For. Sci. 61(2):278–288
http://dx.doi.org/10.5849/forsci.14-112
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operations research

A Chance-Constrained Programming Model to Allocate Wildfire Initial Attack Resources for a Fire Season

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This research developed a chance-constrained two-stage stochastic programming model to support wildfire initial attack resource acquisition and location on a planning unit for a fire season. Fire growth constraints account for the interaction between fire perimeter growth and construction to prevent overestimation of resource requirements. We used this model to examine daily resource stationing budget requirements and suppression resource types and deployments within a fire planning unit. A chance constraint ensures the conditional probability of one or more fire escapes on days with ignitions below a predefined threshold. This chance-constrained approach recognizes that funding for local resources is unlikely to be sufficient for containing all fires in initial attack. For test cases, we used 1,655 fires occurring over 935 historical fire days from the Black Hills Fire Planning Unit in South Dakota. We tested our model under a variety of fire suppression assumptions to estimate appropriate daily stationing budget levels and resource allocations.

Keywords: fire simulation, suppression, exceedance probability, stochastic programming

Increased wildfire activity has been observed in the United States (Westerling et al. 2006) and Canada (Podur et al. 2002) in the past century. This upward trend is likely to continue owing to changing weather conditions associated with climate change (Wotton et al. 2003, Westerling et al. 2006) and development at the wildland urban interface (Snyder 1999, Radeloff et al. 2005). As suppression expenditures continue to rise, government agencies in the United States are seeking economically efficient wildfire management approaches and budget allocation strategies (Venn and Calkin 2011, Petrovic et al. 2012). One aspect of wildfire management that has been studied in this regard is initial attack (IA).

IA includes strategic decisions of deploying resources at fire stations and tactical decisions in dispatching resources to fires (Martell 1982, Ntaimo et al. 2012). Objectives for IA may include but are not limited to minimizing the area burned or suppression cost (Parks 1964, Cumming 2005). The US land management agencies historically have provided effective IA. For example, the US Department of Agriculture (USDA) Forest Service suppressed approximately 98% of all ignitions between 1970 and 2002 before fire sizes exceeded about 121 ha (Calkin et al. 2005). Despite the success of IA on federal lands in the United States, the overall impacts of fires are dominated by the infrequent large events instead of typical medium-sized events (Petrovic et al. 2012). The USDA Forest Service fires that did escape IA represented more than 97% of the total burned area for that agency (Calkin et al. 2005).

Optimization models with different structures and fire containment rules have been developed to improve IA efficiency. Wiitala (1999) converted a mixed-integer nonlinear programming formulation of IA dispatching decisions into a dynamic programming model. This model searches for the lowest cost combination of resources to build enough fire line to contain a fire at a predefined time. Donovan and Rideout (2003) developed a deterministic mixed-integer linear programming (MILP) model to dispatch firefighting resources to a fire across multiple time steps. Kirsch and Rideout (2005) and Rideout et al. (2011) extended this MILP model to address competing IA requirements arising from multiple wildfire ignitions. Containment rules in each of these models compare the total fire line constructed with predicted fire perimeter at each time step while ignoring the possibility that line constructed in earlier periods could retard fire growth in future periods. That is, a free-burning fire is assumed in these models (Wiitala 1999, Donovan and Rideout 2003, Kirsch and Rideout 2005, Rideout et al.

Manuscript received June 18, 2014; accepted August 27, 2014; published online October 2, 2014.

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Acknowledgments: *This research was supported by Agreement 11-CS-11221636-193 between the USDA Forest Service, Rocky Mountain Research Station and Colorado State University. We thank the Northern Great Plains Emergency Coordination Center and the Fire Program Analysis staff for the data they provided.*

2011). Fried and Fried (1996) suggested that modeling suppression based on free-burning fire growth can substantially overestimate final fire sizes and fire line construction resource requirements. Wei et al. (2011) developed an MILP model to allocate control locations on a fire by modeling the interaction between fire spread and line construction across time. Instead of modeling containment, this model minimizes the fire size (or loss) after a predefined period of time.

All of the models introduced above are deterministic. In reality, suppression decisions are made with limited time and information; thus, uncertainty is a crucial component of wildland fire management. Sources of uncertainty include fire weather, fire behavior, inaccurate data, the value of resources at risk, and operational effectiveness (Thompson and Calkin 2011). Stochastic programming models containing multiple fire samples or scenarios have been used to address some of these uncertainties. Haight and Fried (2007) developed a scenario-based two-stage MILP model to position 22 engines among 15 stations to minimize the number of suppression resources deployed and the expected daily number of fires that do not receive a standard response, defined as the "desired number of resources that can reach the fire within a specified response time." The standard response to each fire is derived from the maximum burning index of the fire day and needs to be applied to a fire within 30 minutes after a fire report. This MILP model does not directly model fire containment. Instead, a simulation model CFES2 (Fried and Gilless 1999) was used to evaluate the fire escape probability from engine deployments recommended by the MILP solution. Ntaimo et al. (2012) developed a two-stage stochastic programming model using the sample average approximation approach. Test cases presented in their study deploy 28 dozers to 8 bases in the first stage and dispatch them to each fire at representative fire locations to cover the length of the free-burning fire perimeter for containment at the end of the 6-hour standard response period. Time-specific fire growth (e.g., hour-by-hour) was not modeled in their mathematical formulation. The objective of their model is to minimize the sum of fixed deployment costs and the expected value of suppression cost and net value change. In a following study, Ntaimo et al. (2013) enhanced their model to compare fire line construction and fire perimeter in each fire spread period, declaring containment if the constructed fire line attains a user-defined percentage of the free-burning fire perimeter. Similar types of models can be found for other emergency response systems to deal with risks and solve location and allocation problems, e.g., emergency medical services (for a review, see Li et al. 2011).

The Fire Program Analysis (FPA) system[1] was developed by the USDA Forest Service and the US Department of Interior for modeling federal budget allocation strategies that include the numbers and base locations of various IA resources for a fire season. FPA contains a suite of simulation models to estimate the effects on several performance measures of predefined budget allocations and staffing strategies for 133 fire planning units (FPUs) across the United States. A goal programming model is then used to suggest funding levels and staffing for FPUs based on various sets of national performance measure weights and budget constraints. Within FPA, the Initial Response Simulator models IA on fires for numerous randomly generated fire seasons to estimate containment success and related measures for each budget allocation and staffing strategy in each FPU. The Initial Response Simulator uses the Fried and Fried (1996) method to model containment, which accounts for the interaction between the containment line and a fire's capacity to spread, thereby reducing fire growth.

In this article, we introduce a two-stage stochastic MILP model to station and dispatch hand crews and engines in an FPU. Solutions from the model identify the minimum required resource stationing budget, the number of crews and engines to employ for each fire season, and the fire station assignments for selected resources so as to limit the conditional probability of one or more fire escapes on days with ignitions. We define containment failures here as those fires that are not contained within a prespecified time after ignition. The probability of IA failure is modeled as a chance-constrained program using simulated historical fire events. Our model addresses situations when limited fire resources must be allocated among multiple co-occurring fires (e.g., Arienti et al. 2006), which can increase the likelihood that fires escape IA.

We use a test case from the Black Hills Fire Planning Unit (BHFPU) in South Dakota to test our model under a number of fire suppression assumptions. We parameterized our test model using historic fire ignition records from FireFamilyPlus (Bradshaw and McCormick 2000) and historic daily and hourly fire weather data from multiple remote automated weather stations (RAWS). We simulated each historic fire at its original location and under associated weather in FARSITE (Finney 2004) on a rasterized heterogeneous landscape from LANDFIRE (Rollins and Frame 2006). Simulated hourly free-burning fire perimeters and the average distances between adjacent perimeters are built into mathematical programming equations to track how fire line constructed at earlier hours reduces the length of actively spreading fire perimeters during later hours. This model allows users to adjust how fire line could be built progressively to connect and contain hourly footprints of a free-burning fire. It compares the amount of fire line constructed during each hour with the uncontained fire perimeter to determine containment. Parameters in the model can be set to vary travel time to fires and to account for delays in dispatching IA resources, which can result in dramatic increases in fire loss (MacLellan and Martell 1996) and greater demand for resources (Petrovic et al. 2012). Likewise, the model accounts for changes in the spread rate of fires, which can influence IA success (Haight and Fried 2007), and variations in crew productivity defined as the rate (meters per hour) at which firefighters construct fire line (Hirsch and Martell 1996).

Methods

Birge and Louveaux (1997, p. 84) define a two-stage stochastic linear programming problem as

$$\begin{aligned}
&\text{Minimize } z = \mathbf{c}^T\mathbf{x} + \mathbf{E}_\xi[\min \mathbf{q}(\omega)^T \mathbf{y}(\omega)]\\
&\text{s.t.}\\
&\mathbf{Ax} = \mathbf{b}\\
&\mathbf{T}(\omega)\mathbf{x} + \mathbf{W}\mathbf{y}(\omega) = \mathbf{h}(\omega)\\
&\mathbf{x} \geq \mathbf{0},\ \mathbf{y}(\omega) \geq \mathbf{0}
\end{aligned}$$

where the vector $\mathbf{x}$ contains first-stage decisions that must be made "up front" based on known initial conditions and on the probabilities of subsequent random events, before observing which events occur. The vector $\mathbf{y}$ (ω) contains second-stage "wait and see" recourse decisions that can be made after the random events become known. The "technology" matrix $\mathbf{T}$ (ω) and right-hand side vector $\mathbf{h}$ (ω) contain any random parameters in the constraint set. Matrix $\mathbf{A}$, "recourse" matrix $\mathbf{W}$, and right-hand side vector $\mathbf{b}$ contain only

parameters that are known at the start. The objective function minimizes the sum of known "costs" $\mathbf{c}^T\mathbf{x}$ associated with the first-stage decision plus the expected value of least "costs" associated with second-stage recourse decisions.

In the context of our fire season resource acquisition and location problem, the first-stage decision identifies which suppression resources should be located at each fire station for IA over a fire season. In the USDA Forest Service budget, costs associated with this decision stage are classified as "preparedness" expenditures (USDA Forest Service Manual 1999). Second-stage recourse decisions determine which of the available suppression resources to dispatch to each fire. Costs associated with these second-stage firefighting decisions are classified as "suppression" expenditures (USDA Forest Service Manual 1999). Additional expected losses and benefits from fires might also be accounted for in the second stage, although we do not attempt to do so in this initial investigation. We do consider, however, a third category of costs referred to as "severity" expenditures (USDA Forest Service Manual 1999, 2007), which can be used to preposition resources from other parts of the country in an FPU or take other actions to supplement local resources during periods of exceptional fire danger.

In USDA Forest Service practice, second-stage suppression costs for smaller fires are often aggregated in recordkeeping, in part because detailed accounting is difficult when multiple small fires occur and in part because these costs tend to be dwarfed by suppression costs for the larger fires that escape IA. Consequently, we drop the second-stage expected value term from the objective function in our IA-oriented model, replacing it with a chance constraint aimed to control the probability of experiencing IA days that result in one or more fires escaping containment.

Stochastic programming problems typically are solved using deterministic equivalent mathematical programs, often formulated as approximations or estimates rather than exact models for large, complex problems (Birge and Louveaux 1997). We define our deterministic equivalent formulation and our use of the chance constraint in detail below.

The MILP Model for Seasonal Resource Acquisition and Location (SRAL)

The model contains the following.

Indices

- g index of all sampled fires included in the model.
- k index of fire days. More than one fire could ignite in each fire day.
- i index of fire stations.
- r index of suppression resource types.
- t index of time steps (periods) since the start of a fire.

Sets

- G_k a set of fire ignitions in the same fire day k.
- A the set of fire days (indexed by k) included in the deterministic equivalent MILP model. This set reflects our chance constraint, as described later.

Variables

- $x_{r,i}$ general integer variables denoting how many resources of type r should be stationed at fire station i. These indicate the first-stage decision.
- $y_{r,i,g}$ general integer variables denoting how many resources of type r are dispatched from fire station i to fire g. These indicate the second-stage decision.
- $l_{g,t}$ continuous variables denoting the length of new fire line that can be constructed during period t to suppress fire g. We assume that fire line produced during period t shortens the uncontained perimeter of fire g at the end of period t.
- $f_{g,t}$ binary variables denoting whether fire g will be contained at the end of period t after suppression. $f_{g,t} = 1$ denotes containment of fire g at the end of period t; otherwise $f_{g,t} = 0$.
- $v_{g,t}$ binary variables denoting whether fire line is built to contain fire g in period t. $v_{g,t} = 1$ denotes that fire line is built in period t for fire g.
- $w_{g,t}$ binary variables working as a switch; $w_{g,t-1} = 1$ denotes that the adjusted distance ($\beta_D D_{g,t}$) between two free-burning fire perimeters at periods $(t - 1)$ and t will be added into the uncontained perimeter of fire g at the end of period t.
- $p_{g,t}$ continuous bookkeeping variables used to track the uncontained perimeter of fire g at the end of period t before subtracting the new fire line constructed during period t.
- ρ_g bookkeeping variables tracking the uncontained perimeter of fire g after the T period IA time frame. ρ_g is set to 0 if fire g was contained before or at the end of period T.
- ρ^{Total} bookkeeping variables tracking the total uncontained fire perimeters for all fires in set A after the IA time limit.
- u a bookkeeping variable tracking the total IA resources stationing cost in an FPU (the preparedness cost).

Parameters

- $C_{r,i}$ the cost of stationing resource r at fire station i.
- $L_{r,i,g,t}$ the length of fire line constructed by each resource of type r dispatched from station i to suppress fire g during period t. A dispatch or departure time delay and travel time affecting resource arrival from a station to a fire can also be set through this parameter. For example, $L_{r,i,g,t}$ is set to 0 at period t if a resource of type r cannot arrive at fire g from station i before period t. Likewise, $L_{r,i,g,t}$ can be set to reflect a constant line production rate after the resource has arrived at the fire, or it can be set such that it reflects declines in line productivity after some duration of operations resulting from potential crew fatigue or an out-of-water situation for an engine.
- $P_{g,t}$ the free-burning perimeter of fire g at the end of period t without suppression. This fire perimeter is calculated by simulating fire spread using available fire simulation software, in this case FARSITE (Finney 2004).
- β_D a parameter that reflects the possible line construction paths between two consecutive fire spread periods $(t - 1)$ and t (explained in more detail using Figure 2).
- $D_{g,t}$ the average distance between free-burning fire perimeters at the end of period $(t - 1)$ and t (explained in more detail using Figures 1 and 2).
- M a large positive number used to set the value of logic (switch) variables.

$$\text{Min } Z = u \tag{1}$$

subject to

$$\sum_{g \in G_k} y_{r,i,g} \le x_{r,i} \qquad \forall r,i,k \in A \tag{2}$$

$$\begin{cases} p_{g,t} \ge \dfrac{P_{g,t}}{P_{g,t-1}}(p_{g,t-1} - l_{g,t-1}) \\ \qquad + 2\beta_D D_{g,t} w_{g,t-1} & \forall g,t | P_{g,t-1} \ne 0 \quad (3.1) \\ p_{g,t} \ge P_{g,t} & \forall g,t | P_{g,t-1} = 0 \quad (3.2) \end{cases}$$

$$l_{g,t} \le \sum_r \sum_i L_{r,i,g,t} y_{r,i,g} \qquad \forall g,t \tag{4}$$

$$\begin{cases} l_{g,t} \le M v_{g,t} & \forall g,t \quad (5.1) \\ v_{g,t} \ge v_{g,t-1} & \forall g,t \ge 2 \quad (5.2) \end{cases}$$

$$p_{g,t} - l_{g,t} \le M(1 - f_{g,t}) \qquad \forall g,t \tag{6}$$

$$l_{g,t} - p_{g,t} \le M f_{g,t} \qquad \forall g,t \tag{7}$$

$$w_{g,t} \ge v_{g,t} - f_{g,t} \qquad \forall g,t \tag{8}$$

$$p_{g,T} - l_{g,T} \le \rho_g \qquad \forall g \tag{9}$$

$$\sum_g \rho_g \le \rho^{\text{Total}} \tag{10}$$

$$\rho^{\text{Total}} \le 0 \tag{11}$$

$$\sum_r \sum_i C_{r,i} x_{r,i} \le u \tag{12}$$

Objective function 1 minimizes the total budget for stationing all resources. Constraint 2 enforces that the number of resources of type r dispatched from station i to all fires within any modeled fire day cannot be more than the total number of resources of type r available from station i. This reflects an assumption that any resource which has already been dispatched to a fire cannot be dispatched to another fire within the same day. Resources are assumed to go back to their assigned stations, ready to be dispatched again the next fire day.

Constraint 3.1 calculates the length of uncontained fire perimeter of fire g at the end of period t. It assumes that fire line constructed in earlier periods will halt fire spread at those locations. Whereas the FARSITE simulations used to parameterize our model are run on a spatially defined raster landscape, fire growth in our model is not. Instead, we use the length of the free-burning fire perimeters calculated in FARSITE to calculate a discrete-time linear approximation of perimeter growth reduced by line construction. P_t/P_{t-1} is the ratio between the length of the free-burning fire perimeters at the ends of period t and $(t - 1)$. It reflects the rate of fire perimeter expansion during period t and is used to approximate how much fire line constructed in period $(t - 1)$ can reduce the expansion of fire perimeter during period t. This equation allows us to approximate the effects of fire line construction on perimeter growth by referencing the set of free-burning fire footprints created from FARSITE (demonstrated in Figure 1). We built Constraint 3.2 in place of 3.1 for time steps where FARSITE reported fire perimeter lengths of 0 in the preceding time step. Because Constraint 3.1 represents an approach that discretizes both the continuous fire growth and fire line construction, modeling shorter time steps can help create smoother fire growth and line construction paths. However, shorter periods also require additional variables, creating a more complex model.

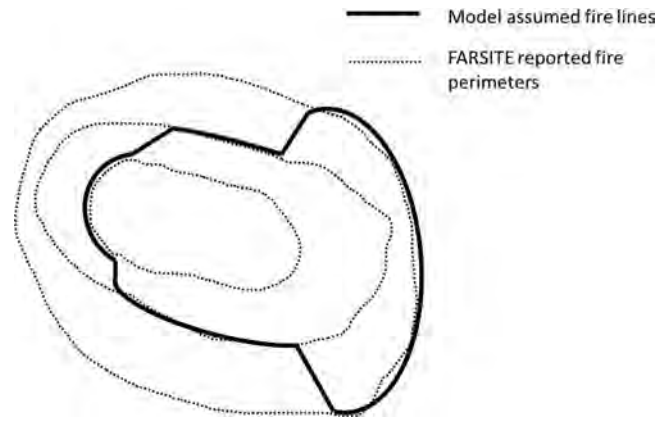


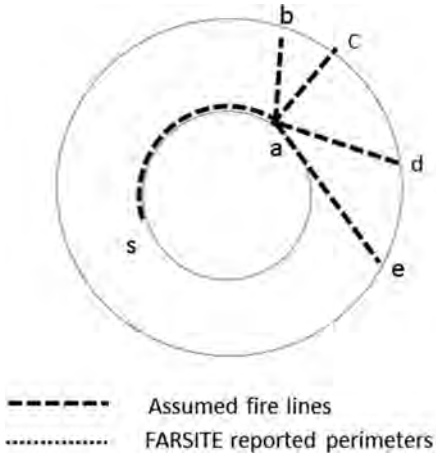
Figure 1. A stylized example of free-burning fire perimeters from fire simulation overlaid by the model-assumed fire line constructed along and across the free-burning perimeters.

We do not model the exact line construction paths in Constraint 3.1. Instead, the value of β_D is set to mimic different fire line construction paths, providing a calibration option in the model. The possible effects of selecting different β_D values to approximate possible fire line shapes are illustrated by Figure 2. Lower values of β_D represent assumptions of more efficient line construction when we connect free-burning fire perimeters (Figure 2). Sensitivity analyses can be used to help select values of β_D.

Constraint 4 tracks the new fire line constructed during period t by all resources dispatched from all stations to a fire g. Constraint 5.1 sets the value of binary variable $v_{g,t}$ to 1 when fire line is first built in period t, and it remains 1 until the end of the IA time limit T (Constraint 5.2) to ensure the continuity of fire line. Constraints 6 and 7 set the binary variable $f_{g,t}$ to indicate whether fire g is contained at the end of period t. If and only if the amount of new fire line constructed during period t is as long as or longer than the uncontained fire perimeter at the end of period t, $f_{g,t}$ is set to 1 to declare containment of this fire. $f_{g,t}$ remains 1 for the time steps after fire containment (e.g., for periods $t + 1$, $t + 2$, … after the fire is contained at period t) because the fire perimeter lower bound (set by Constraint 3.1) remains 0 afterwards. Constraint 8 sets $w_{g,t}$ to 0 before suppression starts (when $v_{g,t} = 0$). Constraint 8 also sets $w_{g,t}$ to 0 in period t when fire g is contained ($v_{g,t} = 1$ and $f_{g,t} = 1$). If $w_{g,t} = 0$, the model excludes $\beta_D D_{g,t}$ from the next-period fire perimeter growth in Constraint 3.1.

Constraint 9 calculates the value of Pg, the perimeter (if any) for fire g remaining after the T period IA time frame. Constraint 10 sums the total uncontained fire perimeter for all fires after the T period IA time frame. Constraint 11 enforces the full containment of all modeled fires. We will describe how Constraint 11 is used with a chance constraint in the following section. If full containment is not required, this constraint could be dropped in concert with other model changes.

Constraint 12 tracks the total resource stationing budget (the preparedness budget). This budget is minimized in the objective function. Recall that the variable cost (suppression cost) of dispatching a resource to a fire is not counted in this study. We assume that



Assumed fire line construction Path	Condition	β_D
s→a→e	ae<ce	< 0
s→a→d	ad=cd	0
s→a→c	Line built perpendicularly across rings	1
s→a→b	Extra line needed	> 1

Figure 2. In this model, we assume that fire lines would first be constructed along each FARSITE-reported free-burning fire perimeter. Lines built along two adjacent fire perimeters also need to be connected by additional fire lines. β_D is used to adjust how to build connecting lines between adjacent perimeters. A larger β_D results in longer fire lines to contain a fire. This example uses circular fire shapes to simplify demonstration.

fire operational costs come from sources different from the stationing costs, although this can vary by firefighting agency. This variable cost could be incorporated into the budget calculation if needed.

Determining the Budget Based on Predetermined Exceedance Probabilities

As stated in the Introduction, we are interested in minimizing the required preparedness budget and identifying fire station assignments for IA resources so as to limit the probability that any day with fire ignitions will result in one or more IA containment failures. In the context of our model, this requires meeting the following chance constraint

$$P_\omega\left(\sum_{g\epsilon G(\omega)}(1-f_{g,T})>0\right)\leq\alpha$$

where the probability that IA fails to contain one or more fires within T time periods on any random fire day ω is no greater than a prespecified exceedance probability α (e.g., $\alpha = 0.05$) (Gumbel 1958, Makkonen 2008). $G(\omega)$ is the set of fires in fire day ω; $f_{g,t} = 1$ denotes that fire g is contained within T time periods.

Use of this chance constraint recognizes that stationing suppression resources for a fire season within an FPU to contain all possible fires can be too expensive to be practical. Instead, the constraint partitions a set of fire days on a planning unit into two groups that we refer to as *group A* and *group B*. Group A contains those fire days in which the budgeted seasonal IA organization is intended to contain all fires in the FPU. Group B contains the remaining fire days in which the budgeted IA organization probably would experience containment failures without supplemental resources. Conceptually, we use the chance constraint to partition the overall FPU planning problem into preparedness and severity subproblems, where group A fire days are intended to be handled by the FPU seasonal (preparedness) IA resources and group B fire days require supplemental (severity) IA resources from outside the FPU. Exceedance probability α sets a breakpoint between these two subproblems. Our SRAL model then identifies an FPU seasonal IA organization that minimizes the preparedness budget required for full containment in all group A fire days.

We employ a sampling procedure to implement this probabilistic constraint. Starting from a large random sample of N fire days, we solve the SRAL model N times, once for each fire day k independently. A minimum resource stationing budget Z_k that provides resources to contain all fires in each fire day k is determined. All fire days {1 to N} are then sorted based on Z_k from the lowest to the highest budget to form an ordered set. An estimated exceedance probability α where $\alpha = 1 - K/(N+1)$ (Gumbel 1958, Makkonen 2008) is achieved if we can contain all fires in K out of N independent fire days. This equation can be used to calculate the value of K with a given α and N, where the resulting K is rounded up to a whole number to identify the breakpoint for partitioning a subset of ordered fire days {1 to N}. The 1st through the Kth fire days are put in group A and used to identify seasonal resource needs and their station assignments to achieve a targeted exceedance probability α. Because our fire day observations are from a historic time series record and are not independent, we used circular block bootstrapping to estimate confidence intervals (Politis and White 2003) on the targeted exceedance probabilities for various group A fire day partitions.

For cases combining large sample sizes with small exceedance probabilities, there could be hundreds or thousands of fire days in group A and substantially more individual fires. Consequently, we used an iterative procedure to avoid having to solve models containing very large numbers of fire days and fires. For this procedure, we observe that if an SRAL model solution is optimal (i.e., achieves minimum cost) for any subset of the group A fire days and is also feasible for the other group A fire days, then that solution is optimal for the full group A fire days problem. Anticipating that the fire days in group A with the largest daily stationing budget requirements are probably the most severe fire days to address, we implemented the following iterative procedure:

1. Begin with the daily stationing budget configuration problem corresponding to the Kth group A fire day, which requires the largest daily stationing budget in all group A fire days, as the master problem and its optimal solution as the master problem solution.

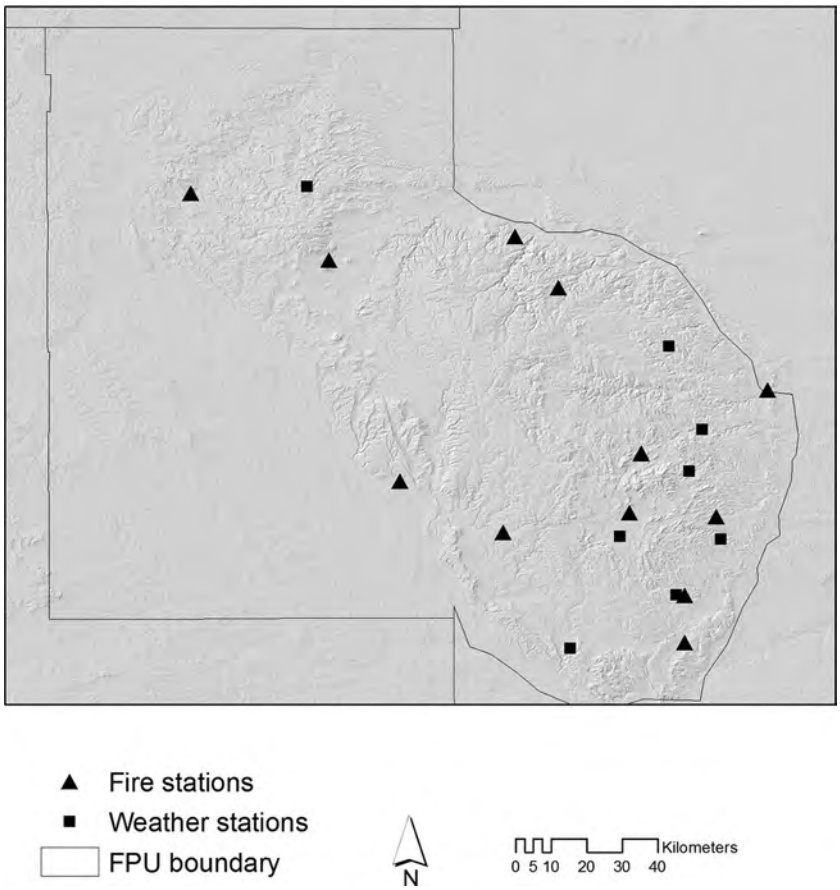


Figure 3. The locations of the RAWS and fire stations in the BHFPU.

2. Impose the master problem solution on the daily budget requirement problem for fire day $K-1$, then for fire day $K-2$, and so on down to fire day 1. If a problem is found to be infeasible, stop at that fire day and go to step 3. When a solution is found that is feasible for all group A fire days, go to step 4.
3. Add the fire day that resulted in infeasibility to the master problem and re-solve. Repeat step 2.
4. Accept this master problem solution as an optimal solution to the full group A problem.

It is also possible that the final solution resulting from this procedure might be a feasible solution for fire days ranked higher (i.e., some of the more costly group B fire days) than fire day K. Such an outcome still conforms to our chance constraint while indicating that the same IA organization can achieve an even smaller exceedance probability than was specified for the problem.

Test Case

We elected to use all federal lands within the BHFPU as the study site to test our model. The BHFPU is located in western South Dakota and northeastern Wyoming, covering an area approximately 200 km long and 105 km wide. Figure 3 shows a map of the BHFPU including the FPU boundary, weather stations, and available fire station locations.

Simulating Perimeters of Free-Burning Fires

We downloaded the BHFPU fire and weather records for years 1993–2010 from FireFamilyPlus (Bradshaw and McCormick 2000). From these records, we selected all federal fire records with sufficient data for our simulations. This process produced records for 1,655 ignitions (Figure 4) occurring in 935 fire days. We collected the hourly and daily weather data associated with these fires and used FARSITE (Finney 2004) to simulate the hourly growth of each fire on raster landscapes with elevation and fuel types downloaded from LANDFIRE (Rollins and Frame 2006). Hourly weather data from seven RAWS (Red Canyon, Elk Mountain, Custer, Custer State Park, Mount Rushmore, Baker Park, and Nemo) were used to support the calculation of spatially varying hourly wind fields (direction and speed) for these simulations, using a "point initialization option" in WindNinja (Forthofer et al. 2009). Perimeters of these simulated free-burning fires were collected for each hour over a 24-hour period to support parameter calculations for our SRAL models. Figure 5 shows a frequency distribution of our 935 daily totals of fire perimeters (summed over all fires in each fire day) from simulating each fire as a free-burning fire for 24 hours.

Suppression Resource Types, Productivity, and Travel Times

Hand crews, fire engines, and water tenders are the three types of fire suppression resources included in this study. We modeled two types of fire engines using data collected for FPA from this FPU: in our models, each small engine, type 5 or type 6, as described by the Federal Fire and Aviation Task Group (2014), dispatches with three crew members; and each large engine, type 3 or type 4, as described by the Federal Fire and Aviation Task Group (2014), dispatches with four crew members. Water tenders transport water from a water source (i.e., lake, stream, well, or other) to the engines at a fire

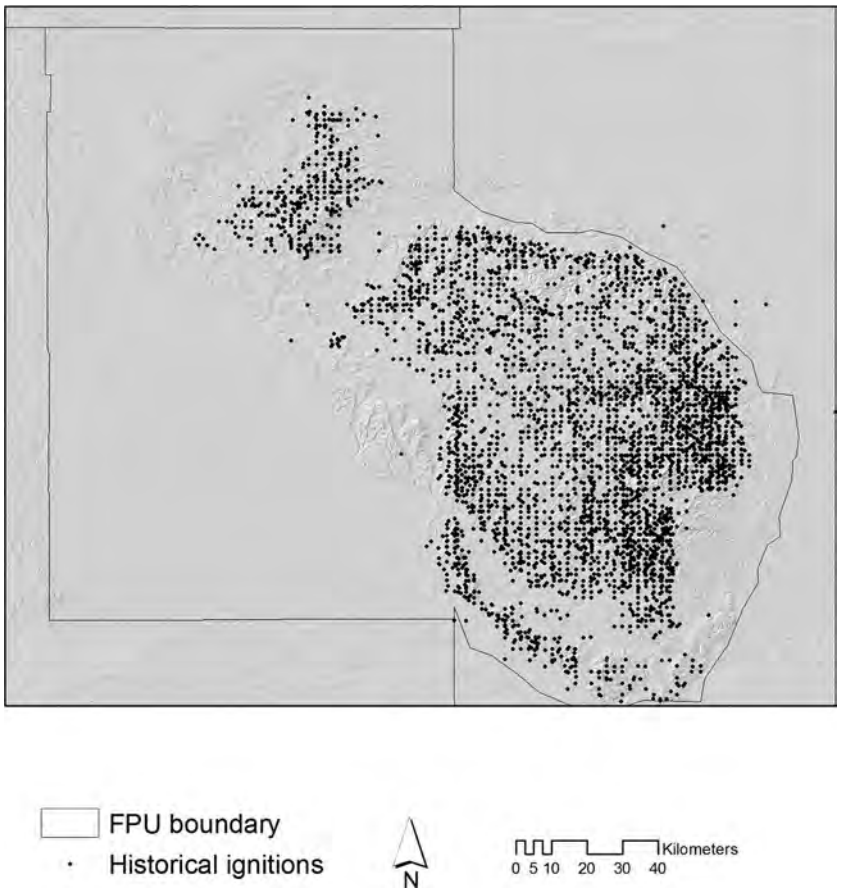


Figure 4. The ignition locations of 1,655 historical federal fires from 1993 through 2010 in the BHFPU.

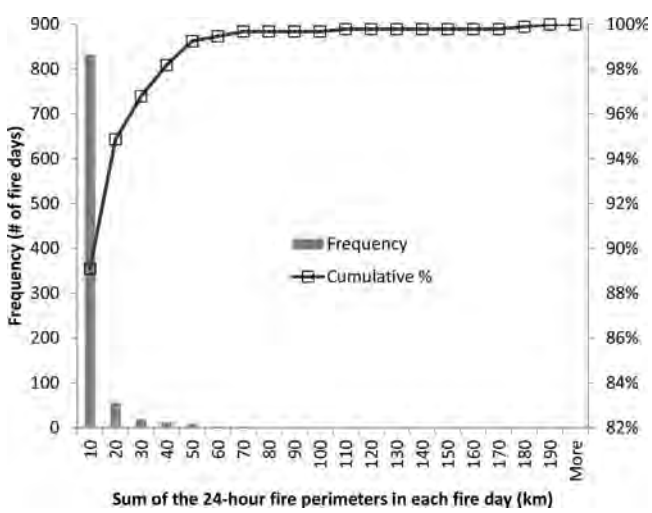


Figure 5. Frequency of the total 24-hour free-burning fire perimeters of all fires per fire day. Fires were simulated by FARSITE from their historical ignition locations using corresponding historical weather.

scene. For simplicity, we allow water tenders to be paired as an option only with the large fire engines to maintain a higher line production rate for this engine type. We assume that the small fire engines will run out of water after an hour of use, and the large fire engines will run out of water every 2 hours without support from a water tender. When an engine without a water tender runs out of water, we assume that one crew member has to drive it to a water source for refilling, while the remaining crew members continue working on the fire at a hand crew rate of line productivity. For example, when a small engine is dispatched to a fire in a location with fuel type 121 (Low Load, Dry Climate Grass-Shrub based on Scott and Burgan 2005), the hourly line production rate would be set to 13.5 chains per hour for odd hours (1, 3, 5, …) after arrival at the fire, reflecting work periods with water available, and would be set to 5.4 chains per hour for even hours (2, 4, 6, …), reflecting the line production rate of a two-person hand crew. When a large engine is dispatched to the same fire without a water tender, the hourly line production rate would be set to 18.9 chains per hour for hours (1, 2, 4, 5, 7, 8, …) after arrival at the fire, reflecting work periods with water available, and would be set to 8.1 chains per hour every third hour (3, 6, 9, …), reflecting the line production rate of a three-person hand crew. If a large engine is operated together with a water tender, the line production rate would be maintained at the higher level of 18.9 chains per hour because we assume the water tender would be able to provide a continuous water supply for this engine. We based these fire line production rates on the average of the resource productivity data found in the FPA system. The fire line production rates used here were collected for FPA modeling purposes and are calibrated specifically for the BHFPU. Other estimates of fire line production rates are available, including a recent national study by Broyles (2011).

In our model, suppression resources can be dispatched from any fire station to any fire in the planning unit. For simplicity in these exploratory tests, we chose not to constrain the numbers of resources that can be located at any fire station. Adding constraints to reflect

Table 1. Daily stationing cost for the four types of resources modeled in the BHFPU.

Resource name	Daily stationing cost ($)
Hand crew	146
Small engine	601
Large engine	1,108
Large engine + water tender	1,300

Table 2. Required daily stationing budget in he BHFPU in a fire season to achieve targeted exceedance probabilities under different assumptions of fire discovery delay, calibration parameter β_D, and IA time limit.

Assumption no.	IA time limit (hr)	Ignition discovery delay (hr)	Value of β_D	Daily preparedness budget ($) required to achieve targeted exceedance probability		
				0.002	0.01	0.05
1	18	0	0	9,739	3,796	2,044
2	24	0	0	8,218	3,796	2,044
3	24	1	0	10,741	3,796	2,044
4	24	0	1	11,749	4,706	2,336
5	24	1	1	13,437	4,706	2,353

capacity limits at various stations would be straightforward. Capacity expansion options and costs also could be included with more work. Likewise, we chose in these tests to calculate travel times from fire stations to fire locations simply as the distance between those points divided by a travel rate from the BHFPU FPA data of 56 km/hour. Modeled average costs per day for stationing each resource type in the planning unit for the fire season (also from FPA) are shown in Table 1.

Results

We tested the effects of four parameters on the minimum daily cost for stationing seasonal fire suppression resources on the BHFPU. Costs are reported on a per day basis. Our primary parameter of interest was α, the exceedance probability that determines how much of the IA workload is intended to be handled by the local planning unit with preparedness funds. We ran tests with three settings for α: 0.05, 0.01, and 0.002 using 890, 927, and all 935 fire days, respectively. Use of the circular block bootstrapping method outlined in Politis and White (2003) indicated 95% confidence intervals for our targeted exceedance probabilities of (0.033, 0.065), (0.005, 0.017), and (0.001, 0.004) at these group A partition points.

Given the recent fire history on the BHFPU, these exceedance probabilities reflect a need for supplemental IA resources ranging on average from about 1 to around 45–50 fire days every few decades. We also ran tests with two settings for the time allowed to contain each fire, T (18 and 24 hours), with two settings for our construction path calibration parameter β_D (0 and 1), and with two settings for delays in fire discovery (or suppression resource travel) times (0 and 1 hour). Table 2 shows the results from these runs. Finally, we conducted an additional analysis that compares the contained perimeters of each fire under different β_D values with FARSITE-reported free-burning fire perimeters at the corresponding hours of containment to directly examine the effect of β_D on fire perimeter growth.

Budget Requirements

The seasonal resource acquisition and stationing budgets vary substantially, depending on the specified exceedance probability. Lowering the exceedance probability target from 0.05 to 0.01 almost doubles the minimum daily stationing budget requirement regardless of the assumed fire discovery delay, IA time limits, or the value of β_D. Further lowering the exceedance probability from 0.01 to 0.002 requires another 2- to 3-fold budget increase, depending on selected fire suppression assumptions. Rather than using such a large increase in the local preparedness budget, fire managers might prefer to handle extreme fire days associated with very low exceedance probabilities using severity funds.

The results in Table 2 show that the daily budget is also sensitive to changes in suppression assumptions. If the targeted exceedance probability α is 0.002, shortening the time frame for IA from 24 hours (assumption 2) to 18 hours (assumption 1) increases the minimum daily stationing budget by about 19%. However, if α is set to 0.01 or 0.05, no additional budget is required to contain all group A fires within 18 hours instead of 24 hours. If the IA duration is set to 24 hours, discovering a fire immediately under assumption 4 decreases the daily stationing budget by about 13% compared with a 1-hour discovery delay (assumption 5) when α is 0.002. More efficient fire line construction (assumption #3 with $\beta_D = 0$) reduces the budget by about 20% compared with runs under assumption 5 with $\beta_D = 1$ when α is 0.002, about 19% when α is 0.01, and about 13% when α is 0.05.

Fire Station Allocations

Our SRAL model allocates crews, engines, and water tenders to different fire stations to provide IA support with the objective of limiting the conditional probability of having a fire day with escapes to be less than or equal to the defined exceedance probability. Figure 6 shows the allocation of fire suppression resources to fire stations for a test case using an exceedance probability of 0.002 under assumption 5 in Table 2. The selection of the best set of resources and the corresponding optimal allocations to fire stations is based on the cost, travel time, and the fuel-type specific line production rates of the four suppression resource types.

Varying the targeted exceedance probability changes not only the total number of resources of each type but also the locations of these resources (Figure 7). For example, with assumption 5 in Table 2 (1-hour discovery time delay, 24-hour IA limit, and $\beta_D = 1$), the model stationed 12 crew members and one small engine in the FPU to limit the exceedance probability to 0.05, 24 crew members and two small engines to limit the exceedance probability to 0.01, and 22 crew members plus 9 small engines, 4 large engines, and two water tenders to limit the exceedance probability to 0.002.

Using the iterative procedure from our Methods section, we found that the number of fire days required in each master problem to obtain an optimal solution varied substantially by exceedance probability. For example, to obtain a stationing solution to contain fires in all 935 fire days ($\alpha = 0.002$), only two of the most severe fire days ($k = 934$ and 935) are needed in the master problem under all tested assumptions (Table 2). With the exceedance probability set to 0.01, as many as 9 of the 927 group A fire days had to be included in the master problem to obtain solutions that contained all fires in fire days from $k = 1$ to 927. Increasing the exceedance probability to 0.05 classified 890 fire days into group A, and as many as 20 fire days had to be included in the master problem to obtain optimal solutions.

Figure 6. The spatial allocation of suppression resources under assumption 5 with a 24-hour IA limitation, 1-hour fire discovery delay time, and $\beta_D = 1$ (Table 2) combined with a targeted exceedance probability of 0.002.

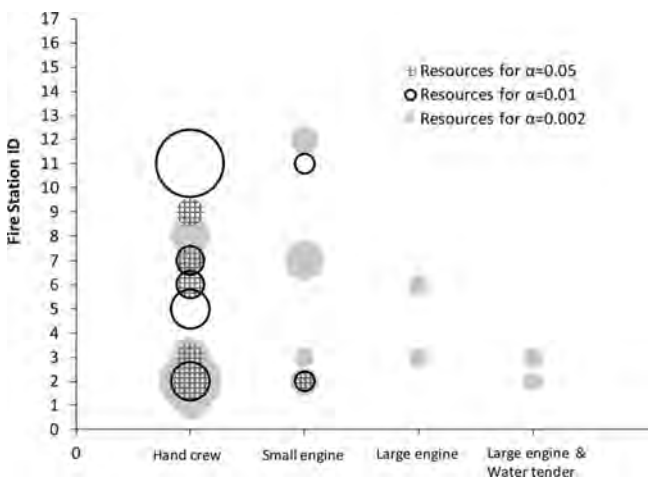


Figure 7. Optimal resource deployment in fire stations under three levels of exceedance probabilities using assumption 5 in Table 2 (1-hour fire discovery delay time and $\beta_D = 1$).

Effect of β_D on Final Fire Perimeters

β_D is the parameter used in our SRAL model to calibrate fire line construction requirements. It affects the degree to which fire line constructed during one period of suppression reduces fire perimeter growth in the subsequent period (see Equation 3.1). We constructed an additional analysis to directly study the impact of β_D on fire perimeters. In these tests, we changed the objective function of the SRAL model to minimize the sum of all contained fire perimeters in each fire day. We compared the contained perimeter of each fire with its FARSITE reported perimeter at the hour of containment under two β_D values ($\beta_D = 0$ and $\beta_D = 1$). In these tests, we assume there is a 1-hour delay before the discovery of every fire; therefore, a fire always grows freely according to the FARSITE-reported fire growth rate until the second hour after ignition. Consequently, the value of β_D has no effect on a fire contained before the end of the second hour after ignition. With $\beta_D = 1$, our analysis indicated that 649 of the 1,655 modeled fires are contained after the second hour after ignition. For these 649 fires, the average ratio of the contained fire perimeter to the free-burning perimeter at the time of containment is about 0.85. With β_D set to 0, 525 of the 1,655 modeled fires are contained after the second hour, and the average ratio between the contained fire perimeter and the free-burning fire perimeter at containment decreases to about 0.26.

Discussion

Our purpose in this article is to develop a new set of methods focused on arriving at first-stage fire season preparedness decisions

regarding how many suppression resources to hire and where to locate them given a preexisting set of fire stations. These first-stage decisions are driven by the IA resource requirements for containing wildfires, modeled here as recourse dispatch decisions given a set of days with fire ignitions. Model decisions were based on simulations of historical fire ignitions using historical fire weather data associated with those ignitions. An implicit assumption here is that fires in the near future will be similar to fires in the recent past. This assumption can be relaxed if we can predict the number of fires and their locations and local weather associated with future fire days. A study in southern Europe demonstrated the potential of using fire ignition indices and forecasted meteorological maps to support decisions for future fire management (Kalabokidis et al. 2012).

We view this study as a foundation for further research. Consequently, we set aside a number of important issues in this article. Perhaps the most notable of these is that second-stage IA dispatch decisions are made with perfect knowledge of the fewest resources required to achieve containment on each fire. Expecting fire dispatchers to have such perfect knowledge is a heroic assumption that we are addressing in a forthcoming study. The extended study will enhance the current SRAL model by integrating the design of standard response rules for different fire dispatch categories into the IA resources deployment plan. Using rule-based dispatching relaxes the perfect knowledge assumption adopted in this study and similar two-stage IA planning models (e.g., Ntaimo et al. 2012, 2013). Dispatch rules selected endogenously through an optimization model may also lead to more efficient resource deployment plans compared with plans based on exogenously generated dispatch rules (e.g., Haight and Fried 2007).

Our current method for approximating fire line production and fire growth interaction might also be enhanced. Test results show that the adjusting factor β_D can have substantial effects on fire line construction requirements. These effects appear to be consistent with the findings of Fried and Fried (1996), although comparisons of our results with theirs are complicated by our focus on perimeter adjustments versus their focus on area adjustment, our use of FARSITE fire growth modeling on heterogeneous landscapes versus their use of geometric fire growth modeling on homogeneous landscapes, and other factors. In future use, our single adjustment factor β_D might be replaced by $\beta_D(g,t)$ to reflect the unique characteristics and suppression tactics associated with each fire g for each suppression period t. For example, this adjustment factor might be used to reflect suppression tactics such as head attack versus tail attack to improve the accuracy of fire growth estimates or to reflect fire suppression practices such as building fire line through lighter fuel instead of following the shortest path.

In this study, we also assume that resources deployed to an FPU are available for IA in the FPU through the entire season. However, resource availability may vary throughout a fire season due to staffing adjustments, reflecting changes in local fire danger as well as reallocations between agencies and FPUs. Changes in resource staffing levels, relocation, and prepositioning represent important and challenging future research. Such research requires an understanding of fire situations in multiple planning areas during different times of the fire season.

Additional simplifying assumptions were adopted in this study. For example, we allow the model to select any configuration of suppression resources for each available fire station; the capacity of each fire station and the cost of opening a fire station were not considered. Although such planning requirements could be incorporated into the model using additional constraints or altering parameter values, they also require the collection of detailed financial data. Research also suggests that crew productivity is inversely related to fire intensity (Hirsch and Martell 1998). We collected data from BHFPU, including the productivities of crews and engines based on fuel types, and used these data to parameterize the model. However, variations in line productivity due to differing fire intensities are not explicitly described by the data nor built into the model. In addition, Arienti et al. (2006) suggest that the IA success rate may be higher for human-caused fires due to factors such as response time, season, and location. We did not distinguish between lightning-caused ignitions and human-caused ignitions in this study. In addition, fire locations, fire simulations, and maps of human and resource values could be combined to provide fire threat information, e.g., the distance from a fire to a town and the time it takes a fire to affect a town. Corresponding IA requirements, e.g., shorter IA time frame for ignitions closer to a town, could be built into the model based on different levels of potential fire threats.

Other issues we think might be addressed using the present work as a foundation include planning for multiple fire ignitions occurring in episodes lasting more than 1 day, extending the set of suppression assets to include aerial resources, and expanding the set of suppression response options to include more than just IA. Within these issues, modeling episodes of fires for multiple days would also help reflect resource shortages at the start of some days. Given sufficient fire forecasting, we anticipate that the model also might be adaptable to the shorter-term problem of prepositioning supplemental resources in anticipation of extreme events.

Resource deployment solutions identified from this model could be reevaluated through postoptimization simulations using historical fires or randomly generated new fires with additional detail and realism that may be difficult to model directly in an optimization model. For example, postoptimization simulations could be used to evaluate the effects of dispatching resources from the closest station. Likewise, we could track the hour a fire is contained, thereby releasing those resources and redeploying them to other fires in the same fire day without sending them back to their fire stations. The IA preparedness budget and resource deployment plan may also benefit from postoptimization simulations to account for the logistic details of employee work hours. In cases where postoptimization simulations result in substantial differences from the optimization model outputs, the optimization model may need to be adjusted and rerun to revise the IA resource deployment plan.

In this study, we introduced a two-stage stochastic programming model to station and dispatch hand crew and engines in an FPU. We used a set of mathematical equations for fire growth and containment parameterized from fire simulations on LANDFIRE landscapes combined with local resource data. These equations approximated the interactions between construction and fire growth, accounting for the average distances between free-burning fire perimeters and average fire perimeter growth rates in each period. The chance constrained approach used in our formulation provides an important method for distinguishing between IA preparedness and severity funding requirements. Despite the limitations of this exploratory model, results from our tests suggest a strong potential for future enhancement and use.

Endnote

1. For more information, see www.forestsandrangelands.gov/FPA/index.shtml.

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