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Modeling Wildland Fire Containment with Uncertain Flame Length and Fireline Width

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Abstract. We describe a mathematical model for the probability that a fireline succeeds in containing a fire. The probability increases as the fireline width increases, and also as the fire's flame length decreases. More interestingly, uncertainties in width and flame length affect the computed containment probabilities, and can thus indirectly affect the optimum allocation of fire-fighting resources. Uncertainty about the fireline width that will be produced can often affect containment chances as much as uncertainty in flame length.

Keywords: Wildland fireline containment probability; uncertainty in flame length; uncertainty in resource productivity.

Introduction

One rarely knows for certain whether a given fireline[1] will succeed in containing a wildland fire. Even if parameters such as the flame length and the width of fireline that will be constructed are known precisely, we cannot hope to know more than the probability of containment under given conditions.

Traditionally, fire modeling (Bratten 1981; U.S. Department of Agriculture 1983; Anderson 1989; Albini et al. 1978) has tended to be deterministic, corresponding to the view that the flame length[2], fireline width produced, and other parameters should be known, and that, given these, one should know for sure whether or not the fire will be contained. We suggest that it is more realistic and productive to begin by modeling the probability of containment, in terms of specified flame length and width produced. One can then proceed to take into account the uncertainty in our knowledge of these parameters, which by itself can substantially affect our probability of containment.

Uncertainty in flame length is implicitly taken into account, since the Fire Management Officer (FMO) has to demand a "target", or "planned" fireline width that is a multiple of the (expected) flame length. Thus uncertainty in flame length is transferred to the targeted width desired. The multiplying factor employed by the FMO is not restricted, and in some instances may be extreme.

To illustrate how this might work, suppose a fire manager is considering how much of the available resources to allocate to a given segment of a fireline. The manager believes that the flame length there will be five feet. This segment is judged to be an important one to hold, and the decision is to plan enough resources to construct a fireline ten feet wide in order to provide a high confidence that the line will be sufficient (containment probability = 99%, let us say). But suppose now that we take into account that the five and ten feet figures are only estimates, either of which may be in error by 40% or more (we will discuss the modeling of these issues shortly). In that case there is a reasonable chance that the actual flame length could, for example, actually exceed the width built, in which case containment prospects are pretty dim. This is, of course, only a possibility, but it serves to reduce the overall containment probability substantially below the original 99%. If it is highly important to hold this fireline segment, with this knowledge the manager may feel impelled to allocate more resources to this segment than the original decision — perhaps to produce an estimated 14 feet width instead of the original 10, even at the cost of a

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[1] A cleared or treated strip used to control a fire's spread; more specifically, that portion of a control line from which flammable materials have been removed (McPherson et al., 1990).

[2] The flame length value used here is discussed by Albini (1976), and is a function of the intensity of the flaming front of a spreading fire. It is used by fire management personnel in estimating the difficulty of controlling a wildfire, including planned fireline width.

lower containment probability at other segments of the fireline. Thus different assumptions about the uncertainty of key fire parameters can lead to differences in the optimum allocation of resources[3].

Mees and Strauss (1992) developed a mathematical model for the optimal allocation of resources (given numbers of ground crews, engine, bulldozers and airdrops) among the segments[4] of a fireline. The approach is to ascribe a utility to the holding of each segment, and model the probability of holding the segment — given uncertainty in fireline width produced — according to the allocation it receives. Traditional operations research methods are then used to determine which allocation maximizes the total expected utility. The present paper, by contrast, focusses on a single segment; we extend the model to incorporate uncertainty in the flame length, and examine the effect of uncertainty both in flame length and in width produced on the containment probabilities. We shall see that while both effects can be substantial, the effect of uncertainty in width produced is as critical to containment chances as that of uncertainty in flame length. This last seems noteworthy, since the very existence of uncertainty in width produced has received scant attention in previous work (Phillips et al.1988; Smith 1986).

The uncertainty in fire behavior applies to both the rate at which the fire may spread and the intensity (flame length) with which it may burn. The uncertainty in the productivity of forces may manifest itself in the length or the width of the fireline that can be constructed in the time available, or both. The analysis presented here is limited to demonstrating the effect of uncertainties in flame length and constructed line width, and assumes that the required line length will be completed prior to the arrival of the fire front at the planned line location, i.e., the flame length and constructed fireline width are independent.

[3] Note that we are not merely saying here that "if the flame length or the constructed width are different from what was expected then the containment chances are affected." This is true, but hardly worth dwelling on. Our point here is rather that simply by acknowledging the uncertainty in these parameters we affect our containment probabilities, and this in turn may well affect our optimal allocation of resources among the fireline segments.

[4] The fireline around a large wildfire can be partitioned into a number of discrete segments represented by rectangles (Bratten, 1970). The planned width of each segment is based on the fire manager's estimate of the width that is needed to hold that particular line (at some implicit level of risk) under the fire intensity conditions forecast. This decision may be affected by factors such as the number and kind of forces that are available and appropriate for use in constructing each line segment.

The Model

We develop a simple model for the containment probabilities, with three components: (a) the probability of holding p(x,m), given a specified width built x, and flame length m; (b), the probability density function f(x) for the random width built, x; and (c), the probability density function g(m) for the random flame length m. Given these, the overall containment probability is

$$P_c = \int_0^\infty \int_0^\infty p(x,m)\, f(x)\, g(m)\, dx\, dm \qquad (1)$$

We now consider the three components of equation (1). In the absence of direct empirical data, we have based our choice of functional forms and parameter values on judgments and suggestions made by wildland fire personnel. (a) The holding probability, given a constructed width x and flame length m, is taken to be

$$p(x,m) = \begin{cases} 0 & \text{if } 0 \le x = < hm, \\ 1-\exp\{(x-hm)*\log(.15)/(Tm-hm)\} & \text{if } x > hm. \end{cases} \qquad (2)$$

The multiple, h (usually less than one), of the flame length is selected such that the chance of holding is zero until the constructed width x exceeds hm. The target width multiplier, T (T is usually greater than 1), is selected such that the probability of holding p(x,m) = .85 when the width constructed equals the target width (Tm) for all values of m. Both T and h are subjectively selected by the user; it is intended that they depend on the expected value of m (average = E(m)). We have adopted the following suggested values for h:

$h = 0$	if $E(m) \le .61$ meters,
$h = (E(m) - .61)/E(m)$	if $.61 < E(m) < 2.44$ meters,
$h = .75$	if $E(m) \ge 2.44$ meters.

Figure 1 illustrates the holding probability function when the target fire line width equals the flame length (T = 1) and E(m) = m = .61 (h = 0.0), 1.22 (h = .5), and 2.13 (h = .71) meters.

(b) The produced width variable is taken to have the gamma distribution (Hogg and Craig 1978, p. 104)

$$f(x) = e^{-x/\beta}\, x^{\alpha-1} \,/\, \beta^\alpha \Gamma(\alpha) \qquad (3)$$

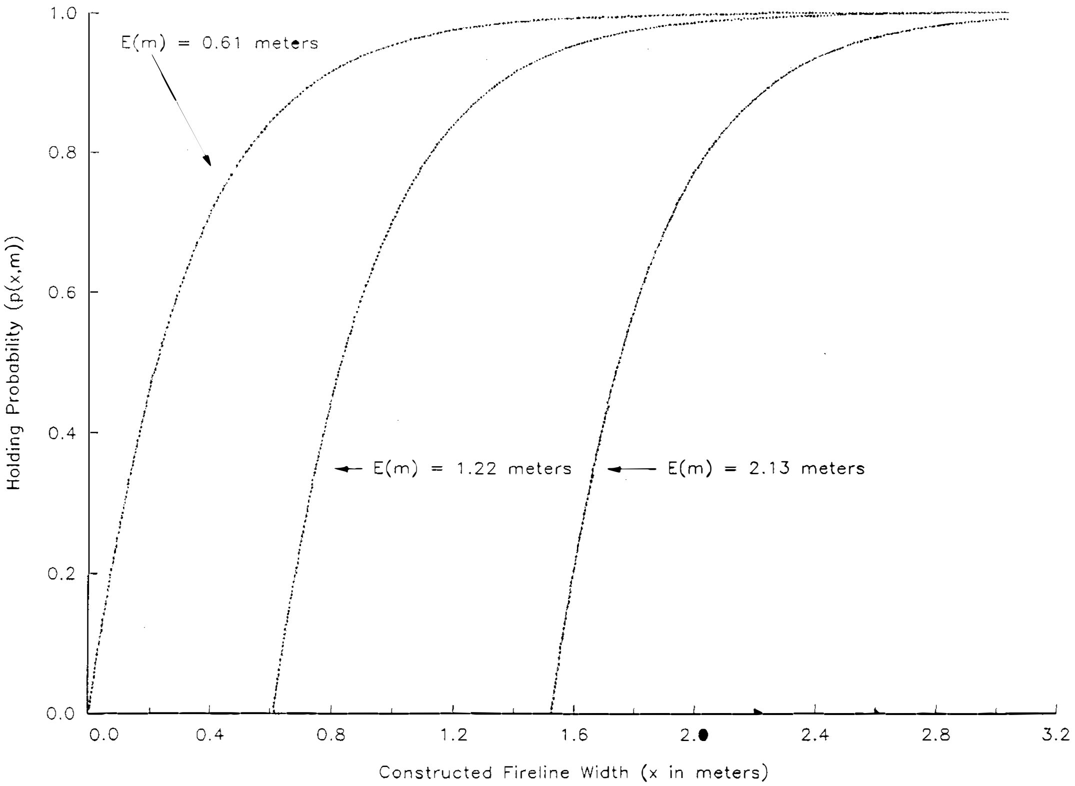

Figure 1. Plotted are the probaility of holding (p(x,m)) for average flame lengths of .61, 1.22, and 2.13 meters as a function of the fireline width constructed (x).

where $\Gamma(\alpha)$ denotes the (complete) *gamma function*:

$$\Gamma(\alpha) = \int_0^\infty e^{-y} y^{\alpha-1} dy.$$

The gamma is probably the most widely used model for a positive-valued random variable on account of its flexibility and convenience. The average and standard deviation of the distribution, $\alpha\beta$ and $\beta\sqrt{\alpha}$ respectively, are set equal to the estimated produced width and its standard deviation (uncertainty). These last two quantities are taken as direct inputs in our work here;[5] given these, the parameters α and β of (3) are easily computed. Figure 2(a) illustrates the probability distribution of the width x for various expected widths when the standard deviation is equal to .4 of the expected width. Figure 2(b) illustrates the gamma density function for fireline width constructed given an expected width of 1.22 meters and various standard deviations.

(c) The flame length random variable m is taken to be uniformly distributed on a certain range (m_0, m_1). That is,

$$g(m) = 1/(m_1 - m_0) \quad \text{if } m_0 \le m \le m_1,$$
$$= 0 \quad \text{otherwise.}$$

We chose the uniform density because field personnel tended to characterize their uncertainty in this form.

The midpoint of the range (i.e., $(m_1+m_0)/2$) is the expected average flame length, while the length of the interval reflects the degree of uncertainty. For our purposes here, the length of the interval is calculated from the standard deviation of m, the latter being a direct input of the model[6].

[5] In practice the estimated area is known. (Lindquist, 1970; Phillips et al., 1988; U.S. Department of Agriculture, 1983). The standard deviation, which has received but little attention in the literature, depends heavily on the fuel type, duration of work, and other factors. Mees and Strauss (1991), using data from Lindquist (1970), suggest an overall typical standard deviation of 40% of the target area.

[6] The relationship of the length of the interval and the standard deviation is given by the well-known formula

$$(m_1 - m_0)^2 = 12 \text{ x (standard devation)}^2.$$

Thus by specifying the expected flame length and the standard deviation (as a percentage of the average), we determine the parameters m_0 and m_1.

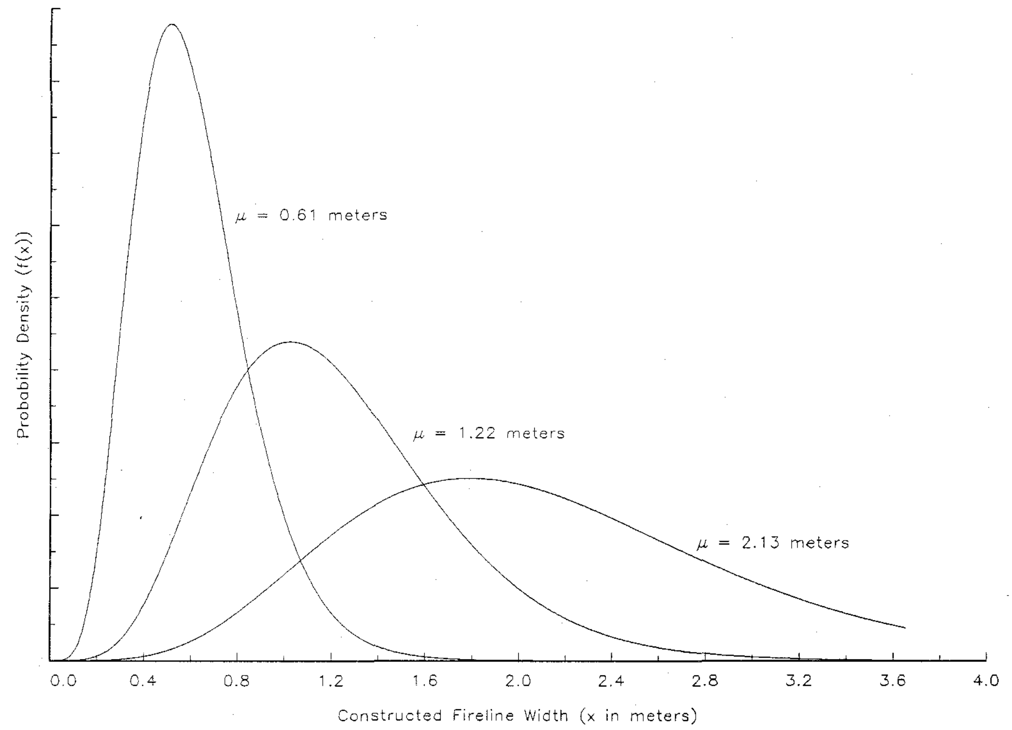

Figure 2(a). The probability density function for the construction of fireline width for 3 expected widths (μ) and standard deviation equals 40% of the expected width. The ordinate is scaled so that the area under the curve is equal to one.

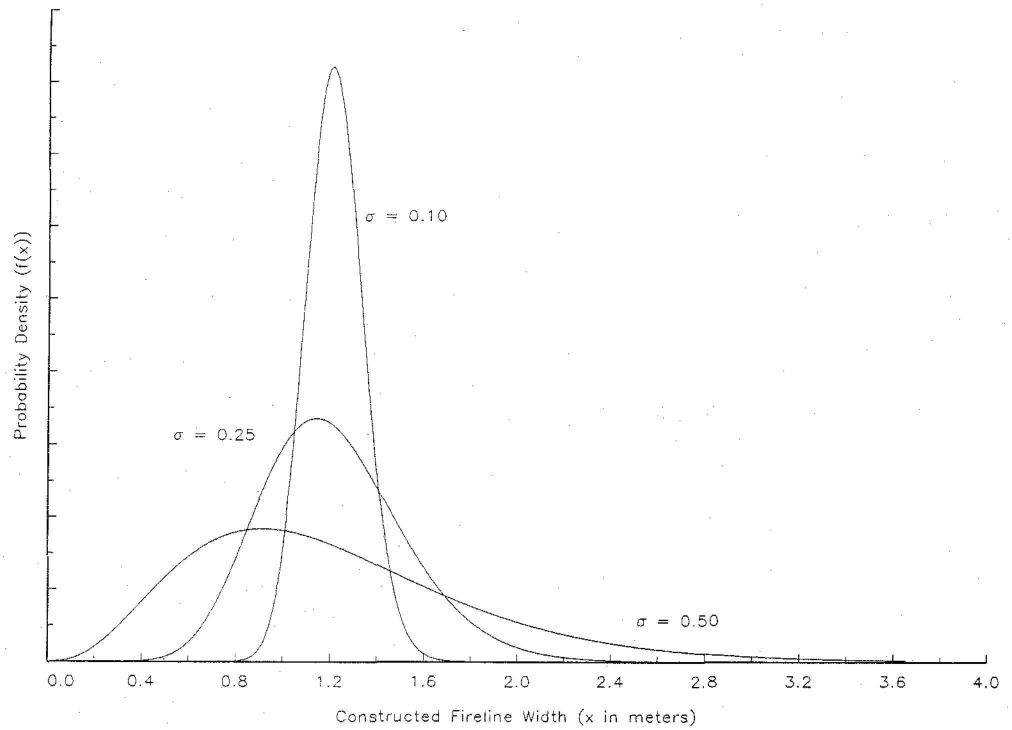

Figure 2(b). The probability density function for the construction of an expected fireline width of 1.22 meters and 3 standard deviations ($\sigma = .1*1.22, .25*1.22$, and $.5*1.22$ meters). The ordinate is scaled so that the area under the curve is equal to one.

We used numerical quadrature to evaluate the probability of containment P_c (Burden et al.1981, p.172).

We remind the reader that the parameters T and h and functional forms used here are subjective selections based on professional judgement. There are no available empirical data to justify the above parameters or forms.

Results

Figure 3 shows results for a fireline segment where the expected flame length is estimated to be 1.22 meters and the expected width produced is 1.22 meters. Since the target width equals the flame length and h = .5, i.e., the width constructed must exceed .61 meters before a non-zero contribution is made to the probability of containment.

The vertical axis is the probability of *not* containing the fire, for various values of the uncertainties. The vertex (0,0), corresponding to certainty about both the flame length and produced width, has a failure probability of 15%. This rises to 36% at the point (50,50), i.e., where both parameters have a standard deviation equal to 50% of their estimated average values. In the case of no uncertainty in production, the chance that the fireline does not hold as flame length uncertainty increases from 0 to 50% of the average increases from 15 to 23%. This occurs because there is now a reasonable chance that the actual flame length is about equal or even greater than the width actually constructed. What is especially noteworthy in the figure is that uncertainty in constructed width alone, with no uncertainty in flame length at all, can increase the failure probability to 35%. In this example, uncertainty in width built affects the containment chances more than uncertainty in flame length.

In Figure 4 the average flame length is .61 meters, and the average width constructed is now .61 meters. As expected, the failure probabilities are systematically smaller than before, ranging from 15 in the certainty case to 21% when both parameters have a standard deviation of 50% of their estimated values. Once again, uncertainty in flame length has only a slight effect on the probabilities (the probabilities of failure have been rounded), while the effect of production width uncertainty is more pronounced.

Finally, Figure 5 shows a situation where the average width built of 2.13 meters is matched against an average flame length of the same size. The (0,0), or

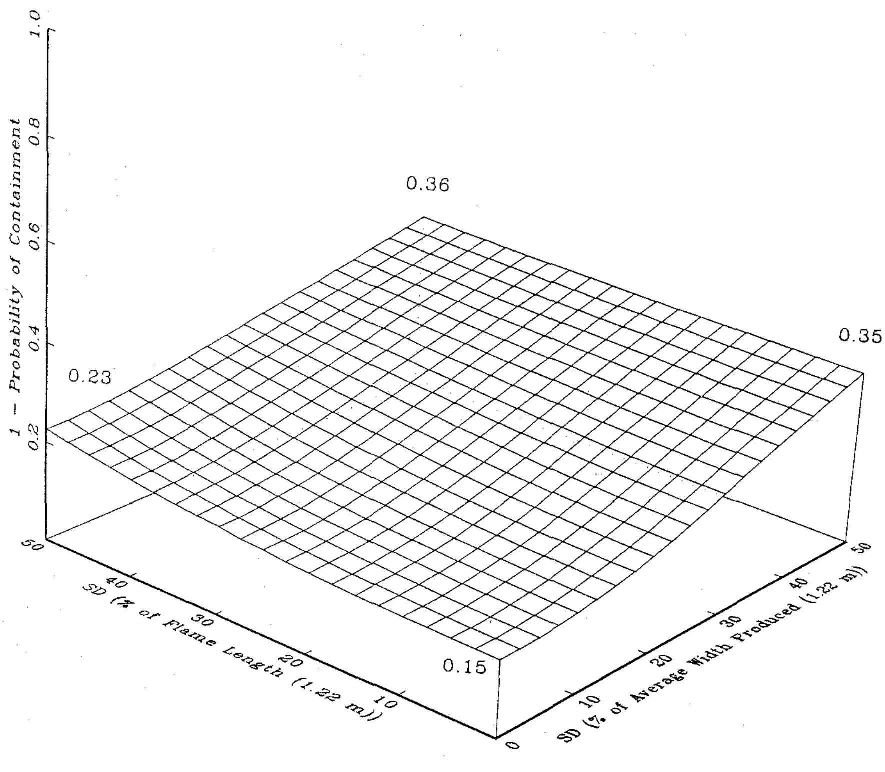

Figure 3. Plotted is (1.0 - Probability of containment) for a 1.22 meters expected flame length and 1.22 meters expected width produced. The standard deviations are expressed as a percentage of the expected flame length and width produced.

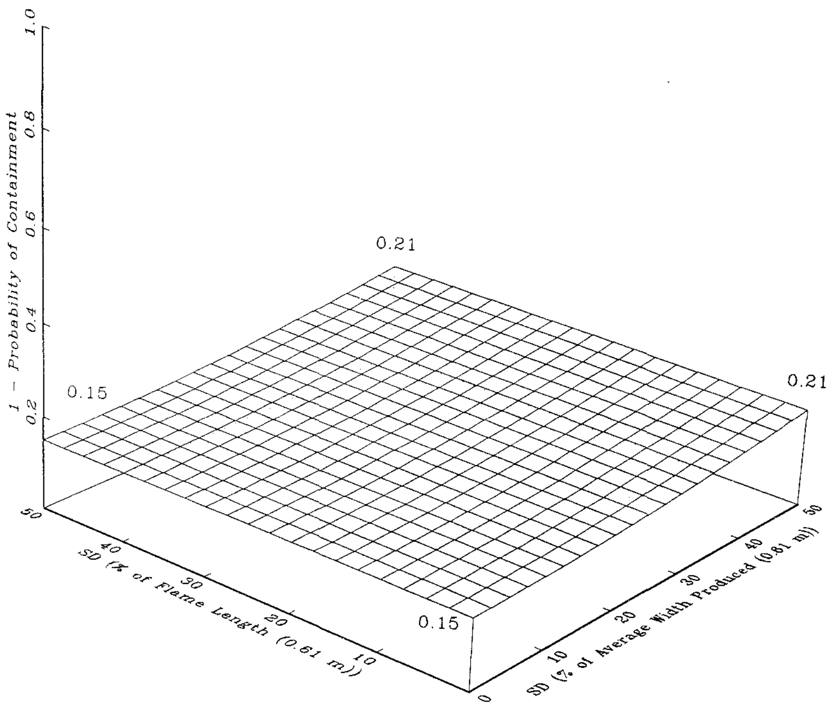

Figure 4. Plotted is (1.0-Probability of containment) for a .61 meters expected flame length and .61 meters expected width produced. The standard deviations are expressed as a percentage of the expected flame length and fireline width produced.

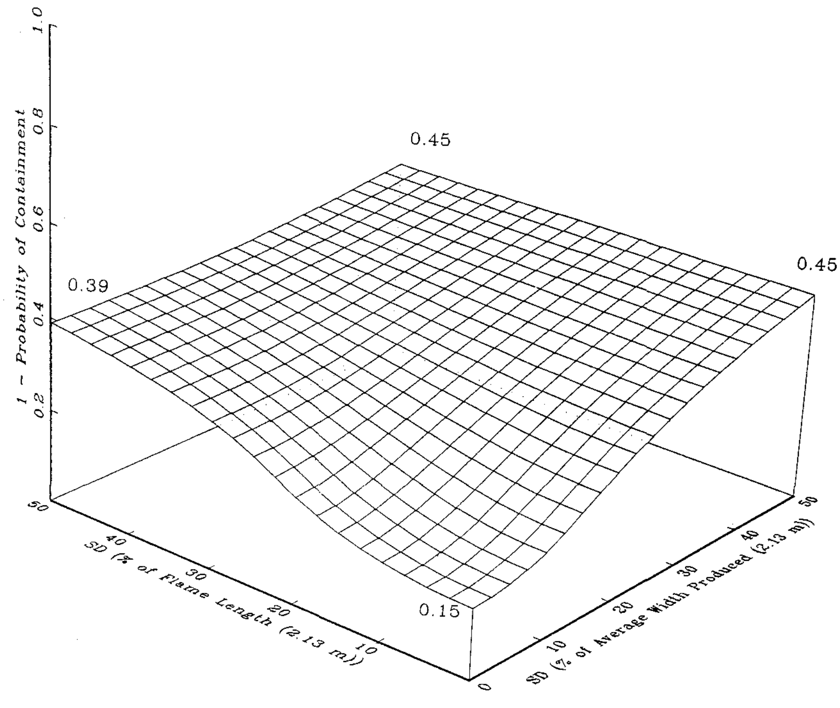

Figure 5. Plotted is (1.0 -Probability of containment) for a 2.13 meters expected flame length and 2.13 meters expected width produced. The standard deviations are expressed as a percentage of the expected flame length and width produced.

"certainty", case gives a failure probability of 15%, as expected. The failure probabilities are at a higher level, ranging from 15 to 45%. Again, the uncertainty in production dominates the uncertainty in flame length. It would, of course, be possible to model the flame length uncertainty with a gamma distribution instead. For comparison we carried out the gamma-based calculations of the containment probability for a number of cases; as expected, the choice of shape of density function made no real difference, at least when the standard deviations were not exceptionally large (less than 50% of the average). For standard deviations in excess of 50% of the average, the choice could matter; the standard deviation of the gamma can be arbitrarily large, whereas for the uniform model there is an upper bound, corresponding to the extreme case where $m_0 = 0$.

The model can also be run with unequal average flame length and fireline width produced. We do not present results for this case, however, because there is then no clear interpretation of " equal uncertainty" for the two factors, and thus no basis for determining which uncertainty has the greater effect on containment probabilities.

Conclusions

Flame length and production uncertainty are largely unaccounted for in currently available operational decision support tools. Uncertainty in these parameters significantly affects the confidence that can (or should) be placed on expectations that any planned fireline will be able to stop the fire's spread. An understanding of the nature and magnitude of that uncertainty is an essential input for the optimal allocation of firefighting resources.

The results of our model also suggest that uncertainty about the productivity of fire fighting resources (as represented in this analysis by width of line produced) can be as critical to the probability of containment as uncertainty about the estimated fire behavior used to determine the planned line width. While considerable research continues to address the understanding and improvement of fire behavior estimates, almost no attention has been given to investigating the variability inherent in fireline production rates for different elements of the suppression force. If significant improvement is to be made in the ability of fire managers to plan effective and efficient fire suppression action, future research must address both parameters.

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