Entrainment regimes and flame characteristics of wildland fires

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Abstract. This paper reports results from a study of the flame characteristics of 22 wind-aided pine litter fires in a laboratory wind tunnel and 32 field fires in southern rough and litter–grass fuels. Flame characteristic and fire behaviour data from these fires, simple theoretical flame models and regression techniques are used to determine whether the data support the derived models. When the data do not support the models, alternative models are developed. The experimental fires are used to evaluate entrainment constants and air/fuel mass ratios in the model equations. Both the models and the experimental data are consistent with recently reported computational fluid dynamics simulations that suggest the existence of buoyancy- and convection-controlled regimes of fire behaviour. The results also suggest these regimes are delimited by a critical value of Byram’s convection number. Flame heights and air/fuel ratios behave similarly in the laboratory and field, but flame tilt angle relationships differ.

Additional keywords: air/fuel mass ratio, combustion regimes, entrainment constant, flame height, flame tilt angle.

Received 18 March 2010, accepted 23 February 2011, published online 24 November 2011

Introduction

An important aspect of wildland fire behaviour deals with whether a surface fire will transition to crown fire, and if so, which type of crown fire will develop (Tachajapong et al. 2008; Cruz and Alexander 2010). The size and shape of the flames are significant factors in this transition because of their influence on important processes such as heat transfer to unburned fuel, scorching of trees, sustained fire due to breaching of firebreaks and fire spread in discontinuous fuels (Lozano et al. 2010).

Flame characteristics have been studied in laboratory tests (Thomas et al. 1963; Thomas 1964; Van Wagner 1968; Fang 1969; Albini 1981; Nelson and Adkins 1986; Fendell et al. 1990; Weise and Biging 1996; Mendes-Lopes et al. 2003; Sun et al. 2006) and experimental field fires (Byram 1959; Thomas 1967; Nelson 1980; Nelson and Adkins 1988; Burrows 1994; Fernandes et al. 2002). Alexander (1998) used results from Fendell et al. (1990) to derive a relationship for fire plume angle from fireline intensity and wind speed in the development of a model to predict crown fire initiation. Anderson et al. (2006) tested currently available flame characteristic models with data from several sources and pointed out the need for standardised measurement methods.

The flame geometry of 2-D wildland fires has been simulated with computational fluid dynamics (CFD). For example, flame characteristics predicted by Porterie et al. (2000) compared favourably with flame models in the literature. Morvan and Dupuy (2004) related heat transfer and fire spread rate in Mediterranean shrub to a flame-length Froude number. Nmira et al. (2010) describe a physical model that produced low- and high-wind regimes of flame characteristic behaviour for stationary area and line fires; predicted values of flame height, flame length and flame tilt angle generally agreed with experimental data. Modelling studies of air flow around fires were reported for laboratory chaparral fires (Zhou et al. 2005; Lozano et al. 2010) and for grass fires in the field by Linn and Cunningham (2005).

Past research has essentially neglected the processes governing movement of air into the fuel-bed combustion zone and its attached flame. We know of only one published report in which the flame air/fuel mass ratio is estimated from air flow measurements; the laboratory data are for fires in alcohol, wood crib and town gas fuels (Thomas et al. 1965). Wildland fire models characterising entrainment are those of Thomas (1963), Fang (1969) and Albini (1981). In the present paper, models of entrainment and flame characteristics are basically thermodynamic, and restricted to head fires of low-to-moderate intensity on flat ground.

Numerical simulations (Porterie et al. 2000; Morvan 2007) suggest that a line of fire spreading in uniform fuel in response to a steady wind may burn in one of several combustion regimes. These regimes are related in part to the processes by which air is entrained into the flame. We hypothesise that when the mean wind speed is zero, the mass of entrained air increases as flame...
height increases and the velocity of this air at a given height is proportional to the upward velocity of the flame fluid at that point (Taylor 1961; Thomas 1967; Fleeter et al. 1984). This process is herein referred to as classical entrainment.

As the wind speed increases, convection begins to influence entrainment; flame tilt angle and rate of fire spread begin to increase significantly; and the flame height to depth ratio begins to decrease. The angle of flame tilt is determined by a momentum flux balance between the transverse components of the horizontally moving ambient air and the buoyant velocity of the flame fluid. This type of entrainment, associated with flame drag and buoyancy forces, is called dynamic entrainment in this paper.

As the wind continues to increase, a point is reached beyond which the mass of air entering the flame disrupts the balance between drag and buoyancy forces. The tilt angle is determined by a ratio between the horizontal and vertical components of the flame fluid mass flux (Albini 1981). This process is referred to in the present paper as accretive entrainment. In some cases of accretion, there may be little suggestion of horizontal inflow at the lee edge of the flame because part of the impinging air flows through the flame, leading to an outflow of unreacted air (air not participating in combustion) at the lee edge. Beer (1991) reported that CSIRO (Australia) researchers did not observe a fire-induced wind in their laboratory and field experimental fires.

The objective of this paper is to describe with mathematical models and experimental data how air entrainment and combustion regimes determine flame characteristics; we use models of flame height and tilt angle to evaluate entrainment constants and air/fuel mass ratios. First, we estimate air/fuel ratios for the combustion zone and flame. Second, we derive a dimensionless criterion that identifies three combustion regimes for head fires of low-to-moderate intensity. Third, we describe mass flow in the combustion zone and then develop flame characteristic equations from a 1-D analysis based on a simplified version of the Albini (1981) flame model. Thus far, we have neglected the differences in the mathematical modelling that follows. The buoyancy flux per unit length of diffuser introduced by Roberts (1979) is

\[ b_R = \frac{g \Delta \rho g}{\rho_e} \]  

with units of cubic metres per second cubed. The corresponding buoyancy flux \( b_F \) for a line fire is

\[ b_F = \frac{\phi_f g l_a}{\rho_s H_c} = \frac{g l_a}{\rho_s c_p T_a} \frac{\mu_e^3 N_c}{2} \]  

where the first equality of Eqn 2 is obtained by analogy with Eqn 1 and \( \phi_f \) is the mean air/fuel ratio (mass of air per mass of original fuel burned) associated with lateral movement of air into the flame. The \( N_c \) criterion is

\[ N_c = \frac{2g l_a}{\rho_s c_p T_a \mu_e^3} \]  

and often is called the convection number (Nelson 1993). In Eqn 3, we assume that ambient wind speed \( (u_0) \) is much greater than the fire spread rate. For a given ocean current of speed \( (u_c) \), Roberts defines a Froude number \( (F_R) \) as

\[ F_R = \frac{u_c^4}{b_R} \]  

and discusses three separate mixing regimes delineated by \( F_R \). For \( F_R < 0.2 \), the flow forms a plume with a strong vertical component. In the intermediate region, \( 0.2 < F_R < 1 \) and the plume is unable to contain all of the incoming flow; it contacts the lower boundary for some distance downstream. For \( F_R > 1 \), the flow is in full contact with the lower boundary and the upper edge of the plume forms a planar interface with ocean water. If similar behaviour occurs when a 2-D fire burns in air of...
horizontal speed \( u_a \), then a fire Froude number \( (F_F) \) may be obtained from Eqns 2 as

\[
F_F = \frac{u_a^3}{H_F} = 2N_c^{-1}
\]  

(5)

We apply the \( F_F \) criteria identified by Roberts (1979) to \( F_F \) so that three combustion regimes are defined by the critical values \( N_c = 2 \) and 10. For a given fireline intensity, the region \( N_c > 10 \) should correspond to weak wind speeds, whereas \( N_c < 2 \) would imply strong winds; the intermediate regime should apply to moderate winds.

A theoretical air/fuel mass ratio \( \phi_f \) for the free flame may be obtained from Eqns 2. If \( H_e = 15 \, 000 \, \text{kJ kg}^{-1}, c_p = 1 \, \text{kJ kg}^{-1} \, \text{K}^{-1} \) and \( T_a = 300 \, \text{K} \), then

\[
\phi_f = \frac{H_e}{c_p T_a} = 50
\]  

(6)

with units of kilogram per kilogram. Support for Eqn 6 comes from the following considerations. Byram and Nelson (1974) found that the steady burning of 1 kg of solid wood expands the atmosphere by 41.8 m\(^3\). Suppose a mixture of combustion products and unreacted air at ambient temperature \( T_a = 1000 \, \text{K} \) enters the flame from the combustion zone along with air entrained laterally from the atmosphere at temperature \( T_a = 300 \, \text{K} \). Mass \( M_e \) of the entrained air has initial volume \( V_e \) and receives heat from the combustion zone fluid. Thus \( M_e \) expands to a larger volume \( V_a \), causing a drop in the mean temperature of the fluid. The mixture of combustion products and entrained air exits the flame tip at temperature \( T_a = 500 \, \text{K} \). If the atmospheric density is 1.2 kg m\(^{-3}\) and air flows in steadily to replace all air leaving the visible flame volume, then the effective air/fuel mass ratio is \( \phi_f = (1.2 \times 41.8) = 50.2 \, \text{kg air kg}^{-1} \) fuel burned, in agreement with Eqn 6.

**Theory of flame characteristics**

Consider a line head fire that burns steadily in response to horizontal wind speed \( u_a \) through fuel distributed uniformly on flat terrain. Modelling of entrainment and flame characteristics requires consideration of both the combustion and flame zones; the overall rates of mass flow associated with these zones are derived below.

**Combustion zone relationships**

The mixture flowing into the flame consists of burned and unburned volatiles, reacted air (air participating in combustion), unreacted air, water vapour formed in combustion, and water not lost during fuel preheating. An approximate mass flow rate through the fuel bed surface per unit length of fireline, \( m_o \) (kg m\(^{-1} \) s\(^{-1}\)), is

\[
m_o = X_b W_a R \left[ \frac{1}{X_b} + N_v + \frac{Z}{e X_b} + 0.56 + \frac{f M}{e X_b} \right]
\]  

(7)

where \( X_b \) is the fraction of volatilised fuel that burns. From left to right, the terms in brackets denote the mass of volatiles produced in the combustion zone, air (reacted and unreacted) present in the combustion zone, and water released from the fuel owing to combustion and evaporation. Thus Eqn 7 describes the stream of combustion products, unreacted air and water vapour entering the flame from the combustion zone.

Fireline intensity is defined as

\[
I_B = H_e X_b W_a R
\]  

(8)

where \( X_b \) is now interpreted as the ratio of the heat release rate \( I_{Bcz} \) in the combustion zone to the heat release rate \( I_B \) of the entire fire. When \( X_b = 1 \), Eqn 8 agrees with the widely accepted definition of fireline intensity, \( I_B = H_e W_a R \).

A different approximation of \( I_B \) calculates the heat required to raise the temperature of the fluid entering the base of the flame from ambient temperature (i.e. before the production of heat by chemical reaction) to the mean temperature at the base of the flame (Albini et al. 1995). Thus when \( X_b = 1 \), \( I_B \) may be written as

\[
I_B = m_o c_p (T_a - T_e)
\]  

(9)

where \( c_p \) is assumed equal for fuel flow and air. In the combustion zone of a wildland fire, \( X_b < 1 \) and the rate of heat release, from Eqns 7–9, is

\[
I_{Bcz} = H_e X_b W_a R \left[ \frac{1}{X_b} + N_v + \frac{Z}{e X_b} + 0.56 + \frac{f M}{e X_b} \right]
\]  

\[
\times c_p (T_a - T_e) \approx X_b W_a R \left[ \frac{1}{X_b} + \phi_{cz} \right] c_p (T_a - T_e)
\]  

(10)

where \( \phi_{cz} \) is the air/fuel ratio of the combustion zone given by the sum of \( N_v \) kg of reacted air and \( Z/e X_b \) kg of unreacted air. If the simplified estimate of \( I_{Bcz} \) in Eqn 10 is reasonable, \( \phi_{cz} \) becomes

\[
\phi_{cz} = \frac{H_e}{c_p (T_a - T_e)} - \frac{1}{X_b} \approx 20
\]  

(11)

When \( X_b \) ranges from 0.5 to 1, \( \phi_{cz} \) ranges from 19.4 to 20.4 kg kg\(^{-1}\) and a mean theoretical \( \phi_{cz} \) may be taken as 20 kg kg\(^{-1}\).

The approximation in Eqn 10 requires \((1/X_b + \phi_{cz}) >> (0.56 + f M/e X_b)\); we make the reasonable assumptions \( \varepsilon > 0.5 \) and \( X_b > 0.5 \) (Albini 1980). For the ordinary laboratory fire \((0.56 + f M/e X_b) < 2\), so Eqn 10 is acceptable. For the green vegetation layers of crown fires, the estimate is less accurate because the moisture term \((0.56 + f M/e X_b)\) could be as large as 9.

**Flame zone relationships**

In the Albini (1981) flame model, air enters the flame by accretion, in which a fraction of the impinging air of speed \( u_a \) becomes incorporated into the flame. Because chemical reactions are neglected in our model, flame temperature is a maximum at the fuel bed surface and decreases upward as air entrainment increases towards the flame tip.
We rewrite the Albini (1981) model equations as follows:

**mass flow:** \[ m = \rho w D \quad (12a) \]

**lateral entrainment:** \[ dm = \rho_a u_e dz \quad (12b) \]

**horizontal momentum:** \[ d(mu) = u_e dm \quad (12c) \]

**vertical momentum:** \[ d(mw) = \rho g D \left( \frac{T - T_a}{T_a} \right) dz \quad (12d) \]

**sensible energy:** \[ d(mcpT) = cpT dm \quad (12e) \]

**flame tilt angle:** \[ A = \tan^{-1} \left( \frac{u}{w} \right) \quad (12f) \]

The flame represented by Eqs 12a–f is presented in Fig. 1. All dependent variables in these equations are regarded as time-averaged values.

Eqs 12a–f may be solved analytically by assuming that entrainment velocity \( u_e \) represents a constant velocity obtained by averaging over flame height \( H \). Quantities \( \rho_a, c_p, T_a \) and \( u_a \) are assumed constant. When the entrainment and sensible energy relations Eqs 12b and 12e are integrated from the flame base (\( z = 0 \)) to the flame tip (\( z = H \)), the flame tip mass flux \( m_t \) becomes

\[ m_t = m_o \left( \frac{T_o - T_a}{T_f - T_a} \right) = m_o + \rho_a u_e H \quad (13) \]

where \( m_o = \rho_o w_o D_o \) is the combustion zone mass flux at \( z = 0 \). If values of 1000, 500 and 300 K are assigned to \( T_o, T_f \) and \( T_a \), then \( m_t = 3.5 m_o \) and Eqs 10 and 13 give

\[ \rho_a u_e H = 2.5 m_o = 2.5 (1 + \chi_b \phi_j) W_j R = \phi_j X_b W_j R = \frac{\phi_j X_b I_B}{H_e} \quad (14) \]

where \( X_b \) may be taken as unity.

The horizontal momentum equation is integrated with limits \( u = u_o \) and \( m = m_o \) when \( z = 0 \) to obtain

\[ mu = m_o u_o + u_a (m - m_o) \quad (15) \]

The vertical momentum equation may be rewritten by multiplying both sides by \( mw \) and substituting the entrainment and integrated sensible energy equations to obtain

\[ d(mw)^2 = \left[ \frac{2g m_o}{\rho_a u_e} \right] \left[ \frac{c_p (T_o - T_a)}{c_p T_a} \right] dm^2 = \left[ \frac{2g I_B}{\rho_a c_p u_e} \right] dm^2 \quad (16) \]

where \( I_B \) is from Eqn 9. Integration of Eqn 16 with \( (mw)^2 = (m_o w_o)^2 \) at \( z = 0 \) leads to

\[ (mw)^2 = (m_o w_o)^2 + \frac{w_o^2}{2u_e} (m^2 - m_o^2) \quad (17) \]

![Fig. 1. A time-averaged visible flame showing mass and energy flow variables. The transverse component of the horizontal drag force \( F_D \) balances the transverse component of the vertical buoyancy force \( F_B \) to determine flame tilt angle \( A \).](image-url)
where the quantity \( w_c \) is a characteristic buoyant velocity (Nelson 2003) representative of the whole fire given by
\[
w_c = \left( \frac{2gL_B}{\rho_c \gamma T_o} \right)^{1/3}
\]
(18)

Boundary velocities \( u_o \) and \( w_o \) depend strongly on wind speed, fuel type and fuel load. Anderson et al. (2010) have shown that at higher speeds, \( u_o \) is a moderate fraction of the free stream speed \( u_o \), for small \( u_o \) values, \( u_o \) ranges from \( u_o \) to \( 2u_o \). However, \( w_o \) is instrumental in the development of model equations for flame height \( H \) and tangent of the tilt angle, \( \tan A \).

**Entrainment velocity equations**

Albini (1981) assumed that entrainment velocities for head fires in moderate winds can be described as
\[
 \dot{u}_e = \eta \dot{u}_o
\]
where \( \eta \) is the fraction of impinging air entering the flame. Because Eqn 19 is not valid as \( \dot{u}_o \) approaches zero, we require an expression for \( \dot{u}_e \) in terms of entrainment constant \( z \) that is applicable for \( \dot{u}_o \geq 0 \). On the basis of exploratory data plots (R. M. Nelson Jr, Missoula Fire Sciences Laboratory, unpubl. data), and because \( w_c \) in Eqn 18 is proportional to \( I_B^{1/3} \) (a function of \( W_a \) and \( \dot{u}_o \)), we infer
\[
 \dot{u}_e = 2w_c
\]
(20)
an equation identical in form to the zero-wind entrainment equation (Taylor 1961). For single fires, Eqn 20 applies when \( \dot{u}_o > 0 \); the equation \( \dot{u}_e = z \dot{u}_o w_c \) applies when \( \dot{u}_o = 0 \). In general, we expect \( z \neq z_c \). Entrainment constants \( \eta \) and \( z \) are quantified in the next section, which compares flame characteristic model equations with our experimental data.

**Flame tilt angle relationships**

The tangent of flame tilt angle \( A \) may be obtained from Eqn 15 and the square root of Eqn 17 in the form
\[
 \tan A = \frac{u}{w_c} = \frac{m(m_a + m_o \dot{u}_o)(m - m_o)}{2u_e}^{1/2}
\]
(21)
If we invoke the assumptions used by Albini (1981) and assume little variation in angle \( A \) from \( z = 0 \) to \( z = H \), then at the flame tip \( m_t \gg m_o, m_a, \dot{u}_o \gg m_o \dot{u}_o, m_t (w_c^2/2u_e) \gg (m_o w_c)^2 \) and \( \tan A \) may be written as
\[
 \tan A = 2^{1/2} \eta^{1/2} \left( \frac{u_o}{w_c} \right)^{1/2}
\]
(22)
Eqns 19 and 22 yield
\[
 \tan A = 2^{1/2} \eta^{1/2} \left( \frac{u_o}{w_c} \right)^{3/2} = 1.414^{1/2} \eta^{1/2} N_c^{-1/2}
\]
(23)
where \( (u_o/w_c)^3 = N_c^{-1} \). The alternative formulation for \( \tan A \) using Eqn 20 in Eqn 22 leads to
\[
 \tan A = 2^{1/2} z^{1/2} \left( \frac{u_o}{w_c} \right) = 1.414 z^{1/2} N_c^{-1/2}
\]
(24)
Eqns 23 and 24 are suitable for evaluating entrainment constants \( \eta \) and \( z \) because the air/fuel ratio \( \phi_f \) is missing from the two equations.

**Flame height relationships**

Anderson et al. (2006) noted that use of Froude number \( F_H \) in flame tilt angle models is problematic from the standpoint of prediction because flame height \( H \) is unknown. Moreover, one can infer from Albini (1981) that \( F_H \) is inversely proportional to convection number \( N_c \). Two additional relationships for \( H \) in terms of fireline intensity \( I_B \) have been applied in various studies (Albini 1981; Anderson et al. 2006), but not in the context of combustion regimes delimited by \( N_c = 10 \). We explore these three flame height relationships using Eqn 14.

First, Eqns 2, 14, and 19 with \( x_0 = 1 \) lead to
\[
 F_H = \frac{u^2}{gH} = \frac{2H \eta}{c_p T_o \phi_f N_c} = 100 \left( \frac{\eta}{\phi_f N_c} \right)^{1/2}
\]
(25)
where \( \eta \) is obtained from a plot of Eqn 23. Second, a dimensional equation for \( H \) comes from combining Eqns 14 and 19 to yield
\[
 H = \frac{\phi_f I_B}{\rho_o H \eta u_o} = 0.0000556 \left( \frac{c_p T_o}{\gamma} \frac{I_B}{\eta} \right)^{1/2}
\]
(26)
where \( \rho_o = 1.2 \). Finally, Eqns 14, 18, and 20 combine to give
\[
 H = \frac{c_p T_o \phi_f w_c^2}{2gH \eta z} = \left( \frac{\phi_f}{c_p T_o} \frac{I_B}{g \rho_o^2 z^2} \right)^{1/3} \left( \frac{\phi_f}{\gamma} \right)^{1/3} \frac{1}{N_c} = 0.000147 \left( \frac{\phi_f}{\gamma} \right)^{1/3}
\]
(27)
where \( z \) is evaluated using a plot of Eqn 24. We note that Eqn 27 is commonly used when \( \dot{u}_o = 0 \); in such cases \( H, w_c, z \) and \( I_B \) should be written as \( H_o, w_{con}, z_o \) and \( I_{Bo} \).

**Comparison of flame characteristics data with model equations**

The laboratory data are from fires in slash pine litter (Pinus elliottii Engelm.) and saw palmetto fronds (Serenoa repens (Bartram) Small) burned in the US Forest Service’s Southern Forest Fire Laboratory (SFFL) wind tunnel in Macon, GA (Nelson and Adkins 1986). The February 1988 field measurements were made in 1-, 2- and 4-year roughs during experimental burns in southern rough fuels of northern Florida (Osceola National Forest) and longleaf pine (Pinus palustris-Mill.) litter–grass fuels of coastal South Carolina (Francis Marion National Forest). The field data, heretofore unpublished, are presented in Appendix A.
Fuel consumption in the field was estimated by weighing oven-dried pre- and post-burn fuels. In some cases, this led to overestimation of the available fuel load. Wind speeds were measured just behind the fireline with a hand-held digital wind meter at mid-flame height. Fire spread rate and flame characteristics were measured using the video methods of Nelson and Adkins (1986).

Theoretically, the data for \( \tan A \) and \( H \) should pass through zero when the independent variable \( X = 0 \), so we set the intercept term to zero and fitted models of the form \( Y = \gamma_1 X^\lambda \) where \( \lambda \) is an exponent determined analytically and \( \gamma_1 \) was estimated statistically by simple linear regression with unweighted least-squares for the model relationships in Eqsns 23–27. Student’s \( t \)-statistic tested significance of the parameter estimate. Overall quality of the regression models was evaluated using root mean squared error (RMSE) and mean absolute error (MAE). We used the Akaike Information Criterion (AICc), adjusted for small sample size (Burnham and Anderson 2004), to compare the different model formulations.

Because the commonly used coefficient of determination \( (R^2) \) can provide spurious information when the intercept term is set to zero (Eisenhauer 2003), we calculated \( R^2 \) as \( R^2 = \sum Y_i^2 / \sum Y_p^2 \) to measure how much of the total variation of the dependent variable (also known as the uncorrected sum of squares) was described by our regression through the origin models. The models and their fit statistics are presented in Appendix B.

**SFFL laboratory fires – flame tilt angle**

The wind tunnel fires in beds of slash pine litter and slash litter under palmetto fronds were treated as coming from a single fuel type (Nelson and Adkins 1986). Initial fuel loads ranged from 0.5 to 1.1 kg m\(^{-2}\), the dead fuel moisture content fraction from 0.09 to 0.13, and wind speed from 0.6 to 2.3 m s\(^{-1}\). Palmetto frond fractional moisture content at the time of burning ranged from 0.09 to 0.13, and wind speed from 0.6 to 2.3 m s\(^{-1}\). 

**Fig. 2.** Southern Forest Fire Laboratory (SFFL) laboratory (a) and field (b) data showing two behaviour regimes for \( \tan A \) separating at a value of \( N_c = 10 \) for the laboratory data. For the field data, regressions (dashed lines) based on Eqn 24 for \( N_c < 10 \) and on Eqn 28 for \( N_c > 10 \) fit the data poorly. We select the regression for all \( N_c \) (solid line) as representative of \( \tan A \) for the field data.

Outlier

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\[
\tan A = 3.85\eta N_c^{-2/3} \quad (28)
\]

Fig. 2\( a \) also includes a plot of the regression of \( \tan A \) on \( N_c^{-2/3} \) for \( N_c > 10 \), \( \tan A = 3.931N_c^{-2/3} \) and suggests that Eqn 28 is a good description of the data. Though not significantly different from zero at the 0.05 level, the slope term was significantly different at the 0.077 level; thus \( \eta = (3.93/3.85)^{1/2} = 1.01 \). The \( \eta = 1 \) estimate for \( N_c > 10 \) implies that fires in light winds entrain a larger fraction of the impinging air (the total amount
Fig. 3. Southern Forest Fire Laboratory (SFFL) laboratory (a) and field (b) data showing that the relationship between \( F_H \) and \( N_c \) is similar for both datasets and that regression suggests a slight difference at approximately \( N_c = 10 \). Solid lines denote fitted equations based on the \( N_c \) criterion; dashed lines illustrate fitted regressions using all \( N_c \) data.

is relatively small) than fires in stronger winds (\( N_c < 10 \)) for which the value \( \eta = 0.71 \) was estimated earlier.

It was not possible to determine \( z \) by using Eqn 20 to derive an equation similar to Eqn 28 because the result would imply \( \tan A = \text{constant} \). Fig. 2a shows that \( \tan A \) for \( N_c > 10 \) is not constant, but described well by Eqn 28. Thus, only Eqn 19 describes the entrainment velocity and flame tilt angle for \( N_c > 10 \). This result may be due to suppression of vertical flow in the SFFL tunnel. We believe a result approximating \( \tan A = \text{constant} \) is representative of fires in large wind tunnels.

**SFFL field fires – flame tilt angle**

For the palmetto–gallberry fuels, flame heights ranged from 0.4 m in 1-year roughs to 5 m in the 4-year roughs; in the litter–grass fuels, flame heights exhibited intermediate values. Fractional moisture content of the dead grass was 0.18; the L and F layers ranged from 0.2 to 0.5, and the live palmetto fronds and gallberry leaves from 1 to 1.4.

Eqns 23–24 describe the \( \tan A \) laboratory data for \( N_c < 10 \), but not the corresponding field data (Fig. 2b); a regression according to Eqn 24 leads to \( \tan A = 1.041 N_c^{-1.3} \), an extremely poor fit (a brief discussion of flame tilt angle in the wind tunnel and field is available in an Accessory publication, see www.publish.csiro.au/?act=view_file&file_id=WF10034_AC.pdf). The linear increase in \( \tan A \) is physically questionable and disagrees with the numerical modelling results of Nmira et al. (2010) and the experimental data of Fendell et al. (1990) discussed by Alexander (1998); these investigators show that \( \tan A \) should be proportional to a reciprocal power of \( N_c \) smaller than unity. Thus we consider the four outermost data points for \( N_c < 10 \) as outliers due to errors in measurement of \( \tan A \) and available fuel load \( W'_{F} \); ignoring these points suggests \( \tan A = \text{constant} \).

To study Eqn 28 for \( N_c > 10 \), we plotted \( \tan A \) v. \( N_c \) in Fig. 2b. An outlying point initially was neglected in both the plot and regression; the result, \( \tan A = 4.119 N_c^{-2.3} \), yields \( \eta = (4.12/3.85)^{1/2} = 1.03 \) and supports the earlier result for the laboratory data, \( \eta = 1 \) when \( N_c > 10 \). Including the outlier in the regression \( \tan A = 4.458 N_c^{-2.3} \) produces a slope estimate not significantly different from 4.119; however, the fit statistics are less desirable for \( N_c > 20 \).

Although Eqn 28 is a possible descriptor of \( \tan A \) for \( N_c > 10 \) and useful for estimating \( \eta \) for the field fires, inspection of all data in Fig. 2b suggests that \( \tan A \) is best described as constant. For example, we consider 26 of the 32 data points in the figure to approximate the horizontal line \( \tan A = 0.65 \). We have regressed all data as coming from a single population of \( \tan A \) values and compared the results with statistics obtained from regressing the data according to \( N_c < 10 \) and \( N_c > 10 \). The statistical fit based on all data (outlier removed) is superior to the fits obtained when the data are separated into two groups (Appendix B). Thus the single-regression equation, \( \tan A = 0.655 N_c^{-0.63} \), indicates that flame tilt angle for the field data is given by \( \tan A = 0.655 \) – a behaviour not seen in the laboratory fires.

The result \( \tan A = \text{constant} \) for all \( N_c \) can be derived from the idea that flame tilt is determined by a balance involving rates at which work is done by rising parcels of flame and parcels of moving air. An equation based on this approach is available in an Accessory publication (see http://www.publish.csiro.au/?act=view_file&file_id=WF10034_AC.pdf). Setting \( \tan A \) in this equation to 0.655,

\[
\tan A = \frac{C_0 \rho_c x^3}{\rho_e} = 3.85x^3 = 0.655
\]

and \( x = 0.55 \), in agreement with the laboratory fire estimate. This result supports our assumption that the laboratory fire
results, $\eta = 0.71$ and $x = 0.55$ for $N_c < 10$ and $\eta = 1$ for $N_c > 10$, can be used to calculate $\phi_f$ for the field fires. This constant-angle regime of burning is referred to as kinetic entrainment.

**SFFL laboratory and field fires – flame height**

The height $H$ of wind-blown flames is described by Eqsn 25–27. For the laboratory data, $F_{H_L}$ and $N_c$ are plotted in Fig. 3a using Eqn 25; the estimated slope is 1.676 for $N_c < 10$. Thus for $\eta = 0.71$, $\phi_f = 42.4$. For $N_c > 10$, the slope is 2.421, so with $\eta = 1$, $\phi_f = 41.3$. A regression using all data yielded $F_{H_L} = 1.726N_c^{-0.04}$, which was significant (Appendix B).

Fig. 3b for the field data shows plots of $F_{H_L}$ vs. $N_c$ according to Eqn 25. For $N_c < 10$, $F_{H_L} = 1.362N_c^{-1}$ and the slope estimate with all data is $\eta = 1$. For $N_c > 10$, the equation $F_{H_L} = 2.310N_c^{-1}$, giving $\phi_f = 43$ for $\eta = 1$. If the $N_c = 10$ criterion is not applied and all data are considered, a model in which both the slope and exponent were estimated from the data, $F_{H_L} = 0.878N_c^{-0.62}$, is a better fit than a model that assumed the $N_c^{-1}$ formulation, $F_{H_L} = 1.397N_c^{-1}$ (not shown in Fig. 3b).

For $N_c = 10$ in Fig. 3, $F_{H_L} \approx 0.25$ for the laboratory and field fires. Pagni and Peterson (1973) and Morvan and Dupuy (2004) state that when flame-length Froude number $F_L < 0.25$, fire spread in pine needle beds is radiation (buoyancy)-controlled and flame tilt is close to vertical; when $F_L > 1$, the spread rate is controlled by a combination of radiation and convection. Neglecting the small difference between $F_L$ and $F_{H_L}$ for our fires, we interpret the intermediate region $0.25 < F_{H_L} < 1$ as one in which radiative preheating decreases as $F_{H_L} \rightarrow 1$ while the convective contribution due to wind increases. For $F_{H_L} > 1$, fire spread becomes increasingly wind-driven. Because $F_{H_L} > 1$ for only two of our laboratory fires, this regime requires further study.

The second relationship for flame height is Eqn 26, which relates $H$ to $I_g\mu_a^{-1}$. The data in Fig. 4a show two burning regimes. For $N_c < 10$, $H = 0.0033I_g\mu_a^{-1}$; thus for $\eta = 0.71$, Eqn 26 yields $\phi_f = 42.4$ – in agreement with the corresponding laboratory value. For $N_c > 10$, $H = 0.0024I_g\mu_a^{-1}$ and $\phi_f = 42.4$ if $\eta = 1$.

The field data in Fig. 4b show three outliers (circled), and preliminary regressions for all data points resulted in poor fits. These outliers were explained by exploratory plots that showed that $W_y$ for the three points was overestimated by a factor of 2. Reduction of fireline intensity $I_g$ by this factor places the data points close to their respective regression lines. When the regressions were repeated with outliers omitted, the equation for $N_c < 10$, $H = 0.0035I_g\mu_a^{-1}$, resulted in $\phi_f = 44.3$; for $N_c > 10$, the equation $H = 0.0024I_g\mu_a^{-1}$ gave $\phi_f = 43.3$. With outliers removed, $\phi_f$ for the laboratory and field fires agreed closely.

The third equation, Eqn 27, describes $H$ in terms of $I_g^{2/3}$. The laboratory data plotted in Fig. 5a are scattered, with a tendency for $N_c > 10$ data to be associated with lower $I_g$. We accepted Eqn 27 as a descriptor of the data in Fig. 5a for two reasons. First, the fitted regression for all laboratory fire data in Fig. 5a, $H = 0.0142I_g^{2/3}$, was not statistically different from the corresponding regression for all field data (Fig. 5b). Second, even though two points from each $N_c$ regime overlap into the other regime, $R^2$ values for both $N_c$ regimes in Fig. 5a exceed 0.95. For $N_c < 10$, $H = 0.0132I_g^{2/3}$ and $\phi_f = 49.4$ when $\eta = 0.71$. For $N_c > 10$, $H = 0.0173I_g^{2/3}$, which gives $\phi_f = 64.7$ if $\eta = 1$.

Plots of the field data according to Eqn 27 are shown in Fig. 5b. Separate regressions for $N_c < 10$ and $N_c > 10$ (not presented) produced slope estimates of 0.0138 and 0.0137 respectively, so there was no suggestion of two burning regimes dependent on $N_c$. A single line with a slope of 0.0137 for all data did not describe the bulk of the data; thus, the most outlying point was not included in a new regression. The slope estimate of
the resulting regression, $H = 0.0155 I_B^{2/3}$, implies $\phi_f = 58$ if $\alpha = 0.55$. This equation has fit statistics similar to the corresponding regression for all laboratory data, $H = 0.0142 I_B^{2/3}$. Apparently, use of Eqn 20 for $u_e$ masks any dependence of $H$ on $u_a$ or $N_c$ under field conditions.

**Summary of results**

Numerical models describing wildland fire (Porterie et al. 2000; Morvan 2007; Nmira et al. 2010) identify a low-wind combustion regime where buoyant forces exceed ambient wind inertial forces, and a moderate-to-high wind regime in which the dominant force is exerted by the wind. To a large extent, these results are supported by findings of the present study. Experimental data for the SFFL laboratory and field head fires showed that the criterion $N_c < 10$ often indicated transition from dynamic entrainment ($N_c > 10$) to accretive entrainment ($N_c < 10$) as wind speed increased. These two burning regimes are likely to appear in analyses of tan $A$ and $H$ involving wind speed $u_a$ – i.e. use of Eqn 19 for entrainment velocity $u_e$. Alternately, when Eqn 20 for $u_e$ was used, tan $A$ in the field data was essentially constant. The presence of fireline intensity $I_B$ in the analysis of

**Table 1.** Individual and averaged entrainment parameters and air/fuel ratios for Southern Forest Fire Laboratory (SFFL) laboratory and field fires

<table>
<thead>
<tr>
<th>Flame characteristics</th>
<th>Model equations</th>
<th>Entrainment constants</th>
<th>Air/fuel ratios</th>
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<td></td>
<td></td>
<td>$N_c &lt; 10$</td>
<td>$N_c &gt; 10$</td>
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<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tan $A$</td>
<td>23</td>
<td>0.71</td>
<td>–</td>
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<tr>
<td>tan $A$</td>
<td>28</td>
<td>–</td>
<td>1.01</td>
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<tr>
<td>$F_H$</td>
<td>25</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$H$</td>
<td>26</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Laboratory average</td>
<td>0.71</td>
<td>1.01</td>
<td>0.55</td>
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<tr>
<td><strong>SFFL field</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>tan $A$</td>
<td>28</td>
<td>–</td>
<td>1.03</td>
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<tr>
<td>tan $A$</td>
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<tr>
<td>$F_H$</td>
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<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$H$</td>
<td>26</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Field average</td>
<td>0.71</td>
<td>1.03</td>
<td>0.55</td>
</tr>
<tr>
<td>Overall average</td>
<td>0.71</td>
<td>1.02</td>
<td>0.55</td>
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</table>

**Fig. 5.** Southern Forest Fire Laboratory (SFFL) laboratory (a) and field (b) data suggesting similar trends for $H$ vs. $I_B$; dark triangles denote $N_c < 10$, open triangles $N_c > 10$. Solid lines are regression results; dashed line for the laboratory data denotes a regression for all $N_c$. Ranges of $H$ and $I_B$ in the laboratory data constitute only a small fraction of the corresponding ranges in the field data. Circled data point is an outlier.
Flame height $H$ in the field fires seemed to incorporate the effects of $u_0$ on $H$ automatically; thus for all values of $N_c$, the fires burned in a single regime. This result differed from laboratory fire data for $H$ vs. $I_B$ (Fig. Sa), which separated according to $N_c$. The difference may be due to experimental design (three fuel groups with constant $W_p$ within groups) and confined buoyant convection in the SFFL wind tunnel.

Entrainment parameters and flame zone air/fuel mass ratios are summarised in Table 1. Accretive and dynamic regimes of entrainment are indicated by $N_c < 10$ and $N_c > 10$ respectively. The combustion zone air/fuel ratio $\phi_{cz}$ is calculated from Eqn 14 as:

$$\phi_{cz} = 0.4\phi_f - \left( \frac{1}{X_b} \right) = 0.4\phi_f - 1.3 \quad (30)$$

with volatile burn fraction $X_b$ taken as 0.75. Table 1 gives an overall air/fuel ratio, $\phi_{cz} + \phi_f$, equal to 67 kg kg$^{-1}$ – a value within the range 60–80 kg kg$^{-1}$ (Thomas et al. 1965). The earlier theoretical estimates, $\phi_f = 50$ and $\phi_{cz} = 20$ kg kg$^{-1}$, compare favourably with the semi-empirical values in Table 1.

The laboratory fires duplicated the field fires with two exceptions. First, for $N_c < 10$, tan $\alpha$ for the laboratory fires was proportional to $N_c^{-1/2}$ or $N_c^{-1/3}$, whereas tan $\alpha$ for the field fires was independent of $N_c$. The Albini (1981) model for flame tilt angle, $\alpha = w_0/w$, was descriptive of only the $N_c < 10$ data from the SFFL wind tunnel, requiring two additional models for describing tilt angle: (1) Eqn 28 for low-wind-speed fires in the tunnel, and (2) Eqn 29 for all data from the field experiments. The second exception was that the laboratory data for $H$ tended to separate according to the $N_c = 10$ criterion, whereas the field data exhibited a similar relationship, but without the $N_c$ separation. These differences among tan $\alpha$, $H$, $I_B$ and $N_c$ seem related to experimental design and the fire environments, rather than to fuel or wind-speed differences. Froude number $F_H$ was proportional to $N_c^{-1}$ for both burning regimes; $H$ was proportional to $I_Bu_0^{-1}$ and to $I_B^{0.67}$ for both the laboratory and field fires.

Conclusions

The objectives of this study were to: (1) develop criteria to determine whether differences in observed flame characteristics can be related to differences in air entrainment mechanisms; (2) derive equations for relating flame height and tilt angle to commonly used fire behaviour variables and entrainment parameters; and (3) develop estimates of entrainment parameters by using the model equations and regression methods to generate statistical fits of the laboratory and field data. Specific conclusions drawn from the present work are:

1. Two burning regimes are found in laboratory wind-tunnel fires in slash pine litter beds; the same regimes are present in field fires in the palmetto–gallberry and longleaf pine litter–grass fuel types. Transition from a low wind speed to a higher wind speed regime is indicated by $N_c = 10$.

2. Equations for flame tilt angle and flame height generally describe the experimental tilt angles and heights well. For the field fires, tan $\alpha$ is constant rather than a power function of reciprocal $N_c$. Kinetic energy fluxes in the ambient air and flame describe the constant tilt angle regime. Laboratory data for the $H$ vs. $F_H^{0.67}$ relationship separate according to the $N_c = 10$ criterion, but the field data for $H$ do not separate.

3. Air enters head fire flames by: (i) dynamic entrainment ($N_c > 10$) in which the entrainment velocity approximates the mid-flame wind speed, or (ii) accretion ($N_c < 10$) in which air is blown into the flame either at a velocity equal to 71% of the mid-flame wind speed, $u_0$, or at a velocity equal to 55% of the characteristic vertical flame velocity, $w_c$.

4. The mean velocity of entrainment, $u_0$, is proportional to either ambient wind speed $u_0$, with proportionality constant $\eta$, or to the characteristic buoyant velocity $w_c$ with proportionality constant $z$. For moderate winds ($N_c < 10$), these semi-empirical constants from the laboratory data are $\eta = 0.71$ and $z = 0.55$ (assumed equal for both laboratory and field fires). For low winds ($N_c > 10$), $\eta = 1.02$; no value is available for $z$.

5. Theoretical flame-zone air/fuel ratio $\phi_f$ is 50 kg kg$^{-1}$; combustion-zone air/fuel ratio $\phi_{cz}$ is 20 kg kg$^{-1}$. Corresponding experimental ratios (averaged for laboratory and field burns over all $N_c$) are 48.5 and 18.2. Thus the theoretical overall air/fuel ratio of 70 compares favourably with the semi-empirical ratio of 67.

6. Field fires in the southern rough and longleaf litter–grass fuel types can be simulated well with laboratory wind-tunnel fires insofar as estimates of the air/fuel mass ratio and flame height are concerned, but tangent of the flame tilt angle is sensitive to environmental conditions.

Symbols used in mathematical models

*Roman symbols*

$A$, flame tilt angle from vertical (degrees of angle)

$A_p$, flow area of flame (m$^2$)

$A_{pp}$, projected flame area (m$^2$)

$B_p$, fireline buoyancy flux (m$^3$ m$^{-3}$)

$B_R$, ocean plume buoyancy flux (m$^3$ m$^{-3}$)

$C_{DF}$, flame drag coefficient

$C_p$, constant-pressure specific heat of burned and unburned volatiles, flame fluid and air (kJ kg$^{-1}$ K$^{-1}$)

$D$, horizontal width of flame at $z$ (m)

$D_{w0}$, flame depth at $z = 0$ (m)

$F_B$, flame buoyant force (kg m s$^{-2}$)

$F_D$, horizontal drag force on flame (kg m s$^{-2}$)

$F_{H}$, flame height Froude number

$F_k$, fire Froude number

$F_L$, flame length Froude number

$F_R$, effluent plume Froude number

$g$, fraction of original moisture remaining after preheating

$g$, acceleration of gravity (m s$^{-2}$)

$H$, flame height (m)

$H_c$, convective low heat of combustion (kJ kg$^{-1}$)

$I_B$, overall fireline intensity (kW m$^{-1}$)

$I_{Bz}$, combustion zone contribution to $I_B$ (kW m$^{-1}$)

$L$, unit length of fireline (m)

$M$, fractional moisture content

$m_e$, mass of an entrained air parcel (kg)

$m_v$, vertical mass flow rate at $z$ (kg m$^{-1}$ s$^{-1}$)

$m_w$, vertical mass flow rate at $z = 0$ (kg m$^{-1}$ s$^{-1}$)
Greek symbols

\( \dot{m}_v \), vertical mass flow rate at \( z = H \) (kg m\(^{-1}\) s\(^{-1}\))

\( N_e \), convection number

\( \lambda_{st} \), stoichiometric air/fuel mass ratio of volatiles (kg kg\(^{-1}\))

\( \Delta p \), pressure drop in flame (kg m\(^{-1}\) s\(^{-2}\))

\( q \), effluent volumetric discharge rate per unit length of diffuser (m\(^3\) s\(^{-1}\))

\( R \), rate of fire spread (m s\(^{-1}\))

\( \sec \), secant of angle \( A \)

\( T_a \), mean flame temperature at \( z = 0 \) (K)

\( T_m \), mean flame temperature at \( z = H \) (K)

\( t \), time (s)

\( u_h \), horizontal component of flame velocity at \( z = (m s^{-1}) \)

\( u_{w0} \), mid-flame ambient wind speed (m s\(^{-1}\))

\( u_{oc} \), horizontal ocean current speed (m s\(^{-1}\))

\( u_{a0} \), mean value of \( u \) at \( z = 0 \) (m s\(^{-1}\))

\( \dot{V} \), volume of heated air parcel of mass \( M_e \) (m\(^3\))

\( \dot{V}_w \), volume of ambient air parcel of mass \( M_e \) (m\(^3\))

\( W \), mean work done by parcels of air or flame (kg m\(^2\) s\(^{-1}\))

\( \dot{W}_e \), available fuel loading (kg m\(^{-2}\))

\( \dot{w} \), vertical component of flame velocity at \( z = (m s^{-1}) \)

\( w_{c0} \), characteristic buoyant velocity (m s\(^{-1}\))

\( w_{w0} \), for zero-wind fires (m s\(^{-1}\))

\( w_{a0} \), mean value of \( w \) at \( z = 0 \) (m s\(^{-1}\))

\( X_{v0} \), fraction of volatiles produced that burns

\( \tilde{Y} \), observed and predicted value of dependent variable respectively

\( Z \), mass of unreacted air in the combustion zone per mass of original fuel

\( z \), vertical distance above fuel bed surface (m)

Acknowledgements

We thank Dale Wade, Ted Ach, Wayne Adkins and Hilliard Gibbs, all formerly of the Southern Forest Fire Laboratory, Macon, GA, for their aid in burn plot preparation and collection of fire behaviour data during the 1988 controlled burns in FL and SC. We also thank anonymous reviewers for their helpful suggestions.

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## Appendix A. Southern Forest Fire Laboratory (SFFL) 1988 field data

<table>
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<tr>
<th>Fire number</th>
<th>$R$ (m s$^{-1}$)</th>
<th>$W_o$ (kg m$^{-2}$)</th>
<th>$u_o$ (m s$^{-1}$)</th>
<th>$D_o$ (m)</th>
<th>$H$ (m)</th>
<th>$I_o$ (kW m$^{-1}$)</th>
<th>$N_c$</th>
<th>$\tan \alpha$</th>
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<td>0.063</td>
<td>0.534</td>
<td>2.03</td>
<td>0.68</td>
<td>0.57</td>
<td>505</td>
<td>3.32</td>
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<td>634</td>
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<td>0.68</td>
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$^A$OS1A1 denotes a fire in the Osceola National Forest, 1-year rough, plot A1.
$^B$FM1A2 denotes a fire in the Francis Marion National Forest, 1-year rough, plot A2.
Appendix B. Regression equations and statistical fits of the model equations to experimental data

t-test results show if a parameter estimate = 0. Y indicates that the estimate is significantly different from zero (rejected null hypothesis). N indicates that the null hypothesis was not rejected. Probability value of t-value # 0.05 defined as significant. Fit statistics are:

$$R^2 = \frac{\sum \hat{Y}_i^2}{\sum Y_i^2} = 1 - \frac{\text{deviance}}{\text{uncorrected sum of squares (USS)}} = 1 - \frac{2K(K+1)}{n-K-1}$$

where K is the number of parameters in a model and n is the number of observations.

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<th>Figure</th>
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Accessory publication

Entrainment regimes and flame characteristics of wildland fires

Ralph M. Nelson Jr^A,D, Bret W. Butler^B and David R. Weise^C

^AUS Forest Service, 206 Morning View Way, Leland, NC 28451, USA. [Retired].

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Herein we report details of the derivation of two supplementary flame characteristic models and a discussion of flame tilt angle in the laboratory and field for which space was not available in the published text.

Background

In the published text, equations for entrainment parameters and flame characteristics of steadily burning 2-D head fires in uniform wildland fuels are derived. The text suggests three separate regimes of flow above such fires, with two of these regimes delineated by a critical value of the Byram convection number \( N_c = 10 \). The starting point for the flame characteristic derivations is a simplified version of the Albini (1981) flame model. The model equations are tested with fire behaviour data from laboratory wind tunnel burns in slash pine litter fuels (Nelson and Adkins 1986) and field data reported in Appendix A of the text. It is shown that flame characteristics derived from the Albini model are descriptive of flame tilt angle only in the laboratory fires and, as expected, only when \( N_c < 10 \). The authors wish to present alternative flame angle models for the \( N_c > 10 \) regime to give the reader a complete report of our work and provide modeling approaches that bring the models into agreement with the experimental data.

\[ \tan \theta \text{ in laboratory and field fires for } N_c > 10 \]

The sketch in Fig. 1 of the text depicts a time-averaged visible flame of height \( H \) tilted at mean angle \( \theta \) from vertical; the flame shape approximates a rectangular solid with flow area \( A_f \) (thickness \( D_c \cos \theta \) by unit width \( L \) of fireline into the page) and length \( H \sec \theta \). A mixture of burning volatiles and combustion-zone air flows steadily along the flame axis with a velocity whose ‘whole fire’ mean vertical component (rather than vertical velocity \( w \) at \( z \)) is the characteristic velocity \( w_c \) (Eqn 18 of the text). The mean flame temperature of 750 K ((1000 +
is computed from previously assumed values for $T_o$ and $T_r$. We assume that viscous forces are negligible and the fluid is incompressible (mean density $\rho_c = 0.48 \text{ kg m}^{-3}$); thus, the integrated form of the Euler equation (Lay 1964) may be used to write the vertical buoyant force as

$$F_B = -A_f \Delta p = gD_o \cos A(\rho_u - \rho_c)HL \sec A = \left(\frac{\rho_c w^2}{2}\right)HL \quad (A1)$$

where $\Delta p$ is the pressure drop in the flame due to buoyancy. The horizontal drag force on the flame, using Eqn 19 of the text, is

$$F_D = \frac{C_D \rho_u u^2 A_p}{2} = \frac{C_D}{2} \rho_u \eta \eta u_c^2 HL \quad (A2)$$

where $C_D$ is the drag coefficient for the inclined flame and $A_p$ is the projected area (the area normal to the direction of air flow). The balance of transverse forces that determines angle $A$ is $F_B \sin A = F_D \cos A$ and leads to

$$\tan A = \frac{F_D}{F_B} = \frac{C_D \rho_u \eta \eta u_c^2}{\rho_c w_c^2} = 3.85 \eta^2 N^{-2/3}_c \quad (A3)$$

where $C_D = 1.54$ (Fang 1969).

**Differences in tan A data for laboratory and field fires**

Fig. 2 of the text indicates that tan $A$ relationships for the laboratory and field fires differ significantly. For the laboratory fires, tan $A$ is proportional to either $N_c^{-1/2}$ or $N_c^{-1/3}$ when $N_c < 10$, and follows Eqn A3 when $N_c > 10$. In the field, tan $A$ is constant for all $N_c$. These differing results may be related to hindered v. freely moving combustion products in and above the flame for the laboratory and field fires respectively. We expect smaller tilt angles and reciprocal $N_c$ values in field measurements than would be observed for the same fire in a wind tunnel. In the field, the reduced influence of wind speed and tilt angle should combine with generally greater fuel loads and an increased rate of spread due to greater fireline length (Cheney and Sullivan 1997) to drive tan $A$ toward a constant value. The dependence of tan $A$ on powers of $N_c$ close to $-1/3$ seems associated with fires in wind tunnels with fixed ceilings (Taylor 1961; Nelson and Adkins 1986); an exception is the study of Weise and Biging (1996) who found a dependence close to $N_c^{-1/3}$ even though their relatively small tunnel was operated with a moving ceiling. However, a tendency toward $N_c$ independence, or at most a weak dependence, seems to occur in relatively large wind tunnels (Anderson et al. 2006) and in tunnels that allow free convection (Fendell et al. 1990).
**tanA in the field fires based on kinetic energy flux**

We assume the flame tilt angle is determined by a balance between the transverse components of the kinetic energy flux of ambient air approaching the flame and the vertical flame fluid kinetic energy flux due to buoyancy. This balance is given by:

\[
\frac{dW}{dt}_{\text{drag}} = \frac{dW}{dt}_{\text{buoyancy}} = F_D \mu_c \cos \alpha = F_B w_c \sin \alpha
\]

where \( W \) is work done and \( t \) is time. With this interpretation, rates at which parcels of air and flame fluid do work apparently govern flame tilt angle for moderate winds in the field, whereas a mass flux balance is operative in wind tunnels such as the SFFL tunnel in which the steady winds are more unidirectional because convection is confined. Use of Eqns 20 of the text and A1 and A2 above leads to

\[
\tan A = C_D \rho_a \alpha^3 / \rho_c = 3.85 \alpha^3 \quad (A4)
\]

This equation gives an estimate of entrainment constant \( \alpha \) identical to that derived for the lab fires from Eqn 23 of the text.

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Accessory publication

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