

Optimal timber harvest scheduling with spatially defined sediment objectives

John Hof and Michael Bevers

Abstract: This note presents a simple model formulation that focuses on the spatial relationships over time between timber harvesting and sediment levels in water runoff courses throughout the watershed being managed. A hypothetical example is developed to demonstrate the formulation and show how sediment objectives can be spatially defined anywhere in the watershed. Spatial sensitivity in the example is shown, indicating that, at least in some cases, tracking sediment levels below a watershed may be inadequate for achieving sediment objectives within the watershed itself.

Résumé : Cette note présente un modèle simple qui met l'accent sur les relations spatiales en fonction du temps entre la récolte de matière ligneuse et les niveaux de sédiments dans le réseau hydrographique du bassin sous aménagement. Un exemple hypothétique est utilisé afin d'illustrer le modèle et montrer comment des objectifs reliés aux sédiments peuvent être fixés partout dans le bassin. L'exemple fait ressortir la sensibilité spatiale du modèle indiquant, qu'au moins dans certains cas, le suivi des niveaux de sédiments à l'exutoire d'un bassin peut s'avérer inadéquat pour rencontrer des objectifs reliés aux sédiments à l'intérieur du bassin lui-même.

[Traduit par la Rédaction]

Introduction

One environmental impact that has often been associated with timber harvesting is increased levels of sedimentation in nearby watercourses just after the harvesting operation and during the subsequent "green-up" period of forest regeneration and growth. Linear programming (and other mathematical programming) models have become standard tools in optimizing the schedule of timber harvests, on the basis of economic returns as well as other objective functions. The typical approach to accounting for sediment effects in these timber harvest scheduling models (as early as Bottoms and Bartlett (1975) or Dane et al. (1977) and as recently as Bettinger et al. (1998)) is to estimate the impact of every potential harvest at some downstream point in or below the watershed being managed without regard for local effects throughout the watershed. If only the sediment exiting the area being managed is of concern, then the traditional approach is reasonable. If, on the other hand, the sediment levels in reaches throughout the managed watershed are of concern, then a more complicated spatial problem is involved.

In this note, we develop a simple model formulation that focuses on the spatial relationships between timber management variables and sediment levels in water runoff courses throughout the watershed being managed, over time. This also allows sediment standards to be set anywhere in the watershed. We will begin by presenting this formulation and then explore it with a hypothetical example.

Formulation

We define the land units as areas that drain into stream sections as a part of a watershed system. The stream sections are defined so that their midpoints are the locations where sediment is potentially regulated and monitored. Typically, the stream sections would be delineated by forks, confluences, and other drainage features that create natural end-points. We will use discrete time periods and a "Model I" formulation (Johnson and Scheurman 1977) for timber harvest scheduling. Timber harvests are then chosen so as to meet prespecified timber targets and minimize sediment objectives that are spatially defined. Sediment levels could also be constrained while maximizing a timber or economic objective function. We employ the following formulation.

Minimize

$$[1] \quad \sum_{j \in \Theta} \sum_t S_{jt}$$

subject to

$$[2] \quad \sum_i \sum_{m=1}^{m_i} v_{imt} Y_{im} = Q_t \quad \forall t$$

$$[3] \quad S_{jt} = \sum_{i \in \Omega_j} \sum_{m=1}^{m_i} b_{ijmt} Y_{im} \quad \forall j, t$$

$$[4] \quad \sum_{m=1}^{m_i} Y_{im} = L_i \quad \forall i$$

$$[5] \quad Q_t \geq R_t \quad \forall t$$

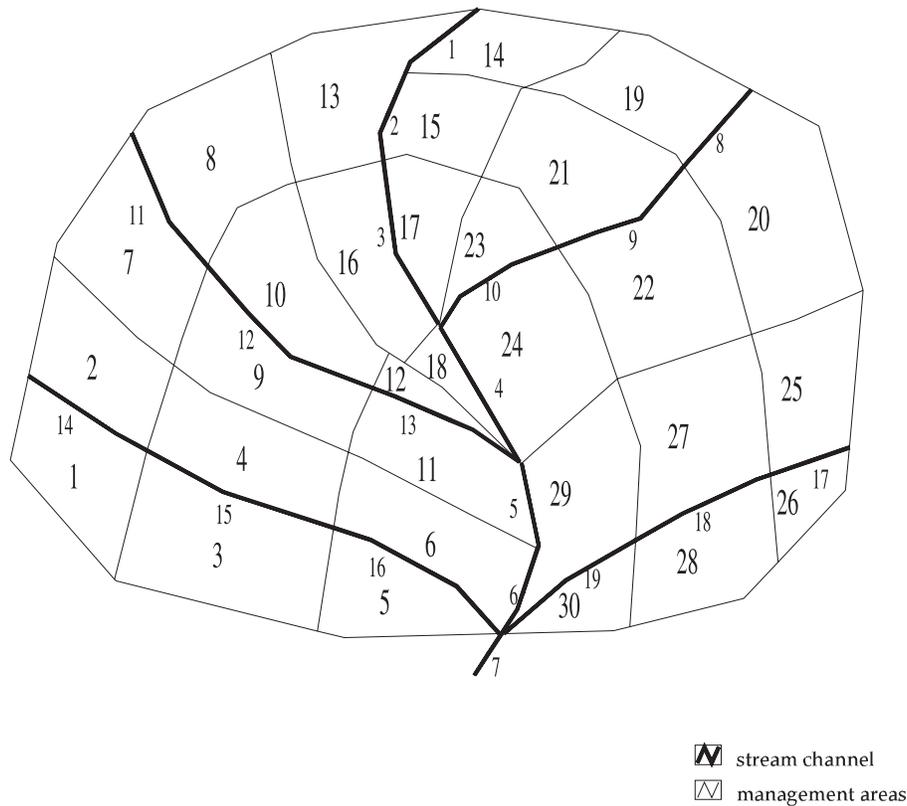
$$[6] \quad Y_{im} \geq 0 \quad \forall i, m$$

where m_i is the number of potential management prescriptions included for land unit i ; v_{imt} is the timber volume ob-

Received August 9, 1999. Accepted April 13, 2000.

J. Hof and M. Bevers, USDA Forest Service, Rocky Mountain Research Station, 2150A Centre Avenue, Suite 361, Fort Collins, CO 80526, U.S.A.

Fig. 1. The hypothetical example watershed, with numbered stream sections (small numbers) and management areas (large numbers).



tained per square kilometre in time period t from implementing management prescription m in land unit i ; b_{ijmt} is the sediment (metric tons) that passes through the midpoint of stream segment j , during time period t , as a result of 1 km² of land unit i being allocated to management prescription m ; θ is the set of stream sections included in the objective function; Ω_j is the set of adjacent and upstream land units that potentially impact sediment levels in stream section j ; L_i is the area (km²) of land in unit i ; R_t is the timber volume required to be harvested in time period t ; Y_{imt} is the area (km²) in land unit i allocated to management prescription m ; Q_t is the total volume harvested (m³) in time period t ; and S_{jt} is the average sediment level (in metric tons per year) at the midpoint of stream section j , during time period t . Equation 1 minimizes the total sediment level across all time periods and the selected stream sections. Including different subsets of the stream sections in the watershed will be explored in the hypothetical example. Equation 2 accumulates the volumes harvested in each time period into Q_t . Equation 3 accounts for the sediment load in each stream section, in each time period, as affected by the harvesting of adjacent and upstream land units (through the management prescriptions included). Equation 3 assumes that the sediment effects of harvesting different land units are additive. These sediment effects are thus totaled for each time period. Equation 4 limits the allocation of each land unit's area (to management prescriptions) by its size. Equation 5 includes the constraints on required timber harvests in each time period, and eq. 6 requires that the area allocated to any management prescription be greater than or equal to zero.

Like many models, the mathematics in eqs. 1–5 are quite

simple, but the determination of the coefficients (in particular, b_{ijmt}) is rather complex. We will discuss this process in the next section, where we develop a hypothetical example.

Hypothetical example

We assume five time periods of 10 years each, with all management actions taking place at the beginning of each time period. We also assume that trees become available for commercial harvest at age 30 and that the forest in all land units is initially 60 years old. For each land unit, we will include nine prescription options numbered in order: cut in time period 1, cut in time period 2, cut in time period 3, cut in time period 4, cut in time period 5, cut in time periods 1 and 4, cut in time periods 1 and 5, cut in time periods 2 and 5, and no harvest.

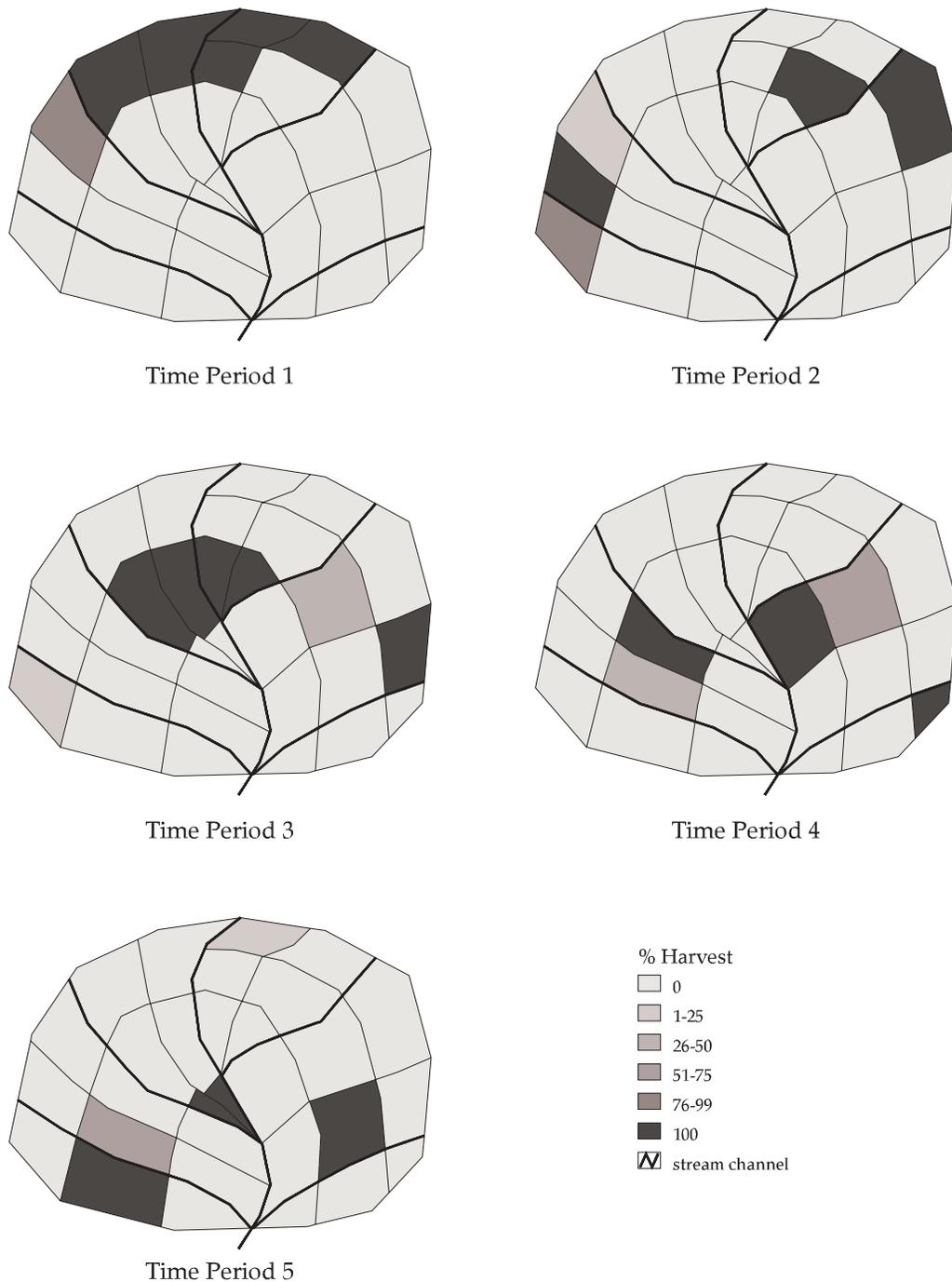
We will assume that artificial regeneration occurs immediately after harvest. For this hypothetical example, we constructed a yield function with a functional form from Rose and Chen (1977):

$$V(t) = 20\,790 (1 - e^{-0.0301t})^{0.6463} (1 - e^{-0.0212t})^{1.243}$$

where $V(t)$ is volume per square kilometre at age $t \geq 30$ years (m³) and t is age in years.

Figure 1 shows a hypothetical landscape with 20.66 km² total area, which we digitized so that a geographic information system (GIS) could be used to calculate land areas (L_i) and distances between land unit centroids and stream section midpoints. To generate the sediment coefficients, we used the following function:

Fig. 2. Harvest solution minimizing sediment in stream section 7, the watershed outlet.



$$[7] \quad b_{ijmt} = 300 \left(\frac{1}{2^{A_{imt}/10}} \right) \left(\frac{2}{D_{ij} + 1} \right)$$

where A_{imt} is the age (since harvest) of land unit i in time period t under management prescription m and D_{ij} is the distance between the centroid of land unit i and the midpoint of stream section j . This function was used to calculate a b_{ijmt} for each i - j pair and for each time period t , where A_{imt} was calculated for each management prescription m . Note that each stream section j is potentially impacted by harvesting

on any adjacent or upstream land unit i . In an application, these b_{ijmt} coefficients would probably be best estimated with a simulation model (as in Bettinger et al. (1998)) or with direct empirical data. For our purposes, we relied on the results from a classic field study (Moring 1975). In that study, the 75-ha Needle Branch watershed was completely clear-cut. Sediment, measured approximately 1 km from the centroid of the clearcut, peaked at 221.4 t/year the year after the harvest and diminished to 127.9 t/year in the 7th year after harvest. Thus, we constructed eq. 7 so that sediment transported from harvest areas declines (exponentially) by

Table 1. Sediment levels in the solutions for Figs. 2–4.

	Fig. 2	Fig. 3	Fig. 4
Minimized sediment in time period 1	543.8	150.7	545.4
Minimized sediment in time period 2	755.1	343.6	1 905.8
Minimized sediment in time period 3	881.5	1 309.5	3 139.6
Minimized sediment in time period 4	976.8	1 425.9	2 933.0
Minimized sediment in time period 5	1065.0	1 911.8	3 656.3
Total minimized sediment	4222.2	5 141.5	12 180.1
Total maximized sediment	7193.8	26 193.8	26 219.1

Fig. 3. Harvest solution minimizing sediment in stream sections 1–6, the main stream channel through the watershed.

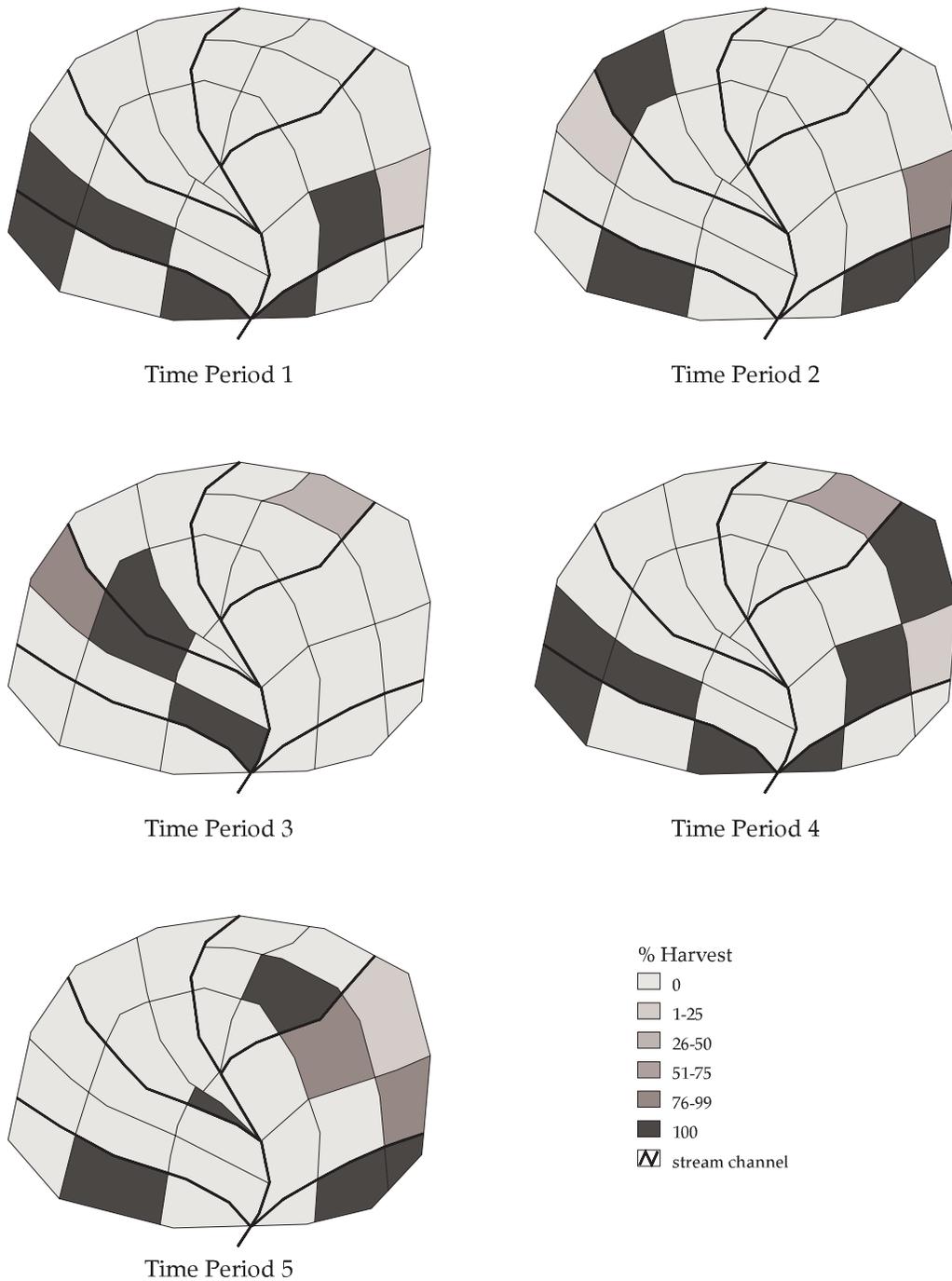
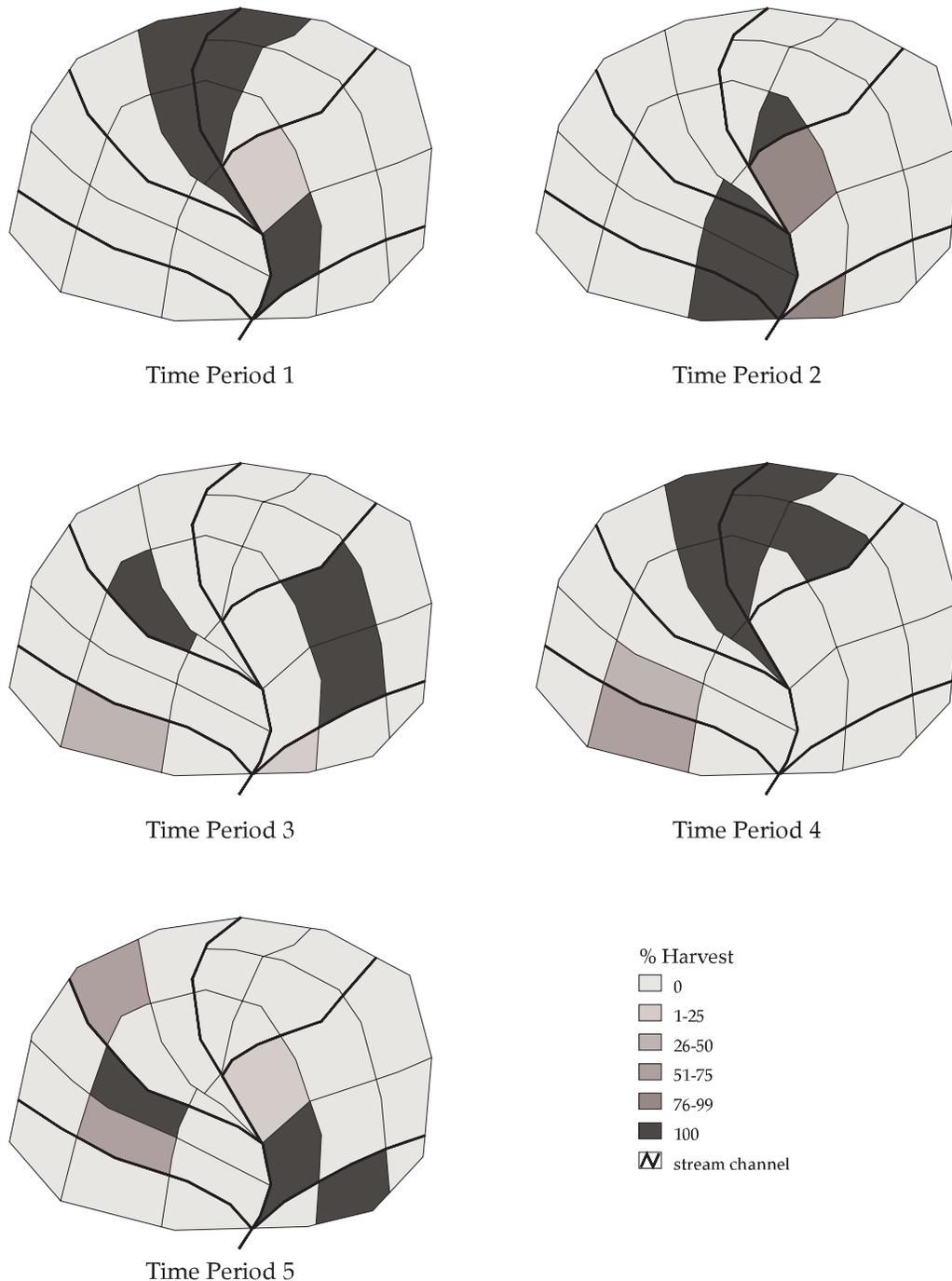


Fig. 4. Harvest solution minimizing sediment in stream sections 8–19, the upper reaches of the watershed.



50% every 10 years. We assumed that sediment will diminish in inverse proportion to distance and scaled eq. 7 so that, at 1 km, the sediment levels observed by Moring (1975) are approximated (221.4 t per 75 ha is approximately equivalent to 300 t/km²).

We built the model with the high-level programming language GAMS (Brooke et al. 1992), which is an algebraic-oriented automated matrix generator. We solved it with LINDO (Schrage 1991), which is a standard simplex-based solver, on a Pentium II (350 MHz, 66 MB RAM) personal computer. The A_{ijmt} coefficients were calculated in the GAMS matrix generator. The D_{ij} coefficients were calcu-

lated in a GIS and input into the matrix generator, where the b_{ijmt} parameters were then calculated for the linear program.

We began the analysis by maximizing the minimum harvest (across all five time periods) with no sediment constraints to determine the maximum even flow for the five time periods (see Hof et al. 1986). This solution indicated a yield of 74 692 m³ in each of the five time periods. We then set our timber targets at 50 000 m³ (approximately two thirds the maximum even flow) in all time periods.

Figure 2 shows the harvest pattern for a model solution with sediment in stream section 7 (the reach just below the managed watershed) minimized over all five time periods.

Table 1 presents the total sediment levels over time (from the objective function) in Figs. 2–4. In this solution, the model tends to avoid recutting young areas and cuts first in the areas least prone to causing sediment in stream section 7; as the remaining timber grows, the timber targets can be met with less area harvested. The areas harvested first are those that are farthest from stream section 7. As time goes on, the model selects areas closer to section 7, with increasing sediment effects despite the growth in timber volume per acre (1 acre = 0.405 ha) over time. This pattern seems inevitable, if sediment effects are negatively related to the distance between the harvested area and the stream section of concern. We tried constraining the sediment levels (to 500 t/year in section 7) and maximizing a discounted economic objective function, and the result was to harvest heaviest in time period 1, followed by reduced harvests with the same basic spatial pattern as seen in Fig. 2. With faster growth, an optimal solution might return to areas 7, 8, 13, 14, 15, and 19 in time period 4 or 5 rather than those areas shown in Fig. 2 but only if the growth was large enough to avoid large increases in area harvested to meet the timber target.

In Fig. 3, the summed sediment levels in stream sections 1–6 are minimized over all five time periods. This solution is much different than that in Fig. 2. As one would expect, early harvests now avoid the upper reaches of the main water course (sections 1–4) and are much closer to section 7. An increasing sedimentation pattern is again displayed, as more sediment-prone areas (relative to sections 1–6) are harvested. The important point is that the spatial selection of the objective function strongly affects the spatial pattern of solution.

This point is further demonstrated in Fig. 4, where the total sediment in stream sections 8–19 are minimized over all five time periods. Here, the harvested areas tend to be close to sections 1–6 but with a preference for areas 13, 14, and 15 (more like Fig. 2) to the point that secondary harvests are taken in time period 4.

At least in our simple example, the spatial pattern of harvests is strongly affected by the spatial definition of the sediment objective, and the individual spatial relationships between management areas and stream sections are key in creating that sensitivity. As another experiment to look at the overall spatial sensitivity of our problem, we maximized the sediment objectives in each of the three models just discussed. The maximum sediment levels are given in the last line of Table 1. Comparing the minimum and the maximum sediment levels for the three models indicates that the potential for spatial misallocation is greatest for the Fig. 3 model, which focuses on segments 1–6 (the maximum sediment level is five times the minimum level). This is interesting because the sediment-minimizing solution in Fig. 3 is the most different from the other two.

Conclusions

It is important not to generalize any results from our simple, hypothetical example. It demonstrates our modeling approach, however, and suggests that, at least in some cases, applying sediment objectives or constraints at the lowest (outlet) reach of a watershed may not be consistent with sediment objectives that apply throughout the watershed. The

need for such objectives has also been indicated empirically (Espinosa et al. 1997). Thus, the choice of spatial sediment objectives may be important, and we provide a simple methodology for including different spatial sediment objectives in linear timber harvest scheduling models. Obviously, measuring sediment at many points within a watershed is more difficult than just focusing on a single downstream reach. Thus, validating modeled sediment projections would be more expensive, the more spatially complex they are. A «minimax» approach (Luce and Raiffa 1957) might also be useful if a more even flow of sediment over time is desired. We should note that we have not accounted for stream temperature or timber transportation system effects. Some combination of the approach in Bettinger et al. (1998) and our spatial relationships within the optimization model might show promise along these lines.

Our final point is that, in viewing the solutions in Figs. 2–4, it appears that in our hypothetical example, sediment minimization tends to be associated with clustered harvests. Some authors (e.g., Roise 1990) have suggested that adjacency constraints would be useful in managing water quality by limiting the effective size of clearcuts. There may be relationships between clearcut size and sediment discharge that our model does not account for, but our results suggest that, in managing the spatial relationships between harvested areas and targeted stream sections, preventing adjacent harvests is not necessarily desirable. This question requires much more evidence for an empirical conclusion than we can provide here, but an interesting hypothesis is suggested: that small, dispersed harvests may not necessarily be best in managing for spatially defined sediment objectives, because concentration of areas that are isolated from targeted stream sections can take advantage of that spatial buffer. Overall, the problem of managing for sediment effects is spatially complex, and this note is just a first step in moving towards a more spatially complete analytical capability to address that problem.

References

- Bettinger, P., Sessions, J., and Johnson, K.N. 1998. Ensuring the compatibility of aquatic habitat and commodity production goals in eastern Oregon with a tabu search procedure. *For. Sci.* **44**: 96–112.
- Bottoms, K.E., and Bartlett, E.T. 1975. Resource allocation through goal programming. *J. Range Manage.* **28**: 442–447.
- Brooke, A., Kendrick, D., and Meerhaus, A. 1992. GAMS: a user's guide, release 2.25 edition. Scientific Press (Boyd and Fraser), Danvers, Mass.
- Dane, C.W., Meador, N.C., and White, J.B. 1977. Goal programming in land-use planning. *J. For.* **75**: 325–329.
- Espinosa, F.A., Rhodes, J.J., and McCullough, D.A. 1997. The failure of existing plans to protect salmon habitat in the Clearwater National Forest in Idaho. *J. Environ. Manage.* **49**: 205–230.
- Hof, J.G., Pickens, J.B., and Bartlett, E.T. 1986. A MAXMIN approach to nondeclining yield timber harvest scheduling problems. *For. Sci.* **32**: 653–666.
- Johnson, K.N., and Scheurman, H.L. 1977. Techniques for prescribing optimal timber harvest and investment under different objectives: discussion and synthesis. *For. Sci. Monogr.* **18**.
- Luce, R.D., and Raiffa, H. 1957. Games and decisions: introduction and critical survey. John Wiley & Sons, New York.

- Moring, J.R. 1975. The Alsea Watershed study: effects of logging on the aquatic resources of three headwater streams of the Alsea River, Oregon, Part II—changes in environmental conditions. Oregon Department of Fish and Wildlife, Clackamas. Fish. Res. Rep. 9.
- Rose, D.W., and Chen, C. 1977. Nonlinear biological yield models for jack pine. Minn. For. Res. Note 262.
- Schrage, L. 1991. LINDO user's manual, release 5.0 edition. Scientific Press, San Francisco, Calif.
- Roise, J.P. 1990. Multicriteria nonlinear programming for optimal spatial allocation of stands. *For. Sci.* **36**: 487–501.