# Assessing Sampling Uncertainty in FVS Projections Using a Bootstrap Resampling Method

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**Abstract**—The Forest Vegetation Simulator (FVS) lets users project changes in forest stands associated with different initial conditions and silvicultural treatments. Our objective is to develop tools that help model users estimate the precision of FVS projections. A technique called bootstrap resampling (bootstrapping) allows us to approximate the sampling distribution of any variable simulated by FVS. To use the technique, the original FVS tree list is sampled repeatedly, with replacement, to build hundreds of bootstrapped tree lists. These bootstrapped tree lists are then used to make several hundred FVS projections. Each projection is thus based on a resample of the original tree list. The resulting empirical distribution provides information on the sampling uncertainty associated with the original tree list, which is important for making statistical inferences about FVS model outcome. This paper introduces a new bootstrapping program (FVSBoot) and describes its purpose and potential value.

*"…The oldest and simplest device for misleading folks is the barefaced lie. A method that is nearly as effective and far more subtle is to report a sample estimate without any indication of its reliability…"* (Freese 1967)

The Forest Vegetation Simulator (FVS) (previously known as the Prognosis Model for Stand Development; Wykoff and others 1982), lets model users project forest stand development through time. To use the FVS model information about the initial condition of a stand must first be formatted into a tree list. A tree list typically is created from a sample of forest conditions (stand exam or forest inventory) and is the basis for projecting the development of vegetation in FVS. Hence, at least two potential sources of uncertainty exist in FVS projections and should concern anyone who uses either the model or its results. The sources of uncertainty are (1) variation in the parameters and model forms that build the simulation and 2) variation in the sample data used to create tree lists.

The FVS model is generally considered deterministic rather than stochastic (Hamilton 1991). It is important to remember that a single realization from a single FVS projection is only one of several possible outcomes for the simulated development of that tree list. To explore "stochastic" (or "random") variation, the FVS user should make several projections (also referred to as "runs" or "simulations"), each of which has the same simulation file tree list but a new random number seed. Model outcomes generated this way can be summarized and analyzed; descriptive statistics include the computation of upper and lower bounds on 95 percent ($\alpha$=0.05) of the FVS outcomes. We define the interval between these bounds as the FVS prediction interval (FVSPI). FVS users have been able to compute FVSPI to characterize uncertainty associated with variation in parameters and models forms for several years.

## The Issue

Results from FVS projections are, in our opinion, too often reported without disclosing distributions of predicted outcomes or associated statistics. At issue is the ability to interpret the precision of FVS simulations.

FVSPI is already available to users via the RANNSEED keyword (Van Dyck 2000). There are several reasons why FVSPI is not often reported, many of which relate to computational intensity. One obvious reason is simply the cost of making a large number of projections. This was especially true prior to the 1990s when FVS users had to pay by the run on time-share systems. Another reason is the time associated with making large numbers of projections on old computers. However, the improved power in desktop systems has made fast and cheap computing possible, and thus these reasons are no longer insurmountable. Users can now easily report FVSPI and simple summary statistics such as the mean and standard deviation.

A different matter altogether is the challenge of reporting distributions that include sampling uncertainty in FVS projections. It hasn't been possible to compute the effects of sampling uncertainty because classical statistical methods are not available to make inferences about FVS projections. A variance estimator is not available for the results of simulation. Our objective is to develop tools that characterize sampling uncertainty in FVS projections so that users can easily report this important component of variation.

## Bootstrapping

Technological advances helped to resolve the problems of time and money in making multiple FVS projections, and we felt technology could also help us to tackle the problem of assessing sampling uncertainty in FVS projections. Indeed,

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a technique called bootstrap sampling ("bootstrapping") allows us to empirically approximate the sampling distribution of any statistic for which we want to make inferences and thus avoid the lack of a variance estimator. In essence, bootstrapping is data-based simulation. The statistical theory underlying bootstrap sampling dates to the 1930s but the computer capacity necessary to apply it to problems such as ours has only recently become widely available. Bootstrapping allows us to substitute computational power for theoretical analysis (Efron 1982).

Starting in the 1970s, Efron introduced bootstrapping as a generalized, computer-based method for estimating standard error for any parameter estimate of interest (Efron 1979 and 1982; Efron and Tibshirani 1993). Since then, a flood of statistical research and reports on practical applications using bootstrapping has been published. The bootstrap method has gained popularity among scientists working with complex problems where estimating standard error and confidence intervals is difficult, inaccurate, or impossible using classical statistical methods. Because of the generalized nature of bootstrapping, it has been applied to a wide range of statistical applications for estimating error rates (Chernick 1999). These include discriminate analysis, multiple, logistic, and nonlinear regression analysis, and complex surveys. It has also been applied in a variety of disciplines, including psychology, all the major physical sciences, economics, medicine, and engineering. Our goal was to apply bootstrapping techniques to FVS simulations. We set three specific tasks:

1. Develop a method using bootstrap sampling techniques for assessing uncertainty in FVS projections caused by variation in input (tree list) data.
2. Create a module compatible with Suppose (Crookston 1997) and FVS to implement the method.
3. Illustrate the method using tree lists from the Gotchen Late Successional Reserve study on the Gifford Pinchot National Forest in Washington State.

## Methods and Results

For the first task, we take our original sample of the stand ($n$) and replace the unknown population characteristics with the known empirical distribution derived from repeatedly resampling $\boldsymbol{n}$ (Chernick 1999). Because $n$ comprises multiple plots (that FVS pools to create a tree list), by resampling with replacement we "reshuffle" the combination of plots that create the FVS tree list for that stand. There are just two steps required:

- Generate a new sample of stand conditions by sampling the original plots with replacement to create a bootstrap sample, also of size $n$.
- Compute a bootstrap mean ($\Theta^*$) from the bootstrap sample for any FVS attribute.

By repeating this process $k$ times, we generate a Monte Carlo approximation of the distribution of $\Theta^*$. The standard deviation of this Monte Carlo approximation is the bootstrap estimate of the true standard deviation for the population.

For the second task, we wrote a program (FVSBoot) to implement the process described above within FVS and Suppose. Output from the bootstrap program includes intervals around a set of FVS predictions. One interval available as FVSBoot output is based on variation from the parameters and model forms of FVS (FVSPI). For example, we made 201 FVS projections of stand 1022144 (using the original tree list and random number seed plus an additional 200 projections seeding new random numbers). Figure 1

```
Stand ID      = 1022144
    Management ID = NONE

    Model output data from FVS SUMMARY Table = Cycle( 3), BA


    FVS-PI mean               =      129.56

    Number of FVS runs        =         200
    Standard Deviation        =        1.30
    Median                    =      130.00
    Max                       =      133.00
    Min                       =      126.00
    Range                     =        7.00


 Frequency distribution for (  201 ) samples for "Cycle( 3), BA" from FVS.

 Interval  Midpoints   Counts
 --------  ---------  ---+-------------------------------------------------------------
     1      126.50    12 |IIIIIIIIIIII
     2      127.50    25 |IIIIIIIIIIIIIIIIIIIIIIIII
     3      128.50    64 |IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
     4      129.50    51 |IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
     5      130.50    37 |IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
     6      131.50    10 |IIIIIIIIII
     7      132.50     2 |II
 --------  ---------  ---+-------------------------------------------------------------
```

**Figure 1**—FVSBoot output of basal area (BA) for stand 1022144 at year 30 based on the same initial tree list projected 200 times with a new random number each time.

illustrates how the FVSBoot program displays FVSPI output for square feet of basal area per acre (BA) after the third cycle (Cycle (3)) for stand 1022144. It shows that the FVSPI mean for BA is 129.56, and the standard deviation is 1.30. The frequency distribution reveals that the midpoint value in 64 of 201 counts was 128.50. Another interval available from FVSBoot is based on variation in the sample data used to create tree lists. We refer to it as the sampling error prediction interval (SEPI). For comparison with FVSPI, figure 2 illustrates how the FVSBoot program displays SEPI

```
Data from FVS Model:  SUMMARY STATISTICS.

    Stand ID      = 1022144
    Management ID = NONE

 FVS Variable           =  Cycle( 3), BA

  FVS-PI
    Mean                =      129.56
    Number of samples   =         201
    Standard Deviation  =       1.295

  SEPI:
    Number of samples   =         500
    Mean                =      131.87
    Standard Deviation  =       14.00
    Bootstrap Median    =      132.00
    Max outcome         =      175.00
    Min outcome         =       93.00
    Range of outcomes   =       82.00

BOOTSTRAP SAMPLING ERROR PREDICTION INTERVALS

     Variable             Mean    Percent  Lower    Upper
  ----------------      --------   -----  -------- --------
  Cycle( 3), BA          129.56     68    118.00    146.00
                                    80    114.00    150.00
                                    90    109.00    157.00
                                    95    105.00    162.00
                                    99     99.00    173.00
  ------------------------------------------------------------

 Frequency distribution for (  500 ) bootstrap samples for "Cycle( 3), BA" from FVS.

 Interval  Midpoints   Counts
 --------  ---------  ---+-------------------------------------------------------------
     1       95.05     2 |II
     2       99.15     3 |III
     3      103.25     9 |IIIIIIIII
     4      107.35    13 |IIIIIIIIIIIII
     5      111.45    20 |IIIIIIIIIIIIIIIIIIII
     6      115.55    31 |IIIIIIIIIIIIIIIIIIIIIIIIIIIII
     7      119.65    44 |IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
     8      123.75    40 |IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
     9      127.85    50 |IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
    10      131.95    83 |IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
    11      136.05    54 |IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
    12      140.15    43 |IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII
    13      144.25    31 |IIIIIIIIIIIIIIIIIIIIIIIIIIIII
    14      148.35    31 |IIIIIIIIIIIIIIIIIIIIIIIIIIIII
    15      152.45    17 |IIIIIIIIIIIIIIIII
    16      156.55     9 |IIIIIIIII
    17      160.65    12 |IIIIIIIIIIII
    18      164.75     4 |IIII
    19      168.85     2 |II
    20      172.95     2 |II
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```

**Figure 2**—FVSBoot output of the mean basal area for stand 1022144 at year 30 based on 500 bootstrapped tree lists projected 500 times.

output for BA after the third cycle (Cycle (3)) for stand 1022144. In this example, the SEPI mean is the mean predicted value basal area per acre (BA) from 500 FVS runs made with 500 bootstrapped samples. The SEPI mean is 131.87, and the standard deviation is 14.00. The 95 percent SEPI for BA ranges from 105-162 $ft^2$ per acre. Except in rare cases, the SEPI interval also includes the variation in the parameters and model forms of FVS. This is because in FVS the sequence of random numbers changes with very small changes in the tree list. Bootstrapping implies changes in the tree list sufficient to generate new random number seeds in FVS over multiple time intervals. For this reason the FVSPI is also reported with the SEPI statistics in the FVSBoot output. In figure 2, the FVSPI summary statistics are based on 201 projections, as described for figure 1.

For the third task, we pilot-tested FVSBoot using stand exam data from the Gifford Pinchot National Forest. We created 200 bootstrap samples for each tree list, ran the East Cascades variant of FVS for 50 years, and evaluated the projected values for variation in quadratic mean diameter. Details, results, and potential implications for two tree lists are reported in Hummel and others (this proceedings).

## Summary

The FVSBoot program offers model users a tool that can strengthen both the analysis of potential treatment effects on stand development and the design of forest inventories.

If users rely on only one FVS projection of a tree list to characterize stand development, then other outcomes possible from different combinations of sample data or from other sequences of random events are not considered. We think it is useful to characterize "extreme" outcomes as well as "average" projected outcomes for FVS variables of interest because analyses that consider the likelihood of different outcomes are more informative than analyses that do not. The value of this additional information will depend on the real or perceived risk associated with rare events. In addition to strengthening FVS analysis for management planning, FVSBoot creates the possibility for computer-based experiments. By generating intervals around FVS output, it is possible to determine whether differences among treatments on projected stand development are statistically significant. Further, it provides a way to investigate assumptions about the distribution of populations and patterns of variation through time.

Another promising area for using FVSBoot is in the evaluation of sample design. We could, for example, characterize the standard deviation from presampling and then calculate the desired number of samples for a given level of acceptable error. We could also ask whether within-stratum variation is greater or less than that among strata to test the value of different stratification methods.

FVSBoot does not cover all potential sources of variation (climate change, for example), nor does it answer whether statistically significant differences are, in fact, biologically significant. Despite these shortcomings, we offer FVSBoot as a new tool that can provide information about important components of uncertainty in FVS model projections.

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