fire probability assumption, no core area should be maintained across the planning horizon (Table 4). By increasing \( N \) to 10, most model runs found no benefit in maintaining any core area (Table 4) and suggest all stands at age class two and three should be harvested during the first period to maximize expected returns (Table 3). The benefits of using larger sample sizes are also reflected by the decreased standard deviations of the average minimum core area, the timber productivity of each period, and the objective function value across the 30 tested SAA model runs (Table 4).

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Comparing the Quality of First-Period Solutions

Forest management occurs across space and time. Management decisions for period one need to be carried out without knowing future fire conditions with certainty. Decisions for later periods, on the other hand, can be adjusted based on the actual management activities and fire occurrences in earlier periods (Savage et al. 2010). A good period one solution should help facilitate adjustment of future forest management activities. Different sample sets used by the SAA model may suggest different period one solutions. Changing sample size \( N \) also causes the model to select different period one solutions as described in Table 3. While we expect better solutions to be derived from models built on larger sample sizes, the problem of selecting a single period one solution from a set of differing solutions remains.

We first revised the SAA model to find the optimal period one solution under the assumption of no fire risk, $500/acre core area price, and no penalty for timber production declines. This leads to a
Influence of the Declining Timber Flow Penalty and Fire Probability

We tested two variations of forest management assumptions regarding the penalty on timber declines and core area prices: (1) $500/acre core area price with a penalty of $90/cord for timber production declines between two consecutive periods; and (2) $500/acre core area price and no penalty for timber declines. In these tests, we also assumed the chance of stand-replacing fire is low (as previously defined). We used two sample sizes to build the SAA model: $N = 6$ and $N = 10$. Increasing $N$ from 6 to 10 created a much larger mixed integer programming (MIP) model. To prevent our computer from running out of memory while using the CPLEX solver, we set the maximum computing time for solving each SAA model to 1 hour and we allowed the solver to stop when a feasible integer solution within 5% of the best possible solution was found.

Results show that a $90/cord penalty effectively prevents timber production from declining between two consecutive planning periods. However, the differences are not statistically significant at the 95% confidence level.

Table 4. A statistical summary of 30 runs based on different random draws. Each run is based on either a core area price of $500/acre or $1,000/acre. No penalty is imposed on declines in timber production. High fire risk is assumed for each stand in the forest.

<table>
<thead>
<tr>
<th>Sample size</th>
<th>Average min core (acres)</th>
<th>Timber yield at three periods</th>
<th>Obj. ($)</th>
<th>Average min core (acres)</th>
<th>Timber yield at three periods</th>
<th>Obj. ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 15$</td>
<td>$500/acre core area price</td>
<td>$P_1$ 4.621 $P_2$ 1.846 $P_3$ 3.886 $63,758$</td>
<td>$P_1$ 0 $P_2$ 165 $P_3$ 97 $165$</td>
<td>$N = 15$</td>
<td>$1,000/acre core area price</td>
<td>$P_1$ 4,621 $P_2$ 1,590 $P_3$ 3,628 $75,732$</td>
</tr>
<tr>
<td>$N = 10$</td>
<td>$500/acre core area price</td>
<td>$P_1$ 4.628 $P_2$ 1.807 $P_3$ 3.889 $63,992$</td>
<td>$P_1$ 1 $P_2$ 288 $P_3$ 149 $288$</td>
<td>$N = 10$</td>
<td>$1,000/acre core area price</td>
<td>$P_1$ 4,328 $P_2$ 1,234 $P_3$ 3,251 $68,006$</td>
</tr>
<tr>
<td>$N = 5$</td>
<td>$500/acre core area price</td>
<td>$P_1$ 4.449 $P_2$ 1.657 $P_3$ 3,622 $64,705$</td>
<td>$P_1$ 4 $P_2$ 491 $P_3$ 326 $491$</td>
<td>$N = 5$</td>
<td>$1,000/acre core area price</td>
<td>$P_1$ 3,992 $P_2$ 1,556 $P_3$ 3,305 $65,623$</td>
</tr>
<tr>
<td>$N = 4$</td>
<td>$500/acre core area price</td>
<td>$P_1$ 3,992 $P_2$ 1,556 $P_3$ 3,305 $65,623$</td>
<td>$P_1$ 6 $P_2$ 638 $P_3$ 465 $638$</td>
<td>$N = 4$</td>
<td>$1,000/acre core area price</td>
<td>$P_1$ 3,849 $P_2$ 1,234 $P_3$ 3,100 $67,059$</td>
</tr>
<tr>
<td>$N = 3$</td>
<td>$500/acre core area price</td>
<td>$P_1$ 3,419 $P_2$ 1,218 $P_3$ 3,100 $67,059$</td>
<td>$P_1$ 8 $P_2$ 928 $P_3$ 523 $928$</td>
<td>$N = 3$</td>
<td>$1,000/acre core area price</td>
<td>$P_1$ 3,690 $P_2$ 1,590 $P_3$ 3,628 $75,732$</td>
</tr>
<tr>
<td>$N = 2$</td>
<td>$500/acre core area price</td>
<td>$P_1$ 3,419 $P_2$ 1,218 $P_3$ 3,100 $67,059$</td>
<td>$P_1$ 8 $P_2$ 928 $P_3$ 523 $928$</td>
<td>$N = 2$</td>
<td>$1,000/acre core area price</td>
<td>$P_1$ 3,690 $P_2$ 1,590 $P_3$ 3,628 $75,732$</td>
</tr>
<tr>
<td>$N = 1$</td>
<td>$500/acre core area price</td>
<td>$P_1$ 3,690 $P_2$ 1,590 $P_3$ 3,628</td>
<td>$P_1$ 14</td>
<td>$N = 1$</td>
<td>$1,000/acre core area price</td>
<td>$P_1$ 4,621 $P_2$ 1,846 $P_3$ 3,886</td>
</tr>
</tbody>
</table>

* The relative gap between the best solution found and the best possible solution is set to 5% to prevent the model from running out of memory. All other runs used a relative gap of 1%.

**Table 5.** Multiple comparison of the performance of different period one solutions discovered from the deterministic approach (denoted by DET) and from stochastic programming with sample sizes $N = 1, 2, or 3$ (denoted by N1, N2, and N3).

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Average difference in 300 samples</th>
<th>Lower bound of 95% confidence interval</th>
<th>Upper bound of 95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obj(N1)–Obj (DET)</td>
<td>23,803</td>
<td>20,526</td>
<td>27,080</td>
</tr>
<tr>
<td>Obj(N2)–Obj (DET)</td>
<td>24,995</td>
<td>21,718</td>
<td>28,272</td>
</tr>
<tr>
<td>Obj(N3)–Obj (DET)</td>
<td>25,845</td>
<td>22,568</td>
<td>29,122</td>
</tr>
<tr>
<td>Obj(N2)–Obj(N1)</td>
<td>1,192</td>
<td>–2,085</td>
<td>4,469</td>
</tr>
<tr>
<td>Obj(N3)–Obj(N1)</td>
<td>2,042</td>
<td>–1,235</td>
<td>5,319</td>
</tr>
<tr>
<td>Obj(N3)–Obj(N2)</td>
<td>850</td>
<td>–4,247</td>
<td>4,127</td>
</tr>
</tbody>
</table>

The objective function values from implementing different period one solutions (Obj(DET), Obj(N1), Obj(N2), and Obj(N3)) are compared through 300 new random samples. Each new sample represents a random sequence of fire events and their interaction with management activities across three 40-year planning periods. In the stochastic model, we assume $500/acre core area price, no penalty for declining timber production, and high fire probability. The confidence interval around the average difference in objective function value is calculated. Tests show the solution by using $N = 3$ gives the highest objective function value.
select the period one decision for each stand. The test cases show, when fire risk is assumed to be low, a decision to clearcut will be preferred at period one only in stand 2 (21 out of 30 runs selected clearcut), stand 5 (21 of 30 runs selected clearcut), and stand 6 (all runs selected clearcut) when timber production declining is not penalized. Under the high fire risk assumption, 7 out of the 11 stands are selected for clearcut at period one (Table 3).

**Conclusion and Discussion**

Stochastic disturbances such as fire, insect, disease, or wind can have a significant impact on long-term forest management. Ignoring the effects of these disturbances in forest plans can lead to suboptimal or incorrect decisions and conclusions. Spatial considerations such as core area, edge effects, adjacency, and patch connectivity pose additional challenges to building disturbance impacts in forest management models. Without spatial considerations, stochastic disturbances might be modeled adequately using mean probabilities of different scenarios implemented as average fractions of land being disturbed. Under a spatial context, using average disturbance rates seems less likely to be adequate for approximating system behavior and optimal management. Even in stochastic programming models it can be difficult to select adequate representative scenarios from the enormous number of possible spatial disturbance patterns and resulting forest spatial structures; sample-based methods might be required, like those used in this study. Spatial forest management models often use binary variables to track and form desired spatial structures such as core area or to prevent undesired spatial conditions such as violations of adjacency constraints or harvest block size restrictions. Both scenario- and sample-based stochastic integer programming approaches can support these binary model formulations.

Harvest decisions for later time periods can often be adjusted through recourse actions, whereas implementation of many first-period decisions must begin immediately. A primary reason for using multistage stochastic programming is to improve the quality of the first stage, or first period, decisions while taking recourse opportunities into account. This research demonstrates that integrating fire risk explicitly in a harvest scheduling model can lead to better first-period decisions than decisions made using deterministic models. Modeling the opportunity to adjust plans through recourse decisions that account for recent fires clearly improved our solutions.

We examined the benefit of using larger sample sizes with a number of test cases. In our models, larger sample sizes improved the persistence of the period-one solutions. Larger sample sizes also reduced the variation in solutions, as indicated by lowering the SD of objective function values across many replicate i.i.d. SAA model runs. However, increasing the sample size also increased model complexity and model size. This study solved relatively small 11-stand and three-period artificial harvest scheduling problems under generous simplifying assumptions such as 40-year planning periods and independent fire events in each stand. These assumptions may not be too unreasonable in forests with long harvest rotations and with dense road networks that support rapid fire response. However, these assumptions do not hold in many other places. We employed these simplifications to allow us to focus on the stochastic model formulation itself. A potential future study might include additional details such as fire spread from stand to stand.

Solving larger forest planning problems with more complicated fire and forest conditions might be possible on computers with more memory by developing more efficient modeling formulations or by employing other solution approaches such as decomposition methods. This model also defines binary variables $W_{jk(n)}$, to add more flexibility in tracking whether a scheduled cut could be implemented before a fire. It would be an interesting future study to compare between modeling detailed fire-management sequences enabled by $W_{jk(n)}$, and using smaller time intervals, e.g., every 10 years.

**Literature Cited**


