

## Effect of a controlled burn on the thermophysical properties of a dry soil using a new model of soil heat flow and a new high temperature heat flux sensor\*

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**Abstract.** Some fires can be beneficial to soils but, if a fire is sufficiently intense, soil can be irreversibly altered. We measured soil temperatures and heat fluxes at several soil depths before, during, and after a controlled surface burn at Manitou Experimental Forest (southern Colorado, USA) to evaluate its effects on the soil's thermophysical properties (thermal conductivity and volumetric specific heat capacity). During the burn the soil was heated to over 400°C at a depth of 0.02 m and to almost 100°C at 0.30 m. Relatively high temperatures persisted for several hours to days even over 1 m deep into the soil. At these intensities and durations significant changes in soil chemistry, structure, and nutrient cycling are likely. However, soil thermophysical properties, estimated before and after the fire with a new model of periodic heat flow in soils, were not significantly changed between the times shortly after sensor installation (October 2001) and 1 month after the fire (February 2002). Estimates of the soil thermophysical properties derived with the new model underestimate laboratory analyses performed on soil samples obtained after the fire. Also presented in this study are some of the first soil heat flux measurements made during a surface fire. Furthermore, data and analyses of the type discussed in this study should aid modeling studies of the soil thermal pulse associated with fire. The ultimate goal of this experiment was to provide tools to assist land managers in the use of prescribed fire to benefit ecosystems and to reduce the potential for harm by examining how the soil's physical properties and different fuel amounts, geometries, and loading densities influence soil recovery and forest regeneration after fires.

**Additional keywords:** fuel management; modeling periodic soil heat flow; slash; soil heat flux measurements during fires; soil thermal conductivity; soil volumetric specific heat capacity.

### Introduction

Both natural and prescribed fires play important positive roles in managing and maintaining many ecosystems throughout the world. However, a sufficiently intense fire can become a major cause of ecosystem disturbance and change. One important aspect of any fire's influence on an ecosystem is the heat pulse and the associated high temperatures that penetrate the soil. High soil temperatures influence an ecosystem's ability to recover after a fire by altering soil properties and structure; killing soil microbes, plant roots, and seeds; destroying soil organic matter; altering soil nutrient availability, water status, and soil nutrient cycling; as well as volatilizing some heavy metals (Giovannini *et al.* 1990; Hungerford *et al.* 1991; Bradstock and Auld 1995; Campbell *et al.* 1995; DeBano *et al.* 1998; Baird *et al.* 1999; Friedli *et al.* 2001; Badía and Martí 2003; Serrano *et al.* 2003). Some

consequences of fire can be subtle and long-term (Sackett and Haase 1992; DeBano *et al.* 1998) while others, such as increasing soil erosion and the concomitant effects on water quality and the hydrologic cycle, are more immediate and obvious (Scott and Van Wyck 1990; DeBano *et al.* 1998).

The present study is part of a larger ongoing study to investigate how different amounts and geometric arrangements of fuel (slash) influence forest regeneration after fires. Here we examine in detail one of seven test burns (Massman *et al.* 2003) with a focus on soil thermophysical properties. Because intense wild fires are an increasingly common component of the landscape (Western Governors' Association 2001; Graham 2003) and because fire is used frequently by land managers to reduce surface fuels, it is important to know if and how soil properties may change as a consequence of the fire-associated soil heat pulse. In particular, it

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is important to know whether the intrinsic (dry) soil thermophysical properties—volumetric specific heat capacity and thermal conductivity—change as a result of soil heating. Significant changes, particularly in the intrinsic thermal conductivity of fire-affected soils, could indicate changes in the soil's structure, because soil thermal conductivity is influenced strongly by soil structure (Farouki 1986). Furthermore, such changes will lead to changes in the daily energy flow through the soil and the associated patterns and magnitudes of soil temperatures, which in turn may affect soil chemistry, soil aggregate stability, soil biota, and ultimately the nature of the soil's recovery from fire.

The primary intention of the present study is to investigate the hypothesis that a controlled surface burn can alter a dry soil's thermophysical properties by comparing *in situ* estimates of the soil's volumetric specific heat capacity and thermal conductivity obtained before and after a large surface fire from profile measurements of soil temperature and heat flux. A similar approach, which exploits models and measurements of the normal 24-h (diel) cycle of soil heating and cooling, was used by Massman (1992) to investigate the temporal variability of the thermophysical properties of a prairie soil.

Although the methods outlined in the present study could be useful for a wet soil as well (Massman 1992), here we examine a dry soil for two reasons. First, the site is located in semi-arid southern Colorado, USA, where much of the soil is usually dry. So most fires and controlled burns are likely to occur when soils are normally dry. Second, we do not wish to confuse any changes in a soil's intrinsic thermophysical properties with changes that result from evaporation or translocation of soil moisture during a fire. In general, soil volumetric specific heat capacity and thermal conductivity are both relatively strong functions of soil moisture (e.g. de Vries 1963; Farouki 1986) and values derived using the *in situ* method can change significantly as a result of soil wetting and drying (Massman 1992). Therefore, any changes in a wet soil's intrinsic thermophysical properties could easily be masked by the reduction (evaporation) or translocation of soil moisture during a fire. Of course, this is not to say that the intrinsic thermophysical properties of a wet soil may not also be affected by fire. Rather it is to acknowledge that *in situ* methods, such as ours, may not be the most appropriate approach for estimating changes in a wet soil's intrinsic thermophysical properties.

A secondary intention of the current study is to present and discuss the soil heat flux measurements made during the controlled burn. This additional goal complements the first because, as will be shown later, estimates of soil thermal conductivity are necessary to obtain the true soil heat flux from the measured soil heat flux (Philip 1961). Furthermore, because of their rarity, any observations of soil heat flux made during fires are valuable contributions to both observational and modeling studies of the

effects of fire on soils (e.g. Steward *et al.* 1990; Clark *et al.* 2003).

There are three novel aspects to the present study. First, to our knowledge this is the first attempt to determine if fire can influence a dry soil's thermophysical properties. Second, although measurements of soil heat flux are fairly routine under normal conditions, they are usually not made during a surface fire because most soil heat flux transducers (HFTs) are not made of materials suitable to high temperatures (e.g.  $\geq 150^\circ\text{C}$ ). Here we employ a new high temperature HFT. (Although Tunstall *et al.* (1976) used a similar transducer design in their study of the thermal pulse associated with an experimental log pile fire, our study provides for the complications that result from the differences between the thermal conductivity of the soil and the HFT (Philip 1961) and for the temperature effects on the HFT's calibration factor and its thermal conductivity.) Third, we develop a new analytical model of the daily thermal pulse into the soil that better describes the observed time lag between the soil heat flux and the soil temperature. The most common model of periodic heat transfer in soils usually assumes that the soil's thermophysical properties are uniform with depth, which yields the result that the daily heat flux 'wave' leads the daily temperature 'wave' by 3 h (e.g. van Wijk and de Vries 1963; Massman 1993). However, for non-uniform soils, this phase lag can differ significantly from 3 h (e.g. Massman 1993; Karam 2000). This new model describes the soil thermophysical properties as simple continuous functions of depth, which is different from the approach taken by either Massman (1993) or Karam (2000), who use piecewise continuous functions to describe the variation of the soil's thermophysical properties with depth.

The remainder of this paper comprises eight sections. These are: site description; instrumentation; the controlled burn; post-burn soil analysis and sensor evaluation; mathematical methods of analysis (which discusses how the soil thermophysical properties are determined from models of periodic heat flow in soils); results; conclusions; and an appendix that discusses and derives the new model for periodic heat flow in soils with non-uniform thermal properties.

### Site description

The burn experiment was conducted in the Manitou Experimental Forest (MEF), in the central Rocky Mountains  $\sim 45$  km west of Colorado Springs, Colorado. The latitude and longitude of the MEF are  $39^\circ 04'$  North and  $105^\circ 04'$  West. The annual mean temperature of the experimental forest is  $\sim 5^\circ\text{C}$  and its mean elevation is  $\sim 2400$  m ASL. The burn area was a large grassy opening that had been created in the surrounding ponderosa pine (*Pinus ponderosa*) forest 2 years prior when several trees were cut in an effort to reduce the amount of mistletoe in the area. At the time of the experiment this opening was covered primarily by senescent bunchgrasses.

Soils within the burn area were Pendant cobbly loam. Soil samples taken at the burn site had a bulk density of  $\sim 1.3 \pm 0.3 \text{ g cm}^{-3}$  and a porosity of  $\sim 0.50 \pm 0.08$  and were either sandy clay loams or sandy loams composed of  $\sim 60\%$  sand, 25% silt, and 15% clay. Soil organic material comprised  $\sim 1\text{--}2\%$  of the soil by volume. Except for a few soils that are derived from red arkosic sandstone, all soils in the MEF were derived from biotite granite and associated igneous rocks of the Pikes Peak batholith.

Annual precipitation on the MEF is  $\sim 400$  mm. However, during 2001 and 2002, the years spanning the experiment, the annual precipitation amounts were much below average. A total of 261 mm of precipitation was recorded at MEF during 2001 and 140 mm during 2002. The detailed precipitation record for this period indicates that precipitation occurred mainly during small sporadic events. In fact, during this 2-year period only two events, one during June 2002 and the other during August 2002, approached 16 mm of rainfall, otherwise most events were less than 5 mm.

### Instrumentation

The chronology of the burn experiment was as follows. In early October 2001 a pit 1.5 m deep and an attached 30 m trench were dug at the site. On 12 October 2001 thermocouples and HFTs were installed at the site at various depths (Table 1), the wires connecting the sensors were laid in the trench and attached to a Campbell Scientific (Logan, UT) CR23X data logger at the end of the trench, and the pit and trench were carefully backfilled. On 18 October 2001 the site was amended by piling slash and brush (by hand) over the instrumentation. On 11 January 2002 the slash pile was ignited and allowed to burn until the fire had totally consumed the fuel. The slash pile itself was conical in shape and  $\sim 6$  m high and 9 m in diameter for a total loading estimated to be  $560 \text{ t ha}^{-1}$ . On 7 March 2002 soil samples were taken inside and outside the burned area for analysis of bulk density, porosity, texture, and percentage composition of sand, silt, and clay. On 8 April 2002 all sensors were dug up and inspected visually for any potential heat damage. The HFTs were then returned to their respective manufacturers for an examination and post-experiment calibration check. On 17 July 2002 soil samples were obtained inside and outside

the burned area and analysed in a laboratory for thermal conductivity, specific heat capacity, and moisture content.

All *in situ* sensor data before and well after the fire were sampled at 1 Hz and the corresponding average for each sensor was recorded every 30 min. Between 2 and 4 h before and during the fire and for 4 days after the fire, data were sampled at 4 Hz and the 15-s averages were recorded for each sensor.

The thermocouples used in this experiment were manufactured by Omega Engineering (Stamford, CT) and all thermocouple junctions were coated before soil insertion with epoxy (Omegabond 101) for electrical isolation.

Two types of soil HFTs were used for this study, a high temperature probe with an alumina core and an exterior ceramic glaze (Thermonetics Corporation, La Jolla, CA) and a more conventional one (Radiation and Energy Balance Systems or REBS, Seattle, WA), which would typically be used for micrometeorological studies of the surface and soil thermal energy flow under normal (non-fire) conditions. The high temperature HFTs are rated to  $775^\circ\text{C}$  and had a nominal sensitivity between  $1250$  and  $1750 \text{ W m}^{-2} \text{ mV}^{-1}$ . These HFTs were positioned nearest the soil surface where the soil temperatures are highest and were attached to a data logger by a cromel extension wire (Omega Engineering TFCH-020, rated to  $260^\circ\text{C}$ ). Table 1 provides more details on these and all other sensors and their deployment.

Because these high temperature HFTs are exposed to such a wide range of temperatures (potentially anywhere between approximately  $-10$  and  $700^\circ\text{C}$ ), it is important to be aware of the effects of temperature on the sensors' thermal conductivity and the calibration factors. The thermal conductivity of these high temperature HFTs,  $\lambda_p$  ( $\text{W m}^{-1} \text{ K}^{-1}$ ), is  $\lambda_p = 0.7 + 0.003T_C$ , where  $T_C$  ( $^\circ\text{C}$ ) is temperature. It is important to know  $\lambda_p$  to correct for the discrepancy between the true soil heat flux,  $G_s$  ( $\text{W m}^{-2}$ ), and the measured soil heat flux,  $G_m$ , that results whenever  $\lambda_p$  differs from the soil's thermal conductivity,  $\lambda_s$  (Philip 1961). Philip's (1961) relationship (or correction) is given as

$$G_s = [1 - \beta r(1 - \varepsilon^{-1})]G_m, \quad (1)$$

where  $\beta$  is a factor related to sensor shape ( $\beta = 1.31$  for the high temperature probes, which are square and  $\beta = 1.70$

**Table 1. Placement and descriptions of soil temperature probes (thermocouples) and HFTs for the controlled burn at Manitou Experimental Forest**

$G_T$  denotes the high temperature HFTs and  $G_B$  denotes the lower temperature REBS sensor. The calibration ratio is the ratio of the HFTs' calibration factor at some temperature,  $T_C$  ( $^\circ\text{C}$ ), to the calibration factor at  $537.8^\circ\text{C}$

Depth (m)	Thermocouple type	Heat flux transducer type	Calibration ratio
0.02	HH-K-24, rated $704^\circ\text{C}$		
0.05	HH-K-24, rated $704^\circ\text{C}$	$G_T$ Transducer no. 11, rated $775^\circ\text{C}$	$0.277 + 0.0012456T_C$
0.10	TT-J-24, rated $260^\circ\text{C}$	$G_T$ Transducer no. 12, rated $775^\circ\text{C}$	$0.328 + 0.0012456T_C$
0.30	TT-T-24, rated $200^\circ\text{C}$	$G_T$ Transducer no. 13, rated $775^\circ\text{C}$	$0.396 + 0.0012456T_C$
0.50	TT-T-24, rated $200^\circ\text{C}$	$G_R$ Transducer model HFT 3.1	Not necessary
1.36	TT-T-24, rated $200^\circ\text{C}$	$G_R$ Transducer model HFT 3.1	Not necessary

HFT, heat flux transducer.

for the REBS sensors, which are circular),  $r$  is the sensor's aspect ratio (the ratio of the sensor's thickness to its horizontal length;  $r = 0.19$  for the high temperature probe and  $r = 0.102$  for the REBS sensor), and  $\varepsilon^{-1} = \lambda_s/\lambda_p$ . If a sensor is perfectly matched to its soil environment, then  $\lambda_p = \lambda_s$ ,  $G_m = G_s$ , and there would be no need to correct for this discrepancy. However, in general, this is unlikely to occur very often at normal day-time or night-time soil temperatures and so is, therefore, even less likely during a surface burn. Because the REBS HFTs are buried so much deeper than the high temperature probes, we will assume that their thermal conductivity is constant with temperature. For the REBS sensors,  $\lambda_p$  is  $1.22 \text{ W m}^{-1} \text{ K}^{-1}$ . Philip's (1961) relationship, equation (1), is very important for the mathematical analysis section where methods are developed for extracting  $\lambda_s$  from measured values of soil temperature and heat flux.

The calibration coefficients of the high temperature HFTs are also a function of temperature. This temperature dependency can be calculated theoretically and is given as follows (H. Poppendiek, Thermonetics Corporation, personal communication 2003) for a sensor that has been calibrated at  $537.8^\circ\text{C}$  ( $1000^\circ\text{F}$ ):

$$K_T/K_{537.8^\circ\text{C}} = 0.330 + 0.0012456T_C,$$

where  $K_T$  is the calibration coefficient at some Celsius temperature  $T_C$ , and  $K_{537.8^\circ\text{C}}$  is the calibration factor at  $537.8^\circ\text{C}$ . In general, the high temperature calibration coefficient varies somewhat ( $\pm 10\%$ ) from one sensor to the next. However, a post-burn low temperature calibration suggested that the theoretical intercept value of 0.330 could also vary somewhat from one sensor to another. The combined high and low temperature calibrations are given in Table 1. As with their thermal conductivity, we assume that the REBS HFTs have a constant calibration factor.

### The controlled burn

Figure 1 shows the soil temperatures during the controlled burn. The burn was initiated about noon on 11 January 2001 and continued to burn or smolder for several hours to days after that time. The maximum 0.02-m soil temperature,  $406.6^\circ\text{C}$ , occurred just before midnight (12:00 a.m.) of 13 January 2001. Assuming that  $300^\circ\text{C}$  is the threshold for possible fire-induced soil structural change (DeBano *et al.* 1998), this figure also indicates that the top 0.05–0.10 m of soil were exposed to conditions that could have altered the soil's structure.

Figure 2 shows the measured (uncorrected) soil heat fluxes during the controlled burn. (Note: the sign convention used for all figures of soil heat flux shown in this study assumes that the heat flux is negative when it is directed downward or into the soil.) The maximum measured soil heat flux at 0.05 m was  $-1141 \text{ W m}^{-2}$ .

Both these figures clearly show that the heat pulse associated with the fire penetrated more than 1 m into the soil and

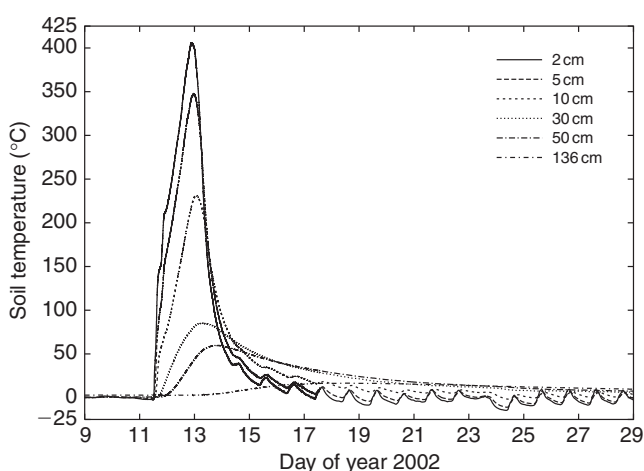


Fig. 1. Measured soil temperatures for 9–28 January 2002 (before, during, and after the controlled burn) at the Manitou Experimental Forest controlled burn site. The burn was initiated about noon on 11 January.

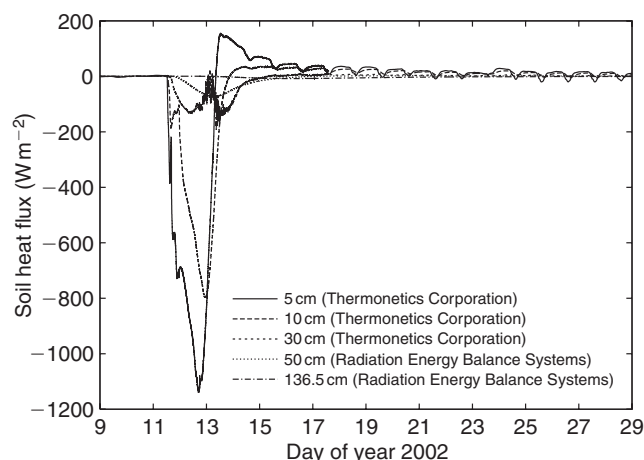


Fig. 2. Measured soil heat fluxes for 9–28 January 2002 (before, during, and after the controlled burn) at the Manitou Experimental Forest controlled burn site. The negative sign of the heat flux indicates that the heat flux is into the soil. The burn was initiated about noon on 11 January.

Fig. 1, in particular, shows that it continued to influence soil temperatures for weeks after the surface fire had gone out. Observations before the fire show very little variation because the slash pile blocks the solar radiation to the soil. Nevertheless, heat from the fire did not penetrate the soil directly below the burning slash pile until the pile had collapsed and combusting material actually came into contact with the soil surface, which did not occur until several hours after ignition.

### Post-fire soil sampling and sensor evaluation

#### Vertical profiles of soil bulk density

Soil bulk density samples were obtained after the fire with the aid of a soil corer and were analysed at Colorado State

**Table 2. Depth profiles of bulk density of soil at or near the slash pile burn site**

Samples were taken on 7 March 2002, ~2 months after the controlled surface burn

Depth of sample (m)	Location relative to burn area	Bulk density (gm cm <sup>-3</sup> )
0.0–0.15	Inside	1.25
0.15–0.30	Inside	1.17
0.0–0.15	Inside	1.27
0.15–0.30	Inside	1.42
0.0–0.15	Edge	1.32
0.15–0.30	Edge	0.95
0.0–0.15	Outside	1.34
0.15–0.30	Outside	1.18

University's Soil, Water and Plant Testing Laboratory (Fort Collins, CO). The profiles of bulk density of soils within and near the slash pile burn area (Table 2) indicate that bulk density does vary vertically. Rather interestingly, there is less vertical variation in bulk density inside (near the center of) the burn area than outside or near the edge of the burn area. These data are discussed in more detail in a subsequent section.

#### Soil thermal conductivity measurements

Soil thermal conductivity ( $\lambda_s$ ) samples at the burn site were also obtained with the aid of a soil corer, and were kept intact by an aluminum cylinder 0.051 m in diameter and 0.152 m in length. The soil the ends of the cylinder were taped shut to ensure sample integrity during transport to the laboratory. The analyses for  $\lambda_s$  were performed with the heated probe method by Thermophysical Properties Research Laboratory (West Lafayette, IN). The heated probe apparatus was checked by an internal standard (sand) and found to be within 2% of the expected value. Two samples were taken for analysis, one inside the burn area and one outside, but within a couple meters of the edge of the burn area. Inside and outside estimates of  $\lambda_s$  were determined at 23°C. A separate estimate of  $\lambda_s$  was also made at 55°C with the outside sample. This sample did show some temperature sensitivity. The temperature dependency of  $\lambda_s$  will also be discussed in a subsequent section.

#### Soil volumetric specific heat capacity measurements

Soil specific heat was determined from samples matched by location to each of the  $\lambda_s$  samples and obtained in a manner similar to those of  $\lambda_s$ . All specific heat measurements were made with a Perkin-Elmer model DSC-2 (Differential Scanning Calorimeter) with sapphire as a reference material. The soil specific heat capacity of each sample was obtained at 23°C and additionally at 10°C increments from 30 to 100°C.

#### Volumetric soil moisture measurements

Samples for soil water content were obtained on 17 July 2002, or ~6 months after the burn, in a similar manner to  $\lambda_s$  and

were likewise matched to  $\lambda_s$ . The volumetric soil moisture,  $\theta_w$ , at that time was  $\sim 0.015 \pm 0.002$ . According to the precipitation record during and after the experiment this particular value of  $\theta_w$  is likely to be a maximal value during much of the experiment. In general, we can fairly safely assume that the soils from October 2001 to April 2002 were extremely dry.

#### Post-fire calibration checks of HFTs

Post-fire calibration checks were performed on both types of HFTs. The high temperature checks, which were performed at  $\sim 538^\circ\text{C}$  by the manufacturer, were within  $\pm 10\%$  of their original calibrations. The REBS sensors were within about  $\pm 5\%$  of theirs. The low temperature calibration for the high temperature probes was performed by the USDA ARS National Soil Tilth Laboratory (Ames, IA) at three different pairs of temperatures and heat fluxes: ( $31.2^\circ\text{C}$ ,  $43 \text{ W m}^{-2}$ ), ( $38.6^\circ\text{C}$ ,  $85 \text{ W m}^{-2}$ ), ( $54.1^\circ\text{C}$ ,  $172 \text{ W m}^{-2}$ ). The sensors performed well during these tests, which, as discussed earlier, were valuable for developing the temperature-dependent calibration curves provided in Table 1. The post-fire visual inspections of the sensors showed nothing abnormal or unexpected. The data from the HFTs during the fire were also subjected to other types of analyses (Massman *et al.* 2003). The major result of these additional analyses suggested that the observed heat flux during the fire compared very favorably with the heat flux observations of Tunstall *et al.* (1976). Furthermore, post-burn examination of cromel extension wires connecting the HFTs to the data loggers did not reveal any high temperature damage. We conclude from this rather disparate set of evaluations that the HFTs and the attached wiring performed as expected before, during, and after the fire.

#### Mathematical method of analysis

The average daily values of the soil thermophysical parameters are estimated from measurements of the daily cycle of soil temperature and heat flux for a few days before and after the controlled burn. The basic approach used in this study exploits the nearly sinusoidal nature of the daily energy flow in and out of the soil and is quite similar to the approach that Massman (1992) used at an eastern Colorado prairie site. Of course the basic presumption is that any significant changes in the soil thermal properties should be detectable from changes in the daily temperature and heat flux waves.

The first step in the analysis uses a Fourier transform for all sensors to determine the amplitude of the daily temperature and heat flux waves and the phase difference between them. This is equivalent to a spectral analysis or to finding a best fit to the daily time series data using

$$\sum_{n=1}^N A_n e^{i(n\omega t + \phi_n)}$$

where  $N$  is the total number of harmonics,  $A_n$  is the amplitude of the  $n$ th harmonic,  $i = \sqrt{-1}$  is the unit imaginary number,

**Table 3. Two examples of observed pre-burn (2001) profiles of amplitudes and phase differences of the daily soil temperature and heat flux waves (see Figs 3, 4)**

Depth (m)	13 October (day 286 of 2001)			17 October (day 290 of 2001)		
	$A_T(z)$ (°C)	$A_{G_m}(z)$ (W m <sup>-2</sup> )	Phase difference (h)	$A_T(z)$ (°C)	$A_{G_m}(z)$ (W m <sup>-2</sup> )	Phase difference (h)
0.02	7.13			7.55		
0.05	5.30	15.8	2.53	5.46	17.9	2.70
0.10	2.88	7.66	2.28	2.94	8.94	2.77
0.30	0.44	1.83	7.58	0.46	2.50	7.85

$t$  is time,  $\omega = 7.272(10^{-5})\text{ s}^{-1}$  is the frequency of the fundamental or diel cycle, i.e.  $\omega = 2\pi/(24\text{ h})$ , and  $\phi_n$  is an arbitrary phase of the  $n$ th harmonic. This approach in essence eliminates the time dependency in each time series and produces an estimate of the amplitude of the temperature and heat flux waves at each sensor depth. Where both temperature and soil HFTs are at the same depth, it also gives an estimate of the phase difference between the two waves.

The second step in the analysis uses the depth profiles of these amplitudes to estimate (1) the soil attenuation depth (for temperature and heat flux) and (2) the amplitudes of the daily temperature and heat flux waves at the soil surface, all of which are then used to estimate the soil's bulk thermal conductivity,  $\lambda_s$ , and volumetric specific heat capacity,  $C_s$  (J m<sup>-3</sup> K<sup>-1</sup>).

A key assumption that is fundamental to this *in situ* model-based approach for estimating  $\lambda_s$  and  $C_s$  is that the thermophysical parameters are independent of time throughout any given day. Of course, strictly speaking this cannot be true for two reasons. First, any change in the soil moisture content, which has its own diel cycle (e.g. Rose 1968), will affect  $\lambda_s$  and  $C_s$  (e.g. de Vries 1963) and second, any change in soil temperature will also cause  $\lambda_s$  and  $C_s$  to change (Kay and Goit 1975; Campbell *et al.* 1994). However, neither of these effects is likely to be very large during any diel cycle typical of this study. As indicated earlier, the soil moisture was very low during the analysis period, so the diel cycle is also likely to be very small, and second, changes in soil temperatures during a typical 24-h period (or the daily amplitude of the daily temperature wave) are also small, being only a few degrees within the soil (e.g. Table 3). Therefore, we do not expect these effects to be significant in our analysis of the 24-h cycle of soil temperatures and heat fluxes. During a fire, however, the variations in soil temperatures can be so large that the temperature effects on  $\lambda_s$  and  $C_s$  cannot be ignored. This issue will be discussed in more detail later.

#### *Uniform soil thermophysical properties: the van Wijk/de Vries model*

The method of analysis employed in this study is now formalized using physically based models of periodic heat flow in soils. We begin with the model of van Wijk and de Vries

(1963), who solved the one-dimensional heat conduction equation (see equation (14) in Appendix 1) for soil temperature and heat flux as a function of time and depth. This model, which assumes that soil thermophysical properties do not vary with either time or soil depth (i.e. the soil properties are uniform) is given as follows:

$$T(z, t) = T_0 + Q_0 z + \sum_{n=1}^N \Delta T_n e^{-z\sqrt{n}/D} e^{i(n\omega t - z\sqrt{n}/D + \phi_n)} \quad (2)$$

and

$$G_s(z, t) = G_0 + \sum_{n=1}^N \Delta G_{sn} e^{-z\sqrt{n}/D} e^{i(n\omega t - z\sqrt{n}/D + \phi_n + \pi/4)}, \quad (3)$$

where  $T(z, t)$  is the soil temperature as a function of depth  $z$  and time  $t$ ;  $T_0$  is the mean daily temperature;  $Q_0$  is the mean daily temperature gradient;  $\Delta T_n$  is the surface amplitude of the  $n$ th harmonic of the daily temperature wave;  $D$  (m) is the soil attenuation depth, which by definition is  $\sqrt{2\lambda_s/(C_s\omega)}$ ;  $G_0$  is the mean daily soil heat flux; and  $\Delta G_{sn}$  is the surface amplitude of the  $n$ th harmonic of the daily heat flux wave. Note that  $z = 0$  is the soil surface.

The mean daily heat flow terms ( $T_0 + Q_0 z$  and  $G_0$ ) and the higher harmonics ( $n > 1$ ) are included in these last two model equations primarily for the sake of completeness. For the purposes of this study they are not necessary. Consequently and henceforth, we focus our discussion and analysis on the fundamental ( $n = 1$ ) or 24-h wave with the understanding that all results can easily be generalized to higher harmonics if necessary. We will also drop the  $n = 1$  harmonic subscript for the remainder of this paper.

The Fourier transform of equations (2) and (3) produces the following depth attenuation functions for the amplitudes of the fundamental temperature and heat flux waves:

$$A_T(z) = \Delta T e^{-z/D} \quad (4)$$

$$A_G(z) = \Delta G_s e^{-z/D}, \quad (5)$$

where  $A_T(z)$  and  $A_G(z)$  are the amplitudes of the associated waves as functions of depth.

Next employing the general relationship between soil temperature gradient and the soil heat flux, which is

$G_s(z, t) = -\lambda_s \partial T(z, t) / \partial z$ , yields the following relationship between the surface amplitudes of the temperature and heat flux waves:

$$\Delta G_s = \lambda_s \Delta T \sqrt{2} / D. \quad (6)$$

These last three equations form the basic method for determining the soil thermophysical properties from the observed daily soil temperatures and heat fluxes. First, the observed 24-h time series of soil temperature and heat flux are transformed with a Fourier transform to produce measured amplitude profiles. Next these measured amplitude profiles are fit to either equation (4) or equation (5), as appropriate, using the Levenberg-Marquardt non-linear least-squares algorithm (Press *et al.* 1992). This yields a daily estimate of  $\Delta T$ ,  $\Delta G_s$ , and two independent estimates of  $D$  ( $D_T$  and  $D_G$ ) if both the temperature and heat flux data are fit independently. For this study only the temperature profile attenuation depth,  $D_T$ , value is used so that, when fitting the heat flux profile data to equation (5), the associated attenuation depth,  $D_G$ , is fixed at the value of the corresponding  $D_T$ . This way the temperature profile is used with a two-parameter model, while the soil heat flux profile is used with a one-parameter model. The compromises introduced by limiting the analysis of the soil heat flux to a single parameter appear to be minimal because the goodness-of-fit, as measured by the residual sum of squares, is virtually identical for both the one-parameter and two-parameter fits. Otherwise, when  $D_G$  was treated as a free (fitting) parameter we always found that  $D_G < D_T$  by a few percent. The reason for this discrepancy is not known but, because of their greater relative size, the soil HFTs are more difficult to locate precisely at their assigned depths in the soil than are the soil thermocouples. Thus at the very least, any values of  $D_G$  and  $\Delta G_s$  are relatively more uncertain than  $D_T$  and  $\Delta T$ .

If the measured soil heat flux were exactly faithful to the true soil heat flux, then equation (6) can be solved directly for  $\lambda_s$  from the estimates of  $D$ ,  $\Delta T$ , and  $\Delta G_s$ , which yields:

$$\lambda_s = \frac{\Delta G_s D}{\Delta T \sqrt{2}}. \quad (7)$$

However, the difference between the thermal conductivities between the HFT and the soil, equation (1), must be taken into account. This is achieved by: (1) combining equation (7) with equation (1); (2) mutually eliminating  $G_s$  from these two equations; and (3) solving the resulting single relation for  $\lambda_s$  in terms of  $G_m$ ,  $\Delta T$ ,  $D$ ,  $\lambda_p$ , and the HFT geometry parameters. For the van Wijk and de Vries (1963) model this yields:

$$\lambda_s = \frac{(1 - \beta r) \frac{\Delta G_m D}{\Delta T \sqrt{2}}}{1 - \beta r \frac{\Delta G_m D}{\lambda_p \Delta T \sqrt{2}}}. \quad (8)$$

Once  $\lambda_s$  has been determined,  $C_s$  and the soil thermal diffusivity ( $\kappa_s = \lambda_s / C_s$ ) can be found as follows:

$$C_s = \frac{2\lambda_s}{D^2 \omega} \quad (9)$$

and

$$\kappa_s = \frac{D^2 \omega}{2}. \quad (10)$$

*Non-uniform soil thermophysical properties: a new analytical model*

One disadvantage of the van Wijk and de Vries (1963) model is the assumption that the soil thermophysical properties are uniform throughout the soil profile, which, as discussed previously, is not always the case. The key diagnostic for uniform soil properties is the phase difference between the 24-h soil heat flux and temperature waves. In this case, the soil heat flux (maximum) will lead the temperature (maximum) by 3 h or  $\pi/4$ , as shown by equations (2) and (3). But, because the data presented in the next section display a phase that is almost always less than 3 h, this study develops and uses a new analytical model of periodic soil heat flow for the case of monotonically increasing soil thermophysical properties. We use a model of increasing soil thermophysical properties because Massman (1993) and Karam (2000) found that the phase should be less than 3 h when  $C_s$  and  $\lambda_s$  increase with depth and more than 3 h when they decrease with depth.

Besides the phase difference, another diagnostic suggesting that soil thermophysical properties may vary with depth can be found in Table 2, which indicates that the soil bulk density varies with depth within and near the slash pile burn area. Therefore, at the very least, we should expect  $\lambda_s$  to vary with depth because it is fairly strongly and positively correlated with bulk density (Farouki 1986). This is especially true in the present study because, as previously discussed, soil moisture and temperature effects are likely to be small. Nevertheless, the specific modeling assumption that the soil thermophysical properties are increasing with depth is not as clearly supported by the results listed in Table 2, because the bulk density of the soil samples taken within the burn area show opposite variations with depth. Consequently, the bulk density measurements are suggestive, but not conclusive, that soil thermophysical properties should increase with depth. However, the observed phase difference determined with the new model is unambiguous about the soil thermophysical properties increasing with depth.

This new model is derived in Appendix 1 and is summarized there by equations (16) and (17). Specifying only the 24-h periodic solution ( $n = 1$ ) and after slightly rephrasing the equations detailed in Appendix 1, this new model can be expressed as

$$T(z, t) = \frac{\Delta T e^{-\gamma_r z / D_0} e^{i(\omega t - \gamma_i z / D_0 + \phi)}}{\sqrt{2 - e^{-(\alpha D_0) z / D_0}}} \quad (11)$$

and

$$G(z, t) = \frac{\lambda_0 \Delta T \sqrt{2} \left( \sqrt{1 + \alpha D_0 / 2 + \alpha^2 D_0^2 / 8} \right)}{D_0 \sqrt{2 - e^{-(\alpha D_0)z/D_0}}} \times e^{-\gamma_r z / D_0} e^{i(\omega t - \gamma_i z / D_0 + \phi + \pi/4 - \alpha D_0 / 4)}, \quad (12)$$

where  $\lambda_s$  and  $C_s$  are assumed to increase monotonically with depth according to  $\lambda_0(2 - e^{-\alpha z})$  and  $C_0(2 - e^{-\alpha z})$ , respectively;  $D_0 = \sqrt{2\lambda_0/(C_0\omega)}$ ;  $\gamma_r \approx 1$  and  $\gamma_i \approx 1$  as discussed in Appendix 1.

In general, except for the depth dependency parameter  $\alpha$  (or equivalently the phase parameter  $\alpha D_0$ ), this new model of the 24-h soil heat wave is very similar to the van Wijk and de Vries (1963) model. The differences may be summarized as: (1) the attenuation of the amplitudes with depth is slightly more complicated (and relatively faster) with the new model than with the previous model; and (2) the phase difference between the soil heat flux and temperature waves is  $(\pi/4 - \alpha D_0/4)$ , rather than  $\pi/4$  which, because  $\alpha > 0$  and  $D_0 > 0$ , indicates that the phase difference is less than 3 h.

The basic procedure for using the new model to infer soil thermophysical properties is quite similar to that outlined above for the van Wijk and de Vries (1963) model. There are only three additional steps involved. First, the phase parameter  $\alpha D_0$  is determined from the phase difference between the Fourier transformations of the observed daily heat flux and temperature waves. After this, the step involving the non-linear least-squares fitting remains the same, except of course that the fitting functions must include the additional  $\sqrt{2 - e^{-(\alpha D_0)z/D_0}}$  term in the denominator of equations (11) and (12). Next, the second additional step involves generalizing equation (8) to include the term  $\sqrt{1 + \alpha D_0/2 + \alpha^2 D_0^2/8}$  as a coefficient (multiplier) of  $\Delta T$ . At this point estimates of  $\lambda_0$  and  $C_0$  are found using the generalized form of equation (8) and the original equation (9). The third and final additional step is to determine  $\alpha$  from  $\alpha D_0$  and  $D_0$  and then to estimate a depth averaged value of  $\lambda_s$  and  $C_s$  from  $\lambda_0$  and  $C_0$  and their depth dependency function  $(2 - e^{-\alpha z})$ . The average is calculated over the top 0.15 m of soil because that is the depth of the sample used in the laboratory determinations of  $\lambda_s$  and  $C_s$ . Consequently, these averaged *in situ* determinations should be directly comparable to the laboratory estimates. Estimating the soil thermal diffusivity from equation (10) remains the same as before.

## Results

### Soil thermophysical properties: analysis of diel thermal waves

The analysis of the *in situ* data for the soil thermophysical properties was performed on three datasets. The first, obtained for the 5 days 13–17 October 2001, was before the controlled burn (see Figs 3, 4). The second and third sets were

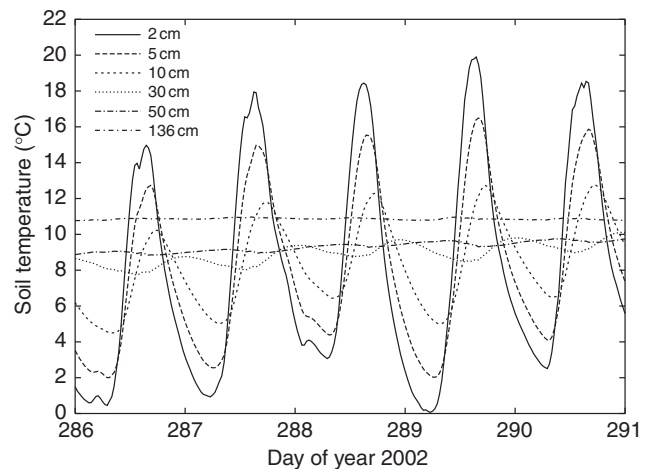


Fig. 3. Pre-burn soil temperatures for 13–17 October 2001 at Manitou Experimental Forest controlled burn site.

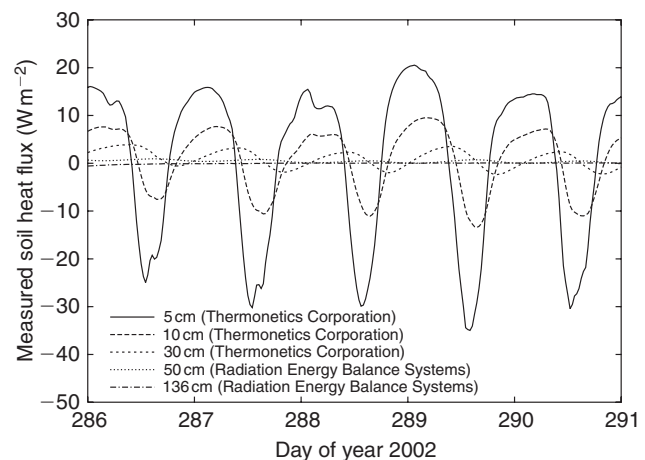


Fig. 4. Pre-burn measured soil heat fluxes for 13–17 October 2001 at Manitou Experimental Forest controlled burn site. The negative sign of the heat flux indicates that the heat flux is into the soil.

after the fire: 25–28 January and 5–8 February 2002. Table 3 lists the amplitudes and lag times between the measured temperature and heat flux waves for the upper four measurement depths. The two lower measurement depths are not included in the daily analysis because the amplitudes of the daily waves were reduced too much for them to be useful in this analysis.

The most important feature here is that the phase lag between the temperature and measured heat flux waves is shorter than 3 h in the top 0.10 m of soil, and is considerably longer at 0.30 m depth. Furthermore, this feature is common to all days analysed for this study (not all data are shown here). Although we cannot be specific about the cause of this dramatic change in the phase difference with depth, it does suggest: (1) that the soil properties may be more strongly and differently stratified with depth than has been anticipated by the models used for this study; (2) that 0.30 m is possibly



**Table 4. Results for the analysis of soil thermophysical properties for selected days of the experiment, assuming uniform soil thermophysical properties**

The first 5 days (October) are pre-burn and the last 8 days (January and February) are post-burn

Date	$\Delta T$ ( $^{\circ}\text{C}$ )	$\Delta G_s$ ( $\text{W m}^{-2}$ )	$D$ (m)	$\lambda_s$ ( $\text{W m}^{-1} \text{K}^{-1}$ )	$C_s$ ( $\text{MJ m}^{-3} \text{K}^{-1}$ )	$\kappa_s$ ( $10^{-6} \text{m}^2 \text{s}^{-1}$ )
13 October	8.92	26.5	0.092	0.153	0.499	0.306
14 October	10.3	30.3	0.094	0.155	0.485	0.320
15 October	9.05	30.6	0.083	0.161	0.637	0.253
16 October	11.9	36.4	0.091	0.159	0.528	0.300
17 October	9.51	30.8	0.088	0.163	0.580	0.281
25 January	10.6	28.0	0.087	0.132	0.463	0.285
26 January	9.59	28.4	0.083	0.139	0.557	0.249
27 January	9.55	28.6	0.082	0.139	0.570	0.243
28 January	8.48	24.6	0.082	0.135	0.547	0.247
5 February	7.56	24.9	0.079	0.147	0.653	0.226
6 February	13.7	38.9	0.084	0.135	0.523	0.259
7 February	13.5	36.3	0.086	0.131	0.484	0.270
8 February	10.3	30.8	0.081	0.136	0.578	0.236

**Table 5. Results for the analysis of soil thermophysical properties for selected days of the experiment, assuming monotonically increasing soil thermophysical properties**The first 5 days (October) are pre-burn and the last 8 days (January and February) are post-burn. Note, for the present purposes it is sufficient to use a single value of  $\alpha D_0$  during each of the three analysis periods. This particular value is the average for the 4 or 5 days comprising each of the analysis periods. For comparisons with laboratory results the values of  $\lambda_s$  and  $C_s$  have been averaged over the top 0.15 m of soil depth

Date	$\Delta T$ ( $^{\circ}\text{C}$ )	$\Delta G_s$ ( $\text{W m}^{-2}$ )	$D_0$ (m)	$\alpha D_0$	$\lambda_s$ ( $\text{W m}^{-1} \text{K}^{-1}$ )	$C_s$ ( $\text{MJ m}^{-3} \text{K}^{-1}$ )	$\kappa_s$ ( $10^{-6} \text{m}^2 \text{s}^{-1}$ )
13 October	9.01	26.5	0.103	0.35	0.198	0.515	0.384
14 October	10.4	30.3	0.105	0.35	0.200	0.499	0.402
15 October	9.15	31.0	0.094	0.35	0.208	0.654	0.319
16 October	12.0	36.9	0.102	0.35	0.206	0.544	0.378
17 October	9.61	31.2	0.098	0.35	0.210	0.596	0.352
25 January	10.8	28.8	0.104	0.63	0.188	0.477	0.393
26 January	9.82	29.2	0.097	0.63	0.196	0.573	0.341
27 January	9.79	29.4	0.096	0.63	0.195	0.584	0.334
28 January	8.69	25.3	0.097	0.63	0.191	0.562	0.339
5 February	7.70	25.4	0.091	0.50	0.195	0.655	0.298
6 February	13.9	39.7	0.097	0.50	0.180	0.525	0.344
7 February	13.7	37.1	0.099	0.50	0.174	0.487	0.358
8 February	10.5	31.5	0.092	0.50	0.180	0.580	0.310

too deep to reliably observe the daily soil heating wave; or (3) both. For any further analyses we will disregard the phase at the lowest level.

Table 4 give the surface amplitudes for the temperature ( $\Delta T$ ) and the (corrected) heat flux ( $\Delta G_s$ ) waves, the attenuation depth ( $D$ ), and the soil thermophysical properties ( $\lambda_s$ ,  $C_s$ ,  $\kappa_s$ ) determined using the van Wijk and de Vries (1963). Table 5 gives the same results (along with the phase parameter,  $\alpha D_0$ ) for the new model of periodic heat flow that incorporates monotonically increasing soil thermophysical properties. To assist in the interpretation of  $\alpha D_0$  it should be noted that a value of 0.35 (October 2001) corresponds to a phase difference of 2.7 h, the value of 0.63 (January 2002) corresponds to a phase difference of 2.4 h, and the value of 0.50 observed during February 2002 corresponds to 2.5 h phase difference between the measured heat flux and the soil temperature.

There are two main conclusions to be drawn from these two tables. First, there is an inherent variability in the daily thermal pulse into the soil, which is reflected in the *in situ* estimates of the thermophysical parameters by a range of variability of about  $\pm 15\%$ . Given that there were no precipitation or other changes in the soil during the individual analysis periods, this percentage variability provides an estimate of the inherent uncertainty in these *in situ* methods. Second, all estimates of the soil thermophysical parameters are consistent with an extremely dry soil (e.g. Bristow 1998; Campbell and Norman 1998; Tarnawski and Leong 2000).

Table 6 compares the thermophysical properties estimated using the van Wijk and de Vries (1963) model and the new soil heat flow model developed for this study with the subsequent laboratory determinations. In general, the new model agrees more closely with the laboratory results than does the van Wijk and de Vries (1963) model. However, both models

**Table 6. Comparison of the thermophysical parameters before and after the controlled burn as determined *in situ* with the new model of heat flow developed for this study, the van Wijk and de Vries (1963) [WV63] model, and the laboratory analysis from a soil sample obtained after the burn and within the burned area**

Laboratory analyses were performed by Thermophysical Properties Research Laboratory (West Lafayette, IN)

Thermophysical parameter [model]	October	January	February	Laboratory
$\lambda_s$ (W m <sup>-1</sup> K <sup>-1</sup> ) [new]	0.204	0.193	0.182	0.32
$\lambda_s$ (W m <sup>-1</sup> K <sup>-1</sup> ) [WV63]	0.158	0.136	0.137	
$C_s$ (MJ m <sup>-3</sup> K <sup>-1</sup> ) [new]	0.562	0.549	0.562	0.92 (±0.03)
$C_s$ (MJ m <sup>-3</sup> K <sup>-1</sup> ) [WV63]	0.546	0.534	0.560	
$\kappa_s$ (10 <sup>-6</sup> m <sup>2</sup> s <sup>-1</sup> ) [new]	0.337	0.352	0.328	0.35
$\kappa_s$ (10 <sup>-6</sup> m <sup>2</sup> s <sup>-1</sup> ) [WV63]	0.292	0.256	0.248	
$D_0$ (m) [new]	0.104	0.098	0.095	0.098
$D$ (m) [WV63]	0.090	0.084	0.082	

tend to underestimate the laboratory results for  $\lambda_s$  and  $C_s$ . This may be explained, at least in part, by the small amount of moisture present in the laboratory sample (discussed earlier), which could cause the laboratory results to be a bit higher than the *in situ* estimates. Another possible contributing factor is that the HFTs are underestimating the heat flux due to poor contact with the soil and a correspondingly high contact resistance to heat flow (Sauer *et al.* 2003). Nevertheless, none of the changes in the thermophysical parameters from before to after the controlled burn appear to be significant, because all variations in the parameters are less than the ±15% inherent variability identified previously in the day-to-day changes. Therefore, we conclude that this controlled burn, which heated the upper centimeters of soil to over 400°C, probably did not affect the thermophysical properties of the soil. On the other hand, there is an apparent change in the phase difference from before and after the burn. The change in  $\alpha D_0$  values (Table 5) before and after the fire suggests that the phase difference was 10–20 min longer before the fire than after. We do not know the cause of this difference, but it may be related to changes in daylength and resulting insolation of the soil, or possibly by compaction of the soil by the slash pile itself. Certainly the greater uniformity with depth in measured soil bulk densities at the center of the pile relative to those at the edge or outside the pile (Table 2) is consistent with the possibility that some compaction may have taken place. If the soil had been compacted by the slash pile, then the increase in soil density could cause the gradients of the soil thermophysical properties ( $\alpha$ ) to increase as well. This change in the phase difference will require further research.

#### Soil heat flux during the fire

We conclude this section by deriving estimates of the true soil heat flux,  $G_s$ , from the measured,  $G_m$ , which requires augmenting the *in situ* estimates of  $\lambda_s$  to include temperature effects. The main source of these temperature effects on  $\lambda_s$  is likely to be the temperature dependency of the thermal conductivity of the soil air (Kadoya *et al.* 1985; Campbell *et al.* 1994).

We begin with the de Vries (1963) model of  $\lambda_s$  for a dry soil, which is

$$\lambda_s = 1.25 \left[ \frac{\phi_m \xi_m \lambda_m + \phi_a \xi_a \lambda_a}{\phi_m \xi_m + \phi_a \xi_a} \right], \quad (13)$$

where  $\phi_a$  is the volume fraction of air or the air filled porosity of the soil ( $\phi_a \approx 0.5$ ),  $\phi_m$  is the volume fraction of soil minerals ( $\phi_m = 1 - \phi_a$ ),  $\lambda_m$  is the thermal conductivity of the dominant soil mineral (which for this study will be taken as granite),  $\lambda_a$  is the thermal conductivity of air, and  $\xi_m$  and  $\xi_a$  are weighting factors. The thermal conductivity for granite is  $\sim 2.25 \pm 0.70$  W m<sup>-1</sup> K<sup>-1</sup> and appears to be independent of temperature, at least in the range  $\sim 20$ – $60^\circ\text{C}$  (Kessler 1927; Maqsood *et al.* 2003).  $\lambda_a$  is temperature dependent and is parameterized for this study, with the aid of the data from Kadoya *et al.* (1985), as

$$\lambda_a = \lambda_0 \left( \frac{T_K}{T_{st}} \right)^{0.8},$$

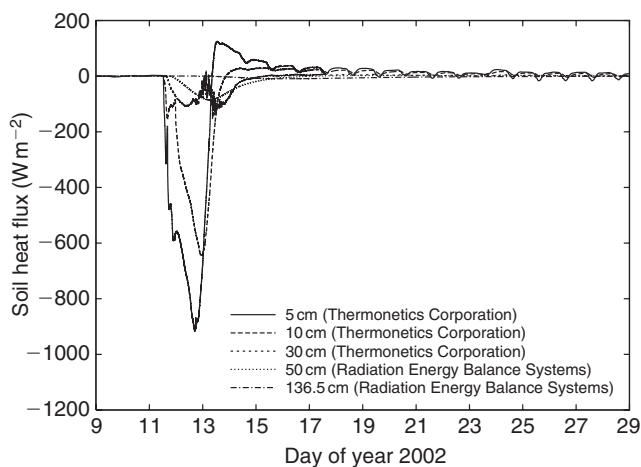
where  $\lambda_0 = 0.02432$  W m<sup>-1</sup> K<sup>-1</sup>,  $T_{st} = 273.15$  K, and  $T_K$  (K) is temperature.

The weighting factors,  $\xi_a$  and  $\xi_m$ , are functions of the shape or geometry of the soil particles and  $\lambda_a$  and  $\lambda_m$  (de Vries 1963; Campbell and Norman 1998). In the case of a dry soil  $\xi_a = 1$  and  $\xi_m$  is given as

$$\xi_m = \frac{2}{3[1 + g_a(\lambda_m/\lambda_a - 1)]} + \frac{1}{3[1 + g_c(\lambda_m/\lambda_a - 1)]},$$

where  $g_a$  and  $g_c$  are the soil particle shape factors with  $g_a \approx 0.1$  for a mineral soil and  $g_c = 1 - 2g_a$  (Campbell and Norman 1998).

At the nominal soil temperature at which the *in situ* method was applied ( $5^\circ\text{C}$ ) the de Vries (1963) model (or equation (13)) predicts that  $\lambda_s \approx 0.22$  W m<sup>-1</sup> K<sup>-1</sup>, which is somewhat lower than the estimate obtained experimentally and somewhat higher than estimated with the *in situ* van Wijk and de Vries (1963) model (Table 6). However, it is in reasonable agreement with the new *in situ* model (Table 6) and so it is used for calculating Philip's (1961) correction to  $G_m$ .



**Fig. 5.** Corrected soil heat fluxes for 9–28 January 2002 (before, during, and after the controlled burn) at the Manitou Experimental Forest controlled burn site. The negative sign of the heat flux indicates that the heat flux is into the soil. The burn was initiated about noon on 11 January. The soil heat flux correction is a result of the difference between the thermal conductivities of the soil and the material comprising the heat flux sensor. See Fig. 2 for the uncorrected (measured) heat flux. See the Instrumentation section of the main text for a discussion of the heat flux correction, equation (1).

Combining this adapted version of the de Vries (1963) model for  $\lambda_s$  with the Philip's (1961) correction, equation (1), yields Fig. 5 for the corrected soil heat flux during the controlled burn. Comparing this figure with its measured heat flux analog, Fig. 2, indicates the importance of the Philip's (1961) correction to measured soil heat fluxes during surface fires. The differences between  $G_m$  and  $G_s$  are highlighted by the difference at the time of maximum heat flow. The maximum corrected soil heat flux at 0.05 m is  $-918 \text{ W m}^{-2}$ , which is  $\sim 20\%$  less (in magnitude) than the measured flux at that depth. These results should be important for future fire/soil model validation studies, which until now have not included any observations of soil heat flux. In addition, observations of soil heat flux during fires could also be used to study the relationships between the rate of thermal energy input into soils and fire-induced changes in soil structure, soil chemistry, and soil biology. Such studies would complement our current understanding of the impact of fire on soils, all of which to date have been based on temperature observations alone (e.g. Hungerford *et al.* 1991; DeBano *et al.* 1998).

## Conclusions

This study explored the possibility that dry-soil thermal conductivity and volumetric specific heat capacity can be altered by fire by combining *in situ* observations of soil temperatures and heat fluxes with models of the daily (periodic) soil heat flow. The analysis was performed using several days of data before and after the controlled burn. Although the experimental burn achieved soil temperatures in excess of  $400^\circ\text{C}$

in the top 0.02 m of soil and over  $300^\circ\text{C}$  within most of the top 0.10 m of soil, it appears that it was not sufficiently intense to have significantly altered the thermophysical properties of the soil at the burn site at least during the month immediately following the fire.

Given that so much of the soil was exposed to temperatures exceeding  $300^\circ\text{C}$ , which is the threshold of expected change in soil structure (DeBano *et al.* 1998), and the close connection between soil structure and thermal conductivity (Farouki 1986), our results may seem at odds with expectations. But there are at least two mitigating issues. First, the soils at Manitou Experiment Forest are extremely poor in organic material (1–2% by volume) and most is located on top of the mineral layer. Therefore, the soil aggregates, which result from the presence and action of organic material (DeBano *et al.* 1998), may not have been significantly affected by the combustion of the soil organic matter. A second consideration is that, for the month or so after the fire, the period examined in this study, the soils were not perturbed. There were no wetting or drying cycles, although the soil did undergo a freeze–thaw cycle almost nightly. But again with virtually no soil moisture, the associated cycle of soil expansion and contraction may not have been enough to have perturbed soil aggregate stability. It is possible, therefore, that the conditions and soils at Manitou Experimental Forest are sufficiently unique, particularly during the period covered by the present study, that only minimal (or undetectable) structural change was possible as a result of the burn. Of course, it is also possible that the duration of the *in situ* soil observations was not long enough to have permitted a perturbation to cause an observable change.

Nevertheless, there is evidence that the soils within the burned area are somewhat different from soils in the surrounding area (see the vertical profiles of soil bulk density, Table 2). The densities within the burned area show less variability with depth, and may even increase with depth, which is the opposite of the outside and edge profiles. But we cannot conclude that this difference is indicative of a fire-induced change in the soil because it may just reflect the natural variability between soil samples or it may have resulted from compaction of the soil by the weight of the slash pile itself. Regardless of possible causes, any alteration of the soil near the center of the burn site does not appear to have been enough to have significantly affected soil thermophysical properties.

Despite this finding, we can conclude that the soils at the burn site were significantly impacted by the fire. Much of the soil's microbial population and other biota are likely to have been eliminated from the upper few centimeters of soil. Even as deep as 0.30 m the soil temperature reached  $\sim 80^\circ\text{C}$ , which would have been enough to have affected most of the biota (DeBano *et al.* 1998). The long-term consequences of changes in soil biota to the thermophysical properties, aggregate stability, and structure of these soils is not known.

However, it is likely that the interplay and feedbacks between the soil biota and the soil physical and thermophysical properties ultimately determine the soil's recovery from fire. The present experiment is the first of several studies intended to examine how the interaction between soil microbial recovery, the soil's physical properties, and different fuel amounts, geometries, and loading densities influence soil recovery and forest regeneration after fires.

The above issues and others raised in this study regarding short- and long-term changes in soil properties caused by fire will require further investigation. The pragmatic goal of these experiments is to provide tools to assist land managers in the use of prescribed fire to benefit ecosystems and to reduce the potential for harm. One such tool should be a better understanding and improved quantification of the threshold temperature for fire-induced changes in soil structure or thermophysical properties, and how that threshold may vary with soil type and fuel loading geometry. Other tools might develop from knowledge of the rate of soil thermal heating and the nature and rate of microbial response to fire. In addition, insofar as models of the soil thermal pulse caused by fire are useful, then experiments like the one discussed here should also prove useful. Such models offer the promise of being more quantitative in our ability to manage the consequences of controlled burns on soils.

Beyond issues directly involving soil thermophysical properties, this study also described and tested a new high-temperature soil heat flux transducer for measurements of soil heat flux before, during, and after fires. With this new sensor it is now possible to obtain (heretofore unavailable) *in situ* data on soil heat flux for model validation studies of the pulse of thermal energy into soils associated with surface fires. To maximize the usefulness of this sensor, however, this study also re-emphasized the need to include the temperature dependencies of: (1) the heat flux transducer's calibration factor; (2) its thermal conductivity; and (3) the soil's thermal conductivity when estimating the true soil heat flux from those measured with the new sensor. These last two features of sensor performance are required to correct the measured heat flux for differences in the thermal conductivity of the soil and the heat flux transducer material (Philip 1961; Figs 2, 5).

Finally, we also developed a new model for describing periodic heat flow in soils and showed that it agreed more closely with the observed time lag between the diel soil temperatures and heat fluxes than did the more familiar model of van Wijk and de Vries (1963). When used with the observed soil temperature and heat flux vertical profile data to obtain *in situ* estimates of the soil thermophysical properties, the new model also agreed more closely with laboratory analyses than did the van Wijk and de Vries (1963) model. The major difference between these two models is that the new model assumes monotonically increasing soil thermophysical properties, whereas the van Wijk and de Vries (1963) model assumes uniform soil thermal properties. Nevertheless, the

new model is (mathematically) very similar to this earlier model and it is also as easy to use.

### Acknowledgements

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## Appendix 1

The purpose of this appendix is to derive an analytical model of heat transfer in soils with non-uniform thermophysical properties. Although the focus here is on modeling heat flow for monotonically increasing soil thermophysical properties, the mathematical development can also be used as a template for monotonically decreasing soil thermophysical properties as well. For the sake of simplicity this discussion is cast in terms of the fundamental or 24-h periodic thermal wave. However, the results can easily be generalized to higher harmonics.

The model described here was developed to meet three specific conditions; therefore, all compromises and approximations used in deriving this model are a result of these imposed conditions. First, it must employ simple continuous analytical expressions for describing depth-dependency of periodic variations in soil temperatures,  $T(z, t)$ , and heat fluxes,  $G_s(z, t)$ . Second, it must permit a simple, but more general, expression for the phase difference between  $G_s(z, t)$  and  $T(z, t)$ , than the more traditional  $\pi/4$  (3 h) for the diel cycle. (Note here that a phase difference of  $2\pi$  corresponds to the 24 h period of the diel cycle.) Third, it should not violate physical reality much more than what occurs by assuming uniform soil thermophysical properties.

The one-dimensional (non-advective) soil heat conduction equation is given as follows

$$C_s(z) \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left( \lambda_s(z) \frac{\partial T}{\partial z} \right), \quad (14)$$

where  $C_s(z)$  is the soil volumetric specific heat capacity ( $\text{J m}^{-3} \text{K}^{-1}$ ), which is considered to be a function of depth  $z$  (m),  $\lambda_s(z)$  is the soil thermal conductivity ( $\text{W m}^{-1} \text{K}^{-1}$ ) also a function of depth,  $T$  is temperature (K), and  $t$  is time (s). Here  $z$  is taken as positive downwards with  $z = 0$  at the soil surface and  $C_s(z)$  and  $\lambda_s(z)$  are functions of depth only and are, therefore, invariant with time and temperature. (Note here that both  $C_s(z)$  and  $\lambda_s(z)$  are, in fact, generally temperature dependent (Kay and Goit 1975; Campbell *et al.* 1994).)

Nerpin and Chudnovskii (1984) showed that equation (14) can be transformed into the following expression

$$\frac{\partial T}{\partial t} = \left( \frac{\lambda_0}{C_0} \right) \frac{\partial^2 \tau}{\partial \zeta^2} + \omega_*(z) \tau, \quad (15)$$

where  $\tau$  is the transformed temperature (K)

$$\tau(\zeta, t) = \left[ \frac{C_s(z) \lambda_s(z)}{C_0 \lambda_0} \right]^{1/4} T(z, t)$$

and  $\zeta$  is the transformed depth (m)

$$\zeta = \sqrt{\lambda_0 / C_0} \int_0^z \sqrt{C_s(z) / \lambda_s(z)} dz,$$

where  $C_0 = C_s(0)$  and  $\lambda_0 = \lambda_s(0)$  are the surface values of the soil thermophysical parameters and  $\omega_*(z)$  ( $\text{s}^{-1}$ ) is given as

follows:

$$\omega_*(z) = \frac{1}{16} \left[ \frac{\lambda_s}{C_s} \right] \left[ \left( \frac{\lambda'_s}{\lambda_s} - \frac{C'_s}{C_s} \right)^2 + 4 \left( \frac{C''_s}{C_s} \right)^2 - 4 \left( \frac{\lambda''_s}{\lambda_s} + \frac{C''_s}{C_s} \right) \right],$$

for which the prime (') denotes differentiation with respect to  $z$ , i.e.  $\lambda'_s = d\lambda_s/dz$ ,  $\lambda''_s = d^2 \lambda_s/dz^2$ , etc.

For periodic forcing, such as the daily 24-h cycle, and for soil thermophysical properties that are constant with depth (i.e.  $\lambda_s(z) \equiv \lambda_0$ ,  $C_s(z) \equiv C_0$ ,  $\zeta \equiv z$ ,  $\tau(\zeta, t) \equiv T(z, t)$ , and  $\omega_*(z) \equiv 0$ ). Equation (14) has the well-known periodic solution (e.g. van Wijk and de Vries 1963: equations (2) and (3) of the main text). However, for a soil with non-uniform thermophysical properties the solution is more complicated. Although some analytical periodic solutions exist for both equations (14) and (15) (van Wijk and Borghorst 1963; Novak 1986; Massman 1993; Karam 2000), they are piecewise continuous (e.g. van Wijk and Derksen 1963; Massman 1993; Karam 2000), which for the present purposes is extremely inconvenient. Here we present a simpler, but approximate, analytical solution to equation (15), which is continuous for all soil depths and which is a relatively simple generalization of the solution to equation (14) for uniform thermophysical properties. These solutions are, however, based on the case that the soil thermophysical properties are either monotonically increasing or decreasing functions of depth into the soil.

Consider the following two cases describing  $C_s(z)$  and  $\lambda_s$ . Case I (monotonically increasing with depth):  $C_s(z) = C_0(2 - e^{-\alpha_c z})$  and  $\lambda_s(z) = \lambda_0(2 - e^{-\alpha_\lambda z})$ , and Case II (monotonically decreasing with depth):  $C_s(z) = C_0(1 + e^{-\alpha_c z})$  and  $\lambda_s(z) = \lambda_0(1 + e^{-\alpha_\lambda z})/2$ . For Case I  $C_s(z)$  increases monotonically from  $C_0$  at  $z = 0$  to  $2C_0$  at  $z = \infty$  (similarly for  $\lambda_s(z)$ ). For Case II  $C_s(z) = C_0/2$  and  $\lambda_s(z) = \lambda_0/2$  at  $z = \infty$ . These two cases are reasonable parameterizations, but they are not the only possibilities. Similar related functions could easily have been constructed to range over more than or less than a factor of 2.0 but, for the present discussion which is largely confined to a dry soil, the above functions are sufficient. For a dry soil  $C_s$  and  $\lambda_s$  are largely functions of the bulk density of the soil (e.g. Farouki 1986; Campbell *et al.* 1994; Campbell and Norman 1998) and, in general, for a dry soil  $C_s$  may vary somewhat less than a factor of two (if the bulk density, for example, varies with depth), whereas  $\lambda_s$  may vary slightly more than a factor of two. Thus the factor of 2 is a compromise.

Two other compromises are needed to make equation (15) amenable to a simple analytical solution. One is to assume that  $\alpha_c = \alpha_\lambda$ . This insures that  $\zeta \equiv z$ . Consequently, we will drop the subscripts from  $\alpha$  when describing the depth variation of  $C_s$  and  $\lambda_s$ . The second approximation involves  $\omega_*(z)$ . Focusing on Case I (monotonically increasing thermophysical

properties) yields the following expression for  $\omega_*(z)$ :

$$\omega_*(z) = \frac{1}{4} \frac{\lambda_0}{C_0} \alpha^2 \left[ \frac{e^{-\alpha z}}{2 - e^{-\alpha z}} \right] \left[ 1 + \frac{2}{2 - e^{-\alpha z}} \right]$$

which leads to the following inequality valid for all depths:

$$0 < \omega_*(z) \leq \frac{3}{4} \frac{\lambda_0}{C_0} \alpha^2$$

For periodic solutions the time dependency of both  $\tau(z, t)$  and  $T(z, t)$  can be described by  $e^{i\omega t}$ , which allows the preceding inequality to be expressed as

$$0 < \frac{\omega_*(z)}{\omega} \leq \frac{3}{8} \alpha^2 D_0^2$$

where  $\omega = 7.272(10^{-5}) \text{ s}^{-1}$  is the frequency of the diel cycle,  $i = \sqrt{-1}$  is the unit imaginary number, and  $D_0 = \sqrt{2\lambda_0/(C_0\omega)}$  (m) is the attenuation depth for the new model and is the analog to the attenuation depth for the uniform soil thermophysical properties model. The second approximation simply replaces  $\omega_*(z)$  in equation (15) by its maximum value, as given in the last inequality. Incorporating the above approximations into equation (15) yields the following solution (model) for periodic heat flow in soils with monotonically increasing thermophysical properties

$$\tau(z, t) = \Delta\tau e^{-\sqrt{\frac{C_0}{\lambda_0}(i\omega - \omega_e)z}} e^{i\omega t},$$

where  $\omega_e = 3\alpha^2 D_0^2 \omega / 8$  and  $\Delta\tau$  is the amplitude of the periodic  $\tau$ -wave at the soil surface. Further algebraic manipulation yields

$$\tau(z, t) = \Delta\tau e^{\gamma z / D_0} e^{i\omega t},$$

where  $\gamma = \sqrt{1 + \omega_e^2 / \omega^2} - \omega_e / \omega + i \sqrt{1 + \omega_e^2 / \omega^2} - \omega_e / \omega$ . But,  $\gamma$  can be further simplified by assuming that  $\omega_e / \omega = 3\alpha^2 D_0^2 \omega / 8 < 1$ . At this point this purely a heuristic assumption, which is justified only by the limited data presented in this study (see the Results section) and some unpublished soil data obtained at a site near where the present experiment was performed. In fact, the data presented in this study suggest that  $\omega_e / \omega < 0.2$  is more precise. Consequently,  $\gamma = (1 - 3\alpha^2 D_0^2 / 16) + i(1 + 3\alpha^2 D_0^2 / 16)$  appears to be a good approximation. Furthermore, if  $\omega_e / \omega < 0.2$  proves universally true, then  $\gamma$  can be further simplified to  $\gamma = 1 + i$ . For the soil data presented in this study, this last simplification is reasonable and will be adopted for the remainder of this work.

The resulting new model for periodic soil temperature variations is

$$T(z, t) = \Delta T \frac{e^{-\gamma z / D_0} e^{i\omega t}}{\sqrt{2 - e^{-(\alpha D_0)z / D_0}}}, \quad (16)$$

where  $\Delta T$  is the surface amplitude of the diel soil temperature wave. Note here (1) that  $\Delta\tau \equiv \Delta T$ , which follows from the preceding relationship between  $\tau(z, t)$  and  $T(z, t)$ ; and (2) that we have intentionally expressed the exponent in the denominator as  $(\alpha D_0)z / D_0$  because, as shown next, the product,  $(\alpha D_0)$ , can be determined directly from the phase relationship between the diel soil heat flux and temperature waves.

The soil heat flux,  $G_s(z, t)$  ( $\text{W m}^{-2}$ ), is given as

$$G_s(z, t) = -\lambda_s(z) \frac{\partial T(z, t)}{\partial z}$$

which yields

$$G_s(z, t) = \frac{\Delta T \lambda_0 (\sqrt{2 - e^{-(\alpha D_0)z / D_0}})}{D_0} \times \left[ \gamma + \frac{\alpha D_0}{2} \left( \frac{e^{-\alpha z}}{2 - e^{-\alpha z}} \right) \right] e^{-\gamma z / D_0} e^{i\omega t}$$

or

$$G_s(z, t) = \frac{\Delta T \lambda_0}{D_0 (\sqrt{2 - e^{-(\alpha D_0)z / D_0}})} \times \left[ \gamma (2 - e^{-\alpha z}) + \frac{\alpha D_0}{2} (e^{-\alpha z}) \right] e^{-\gamma z / D_0} e^{i\omega t}.$$

The term in the square brackets determines the phase difference between the soil heat flux wave and the temperature wave, which not surprisingly is a function of depth. Here we simplify the phase component only by replacing the terms  $e^{-\alpha z}$  and  $(2 - e^{-\alpha z})$  with their maximum values, which is equivalent to approximating the phase term in either of these two models of  $G_s(z, t)$  by  $[\gamma + \alpha D_0 / 2]$ . With this assumption the only difference between the last two expressions for  $G_s(z, t)$  is the  $\sqrt{2 - e^{-(\alpha D_0)z / D_0}}$  term, which is part of the numerator in the first relationship and part of the denominator in the second. Comparisons with observed profiles of heat fluxes suggested that incorporating it in the denominator produced a better model. Therefore, the second expression is the preferred model for this study. After some further complex arithmetic and using a Fourier expansion of  $\tan^{-1} x$  about  $\pi/4$ , the final expression for the new soil heat flux model is

$$G_s(z, t) = \frac{\Delta T \lambda_0 \sqrt{2} \left( \sqrt{1 + \alpha D_0 / 2 + \alpha^2 D_0^2 / 8} \right)}{D_0 (\sqrt{2 - e^{-(\alpha D_0)z / D_0}})} \times e^{-\gamma z / D_0} e^{i(\pi/4 - \alpha D_0 / 4)} e^{i\omega t}.$$

Equations (16) and (17) are the new model for periodic heat flow in a soil with monotonically increasing thermophysical parameters. They are relatively simple continuous functions, which generalize the standard (uniform thermophysical

properties) model for periodic diel soil heat flow. This new model differs from the standard model primarily with the introduction of one additional parameter,  $\alpha$ , which when combined with the attenuation depth,  $D_0$ , essentially determines the phase difference between the daily soil heat flux and the temperature waves. In equation (17) this phase relation is expressed through the term  $e^{i(\pi/4 - \alpha D_0/4)}$  which, because

$\alpha > 0$  and  $D_0 > 0$ , is consistent with Massman's (1993) prediction that the soil heat flux wave leads the temperature wave by less than 3 h when soil thermophysical properties increase with depth. Although not discussed here, the case for monotonically decreasing soil thermophysical properties results in a lag time that exceeds 3 h, also in agreement with Massman (1993).