Effects of Registration Errors Between Remotely Sensed and Ground Data on Estimators of Forest Area

Paul L. Patterson and Michael S. Williams

ABSTRACT. The estimation of area by land cover type is a key component of most large-scale forest inventories. Historically, these estimates were derived from a large sample of points, taken from aerial photos, followed by a smaller sample of ground points, which were used to correct errors in the classification of the aerial photo points. There has been interest in replacing aerial photography with satellite imagery. One problem with using satellite imagery is the registration errors between a pixel and a plot or a point on the ground. The estimators and modes of inference can differ substantially depending on whether the sample unit is a plot or point on the ground. Two terms, tessellated and point paradigm, are used to differentiate between the two approaches when the ground data consists of plots or points, respectively. The effects of registration errors are compared using the two paradigms for estimating the area of land by cover type. The effects of registration errors on the expected value and variance of the forest area estimator under both the tessellated and point paradigm were studied using a simulation study, where a percentage of the samples had a random one- or two-pixel registration error. The simulation study shows that registration errors increased the variance of the estimator of forest area from 4% to 434%. The estimator of forest area under both the tessellated and point paradigm exhibited no detectable bias. In the presence of registration errors, the estimated variance under the tessellated paradigm tended to overestimate the true variance with the achieved coverage rate for a nominal 80% confidence interval ranging from about 81% to 86%. For sample sizes of fewer than 100 ground points, the estimated variance under the point paradigm tended to underestimate both the 80% confidence interval and the true variance, regardless of whether there were registration errors. Further testing showed that sample sizes of between 100 and 250 ground points were needed before the estimator of the variance under the point paradigm converged to within 5% of the true variance of the estimator of forest area. Thus, we conclude that registration errors can drastically increase the variance of area estimators and the resulting confidence intervals will not achieve their nominal coverage rates. In this study, the forest area estimator under the tessellated paradigm was clearly more robust to registration errors than the area estimator under the point paradigm. For. Sci. 49(1):110–118.

Key Words: Aerial photography, satellite imagery, variance estimator, design- and model-based inference.

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For simplicity, this article will focus on the estimation of the area of forest within a larger area. The area of other land cover types can be estimated using these techniques without loss of generality.

In the past, the estimation of forest area was performed using a combination of aerial photography and ground points (see de Vries 1986, p. 133–140). A double sampling scheme was employed where n' first phase points were classified on aerial photos and a smaller second phase sample of n ground points was used to determine if the classification of the point on the photo matched the classification of the point on the ground. Thus, the second phase information was used to correct the estimated area from the first-phase sample. This approach assumes the first-phase sample points are distributed according to a multinomial distribution. This reliance on an assumed model means that inference under this paradigm is model-based (see Gregoire 1998). The statistical theory for this estimation problem originated in the quality control literature, where the proportion of photo points classified as forest is an inexpensive but potentially biased estimator of the true proportion. Ground points are a more expensive measurement of a subsample of the same points used to correct the bias. Tenebein (1972) derives the maximum likelihood estimator of the proportion of forested area and its asymptotic variance.

We shall refer to this approach as the point paradigm because the sample unit is a point on both the image and ground and the datum value indicates the cover type classification. One of the key assumptions of the point paradigm is that a point on an aerial photograph can be co-located with its true location on the ground, which gives a true one-to-one correspondence between the first and second phase of sampling. This is a reasonable assumption with aerial photography because field crews can usually determine almost exactly where the photo point falls on the ground.

The advent of readily available satellite imagery has had a number of distinct effects on the estimation of forest area. The first is that when the pixels of a satellite image are used, there is no natural one-to-one correspondence between a pixel and a point on the ground because the pixel contains an infinite number of possible points. A simple solution is to realize that all points in a pixel have a single classification. By doing this, every point within the pixel has the same classification, and the desired one-to-one correspondence can be achieved without actually having to identify a single point on the satellite image.

A second significant change because of the use of satellite imagery is that the first phase estimator of the proportion of the area that is forested is replaced by the true proportion. Card (1982) derives the maximum likelihood estimator of the proportion of forest area and its asymptotic variance making the adjustments for the known proportions.

A third change brought about by satellite imagery is the possibility of using a completely different approach to the estimation problem. To do so requires that the pixels of the satellite image are used to tessellate the land base into an area frame containing N sampling units (see Särndal et al. 1992, p. 12). The other requirement is that the area of forest within each sample unit can be measured or estimated. Thus, the datum value for each sample unit is an area and must be measured on a plot rather than a point. This approach is based on finite population sampling theory, where the mode of inference is design-based (see Gregoire 1998). It will be referred to as the tessellated paradigm. The Forest Inventory and Analysis (FIA) program in the United States is a good example of a survey where the use of the tessellated paradigm is possible with some simplifying assumptions (for a discussion of assumptions see de Vries 1982, p. 161–168, Williams and Patterson 2003, Williams and Eriksson 2002). This inventory uses a cluster plot of four 1/60 ha fixed area plots, where three of the plots are located about a center plot with a separation distance of 36.6 m and 120°. Using 28.5 m Landsat pixels, the logical primary sampling unit for this plot is a 3×3 block of pixels. Williams and Patterson (2003) derive the estimator for the area of forest and its variance under the tessellated paradigm and compared the estimators under the point and tessellated paradigms in a simulation study. They found that neither estimator was consistently superior. The estimator under the point paradigm had the smallest variance when the forest/nonforest area was highly fragmented, and the accuracy of the satellite imagery was high (0.9 for the populations studied). The estimator under the tessellated paradigm had a smaller variance when the accuracy was lower and when the percentage of pixels that could not be reliably classified as either forested or nonforested was relatively large (>10%).

The use of satellite imagery is not without its own problems. One disadvantage is that the resolution of satellite imagery is generally inferior to that of aerial photography, increasing misclassification in some types of areas. The other disadvantage is the registration error between a pixel and a point or plot on the ground. These errors have been studied by Cooke (2000). He looked at the potential errors associated with using Landsat 7 thematic mapper (TM) data for the FIA program. The first source of registration errors is the error in the GPS coordinates, which were found to be accurate to within ±5 to 20 m (16–64 ft). The accuracy of the coordinates depends on the time of acquisition, number of GPS locator satellites acquired, and interference caused by the forest canopy. The second source is the registration error of the pixel to its true location on the ground. These errors tend to be ±1 pixel in flat terrain, which is 28.5 m (93.5 ft). Steeper terrain can cause errors as large as 2 pixels. Cooke concludes that in the worst case, the total registration error in flat terrain is additive and nearly 50 m (20 + 28.5 = 48.5 m). Hoppus et al. (2001) and Riemann et al. (2001) also studied the relationship between the FIA ground plot and Landsat data. In these studies they used aerial photos, with the FIA plot location pin-pricked on the photo, overlaid on the Landsat image to correct for registration error. They found that even when using 3×3 blocks of pixels, 31.5% of all classification errors appeared to be due to registration errors between the ground and satellite data. Correcting these errors increased the accuracy of a forest area classification from 80% to 87%. Because only misclassified

ground plots were checked for registration errors, the true percentage of registration errors cannot be determined from this study.

The common practice is to assume that errors caused by misclassification and registration errors are equivalent. However, the effects of registration errors are unique, and their influence on the estimators under the tessellated and point paradigms are different. For the former, registration errors cause the sample unit information on any given unit to be randomly assigned to one of the strata defined by the satellite image. Hence, the reference distribution (see Gregiore 1998) differs from the one defined by the area frame. This type of error is similar to the problem of coverage errors in the sampling frame (Särndal et al. 1992, p. 540–543), with the difference being that the assignment to the wrong element in the frame is random. For the estimator under the point paradigm, it is assumed that there is a one-to-one correspondence between the point on the satellite map and the ground, with the point on the ground being the true value. In the presence of registration errors, this one-to-one correspondence may no longer hold for all points. Thus, registration error under both paradigms is an additional random process that is not accounted for in the derivation of both the expected value and variance of the estimators. While it may be possible to derive new estimators that account for registration error, it seems unlikely that the parameters of a registration error model could be estimated without some additional data source, such as aerial photos.

What is not clear from the literature is how these violations of the underlying assumptions affect the estimators and whether one estimator is more robust than the other. In practice, registration errors are often ignored, which we will show can have a number of undesirable consequences. Thus, the goals of this article are to:

- determine the effects of registration errors on the expected value and variance of the estimator of forest area under both the tessellated and point paradigms,
- determine if the estimator of the variance is biased in the presence of registration errors, and if so, determine the magnitude and direction of the bias,
- compare the nominal and achieved coverage rates of 80% confidence intervals, and
- attempt to set some guidelines for the minimum sample size in order for the asymptotic estimators under the point paradigm to converge to the true mean and variance.

These goals were accomplished using a simulation study that studied a series of artificial populations. The simulation study was loosely based on the FIA sampling design. For more details on the sampling design see Williams and Patterson (2003).

The Estimators

Common features to both paradigms are the following: Assume the area of interest is a flat plane denoted by A, with known area $|A|$ in hectares. It is assumed that the satellite map has already been classified and geo-referenced. For example, the satellite image may have been classified using the iterated conditional mode algorithm of Besag (1986) or a similar algorithm intended to achieve the best possible image. For our purposes, the satellite map must be fixed prior to sampling. In producing the satellite map, registration errors can be confounded with classification errors. This can happen if ground plots are used for validation and training. This confounding will be ignored in this study. The plots used for estimation should not be used as training/validation plots and vice versa.

Given a satellite image, the goal is to estimate the area of forest, denoted by A_F, using the satellite imagery and a sample of n locations on the ground. It suffices to estimate the proportion of A that is covered by forest, denoted by p_F, since $A_F = |A|P_F$. Each pixel of the image is classified into one of L different classes. The classification scheme used here assumes $L = 3$ classes, these being F, I, and NF to denote forest, indeterminate, and nonforest classes. The indeterminate class contains those pixels whose reflectance values do not strongly indicate either forest or nonforest cover. Examples of areas that are likely to be assigned to this class are the forest/nonforest interface and areas where the tree cover is sparse. Methods to derive an indeterminate class are proposed by Hjort and Mohn (1984).

The Tessellated Paradigm

The pixels of the satellite image are used to construct an area frame (Särndal et al. 1992, p. 12), with every element in the frame having an assigned stratum value. Under this paradigm it is assumed that A is divided into N plots of equal area, which are assumed to be the size of the primary sampling unit (PSU). Thus, the size of population is $N = A / a_{PSU}$, where a_{PSU} is the area of the PSU. This assumption does not account for the fact that some PSUs along the boundary of a population only partially cover A, but the number of straddler to interior PSUs is small for a large-scale survey, generally less than 0.5% for the FIA inventory.

The linkage between Landsat TM pixels and the FIA ground plot is such that the individual pixels must be grouped to completely encompass the plot. Ignoring locational errors, a logical PSU is a 0.81 ha square comprised of 9 pixels. Thus, the number of PSUs is $N = |A|/0.81$. Multistage cluster plot sampling results can be used to approximate the division of each PSU into M secondary sampling units (SSU), from which a sample of size m is chosen. Because the subplot locations are fixed and the subplots are circular, there can be no true randomization within the PSU. To approximate two-stage cluster sampling, each subplot can be viewed as a secondary sampling unit. Assuming each subplot represents the area covered by a pixel, this yields $m = 4$ and $M = 9$. This is a higher sampling rate than for FIA, which is approximately $m = 4$ and $M = 49$ for the same size PSUs.

The L pixel classes are used to define H strata. To reduce the variance of the estimator, each PSU is assigned to one of H strata. It may be that L is not equal to H. One way to define strata would be to assign a score to each class based on whether the pixel appears to be forested and make stratum assignments based on

the total of the score for the 9 pixels. For this study the $L = 3$ satellite classes were forest, indeterminate, and nonforest, and were assigned scores 2, 1, and 0, respectively. $H = 3$ strata were used based on total scores of 0–3, 4–14, and 15–18 points. [As suggested by Rao (1984), these scores were arbitrarily chosen prior to observing data from the simulation study.] Regardless of the method of assignment for each PSU, the goal is to estimate the proportion of forest in each stratum. To accomplish this, the n ground plots are treated as a poststratified sample with a random sample size n_h in each stratum. For each ground plot, the proportion of the 0.81 ha PSU covered by forest is estimated by mapping the forest/nonforest boundary on each subplot and measuring the proportion of the 4-point cluster plot covered by forest. Thus, given that the assumptions are reasonable, the estimator of forest area is a poststratified, two-stage cluster sample (Cochran 1977, Chapters 5 and 10), given by

$$\hat{A}_F = |A|\hat{p}_F = |A|\sum_{h=1}^{H}\frac{N_h}{N}\left[\sum_{i=1}^{n_h}\sum_{j=1}^{4}\frac{\hat{p}_{hij}^F}{4n_h}\right] \tag{1}$$

where $\hat{p}_{hij}^F$ is the percentage of forest in the jth subplot of the ith PSU within the hth stratum. For n_h random, an approximate variance is given by

$+\frac{5}{36}S_{2h}^2$

$$\operatorname{var}\left[\hat{A}_F\right] \approx |A|^2\left[\begin{array}{l}\displaystyle\sum_{h=1}^{H}\frac{1}{n}\frac{N_h}{N}\left[\left(1-\frac{n}{N}\right)S_{1h}^2\right] \\ \displaystyle+\left(1-\frac{N_h}{N}\right)\frac{1}{n^2}\left[S_{1h}^2-\frac{5}{36}S_{2h}^2\right]\end{array}\right] \tag{2}$$

where S_{1h}^2 and S_{2h}^2 are the between cluster and within cluster variances for stratum h [see Appendix 2, Williams and Patterson (2003)]. Using results derived in Theorem 10.2 of Cochran (1977), it can be shown that an unbiased estimator for the above approximation of $Var\left[\hat{A}_F\right]$ is given by

$$\operatorname{var}\left[\hat{A}_F\right] = |A|^2\left[\begin{array}{l}\displaystyle\sum_{h=1}^{H}\frac{1}{n}\frac{N_h}{N}\left[\left(1-\frac{n}{N}\right)s_{1h}^2+\frac{n}{N}\frac{5}{36}s_{2h}^2\right] \\ \displaystyle+\left(1-\frac{N_h}{N}\right)\frac{1}{n^2}s_{1h}^2\end{array}\right] \tag{3}$$

where s_{1h}^2 and s_{2h}^2 are the between cluster and within cluster sample variances for stratum h (Cochran, Chapter 10). The above sampling strategy and estimator will be referred to as the tessellated paradigm and tessellated estimator, respectively.

The Point Paradigm

From the satellite data, the true proportion of points in each of the L classes is derived from a count of the pixels in each class divided by the total number of pixels covering A. Thus, the first phase estimator of the proportion of points in each remotely sensed class is

$$\pi_l = \frac{N_l'}{N'}$$

where N' is the number of pixels in A and N_l' is the number in class l.

The ground points are a random sample of size n for which both the ground and satellite classification are known. This allows the true multinomial proportions of all n points to be estimated. That the location of each ground plot for an inventory such as FIA has been predetermined means that the number of ground points that fall in each of the L classes is random. The ground points are broken down into classes based on both their ground and satellite classifications. The counts are denoted by n_{kl}, where $k = 1,\ldots,K$ indexes the true class determined on the ground and $l = 1,\ldots,L$ is the satellite class, with

$$n = \sum_{k=1}^{K}\sum_{l=1}^{L} n_{kl}$$

This leads to the unbiased estimator of forest area

$$\hat{A}_F = |A|\hat{p}_F = |A|\sum_{l=1}^{L}\pi_l\hat{\lambda}_{F,l}, \tag{4}$$

where

$$\hat{\lambda}_{F,l} = n_{Fl} / \sum_{k=1}^{K} n_{kl}$$

is an estimate of the proportion of the points assigned to class l on the satellite image that were truly forest land. Using the results of Card (1982), the asymptotic variance is

$$Var\left[\hat{A}_F\right] = |A|^2\sum_{l=1}^{L}\lambda_{F,l}(1-\lambda_{F,l})\pi_l / n. \tag{5}$$

The asymptotic variance estimator is

$$\operatorname{var}\left[\hat{A}_F\right] \approx |A|^2\sum_{l=1}^{L}\hat{\lambda}_{F,l}(1-\hat{\lambda}_{F,l})\pi_l / n. \tag{6}$$

The above sampling strategy and estimator will be referred to as the point paradigm and point estimator, respectively.

Simulation Study

There are no real data sets available where the amount of registration error is known or can be controlled. Thus a major part of the simulation study was the creation of landscape and satellite maps that we felt could be realistic approximations of actual conditions. Thus the goal was to simulate a very large number of artificial populations where the degree of registration error, strata size, fragmentation, and classification accuracy could be varied. Then patterns could be established between the performance of both estimators by looking for similarities across all of the artificial populations.

Each population consisted of two square arrays, one representing the satellite image and the other representing the ground "truth." Each array element represented a pixel on the satellite map, and the corresponding array element of the ground represented the actual ground condition. For simplic-

ity, a ground element will be referred to as a pixel. The satellite map had three types of pixels: forested, indeterminate, and nonforested. The ground map should be coded with the percentage of forest, but for simplicity, each pixel was either 100% forest or 100% nonforest. The two arrays will be referred to as a pair of maps. The array sizes used were always a multiple of 9 so that all PSUs were equal in size for the tessellated paradigm.

Four aspects were involved in the simulations: fragmentation, indeterminate pixels, misclassification, and registration error. The starting point was to code both the ground map and the satellite map as completely nonforested. Fragmentation was accomplished by placing uniformly distributed irregularly shaped patches of forested pixels within the nonforested region. The shape and size of the patches were random. Indeterminate pixels were placed on the satellite along randomly selected edges of the forested patches.

Misclassification was accomplished by changing the classification on the ground map of randomly placed and shaped patches of pixels. The number of such patches was fixed. These randomly shaped patches were generated from a "center" point. A proportion of these misclassified patches were associated with randomly shaped patches of indeterminate pixels on the satellite map. The indeterminate patches were generated about the same "center" point as the associated misclassified patch. This was done because we felt that any grouping of misclassified pixels was likely to have some indeterminate pixels in the general vicinity. The misclassified and indeterminate patches about each point were generated from different random processes.

The parameters that controlled the generation of each pair of maps were the number of forested and nonforested patches, the distribution of size and shape of these patches, the distribution of the number of sides to which indeterminate pixels were added, the number of misclassified forest and nonforest patches, the distribution of the size and shape of these patches, the distribution of the number of indeterminate patches associated with the misclassified patches, and the distribution of the size and shape of these indeterminate patches.

Once a pair of maps was generated from a set of map parameters, 1,000 samples were drawn for both the point paradigm and the tessellated paradigm. A sample for the point paradigm was produced by drawing a simple random sample without replacement of size n of pixels on the ground and the associated pixels on the satellite map. This is equivalent to a noninformative sample of ground points with the restriction that at most one point can be sampled from area covered by a pixel (for a discussion of noninformative sampling see Särndal et al. 1992 or Gregoire 1998). For the tessellated paradigm, a sample consists of a simple random sample of size n drawn from the PSUs and then from each of the sampled PSUs a simple random sample without replacement of 4 ground pixels. For each of the 1,000 samples a "parallel" sample with registration errors was created by replacing the ground pixel with a ground pixel a distance of either 0, 1, or 2 pixels away. The stratification was determined using the original satellite pixels. For the tessellated paradigm the same registration error was added to each ground sample unit within a sampled PSU.

The parameters that controlled the registration error were the probability that a registration error occurred, and given that an error occurred, the probability that either a 1 pixel shift or a 2 pixel shift occurred. Once the number shift was determined, any pixel at that distance was equally likely (there are 8 or 16 pixels a distance 1 or 2, respectively, from a center pixel).

The simulation was implemented in FORTRAN. Additional details of the implementation and the computer program are available from the authors.

The 1,000 samples were drawn from a fixed map and a fixed set of registration error parameters. For the map, both the true area of forest and the variance of the estimators was calculated for both paradigms. For each sample, the estimator $\hat{A}_F$ and its sample variance were calculated for both paradigms. From $\hat{A}_F$ and its variance estimator, a two-sided 80% confidence interval for the percentage of forest was calculated using a t-distribution with degrees of freedom equal to 1 less than the number of sample points. From the 1,000 samples, an actual coverage rate was calculated. The difference between the sample estimates with and without registration error was quantified by calculating the ratio of the means of the 1,000 sample variances, which was expressed by

$$R_* = \frac{mean(\mathrm{var}[\hat{A}_F(\text{with registration error})])}{mean(\mathrm{var}[\hat{A}_F(\text{without registration error})])}$$

where R_t and R_p denote using the tessellated and point estimator, respectively. An advantage of this metric is that R_* gives an estimate of the increase in n needed to reduce the variance of an estimator when registration errors are ignored. Thus, it also gives a direct estimate of the cost of registration errors. It should be noted that for the tessellated paradigm, increasing n does not reduce the within-cluster variance. Thus, R_t is only a rough approximation of the increase in n needed to correct for the increase in the variance due to registration errors.

Several populations were created that represented a wide range of patterns. The simulation attempts to imitate the wide variety of landscape patterns that might be encountered by a program such as FIA. Proving that any given simulated landscape mimics reality is next to impossible, but we contend that the simulated landscapes are similar enough that running simulation over a varied set of input parameters would produce patterns of problems that arise from registration errors in "reality." Over all the sets of input parameters, the same pattern(s) occurred. In the Results section, these patterns are summarized, and the results of two of the simulations are presented in detail. For several sets of input parameters, a parallel simulation was run with the only difference in the input parameters being that no indeterminate pixels were created. The results from these simulations are also summarized in the Results section.

The common factor of the simulations within a set was that the parameters used to generate the map were held constant. For a set of map parameters, simulations were

run with differing amounts of registration error. For each set of registration error parameters, three to five 1,000-run simulations were conducted. Since the maps were generated using random processes, maps generated from the same parameters were not equal, thus several runs were completed to see how robust the results were. Three figures depicting the map pairs are given for each of the map parameter sets so the degree of fragmentation can be better understood. The first is the satellite map (Figures 1a and 2a). On these maps there are three types of pixels: forested, indeterminate and nonforested. The second is the ground map (Figures 1b and 2b), which contain only forested and nonforested pixels. The third is a map of the classification of the 3 × 3 pixel blocks (Figures 1c and 2c). Again, there are three types of pixels on these maps indicating the strata assignment of the PSUs for the tessellated estimator. The first two maps are three pixels larger on all sides than the third map to accommodate registration errors. The parameters used to create these sets of maps will be discussed in the next section.

Results

First we present the results that were common to all the simulation runs, then the details from two specific popula-

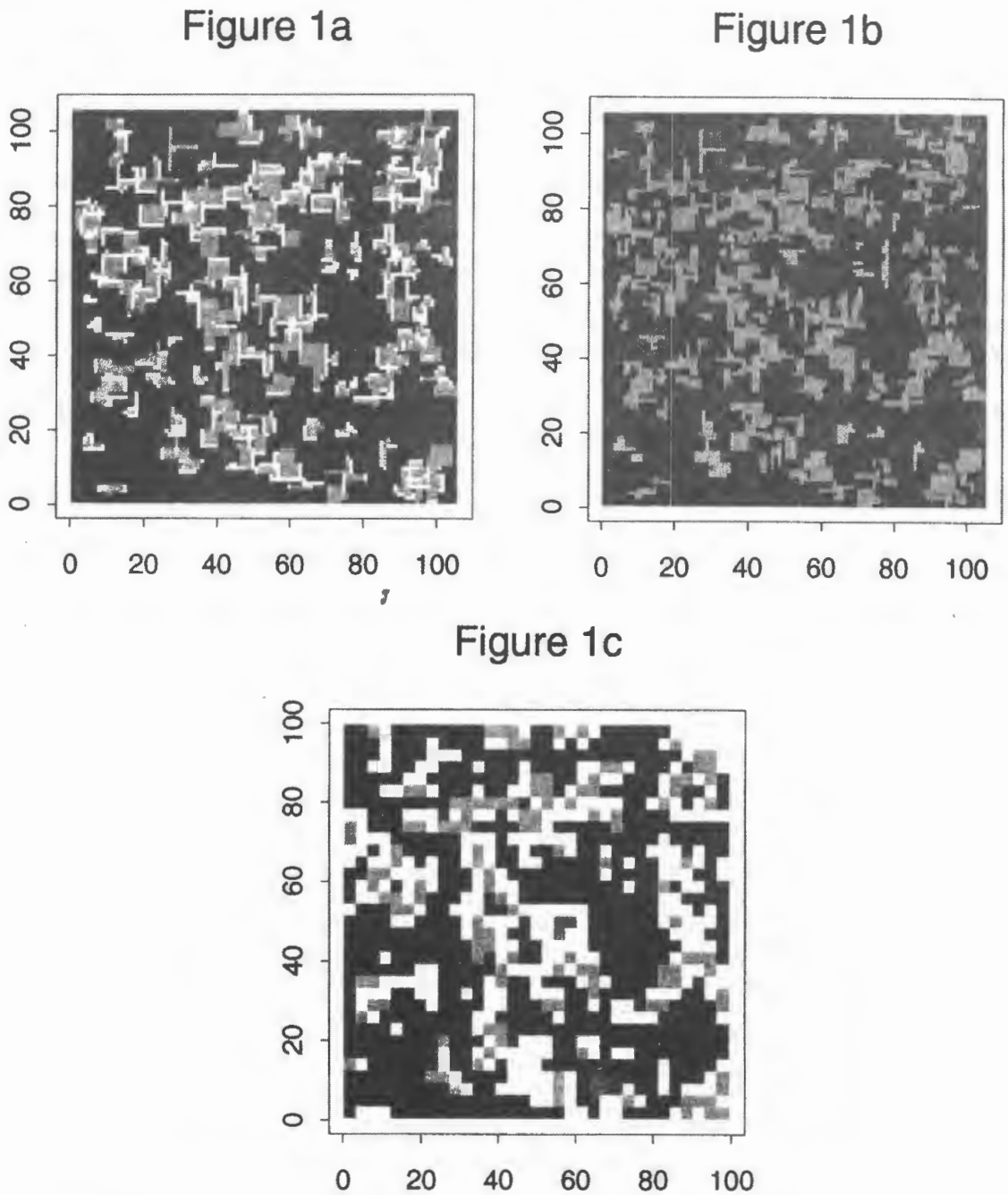

Figure 1. (a) Satellite map, (b) ground truth map, and (c) map of strata classification for 3 × 3 tessellation, for where the tessellated estimator has a smaller variance. There is a smaller degree of fragmentation than in Figure 2. Note the larger patches on nonforest in 1b than in Figure 2b. Forest is light gray, indeterminate is white, and nonforest is dark gray.

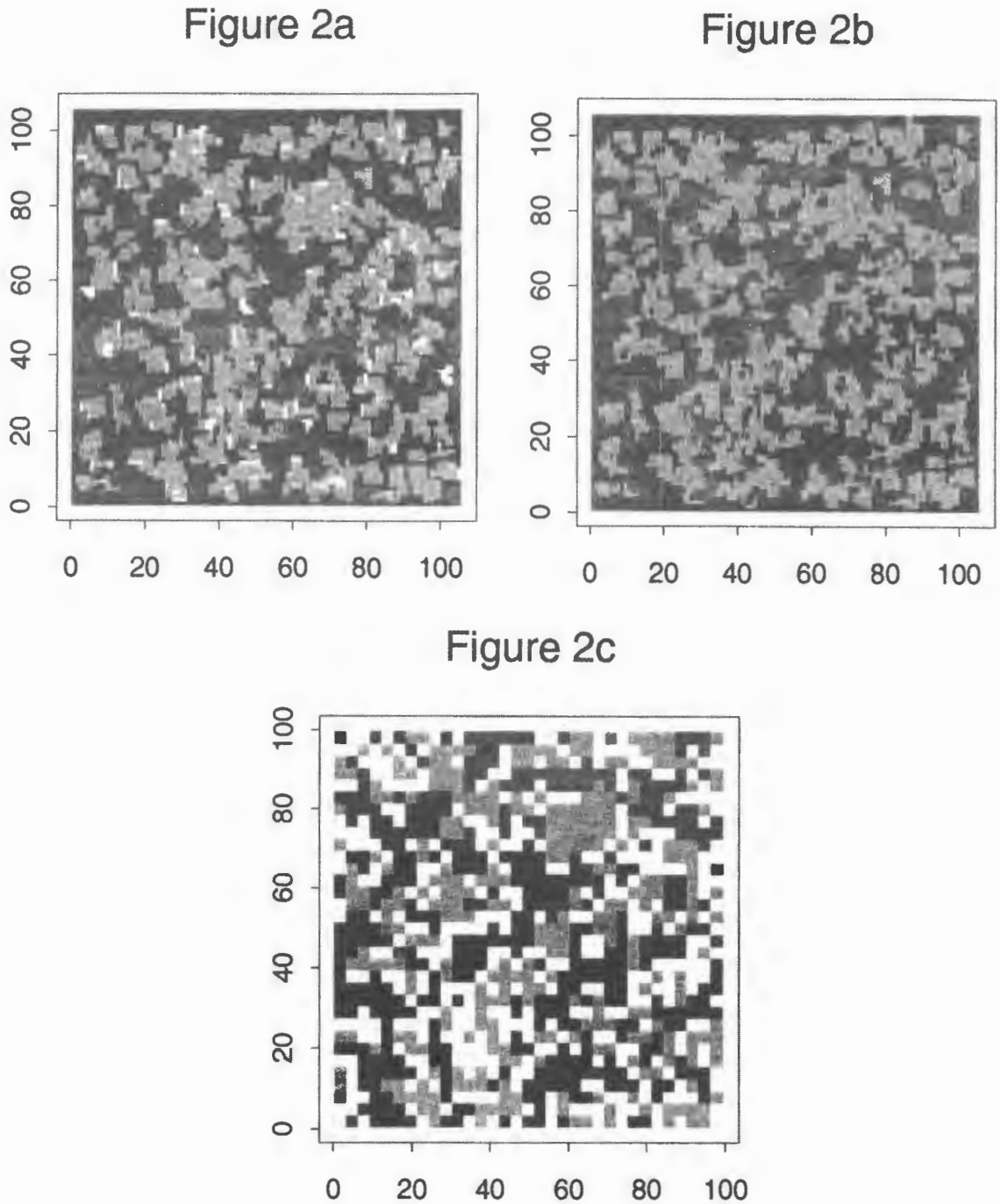

Figure 2. (a) Satellite map, (b) ground truth map, and (c) map of strata classification for 3 × 3 tessellation, for a set of parameters where the point estimator has a smaller variance. Note the small number of indeterminate pixels in the satellite map (a) and the high degree of fragmentation in the ground map (b). Forest is light gray, indeterminate is white, and nonforest is dark gray.

tions are given. The first population depicts a situation where the tessellated estimator had a smaller variance than the point estimator when no registration errors existed. The second population depicts a situation when the point estimator had the smaller variance.

In none of the simulations was there a detectable bias in the presence of registration errors for either the tessellated or point estimators of A_F. The most extreme case of registration error used a 50% chance of a sample point having a registration error. For this case, if a point had an error there was a 70% chance the location was shifted by 1 pixel and a 30% chance the point was shifted 2 pixels. Because there was no detectable bias associated with registration errors, the variances of the estimators are the primary concern.

When the number of indeterminate pixels was small, the classification accuracy was high in both the forested and nonforested map classes and the sample size was 70, there appears to be a substantial bias in the variance estimator under the point paradigm. This is manifest in the actual coverage rate for the confidence interval being less than 80%.

One reason for this is that when the accuracy of the image is very high, a significant percentage (as high as 7–10%) of the simulations had a variance estimate of zero (if all the pixels within a sample are correctly classified then the sample variance will be zero). The bias decreased as the sample size was increased. However, for the coverage rates to approach the nominal 80% value, the sample size had to be increased sometimes to as high as $n = 250$. One of the most significant findings was that for any amount of registration error the point estimator always had a greater percentage increase in the mean of the variance estimates than the tessellated estimator.

As pointed out in Williams and Patterson (2003), the point estimator can have a smaller variance than the tessellated estimator when the classification accuracy is high in the forested and nonforested strata, size of the indeterminate strata is small, and the forest area is highly fragmented. When there is a high degree of fragmentation, the mean of the sample variances for point estimator showed large increases in variance for even small amounts of registration error (as little as 5%). While the amount the variance increased due to registration errors also increased with an increase in fragmentation for the tessellated estimator, the increase was smaller.

With respect to the confidence intervals, the presence of registration errors always increased the actual coverage, but never by more than 4%. When the classification accuracy was high in both the forested and nonforested strata and the size of the indeterminate class was small, the actual coverage rate for the point paradigm was much less than 80%. This is probably caused by the presence of samples which have zero sample variance.

As mentioned in the simulation section, for several sets of input parameters, another simulation was run with the same input parameters, except the parameters for generating the indeterminate pixels were set to zero. For both estimators there was at least some increase in R when there were no indeterminate pixels. At all levels of registration error the amount of increase in R for the point estimator was greater than for the tessellated estimator.

Tessellated Estimator Had Smaller Variance

In this example, the degree of fragmentation was smaller than in the next example. This was accomplished by using a smaller number of patches of forest which are larger in size. The other main differences were that the number of indeterminate pixels, and the number of misclassified pixels was greater than in the next example. The number of indeterminate pixels was approximately 13%, with the number of 3 × 3 pixel blocks (PSUs) classified as indeterminate being 30% (Figures 1a–c). Approximately 21% of pixels were classified as forested, while the percentage of 3 × 3 blocks classified as forested was approximately 12%. The variance of the point estimator was approximately 50% larger than the variance of the tessellated estimator. For each pair of registration error probabilities, results for one of the five runs of 1,000 samples are presented (see Table 1). The results for the other runs were within two to three percentage points for the nominal coverage rates and within 0.005–0.01 for R_t and 0.03–0.04 for R_p. The actual coverage rates for the point estimator were slightly lower than the nominal rate. At all levels of registration error R_p was consistently greater than R_t.

Point Estimator Had Smaller Variance

In this example, there was a high degree of fragmentation, and only 3% of the pixels were classified as indeterminate (see Figures 2a and 2b). Forty-five percent of the pixels were forested and the remaining 52% nonforested. The high degree of fragmentation led to a high percentage (33%) of 3 × 3 pixel blocks being classified as indeterminate (see Figure 2c), while 29% of the 3 × 3 pixel blocks were classified as forested. For each pair of registration error probabilities, results for one of the five runs of 1,000 samples are presented (see Table 2). The results for the other runs were within two to three percentage points for the nominal coverage rates and within 0.02–0.03 for R_t and 0.04–0.05 for R_p. Note how liberal the confidence intervals were for the point estimator. This occurred because there were a large number of samples (7%–9%) where the point sample variance was zero. When there were no registration errors, the point estimator's variance was approximately 12% smaller than the variance of the tessellated estimator. However, when there was even a small percentage of registration errors (5% of samples), the mean of the sample variance for the tessellated estimator was consistently smaller than the mean of sample variance for the point estimator. Another point of interest is the conservative coverage of the tessellated estimator when there were registration errors.

Discussion and Conclusions

An important observation is that across all the simulated populations, in the presence of registration errors the vari-

Table 1. Statistics for simulation runs where the design-based estimator has a smaller variance when no registration errors are present. The first column is the probability a sampled unit has registration error. The second column is the probability that a unit with registration error has a 1 pixel shift. The third through sixth columns are the percentage of confidence intervals that contained the true percentage of forest. The last two columns contain the ratios defined at the end of the simulation study section.

		% actual coverage for an 80% nominal coverage					
		Tessellated		Point			
Probability sample unit is shifted	Probability shift is 1 pixel	Unshifted	Shifted	Unshifted	Shifted	R_t	R_p
5	100	80.1	81.0	77.8	77.5	1.02	1.06
10	100	80.7	81.1	77.2	78.4	1.04	1.13
20	100	79.3	80.7	76.9	78.5	1.07	1.25
50	100	79.6	83.3	75.8	76.2	1.20	1.61
50	70	79.6	83.8	75.8	76.6	1.29	1.70

Table 2. Statistics for simulation runs where the model-based estimator has a smaller variance when no registration errors are present. The first column is the probability a sampled unit has registration error. The second column is the probability that a unit with registration error has a 1 pixel shift. The third through sixth columns are the percentage of confidence intervals that contained the true percentage of forest. The last two columns contain the ratios defined at the end of the simulation study section.

		% actual coverage for an 80% nominal coverage					
		Tessellated		Point			
Probability sample unit is shifted	Probability shift is 1 pixel	Unshifted	Shifted	Unshifted	Shifted	R_t	R_p
5	100	80.4	80.7	71.7	73.2	1.04	1.29
10	100	81.6	82.0	70.2	74.5	1.07	1.56
20	100	80.6	83.0	71.2	74.2	1.17	2.22
50	100	78.2	82.6	71.4	77.9	1.36	3.68
50	70	80.0	86.2	71.1	76.9	1.54	4.34

ance of the tesselated estimator was smaller than the variance of the point estimator. Thus, any advantage the point paradigm may have is probably lost when registration errors are not accounted for. While it is possible that this pattern could be reversed in some situations by reducing the size of the second phase sample ($m = 4$ and $M = 9$ in this study), we still conclude that the tessellated estimator is generally more robust in the sense that it is not affected as severely in the presence of registration errors, even for small sample sizes. A somewhat surprising result was that the tessellated paradigm variance estimator performed reasonably well, although the effects of registration errors were not considered in the derivation.

Another important point is that there are two obvious methods of counteracting the increase in the variance of estimators due to registration error. One is to increase the number of ground samples, which is extremely costly. The other is to correct for the registration errors between the satellite and ground data. One way this has been accomplished is to use an aerial photo, with the ground plot pinpricked and overlaid on the satellite image (Befort and Mellin 1994). The center of the plot can then be moved to the correct pixel when there is an obvious registration error. This presents a rather humorous predicament because one of the primary selling points of using satellite imagery was that it would be less expensive than acquiring aerial photography. However, a careful cost-benefit analysis may show that the most effective method of reducing the variance could be to acquire aerial photography in addition to satellite imagery.

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