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Discussion

Reply to comment by Rannik on "A simple method for estimating frequency response corrections for eddy covariance systems"

W.J. Massman*

USDA/Forest Service, Rocky Mountain Research Station, 240 West Prospect, Fort Collins, CO 80526, USA

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1. Introduction

First, my thanks to Dr. Üllar Rannik for his interest and insights in my recent study of spectral corrections and associated eddy covariance flux loss (Massman, 2000, henceforth denoted by M2000). His comments are important and germane to the attenuation of low frequencies of the turbulent cospectra due to recursive filtering and block averaging. Dr. Rannik addresses two specific issues. The first concerns my formulation of the transfer function associated with the high-pass recursive filter (McMillen, 1988) and the second involves the appropriateness of the block averaging filter, $H_{\text{block}}(f) = 1 - \sin^2(\pi f T_b)/(\pi f T_b)^2$ (Panofsky, 1988; Kaimal et al., 1989), when removing the running means from two covarying time series using a recursive filter. My response is divided into two sections in which each issue is discussed in turn.

2. Recursive high-pass filter

Dr. Rannik's assertion is correct; there is an error in my original formulation of the recursive high-pass filter. This section derives the correct transfer function and updates the results of M2000. (Note that throughout this discussion, I will employ the same notation as in M2000.)

The Fourier transform of the digital high-pass recursive filter suggested by McMillen (1988) is given correctly by Eq. (4) of M2000 and is repeated below:

$$h_r(\omega) = \frac{[A + A^2][1 - \cos(\omega/f_s)] + j[A - A^2][\sin(\omega/f_s)]}{1 - 2A\cos(\omega/f_s) + A^2} \quad (1)$$

where $j = \sqrt{-1}$, $\omega = 2\pi f$, f is the frequency, f_s the sampling frequency, and $A = e^{-1/\tau_r f_s}$ with τ_r as the filter's time constant. This filter is usually applied to each of the covarying time series so that the correct cospectral transfer function should be $h_r(\omega)h_r^*(\omega) = |h_r(\omega)|^2$ where * denotes complex conjugation. After some complex arithmetic this yields the following cospectral transfer function

$$H_r(\omega) = \frac{2A^2[1 - \cos(\omega/f_s)]}{1 - 2A\cos(\omega/f_s) + A^2} \quad (2)$$

From $A \approx 1$, it follows that $2A^2 \approx A + A^2$, which in turn implies that $H_r(\omega) \approx \text{Re}[h_r(\omega)]$. Therefore, the correct transfer function, Eq. (2) above, more closely resembles $\text{Re}[h_r(\omega)]$ than $\{\text{Re}[h_r(\omega)]\}^2$ as I originally asserted. Consequently, the results shown in M2000 are over-filtered in the low frequencies. Also note (a) that the true transfer function associated with high-pass recursive filtering is real so that

* Fax: +1-970-498-1314.
E-mail address: wmassman@fs.fed.us (W.J. Massman).

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any phase shift associated with recursively high-pass filtering each individual time series is of no concern when forming the flux, and (b) that the equivalent time constant for $H_r(\omega)$ above remains τ_r as asserted by M2000.

The major consequence of Eq. (2) to M2000's results is that, in essence, the effects of the high-pass recursive filter have been double counted in both the exact (numerically integrated) and analytical approximations to the flux loss calculations. Consequently, Eq. (10) of M2000 for the ratio of the measured (attenuated) flux to the true flux, $(\overline{w'\beta'})_m/(\overline{w'\beta'})$, should read as

$$\frac{(\overline{w'\beta'})_m}{\overline{w'\beta'}} = \frac{2}{\pi}\int_0^\infty \left(\frac{1}{1+x^2}\right)\left(\frac{a^2x^2}{1+a^2x^2}\right) \times \left(\frac{b^2x^2}{1+b^2x^2}\right)\left(\frac{1}{1+p^2x^2}\right) \mathrm{d}x \qquad (3)$$

Note that Eq. (3) has only one term associated with high-pass recursive filtering, $a^2x^2/(1+ax^2)$, rather than the two terms shown in Eq. (10) of M2000. Performing the integration shown on the right-hand side of Eq. (3) yields the following replacement of Eq. (11) of M2000:

$$\frac{(\overline{w'\beta'})_m}{\overline{w'\beta'}} = \left[\frac{ab}{(a+1)(b+1)}\right]\left[\frac{ab}{(a+p)(b+p)}\right] \times \left[\frac{1}{p+1}\right]\left[1+\frac{p+1}{a+b}\right] \qquad (4)$$

where the basic change to M2000 is the replacement of the term $[1-p/(a+1)(a+p)]$ in the analytical result with $[1+(p+1)/(a+b)]$. (See M2000 for a discussion of the notation and other terms.) This result also obviates the discussion and approximation developed in Appendix B of M2000 because the term $[1-p/(a+1)(a+p)]$ is itself an approximation, whereas the term $[1+(p+1)/(a+b)]$ is exact. Note for computational efficiency, it is possible to simplify the product $[1/(p+1)][1+(p+1)/(a+b)]$ of Eq. (3) to $[(1/(p+1))+(1/(a+b))]$.

Figs. 1 and 2 of the present discussion show the "corrected" integral correction factors and replace Figs. 1 and 2 of M2000. The basic difference between these new figures and the original ones presented by M2000 is that for horizontal wind speeds of $1\,\mathrm{m\,s^{-1}}$ or less, the new correction factors are smaller than the original ones. For the present study, lower wind speeds are associated with more cospectral power in the lower frequencies and higher wind speeds are associated with more power in the higher frequencies. This is a consequence of the parameterization of f_x, the frequency at which the frequency-weighted cospectra, $f\mathrm{Co}_{\omega\beta}(f)$, achieves a maximum (see M2000). A comparison of the differences between the new numerically integrated correction factor and the new analytical results is almost identical to those presented in Figs. 3 and 4 of M2000 and need not be repeated here. In fact, the new analytical results compare slightly better than the original results because

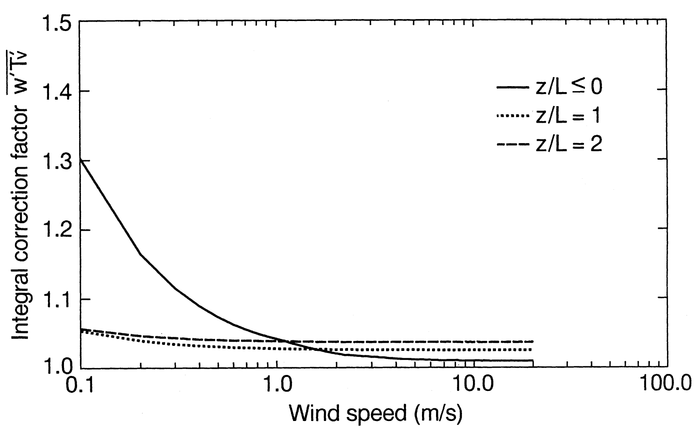

Fig. 1. Revised correction factors for Fig. 1 of Massman (2000). Numerically integrated correction factors for $\overline{w'T_v'}$ measured using sonic thermometry. Correction factors are given for neutral and unstable atmospheric conditions (solid line, $z/L \le 0$) and for stable conditions (dotted line, $z/L = 1$; dashed line, $z/L = 2$).

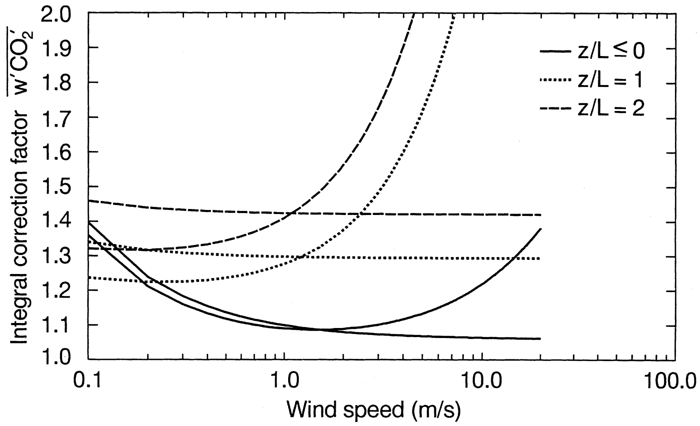

Fig. 2. Revised correction factors for Fig. 2 of Massman (2000). Numerically integrated correction factors for open- and closed-path CO_2 eddy covariance systems. The open-path instrument is taken to be the instrument developed at NOAA/ATDD (Auble and Meyers, 1991). Correction factors are given for neutral and unstable atmospheric conditions (solid line, $z/L \leq 0$) and for stable conditions (dotted line, $z/L = 1$; dashed line, $z/L = 2$). Regardless of atmospheric stability, for wind speeds greater than $2\,\mathrm{m\,s^{-1}}$, the correction factor for the closed-path system exceeds that for the open-path system.

fewer and more precise approximations are needed for the new results than with the original.

Table 1 now replaces and updates Table 2 of M2000. All other terms of the approximations developed by M2000 remain valid and so the only difference between the new analytical formulations and the originals is, as above, the replacement of the term $[1 - p/(a+1)(a+p)]$ in the analytical result with $[1 + (p+1)/(a+b)]$.

In general, except for this change in the treatment of the high-pass recursive filter all other results and conclusions of M2000 were unaffected. The major consequence to my original study is that the low frequency attenuation is now treated more precisely.

3. High-pass block averaging filter

I agree with Dr. Rannik that in order to apply the block averaging filter, $H_{\mathrm{block}}(f)$, after high-pass filtering, the time series with a recursive filter that the mean of each of the resulting time series must be removed.

In practice, even though these two time series are filtered with a high-pass running mean filter, there is often a small residual mean associated with each of these time series. However, the magnitude of this residual mean is in part influenced by the choice of τ_r and in part by the amount of cospectral power in the low frequencies. Dr. Rannik correctly points out that for $\tau_r \leq \frac{1}{4}T_b$, the influence of block averaging

Table 1
Corrected version of Table 2 of Massman (2000)

Stable atmospheric conditions ($0 < z/L \leq 2$)

Fast-response open path systems

$$\frac{(\mathrm{Flux})_m}{\mathrm{Flux}} = \left[\frac{ab}{(a+1)(b+1)}\right]\left[\frac{ab}{(a+p)(b+p)}\right]\left[\frac{1}{p+1}\right]\left[1+\frac{p+1}{a+b}\right]$$

Scalar instrument with 0.1–0.3 s response time

$$\frac{(\mathrm{Flux})_m}{\mathrm{Flux}} = \left[\frac{ab}{(a+1)(b+1)}\right]\left[\frac{ab}{(a+p)(b+p)}\right]\left[\frac{1}{p+1}\right]\left[1+\frac{p+1}{a+b}\right]\left[\frac{1+0.9p}{1+p}\right]$$

Unstable atmospheric conditions ($z/L \leq 0$)

$$\frac{(\mathrm{Flux})_m}{\mathrm{Flux}} = \left[\frac{a^\alpha b^\alpha}{(a^\alpha+1)(b^\alpha+1)}\right]\left[\frac{a^\alpha b^\alpha}{(a^\alpha+p^\alpha)(b^\alpha+p^\alpha)}\right]\left[\frac{1}{p^\alpha+1}\right]\left[1+\frac{p^\alpha+1}{a^\alpha+b^\alpha}\right]$$

is under most conditions negligible. For example, my experience is that neither the residual means nor the block averaging filter influence the fluxes (or the flux correction factor) by more than 1% for horizontal wind speeds greater than $0.5\,\mathrm{m\,s^{-1}}$. But, these small corrections could become significantly larger as cospectral power shifts to lower frequencies. This concern about the low frequency power takes on even more importance if the flat terrain frequency-weighted spectra (and by implication the cospectra) of Kaimal et al. (1972) actually result from too much high-pass filtering (Högström, 2000). Given the potential importance of these low frequency flux issues to long-term energy and carbon balances, it may, therefore, be useful to expand upon the results of M2000 under the assumption of less or different high-pass filtering with eddy covariance than originally examined.

Here I consider two cases: the case where no recursive filtering is done and the case where linear detrending is done to ensure stationarity rather than a high-pass recursive filter. The first case is equivalent to ignoring the $a^2x^2/(1+a^2x^2)$ term in Eq. (3). The corresponding analytical expression to Eq. (4) can easily be derived by taking the limit as $a \to \infty$ of the right-hand side of Eq. (4). The result is

$$\frac{(\overline{w'\beta'})_\mathrm{m}}{\overline{w'\beta'}} = \left[\frac{b}{b+1}\right]\left[\frac{b}{b+p}\right]\left[\frac{1}{p+1}\right] \tag{5}$$

(Note that implicitly Eq. (5) requires the covarying time series be very nearly stationary, which reduces the need for the recursive filter in the first place.) Now if the flat terrain cospectra used by M2000 actually are distorted by too much high-pass filtering, Eq. (5) may not give a very precise estimate of the integral correction factor. In this case, Eq. (5) can be adjusted to the true cospectra (if known) by introducing the parameter α (M2000) which applies for cospectra which are broader than the flat terrain cospectra for stable atmospheric conditions. Therefore, for cospectra with power more uniformly distributed across frequencies near f_x than the stable flat terrain cospectra, Eq. (5) can be replaced with Eq. (6).

$$\frac{(\overline{w'\beta'})_\mathrm{m}}{\overline{w'\beta'}} = \left[\frac{b^\alpha}{b^\alpha+1}\right]\left[\frac{b^\alpha}{b^\alpha+p^\alpha}\right]\left[\frac{1}{p^\alpha+1}\right] \tag{6}$$

where $\alpha < 1$ would be determined by the shape of the cospectra as done by M2000.

Second, assuming the cospectral integral time scale is much smaller than the block averaging period, T_b, the high-pass transfer function for linear detrending with mean removal, $H_\mathrm{hp}(f)$, is (Rannik and Vesala, 1999; Kristensen, 1998)

$$H_\mathrm{hp}(f) = 1 - \frac{\sin^2(\pi f T_\mathrm{b})}{(\pi f T_\mathrm{b})^2} - 3\frac{[\sin(\pi f T_\mathrm{b}) - \pi f T_\mathrm{b}\cos(\pi f T_\mathrm{b})]^2}{(\pi f T_\mathrm{b})^4} \tag{7}$$

It should be pointed out that linear detrending with mean removal combines aspects from both the block averaging filter and the recursive filter. The equivalent time constant of this filter is $T_\mathrm{b}/5.3$ (M2000) and the appropriate analytical approximation to the integral correction factor is given by Eqs. (5) and (6), because only one high-pass filter is used rather than two as originally formulated by M2000.

Eqs. (5) and (6) of the above discussion of linear detrending serve to correct the claim I made in M2000 that the block averaging filter, $1-\sin^2(\pi f T_\mathrm{b})/(\pi f T_\mathrm{b})^2$, was “inescapable”. In fact what is inescapable, assuming some form of mean removal associated with either block averaging or linear detrending, is high-pass filtering of the cospectrum. But the functional form associated with any one type of high-pass filter is not inescapable because it can vary with the choice of filtering technique.

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