BOUNDARY SHEAR STRESS ALONG RIGID TRAPEZOIDAL BENDS

Prepared for the

U. S. Department of the Interior
Bureau of Reclamation
Albuquerque Area Office
555 Broadway N.E., Suite 100
Albuquerque, New Mexico 87102-2352

This research was supported in part by funds provided by the
Rocky Mountain Research Station, Forest Service, U. S. Department of Agriculture.

Prepared by

Christopher I. Thornton, Kyung-Seop Sin, Paul Sclafani, and Steven R. Abt

June 2012

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EXECUTIVE SUMMARY

A study was conducted in which the results of four laboratory investigations that determined the ratio of shear stress in a channel bendway to the straight channel approach shear stress (K_b), were compiled and related to the channel radius of curvature and channel top width. The composite data were compared to procedures presented by the U. S. Department of Transportation *Hydraulic Engineering Circular No. 15* (*HEC-15*) for determining K_b. Then, four alternative approaches were developed and evaluated for determining K_b based upon the composite data. The four proposed K_b prediction approaches yield more conservative K_b values than the *HEC-15* predictions by 15 to 50%. Unique, more conservative approaches for predicting K_b are recommended for routine river bendway evaluations as well as for bendway applications that influence high property values and/or a threat to life. These findings are applicable to rigid boundary, trapezoidal channels.

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LIST OF SYMBOLS, UNITS OF MEASURE, AND ABBREVIATIONS

K_b	=	ratio of shear stress in a channel bendway to the straight channel approach shear stress [dimensionless]
$K_{Bend\text{-}Shear}$	=	ratio of shear stress in channel bend to straight channel approach shear stress at maximum depth
R^2	=	coefficient of determination
R_c	=	radius of curvature of the bend to the channel centerline [L]
R_c/T_w	=	ratio of radius of curvature to top width
T_w	=	channel top width (water surface) [L]

Units of Measure

º	degree(s)
ft	foot or feet
ft/ft	feet per foot
m	meter(s)
%	percent

Abbreviations

CSU	Colorado State University
FHWA	Federal Highway Administration
H:V	Horizontal:Vertical
HEC-15	*Hydraulic Engineering Circular No. 15*
USBR	U. S. Bureau of Reclamation

1 INTRODUCTION

The migration of alluvial channels through the geologic landform is an outcome of the natural erosive processes. Mankind continually attempts to stabilize channel meandering processes, both vertically and horizontally, to reduce sediment discharge, provide boundary definition, and enable economic development along the river's edge. A critical component in the reduction of bank erosion and subsequent channel migration is to stabilize river bendways. The mitigation of erosion is addressed through the damping of the magnitude and orientation of the applied shear stresses developed by the river's flow (Julien 2002). Therefore, bank-protection measures are required to resist the erosive forces imparted by applied shear stresses, but often at significant expense.

In order to properly size bank protection with an acceptable level of risk, the engineer requires a design tool that appropriately predicts the applied shear stresses that develop at the river bendway – both bed and bank. The literature adequately provides insight into estimating the shear stresses attributed to the imbalance in radial pressure around bends resulting in transverse velocity distribution, to flow super-elevation impacts on outer banks in bendways, and to cross flows and secondary circulation as the discharge transitions from straight to bendway sections of the channel (Federal Highway Administration (FHWA) 2001). One of the most common empirically-derived approaches to estimating the increase in shear stress associated with a meander bend is presented by the FHWA (2005), in which they develop a series of relationships between the ratio of bend shear stress to the straight channel approach shear stress and the ratio of radius of curvature to top width (R_c/T_w) presented as:

$$K_{Bend\text{-}Shear} = 2.0 \qquad [R_c/T_w] \leq 2 \qquad \textbf{Equation 1.1}$$

$$K_{Bend\text{-}Shear} = 2.38 - 0.206[R_c/T_w] + 0.0073[R_c/T_w]^2 \qquad 2 < [R_c/T_w] < 10 \qquad \textbf{Equation 1.2}$$

$$K_{Bend\text{-}Shear} = 1.05 \qquad 10 > [R_c/T_w] \qquad \textbf{Equation 1.3}$$

where

$K_{Bend\text{-}Shear}$ (K_b) = ratio of shear stress in channel bend to straight channel approach shear stress at maximum depth [dimensionless];

R_c = radius of curvature of the bend to the channel centerline [L]; and

T_w = channel top width (water surface) [L].

Equations 1.1 through 1.3 have been adopted by the FHWA (2005) as a recommended approach for predicting the bend shear stress thereby providing guidance as to the appropriate level of protection required along the outer channel to stabilize the bank and slope toe.

The objective of this report is to evaluate the boundaries of application and the degree of conservatism for determining the applied shear stresses developed in channel bends using the FHWA (2005) procedure. Their approach will be assessed by obtaining laboratory data from multiple published laboratory studies and comparing these data to the FHWA (2005) guidelines.

2 BACKGROUND

The determination of shear stress(s) in a channel bendway is different in both distribution and magnitude than in a straight channel reach (FHWA 2005). Usually, higher than average shear stresses (i.e., bed shear stress in straight channels) occur at both the entrance and exit of the outer bend. At the inside of the bend curve entrance, an uplifting force may also occur that is not accounted for in many shear stress analyses. Generally, short radius bends experience greater increases in shear stress than longer radius bends.

Four approaches are often considered to determine bed shear stress in open-channel bendways: 1) linear regression of velocity profile (Clauser 1956, Schlichting 1987, Smart 1999); 2) Rozovskii Method (Rozovskii 1961); 3) Reynolds shear stress extrapolation (Julien 1998, Wahl 2000, Baird 2004); and 4) boundary shear stress determination with a Preston tube. In each of the first three procedures, the shear stress is computed using a series of flow variables as stipulated by each investigator and may be considered non-simplistic by the applied engineer. The Preston tube is a direct measurement approach using Pitot tubes to determine the differential pressure along the boundary of the channel; the differential pressure is then calibrated to a shear stress. Sin (2010) compared these four approaches to determine shear stress using a common laboratory data set, and concluded that the most consistent and accurate approaches to determining shear stress in a bendway channel are the Reynolds and Preston-tube techniques. It was further indicated that once the Preston tube is calibrated, it represents the most direct means of determining boundary shear stress. The literature provides at least four studies in which bendway shear stress is determined using the Preston tube.

In 1962, Ippen and Drinker conducted a series of sixteen experiments measuring the boundary shear stress in rigid, trapezoidal-shaped channel bends with 1-ft (0.3-m) and 2-ft (0.61-m) bottom widths in a singular bendway. A calibrated Preston tube was utilized to determine the boundary shear stresses throughout each channel under a variety of flow conditions. The channel walls (2H:1V (Horizontal:Vertical)) and floor were smooth for all experiments. Table

2.1 presents a summary of the channel bottom widths, R_c/T_w, and resulting K_b values. K_b values ranged from 1.59 to 3.00. Ippen and Drinker (1962) concluded that for shallow channel flows, the location of maximum shear stress was on the outer bend, downstream of the apex. For deep channel flow, the maximum boundary shear stress occurred on the inner bank near the upstream end of the bend.

Table 2.1: Composite Preston-tube Determined K_b Data

	Investigator(s)	Channel Width (ft)	R_c/T_w	K_b	Side Slope (H:V)
1	Ippen and Drinker (1962)	2.0	1.67	2.00	2:1
2	Ippen and Drinker (1962)	2.0	1.52	1.78	2:1
3	Ippen and Drinker (1962)	2.0	1.50	1.78	2:1
4	Ippen and Drinker (1962)	2.0	1.35	2.20	2:1
5	Ippen and Drinker (1962)	2.0	1.35	2.20	2:1
6	Ippen and Drinker (1962)	2.0	1.25	2.40	2:1
7	Ippen and Drinker (1962)	2.0	1.25	2.40	2:1
8	Ippen and Drinker (1962)	1.0	3.49	1.59	2:1
9	Ippen and Drinker (1962)	1.0	2.91	1.60	2:1
10	Ippen and Drinker (1962)	1.0	2.51	1.76	2:1
11	Ippen and Drinker (1962)	2.0	1.47	2.51	2:1
12	Ippen and Drinker (1962)	2.0	1.22	2.80	2:1
13	Ippen and Drinker (1962)	2.0	1.52	2.22	2:1
14	Ippen and Drinker (1962)	2.0	1.25	2.40	2:1
15	Ippen and Drinker (1962)	2.0	1.52	2.86	2:1
16	Ippen and Drinker (1962)	2.0	1.25	3.00	2:1
17	U. S. Bureau of Reclamation (USBR 1964)	2.0	3.76	1.35	1.5:1
18	Yen (1965)	6.0	4.18	1.20	1:1
19	Yen (1965)	6.0	4.00	1.30	1:1
20	Yen (1965)	6.0	3.99	1.30	1:1
21	Yen (1965)	6.0	3.98	1.30	1:1
22	Yen (1965)	6.0	3.73	1.30	1:1
23	Colorado State University (CSU)	10.2	2.82	1.79	3:1
24	CSU	6.0	6.91	1.78	3:1
25	CSU	10.2	2.62	1.78	3:1
26	CSU	6.0	6.20	1.88	3:1
27	CSU	10.2	2.48	1.94	3:1
28	CSU	6.0	5.72	1.99	3:1
29	CSU	10.2	2.41	1.99	3:1
30	CSU	6.0	5.44	1.68	3:1

The USBR (1964) investigated the shear stress distribution of a trapezoidal-shaped, single-bend channel with 2-ft (0.61-m) bottom width, 1.5H:1V rigid side slopes, and central bend angle of 15°. The boundary shear stress was measured with a Preston tube. The maximum shear stress was located on the inner bank near the entrance to the bend. The resulting K_b from the single test is 1.35 as presented in Table 2.1.

Yen (1965) studied the shear stress distributions in a smooth, rigid boundary, trapezoidal-shaped channel with a 90º bend transitioning to an adjacent 90º bend downstream, 6-ft (1.82-m) bottom width, and 1H:1V side slopes. Shear stresses were measured at the entrance to the bend, through the tangent reach, and through the downstream bend. The boundary shear stress distribution was determined relative to the average boundary shear stress. Areas of maximum boundary shear stress were located along the inner wall of the curve entrance. Table 2.1 portrays the K_b values for the five experiments with values of 1.20 to 1.30.

Heintz (2002) constructed a smooth, rigid boundary, trapezoidal-shaped compound channel with bed slope of 0.00086 ft/ft and 3H:1V slide slopes. The channel was configured such that flow entered an upstream bend with curvature angle (bending right) of 125º (10.2-ft (3.1-m) bottom width) exiting into a straight transition. The transition section then entered a bend of curvature angle (bending left) of 73º (6-ft (1.82-m) bottom width). A Preston tube was calibrated and used to determine shear stress distributions throughout the channel under varied discharges. Heintz (2002) found that the maximum shear stresses were located at the entrance of the upstream bend near the inner bank, then migrated toward the outer bank at the bend exit, similar to Ippen and Drinker (1962). Inner bank shear stresses were measured to be approximately 50% higher at the bend entrance than at the same bend exit. Further, the outer bank shear stress at the entrance to the downstream bend increased more significantly than the entrance to the upstream bend. Sclafani (2010) and Sin (2010) evaluated the Heintz (2002) shear stress database for four discharges throughout the channel reach to include the two-bendway and transition sections. The resulting K_b values of the eight tests are presented in Table 2.1 (referenced as CSU for the combined efforts of Heintz (2002), Sclafani (2010), and Sin (2010)) range from 1.78 to 1.99.

3 ANALYSIS

The studies of Ippen and Drinker (1962), the USBR (1964), Yen (1965), and CSU present a composite of thirty unique K_b values (Table 2.1) derived using calibrated Preston tubes in rigid boundary, trapezoidal-shaped channels (bottom widths of 1 (0.3 m) to 10.2 ft (3.1 m)) with one or more channel bendways. The composite data for K_b and R_c/T_w from Table 2.1 were plotted with the *HEC-15* (FHWA 2005) relations expressed in Equations 1.1 through 1.3, as shown in Figure 3.1. It is observed that the Ippen and Drinker (1962) K_b values lie predominately above *HEC-15* guideline for R_c/T_w values between 0 and 2. The average K_b for R_c/T_w between 0 and 2 is 2.35 with a maximum K_b of 3.0. The *HEC-15* relation yields a K_b of 2.0.

The K_b values (CSU and Ippen and Drinker (1962)) for R_c/T_w between 2 and 3 cluster relatively tightly about the *HEC-15* relation. However, K_b values (Ippen and Drinker 1962, Yen 1965) begin to fall below the *HEC-15* guideline as R_c/T_w ranges from 3 to 5, indicating approximately a 24% drop in K_b. For R_c/T_w between 5 and 7, the CSU K_b values are elevated above the *HEC-15* relation approximately 25% at an R_c/T_w of 6. The CSU data are the first points reported for an $R_c/T_w > 5$. The data displayed in Figure 3.1 indicate that the *HEC-15* relation is reasonable for estimating K_b for R_c/T_w between 2 and 5. However, it appears that the *HEC-15* relations are not conservative for $R_c/T_w \leq 2$ or $R_c/T_w \geq 5$. Therefore, a more conservative means of estimating K_b for $R_c/T_w \leq 2$ and $R_c/T_w \geq 5$ may be warranted.

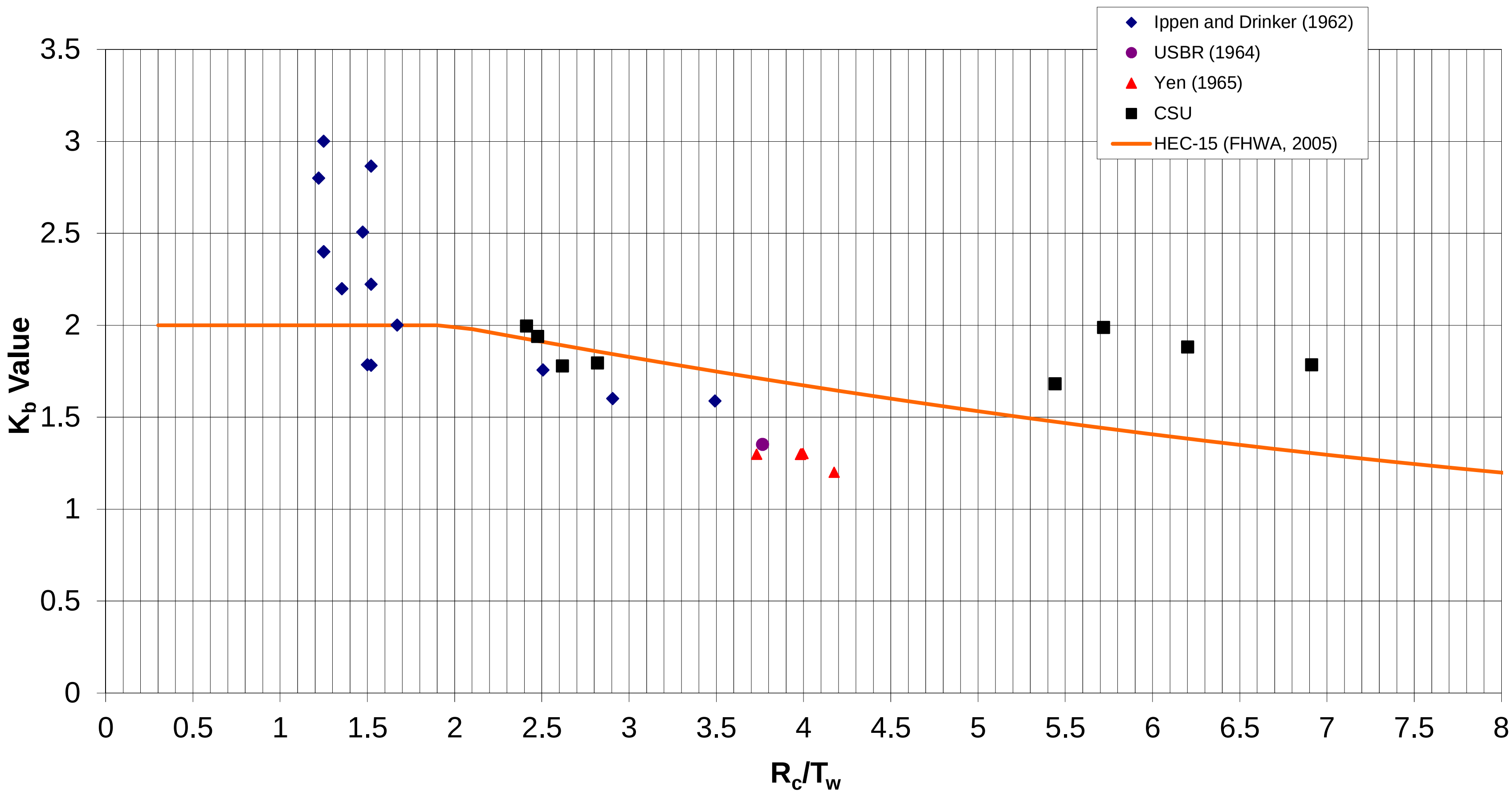

Figure 3.1: K_b Composite Database with *HEC-15*

3.1 Approach 1

A linear regression analysis was performed using only the Ippen and Drinker (1962) K_bs for $R_c/T_w \leq 2$ and the CSU K_bs for $R_c/T_w > 5$, as shown in Figure 3.2. The analysis yielded a predictive relationship for R_c/T_w values ranging from 2 to 7 as:

$$K_b = -0.113\,[R_c/T_w] + 2.51 \qquad \textbf{Equation 3.1}$$

where

K_b = ratio of shear stress in a channel bendway to the straight channel approach shear stress [dimensionless];

R_c = radius of curvature of the bend to the channel centerline [L]; and

T_w = channel top width (water surface) [L].

The coefficient of determination (R^2) is 0.33. In a fashion similar to the FHWA (2005), K_b is determined using Equation 3.1 applicable for R_c/T_w values ranging from 2 to 10. K_b is a constant of 2.29 for $R_c/T_w < 2$, as portrayed in Figure 3.2. It is apparent that Equation 3.1 does not provide the most desirable fit to the composite data, but the relation incorporates a more conservative approach to determining K_b without enveloping all of the data. K_b is thereby determined in a three-step model similar to *HEC-15* (FHWA 2005) where:

$$K_b = 2.29 \qquad [R_c/T_w] \leq 2 \qquad \textbf{Equation 3.2}$$

$$K_b = -0.113\,[R_c/T_w] + 2.51 \qquad 2 < [R_c/T_w] < 10 \qquad \textbf{Equation 3.3}$$

$$K_b = 1.05 \qquad [R_c/T_w] \geq 10 \qquad \textbf{Equation 3.4}$$

where

K_b = ratio of shear stress in a channel bendway to the straight channel approach shear stress [dimensionless];

R_c = radius of curvature of the bend to the channel centerline [L]; and

T_w = channel top width (water surface) [L].

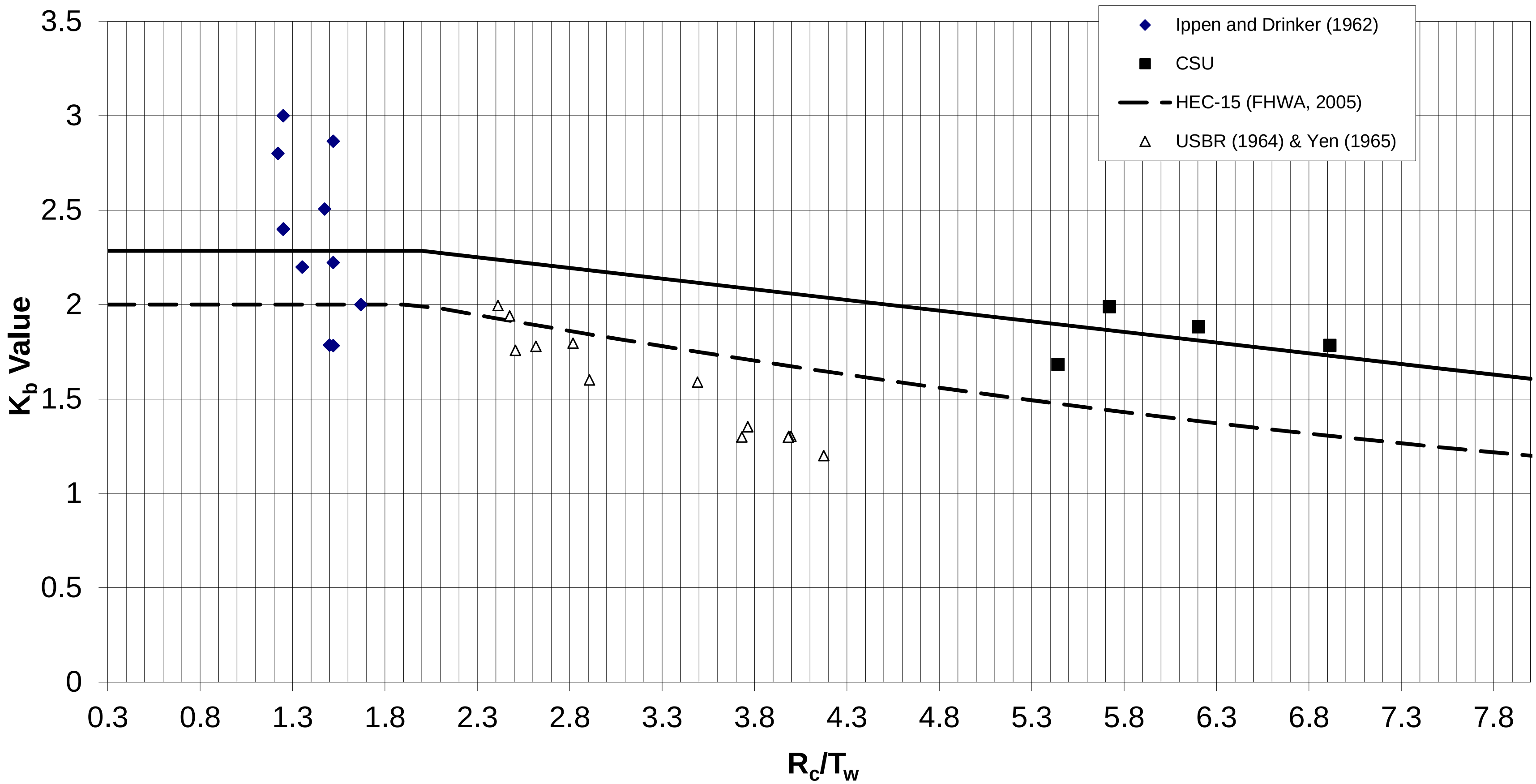

Figure 3.2: Best-fit Relation for Determining K_b, Using Only the Ippen and Drinker (1962) and CSU Data (Approach 1)

3.2 Approach 2

In an attempt to synthesize a multi-step model approach for determining K_b, as provided by the FHWA (2005), while incorporating the predictive segments that best fit the composite data; it is possible to transform the three-step *HEC-15* model to a four-step model as indicated in Figure 3.3. Therefore, the proposed K_b predictive steps are as follows:

$$K_b = 2.29 \qquad [R_c/T_w] \leq 2 \qquad \textbf{Equation 3.5}$$

$$K_b = 2.38 - 0.206[R_c/T_w] + 0.0073[R_c/T_w]^2 \qquad 2 < [R_c/T_w] < 5 \qquad \textbf{Equation 3.6}$$

$$K_b = -0.113[R_c/T_w] + 2.51 \qquad 5 \leq [R_c/T_w] < 10 \qquad \textbf{Equation 3.7}$$

$$K_b = 1.05 \qquad [R_c/T_w] \geq 10 \qquad \textbf{Equation 3.8}$$

where

K_b = ratio of shear stress in a channel bendway to the straight channel approach shear stress [dimensionless];

R_c = radius of curvature of the bend to the channel centerline [L]; and

T_w = channel top width (water surface) [L].

Although Approach 2 is slightly more complex than Approach 1, the level of conservatism is increased for R_c/T_w ranging from 0 to 5. Further, the predictive relations derived from Equations 3.6 and 3.7 (Approach 2) more closely align to the existing data.

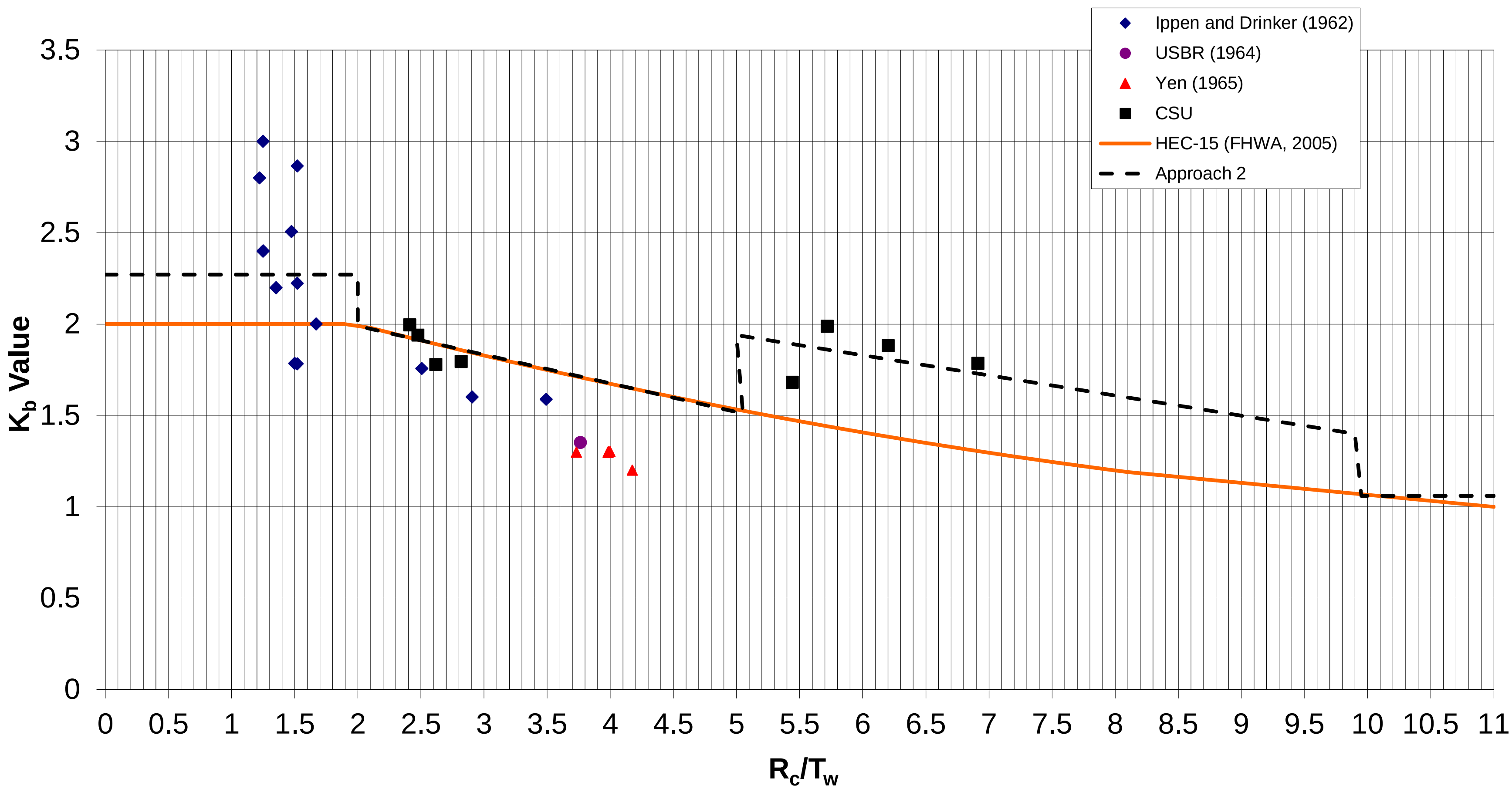

Figure 3.3: Multi-step Relation for Determining K_b

3.3 Approach 3

A power regression analysis was performed using the thirty data points from Table 2.1 to prescribe a best-fit relation for determining K_b across the spectrum of $R_c/T_{w,}$ as presented in Figure 3.4. The resulting relationship is expressed as:

$$K_b = 2.4992\,[R_c/T_w]^{-0.321} \qquad \textbf{Equation 3.9}$$

where

K_b = ratio of shear stress in a channel bendway to the straight channel approach shear stress [dimensionless];

R_c = radius of curvature of the bend to the channel centerline [L]; and

T_w = channel top width (water surface) [L].

The R^2 is 0.51. It is apparent that Equation 3.9 provides an improved fit to the data in predicting K_b for $R_c/T_w < 3.5$, but poorly aligns with the data for $R_c/T_w > 5$, similar to *HEC-15* (FHWA 2005). Equation 3.9 and *HEC-15* yield similar K_b predictions for R_c/T_w between 3.5 and 6. Projecting Equation 3.9 K_b estimates beyond R_c/T_w of 10 yields a K_b of approximately 1.16, an increase of 0.11 over the *HEC-15* guidance. Equation 3.9 presents a simplified means (single expression) of estimating K_b compared to *HEC-15*, Approach 1 and Approach 2.

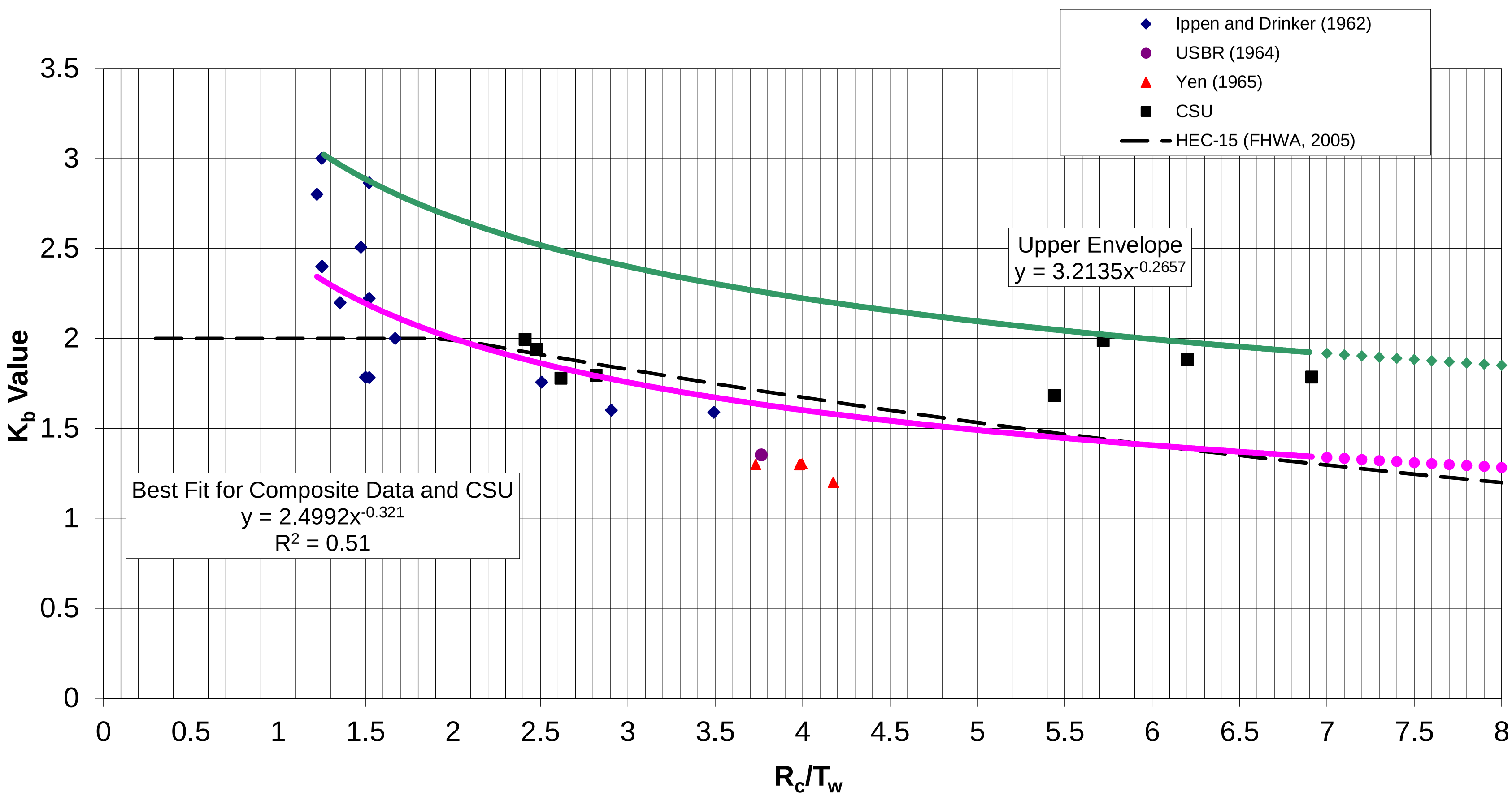

Figure 3.4: K_b **Determination, Utilizing the Composite Data**

3.4 Approach 4

A regression analysis was performed enveloping all thirty data points, as portrayed in Figure 3.5. The envelope relation is expressed as:

$$K_b = 3.2135 \, [R_c/T_w]^{-0.2657} \qquad \textbf{Equation 3.10}$$

where

K_b = ratio of shear stress in a channel bendway to the straight channel approach shear stress [dimensionless];

R_c = radius of curvature of the bend to the channel centerline [L]; and

T_w = channel top width (water surface) [L].

Equation 3.10 presents an extremely conservative approach to predicting K_b values. The envelope relation yields K_b values approximately 40% higher than Equation 3.9, using a best-fit approach. Although Equation 3.10 yields conservative K_b values, circumstances may be warranted for this approach if bendway migration potentially results in an extreme risk to high-valued property or human life.

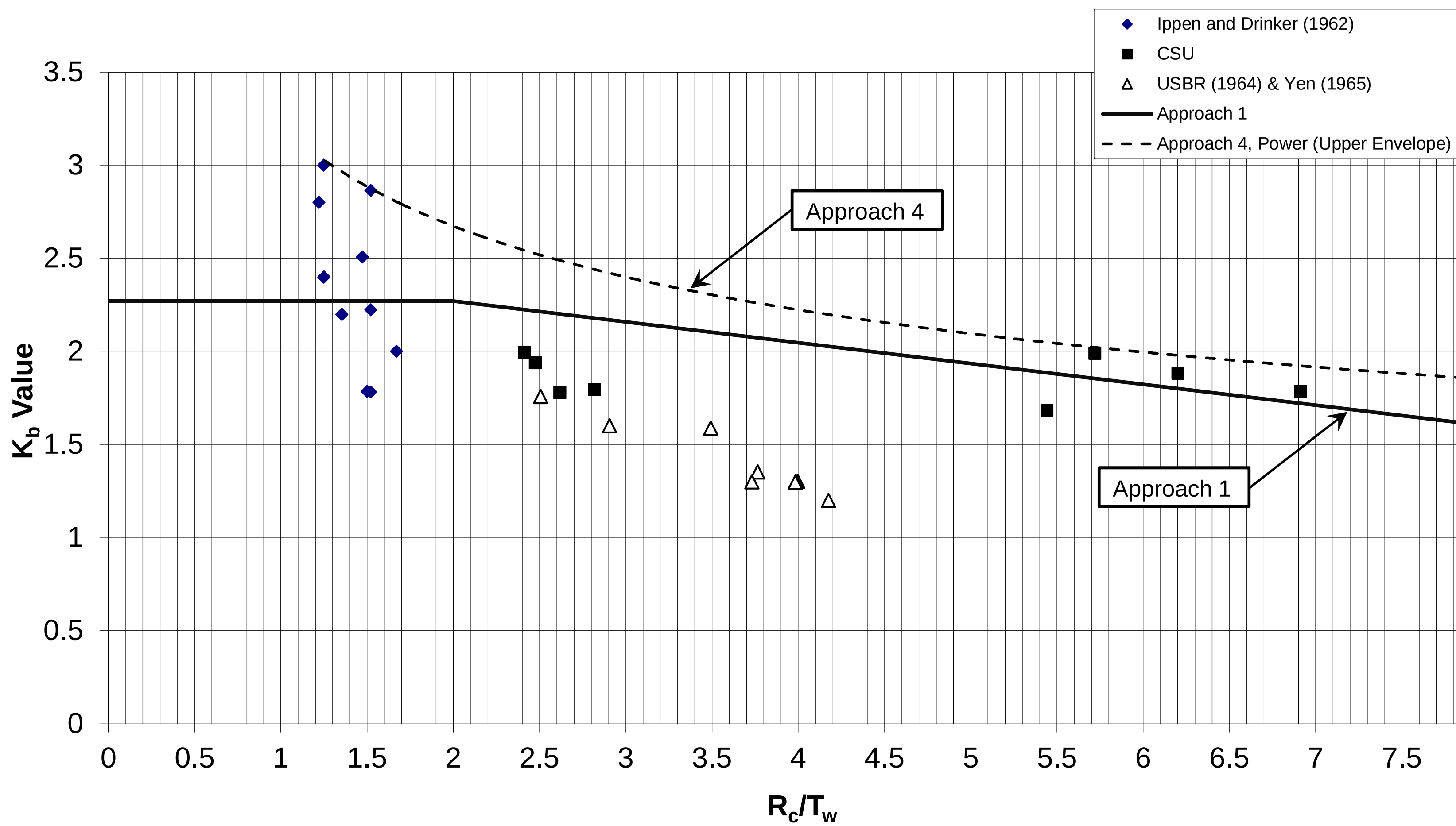

Figure 3.5: Recommended Approaches for Determining K_b, Utilizing the Composite Data

4 DISCUSSION AND RECOMMENDATIONS

It is apparent that each of the four approaches presented herein prescribe a means for increasing the estimate of K_b over the values determined using *HEC-15* based upon laboratory data presented. Approaches 1 and 2 segment the estimate of K_b based upon the value of R_c/T_w in a manner similar to *HEC-15*. The segmented approaches, particularly Approach 2, yield the best fit to the existing database but increases the guidance complexity for determining K_b. Further, Approach 2 provides a series of expressions that best fit the data along the entire spectrum of R_c/T_w.

Approach 3 (Equation 3.9) provides a best-fit, unique relationship for estimating K_b along the R_c/T_w spectrum, closely aligns with the *HEC-15* relation for R_c/T_w ranging from 2 to 6, provides a slightly more conservative K_b for $R_c/T_w > 10$, and presents an improved fit to the data for $R_c/T_w < 2$. However, Approach 3 does not fully incorporate the influence of the CSU data for $R_c/T_w > 5$ into the K_b determination. Approach 4 (Equation 3.10) presents a conservative, envelope relation that minimizes the risk of design but potentially increases construction costs. Application of Equation 3.10 may be best applied to bendways where the losses of extremely high-valued property or human life are eminent. However, the conservatism of Approach 4 may not be warranted for routine determination of K_b.

The composite database and each of the proposed approaches presented herein indicate that *HEC-15* is not a conservative means of estimating K_b. Therefore, it is recommended that a combination of Approach 1 and Approach 4 be considered for determining K_b and subsequent bendway stability, as portrayed in Figure 3.5. Approach 1 (Equations 3.2 through 3.4) should be applied in determining K_b where high-valued property degradation or loss of life are not focal to the analysis. Approach 1 increases the K_b values over those predicted using *HEC-15* by approximately 15% for R_c/T_w values of 0 to 2, by approximately 24% for R_c/T_w values of 2 to 5, and by approximately 27% for R_c/T_w values for 5 to 10. However, when the degradation of high-valued property or loss of life must be considered, it is recommended that Approach 4 (Equation

3.10) be considered to determine K_b. Equation 3.10 increases K_b values over *HEC-15* predictions by approximately 50% for R_c/T_w values of 0 to 2, approximately 34% for R_c/T_w values from 2 to 5, and approximately 44% for R_c/T_w values from 5 to 10. Depending upon the approach used, *HEC-15* appears to potentially underestimate K_b values from 15 to 50%, depending upon the criticality of the bendway being evaluated. The application of either Approach 1 or Approach 4, depending upon the circumstances, provides the user a more conservation assessment of bendway stability.

It is imperative that the K_b database be expanded, particularly for R_c/T_w values greater than 5. Further, it is anticipated that these relations may significantly alter as channel shapes transition toward a natural channel configuration. These analyses and outcomes are based solely from rigid boundary, trapezoidal channels.

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