

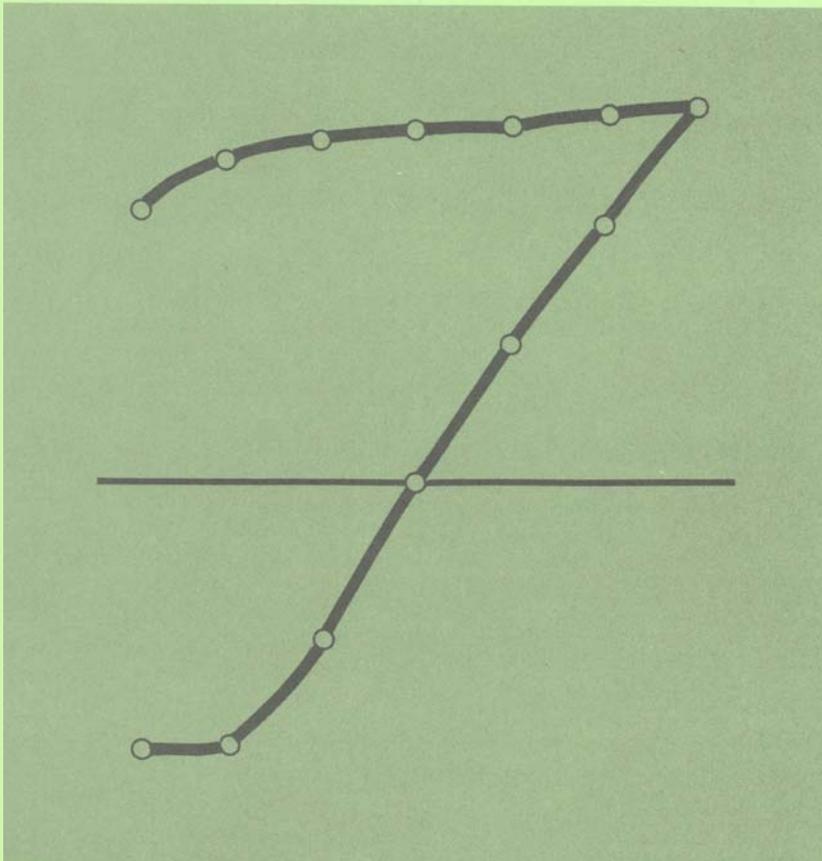
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ONE-SIDED TRUNCATED SEQUENTIAL t-TEST: application to natural resource sampling

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GLOSSARY

H₀:	null or test hypothesis
H₁:	alternative hypothesis
α:	level of significance (probability of Type I error)
δ:	non-centrality parameter
β:	probability of Type II error, at a specified value of δ
n₀:	truncation point
ADS:	average decision stage
ASN:	average sample number
BSTT:	Barnard's open one-sided sequential t-test
CALPRO:	computer program which calculates the decision probabilities for TSTT
OC:	operating characteristic (1-power)
OCASN:	computer program which calculates the OC and ASN functions of TSTT
power:	probability of rejecting H ₀ . For a given test, power is expressed as a function of δ
RAND:	fixed sample size t-test
SN:	sample number
SPRT:	sequential probability ratio test
STTEST:	computer program which approximates the decision boundaries for TSTT
TSTT:	one-sided truncated sequential t-test

SUMMARY

Fowler, Gary W., and William G. O'Regan

1974. **One-sided truncated sequential t-test: application to natural resource sampling.** USDA Forest Serv. Res. Paper PSW-100, 17 p., illus. Pacific Southwest Forest and Range Exp. Stn., Berkeley, Calif.

Oxford: 524.63-0.15.5.

Retrieval Terms: biometrics; sampling; sequential sampling; t-test.

The high degree of approximation in current sequential tests, the emphasis on two-sided tests, and the unpleasant possibility of large sample sizes have led us to develop a new procedure for constructing one-sided truncated sequential t-tests of the hypothesis

$$H_0: E(X) = \mu = \mu_0$$

$$H_1: \mu > \mu_0$$

$$X \sim \text{NIID}(\mu, \sigma^2)$$

$$\sigma^2 \text{ unknown}$$

Current tests specify α and β in advance, with α being the probability of rejection when $\mu = \mu_0$ and β being the probability of acceptance when $\mu = \mu_0 + \delta\sigma$, and use maximum likelihood procedures weakened by various approximations to obtain decision boundaries.

The truncation point (n_0) is determined by α , β , and the approximation procedure and is not necessarily an integer. The actual values of α and β for such tests when estimated by Monte Carlo procedures seem to be distinctly different from the nominal values. Little is known about OC and ASN functions of these tests except for some empirical studies, due to the complexity of the mathematics involved.

Thinking that it might be more practical to specify n_0 in advance rather than β , we have developed a one-sided truncated sequential Nest based on a specified α , n_0 , and probability boundary pattern. The probability boundary pattern sets the probabilities of accepting and rejecting H_0 (H_0 being true) at each stage of the test such that the overall probability of rejecting H_0 (H_0 being true) is equal to α . An algorithm was constructed to determine these acceptance and rejection probabilities at each stage of the test for a specified α , n_0 , and probability boundary pattern. Three intuitively appealing boundary patterns were considered.

Monte Carlo procedures based on a pseudo-normal deviate generator were used to approximate the

distributions of the conditional test statistic at each stage of the test. Acceptance and rejection points associated with the acceptance and rejection probabilities at each stage of the test were then determined to obtain the approximate decision boundaries of a specific one-sided truncated sequential t-test. For a given value of α and n_0 , the power function of the test depends upon the probability boundary pattern.

The observed value of α when estimated by Monte Carlo procedures is quite close to the nominal value.

Five examples are presented. Each has α set at 0.05. Three have $n_0 = 10$, each with a different probability boundary pattern. One boundary pattern has two additional examples with $n_0 = 4$ and $n_0 = 7$.

The operating characteristic and average sample number functions were approximated for each of the tests. Monte Carlo procedures based on a pseudo-normal deviate generator were used to simulate sampling normal distributions 1000 times with each test for values of the non-centrality parameter (δ) varying from 0.0 to 4.0 with an interval of 0.5. These points give an adequate description of each function. Monte Carlo estimates of α were very close to the nominal α (0.05) and varied from 0.046 to 0.051.

The properties of a specific test are compared with the fixed sample size t-test and Barnard's one-sided sequential t-test having approximately the same reliability. The one-sided truncated sequential t-test yielded an estimated α and associated ASN (average sample number) of 0.047 and 5.185 (based on 20,000 samplings of the null distribution) with the nominal values being 0.050 and 5.180, respectively. This test yielded a Monte Carlo estimate of $\beta = 0.057$ at $\delta = 1.5$ (based on 1000 samplings of a normal distribution with $\delta = 1.5$). The fixed sample size test yielded an estimated α and β at $\delta = 1.5$ of 0.048 and 0.041 with the nominal values being 0.050 and 0.039, respectively. Barnard's test yielded an estimated α and β at $\delta = 1.5$ of 0.032 and 0.016, with the nominal values both being 0.050. The ASN function of the fixed sample size test is uniformly larger than that of the

new test while the ASN function of the new test is uniformly lower than that of Barnard's test for $\delta > 0.3$ with the reverse being true for other values of δ . Given the absolute guarantee of a limiting sample size, the new test compares quite favorably with the other two tests.

Since there have been no applications of sequential t-tests to natural resources sampling to date, examples

of possible natural resources applications are considered and field procedures are outlined for the new test. The original test statistic is transformed to one that is easier to calculate in the field, and the possibility of a series of observations having the same value is considered. Recommendations are given to the potential user. The upper one-sided test presented can easily be modified to a lower one-sided test.

The researcher in natural resources is commonly faced with the problem of sampling populations that have unknown underlying distributions with the variance almost always being unknown. One- and two-sided open sequential t-procedures have been developed by Barnard (1952), Rushton (1950, 1952), and Wald (1947), and can be used for situations in which the variance is unknown. But these tests possess the major weakness of all open sequential procedures – the possible occurrence of very large sample sizes. To date, we know of no applications of these procedures to the field of natural resources.

Schneiderman and Armitage (1962b) and Meyers, Schneiderman, and Armitage (1966) developed two forms of two-sided closed sequential t-tests. Suich and Iglewicz (1970) and Alexander and Suich (1973) proposed a truncated sequential t-test for the one- and two-sided case based on the method of Anderson (1960). These truncated procedures have been developed for nominal α (level of significance or probability of Type I error) and β (probability of a Type II error) at a specified value of δ , without fixing the truncation point of the test in advance. Because the truncation point is a function of α , β , and one or more mathematical approximations, it is not necessarily an integer.

In sequential hypothesis testing, the test procedures terminate according to some stopping rule related to the sequence of observations. These tests usually require, on the average, fewer observations than do equally reliable tests based on fixed sample size procedures. A major portion of the literature and application of sequential hypothesis tests is related to Wald's Sequential Probability Ratio Test (SPRT). Early applications of this test in the biological sciences were reported by Oakland (1950) and Morgan, *et al.* (1951). Such applications assume prior knowledge of the underlying distribution and related non-test parameters and are open in that they have parallel decision boundaries.

Applications of the SPRT in natural resources, and specifically forestry, can be found related to reproduction surveys and insect or disease control programs for the binomial, negative binomial, normal and Poisson distributions. Tests based on these

distributions have been reported for classifying the following populations:

	<u>Population</u>
<i>Distribution:</i>	
Binomial:	Larch sawfly (Ives 1954; Ives and Prentice 1964)
Negative binomial:	Spruce budworm (Morris 1954; Waters 1955; Cole 1960) Forest tent caterpillar (Connola, Waters, and Smith 1957) Red-pine sawfly (Connola, Waters and Nason 1959) Engelmann spruce beetle (Knight 1960a) Black Hills beetle (Knight 1960b; Knight 1967) Cone and seed insects (Kozak 1964) White grubs (Ives and Warren 1965) Gooseberry or currant bushes or both (Offord 1966) Jack pine sawfly (Tostowaryk and McLeod 1972) Lygus bugs (Sevacherian and Stern 1972)
Normal:	Lodgepole needle miner (Stark 1952; Stark and Stevens 1962)
Poisson:	Winter moth (Reeks 1956) Spruce budworm (Cole 1960)

Smith and Ker (1958) described the application of a test based on the Poisson distribution in reproduction surveys.

Approximations to the operating characteristic (OC) and the average sample number (ASN) functions have been developed for the SPRT (Wald 1947). And while little is known about the OC and ASN functions of the sequential t-tests, Monte Carlo simulation indicates that their actual α and β are usually quite divergent from the nominal α and β .

We were led to obtain boundaries for one-sided truncated sequential t-tests by Monte Carlo procedures because of these conditions: the high degree of approximation inherent in current sequential procedures, the undesirable possibility of large sample sizes, the emphasis on two-sided tests, and the dependence of the truncation point (n_0) on α and β . The decision boundaries of a specific test are constructed for a given α , n_0 , and specific probability

boundary pattern. The value of β at a particular alternative depends on the probability boundary pattern for given values of α and n_0 . We do not set a nominal value of β at a particular alternative. However, Monte Carlo approximations to the OC and ASN functions are presented.

This paper describes a procedure for constructing a

one-sided truncated sequential t-test, constructs a series of tests for a given α with different n_0 's and probability boundary patterns, compares a specific test with the fixed sample size t-test and Barnard's one-sided sequential t-test, considers possible applications of the test to natural resources, and outlines field procedures for the new test.

TEST PROCEDURES

The decision boundaries of a specific one-sided truncated sequential t-test are constructed with the prior knowledge of α , n_0 , and a specific pattern of acceptance and rejection probabilities (probability boundary pattern) that yield the given value of α . The OC and ASN functions of such a test are controlled by varying α , n_0 , and the probability boundary pattern.

In developing the test,¹ we consider the following problem:

Test

$$H_0: E(X) = \mu_0 \text{ or } \delta = \delta_0 = 0$$

against

$$H_1: E(X) > \mu_0 \text{ or } \delta = \delta_1 \text{ (} \delta_1 > 0 \text{)}$$

$$X \rightarrow \text{NIID } (\mu, \sigma^2)$$

in which $\delta = (\mu - \mu_0)/\sigma$ (non-centrality parameter), and σ is unknown.

We would like to choose

- d_s the decision statistic at stage s , $s = 1, \dots, S$
- d_s^r the rejection point in the distribution of d_s
- d_s^a the acceptance point in the distribution of d_s
- n_s the number of observations at stage s
- S the upper limit in the number of stages so that the OC and ASN functions of the test have specified qualities and such that

$$n_0 \geq \sum_{s=1}^S n_s$$

¹ In the process we had frequent and helpful reference to: Alexander and Suich (1973), Ailing (1966), Anderson (1960), Armitage (1957), Aroian (1968), Barnard (1952), Davies (1954), Ghosh (1970), Hall (1962), Jackson (1960), Johnson (1961), Meyers, Schneiderman, and Armitage (1966), Rushton (1950, 1952), Samuelson (1948), Schneiderman and Armitage (1962a, 1962b), Stockman and Armitage (1946), Suich and Iglewicz (1970), U. S. Department of Commerce (1951), Wald (1947), and Wetherill (1966).

with n_0 , the truncation point or maximum number of possible observations, being specified in advance.

We have been unable to obtain an analytical solution to this problem. However, we have obtained a test based on Monte Carlo procedures.

The test is developed as follows. We define

- α_s = probability of rejecting H_0 at stage s , given $\mu = \mu_0$
- γ_s = probability of accepting H_0 at stage s , given $\mu = \mu_0$
- α = overall probability of rejecting H_0 , given $\mu = \mu_0$

and set

$$\gamma_s = \frac{\gamma_s}{\alpha_s} \alpha_s \text{ with } \alpha_s < \alpha_s, \text{ for } s = 1, \dots, S - 1$$

Because the test is truncated at stage S , we know that

$$\gamma_S = 1 - \alpha_S \text{ and } \gamma_s = \frac{1 - \alpha_S}{\alpha_s} \alpha_s$$

We let

$$D_s = \alpha_s + \gamma_s = \frac{\alpha_s}{\alpha_s} \text{ probability of a terminating decision at stage } s$$

$$C_s = 1 - D_s = 1 - \frac{\alpha_s}{\alpha_s} = \frac{\alpha_s}{\alpha_s} \text{ probability of continuing at stage } s$$

$$P_s = \prod_{j=0}^{s-1} C_j \text{ probability of reaching stage } s$$

$$C_0 = 1$$

and note that

$$\alpha = \sum_{s=1}^S P_s \alpha_s$$

Since $\alpha_s = D_s \alpha_s$ from above,

$$\alpha = \sum_{s=1}^S P_s D_s \alpha_s = \alpha_s \sum_{s=1}^S P_s D_s$$

and, as the test is closed at S , we have

$$\sum_{s=1}^S P_s D_s = 1 \text{ and } \alpha = \alpha_s$$

Thus, any probability boundary pattern that satisfies the above criteria can be used to develop the exact acceptance and rejection probabilities at each stage for a test with a given α and n_0 .

However, given S and the probability boundary pattern, we have neither a decision statistic with known probability distribution nor an analytical procedure for setting n_s . We proceed arbitrarily along the following lines:

Set $n_1 = 2$ and $n_s = 1$ for $s = 2, 3, \dots, S$, and let

$$d_s = \frac{(\bar{x}_s - \mu_0)}{\sqrt{\sum_{j=1}^s \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_s)^2 / (\sum_{j=1}^s n_j)(\sum_{j=1}^s n_j - 1)}}$$

in which

$$\bar{x}_s = \frac{\sum_{j=1}^s \sum_{i=1}^{n_j} x_{ij}}{\sum_{j=1}^s n_j}$$

The statistic d_1 has a t-distribution with one degree of freedom, which allows us to set the points d_1^a and d_1^r where the probabilities of accepting and rejecting H_0 are

$$\gamma_1 = P(d_1 \leq d_1^a)$$

and

$$\alpha_1 = P(d_1 \geq d_1^r)$$

The statistic d_s ($s > 1$) has an unknown conditional distribution. The conditional probabilities of accepting and rejecting H_0 are

$$\gamma_s = P(d_s \leq d_s^a \mid d_1^a < d_1 < d_1^r, \dots, d_{s-1}^a < d_{s-1} < d_{s-1}^r)$$

and

$$\alpha_s = P(d_s \geq d_s^r \mid d_1^a < d_1 < d_1^r, \dots, d_{s-1}^a < d_{s-1} < d_{s-1}^r)$$

For the values of n_s considered, the statistic d_s reduces to the statistic d_n where

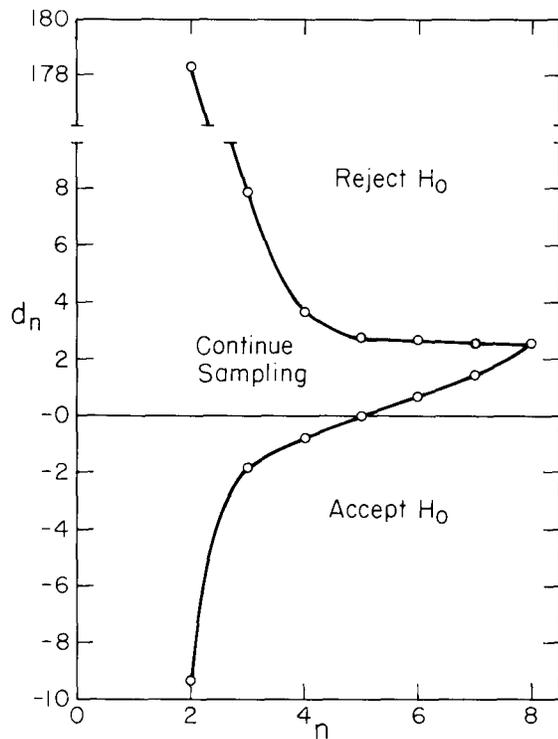
$$d_n = (\bar{x}_n - \mu_0) / \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 / (n)(n-1)}$$

and $\bar{x}_n = \sum_{i=1}^n x_i / n$, $n = s + 1$, and $n_0 = S + 1$

Interpretation of the acceptance and rejection probabilities using the statistic d_n can be facilitated graphically (*fig. 1*). Even though we have developed our test for $n_1=2$ and $n_s=1$ for $s > 1$, the procedure can easily be modified to handle the more general case where $n_s > 1$ for all s .

We have approximated the decision points d_n^a and d_n^r ($n > 2$) for several cases by Monte Carlo procedures.

Figure 1—Acceptance and rejection boundaries in terms of d_n for a Pattern 3 test with $S = 7$, $n_0 = 8$, $n_1 = 2$, $n_s = 1$ ($s > 1$), and $\alpha = 0.05$.



Examples of Proposed Test

In constructing examples of the proposed test, we utilized three probability boundary patterns. Each is intuitively appealing.

Pattern 1 sets $\alpha_s = \alpha \left[\frac{\sum_{i=1}^s (S+1-i)}{\sum_{i=1}^S i} \right]$ (probabilities increase at a constantly decreasing rate), Pattern 2 sets $\alpha_s = \alpha [s/S]$ (probabilities increase at a constant rate), and Pattern 3 sets $\alpha_s = \alpha^x \left[\frac{\sum_{i=1}^s i}{\sum_{i=1}^S i} \right]$ (probabilities increase at a constantly increasing rate).² In each case, $\gamma_s = \frac{1 - \alpha_s}{\alpha_s}$.

Five examples are presented. Each has α set at 0.05. Three have $n_0 = 10$, each with a different probability boundary pattern. One boundary pattern has two additional examples with $n_0=4$ and $n_0=7$. The decision points at each stage of a given test were obtained by approximating the unknown probability distribution of d_n with 1000 iterations of the statistic $d_n(n = 2, 3, \dots, n_0)$.

Computer program CALPRO³ calculates the

³ Fowler, G. W. *An investigation of some new sequential procedures for use in forest sampling*. 1969. (Unpublished Ph.D. thesis on file at University of California, Berkeley)

³ All computer programs are on file at the Pacific Southwest Forest and Range Experiment Station, Berkeley, California 94701.

acceptance and rejection probabilities for Patterns 1, 2, and 3 and for any combination of α and n_0 . Computer program STTEST approximates the decision boundary (acceptance and rejection points) for any test with α , n_0 , and probability boundary pattern satisfying the specified criteria.

The probability boundary patterns and decision boundaries of tests with $n_0 = 4, 7$, and 10 for Pattern 1 and for Patterns 1, 2, and 3 with $n_0 = 10$ are given in *tables 1* and *2*.

The OC and ASN functions of each test were approximated by sampling normal distributions 1000 times each for $\delta = 0.0(0.5)4.0$. Computer program OCASN approximates the OC and ASN points of a given test for any range of and interval between values of δ . The approximate OC and ASN functions for the tests described in *table 1* and *table 2* are given in *table 3* and *fig 2*.

It should be recalled that the proposed test is constructed given α , n_0 , and a probability boundary pattern—a particular value of β at a chosen value of δ is not specified in advance as in other sequential t-procedures. In other words, for n_0 given, the OC function depends on the probability boundary pattern.

Once the OC function of a given test has been obtained, a value of β can be approximated for any desired value of δ . If $\delta = 2.0$ represents a critical alternative in hypothesis testing, *table 3* will yield the approximate value of δ for a given n_0 and probability boundary pattern. By using a much more expanded set of tables, the researcher could choose that test

Table 1—Probabilities of acceptance (γ_n) and rejection (α_n) at all possible sample points (n) for boundary patterns 1, 2, and 3 and indicated truncation points (n_0), $\alpha = 0.05$.

n	P a t t e r n 1				P a t t e r n 2		P a t t e r n 3			
	n ₀									
	4		7		10		10		10	
	γ_n	α_n	γ_n	α_n	γ_n	α_n	γ_n	α_n	γ_n	α_n
2	0.475	0.025	0.271	0.014	0.190	0.010	0.106	0.006	0.021	0.001
3	.792	.042	.498	.026	.359	.019	.211	.011	.063	.003
4	.950	.050	.679	.036	.507	.027	.317	.017	.127	.007
5			.814	.043	.633	.033	.422	.022	.211	.011
6			.905	.048	.739	.039	.528	.028	.317	.017
7			.950	.050	.823	.043	.633	.033	.443	.023
8					.887	.047	.739	.039	.591	.031
9					.929	.049	.844	.044	.760	.040
10					.950	.050	.950	.050	.950	.050

Table 2 –Decision values (d_n^a, d_n^r) at all possible sample points (n) for bound-
ary patterns 1, 2, and 3 and indicated truncation points (n_0), $\alpha = 0.05$.

n	P a t t e r n 1					P a t t e r n 2		P a t t e r n 3		
	n_0									
	4		7		10		10		10	
	d_n^a	d_n^r	d_n^a	d_n^r	d_n^a	d_n^r	d_n^a	d_n^r	d_n^a	d_n^r
2	-0.079	12.706	-0.874	22.266	-1.472	31.820	-2.904	57.295	-15.057	286.506
3	1.705	4.289	.300	4.550	-.142	5.006	-.822	8.085	-2.410	16.416
4	4.243	4.243	1.414	3.194	.630	3.570	-.067	4.105	-1.048	5.164
5			2.556	3.391	1.470	3.259	.531	3.044	-.425	3.453
6			3.645	3.869	2.255	3.301	1.158	2.810	.041	2.883
7			4.681	4.681	3.108	3.510	1.743	2.707	.583	2.506
8					4.000	4.167	2.354	2.926	1.260	2.518
9					4.942	5.011	3.006	3.236	1.906	2.503
10					5.945	5.945	3.731	3.731	2.742	2.742

which was "best" in terms of α , n_0 , OC function, and ASN function for a given problem.

The Monte Carlo estimates of α ($\hat{\alpha}$) are close to the nominal α ; for, $0.046 \leq \hat{\alpha} \leq 0.051$ (table 3). The Monte Carlo estimates of β at $\delta = 2.0$ are 0.368 for $n_0=4$, 0.122 for $n_0=7$, and 0.052 for $n_0=10$ with Pattern 1 and 0.052 for Pattern 1, 0.008 for Pattern 2, and 0.000 for Pattern 3 with $n_0 = 10$.

The Monte Carlo estimates of α at $\delta = 0$ and β at the chosen value of δ for all other sequential procedures (Alexander and Suich 1973; Meyers, Schneiderman, and Armitage 1966; Schneiderman and Armitage 1962b; Suich and Iglewicz 1970; Wetherill 1966) are distinctly different from the respective nominal values due to the approximations involved in their development. Our research indicates

Table 3 –Monte Carlo approximations of the probability of acceptance (OC) and the average sample number (ASN) as a function of the non-centrality parameter (δ) for Pattern 1 with $n_0 = 4, 7, 10$, and for Patterns 2 and 3 with $n_0 = 10$, $\alpha = 0.05$.

δ	O C					A S N				
	P a t t e r n					P a t t e r n				
	1		2		3	1		2		3
	n_0					n_0				
	4	7	10	10	10	4	7	10	10	10
0.0	0.949	0.954	0.951	0.952	0.950	2.57	3.11	3.57	4.52	6.05
.5	.878	.829	.817	.767	.687	2.94	3.84	4.59	5.74	7.37
1.0	.730	.594	.539	.369	.217	3.21	4.20	5.08	5.94	6.84
1.5	.553	.325	.211	.077	.020	3.32	4.14	4.67	5.14	5.54
2.0	.368	.122	.052	.008	0	3.26	3.74	4.06	4.36	4.78
2.5	.203	.033	.007	0	0	3.12	3.40	3.58	3.94	4.42
3.0	.097	.008	0	0	0	2.97	3.16	3.29	3.68	4.19
3.5	.046	0	0	0	0	2.84	3.02	3.12	3.54	4.02
4.0	.018	0	0	0	0	2.74	2.94	3.01	3.44	3.88

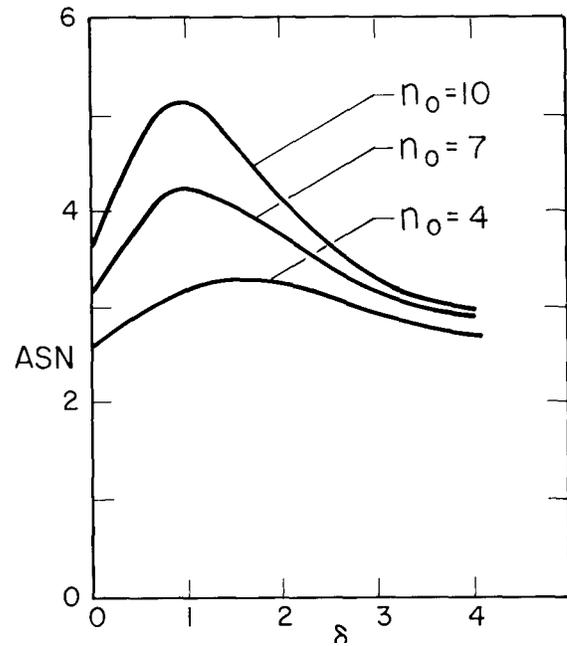
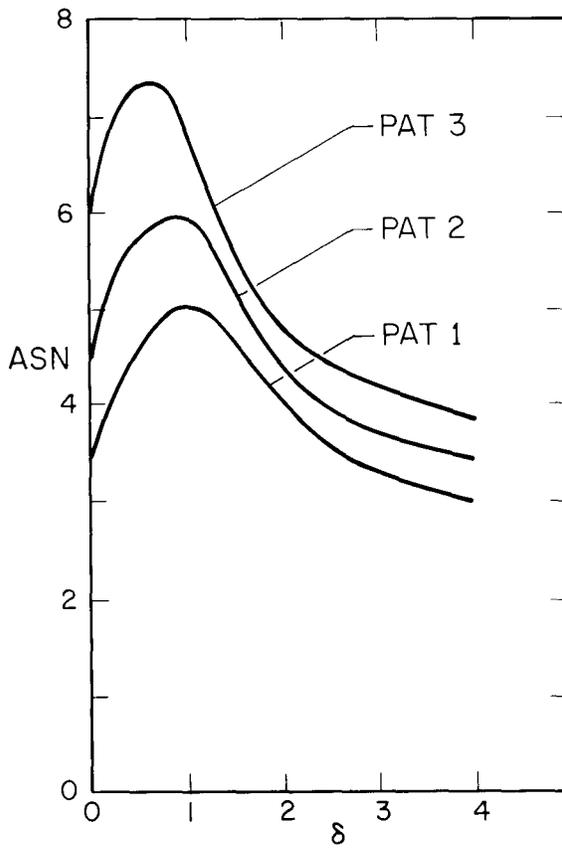
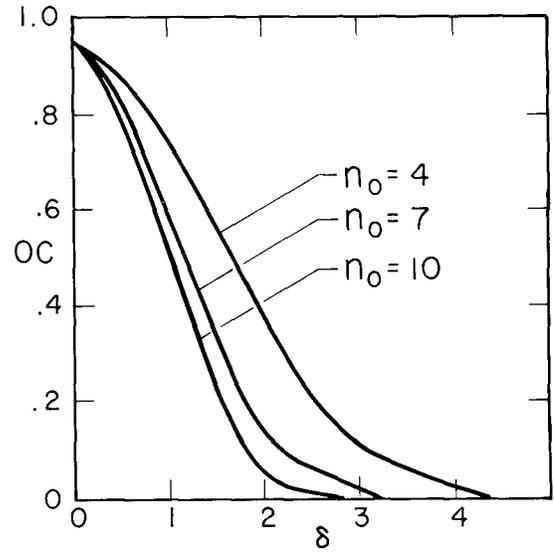
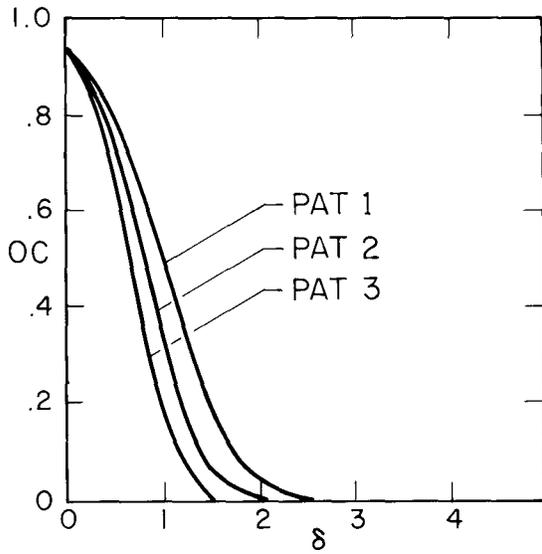


Figure 2—With $n_0 = 10$, the approximate probabilities of acceptance of H_0 (OC), *top*, and the approximate average sample numbers (ASN), *bottom*, as functions of the non-centrality parameter (δ) are illustrated for three specified boundary patterns, $\alpha = 0.05$.

Figure 3—For Pattern 1, the approximate probabilities of acceptance of H_0 (OC), *top*, and the approximate average sample numbers (ASN), *bottom*, as functions of the non-centrality parameter (δ) are illustrated for chosen values of n_0 (4, 7, 10), $\alpha = 0.05$.

Table 4 – Probability boundary pattern and decision points for a Pattern 3 test with $S=7$, $n_0=8$, $n_1=2$, $n_s=1$ ($s > 1$) and $\alpha=0.05$.

S	n	α_n	γ_n	d_n^r	d_n^a
		$P(R \mu=\mu_0)$	$P(A \mu=\mu_0)$	Rejection Point	Acceptance Point
1	2	0.0018	0.0339	178.223	-9.345
2	3	.0054	.1018	7.949	-1.828
3	4	.0108	.2036	3.646	-.777
4	5	.0179	.3393	2.966	.001
5	6	.0268	.5089	2.683	.712
6	7	.0375	.7125	2.500	1.482
7	8	.0500	.9500	2.588	2.588

that our Monte Carlo estimates of α and β are quite close to the true unknown values of α and β . The differences are strictly due to the sampling errors involved in the Monte Carlo procedure.

The ASN function increases and the peak of the ASN function increases and becomes closer to $\delta = 0$ as n_0 increases for a given probability boundary pattern (table 3 and fig. 3). The ASN function tapers slowly from the peak as δ increases for a given n_0 . The ASN function also increases as the peak increases and becomes closer to $\delta = 0$ as the probability boundary pattern goes from Pattern 1 to Pattern 2 to Pattern 3. The variances of the Monte Carlo estimates of the ASN and OC points indicate that these estimates are highly reliable and probably very close to the unknown parametric values.

All examples of the new test are upper one-sided tests. Lower one-sided tests can easily be developed from boundaries of the upper one-sided tests.

Approximate two-sided tests can also be developed.

Examination of Given Test

Table 4 gives α_n , γ_n , d_n^r , and d_n^a for a test with $S=7$, $n_0=8$, $\alpha=0.05$, and Pattern 3, with n_s , d_s , and d_n as defined earlier. d_n^r and d_n^a ($n > 2$) were based on 1000 computed d_n at each stage. Fig. 1 shows the acceptance and rejection boundaries of the test.

The true average decision stage (ADS), average sample number (ASN), and the level of significance at H_0 of the test listed in table 4 are:

$$ADS = 4.180$$

$$ASN = 4.180 + 1.000 = 5.180$$

$$\alpha = P(R|\mu=\mu_0) = 0.050$$

Table 5 presents the results of 20,000 simulated tests from a $N(\mu_0, \sigma^2)$ distribution.

Table 5 – Probabilities observed in 20,000 Monte Carlo trials compared to exact probabilities for a Pattern 3 test with $S=7$, $n_0=8$, $n_1=2$, $n_s=1$ ($s > 1$) and $\alpha=0.05$.

S	n	α_n		γ_n	
		$P(R \mu=\mu_0)$		$P(A \mu=\mu_0)$	
		Exact	Observed	Exact	Observed
1	2	0.0018	0.0010	0.0339	0.0318
2	3	.0054	.0065	.1018	.1016
3	4	.0108	.0122	.2036	.2072
4	5	.0179	.0150	.3393	.3479
5	6	.0268	.0200	.5089	.5096
6	7	.0375	.0433	.7125	.7039
7	8	.0500	.0529	.9500	.9471

The observed probabilities of *table 5* allow us to calculate the Monte Carlo estimates

$$\begin{aligned}\widehat{ADS} &= 4.185 \\ \widehat{ASN} &= 5.185 \\ \hat{\alpha} &= 0.047\end{aligned}$$

Some discrepancies between the observed and exact probabilities at each stage are evident, but the Monte Carlo estimates of α and ASN are in relatively close agreement with the true values. These discrepancies are due to (a) the Monte Carlo procedures used in estimating the decision points at each stage of the test, and (b) the Monte Carlo procedures used to obtain the observed probabilities themselves. The exact OC and ASN functions of the test are predetermined (but unknown) as a function of α , n_0 , and the probability boundary pattern. The actual OC and ASN functions are determined by the estimated decision points, and are unknown but different from the exact functions. The observed OC and ASN functions are estimates of the actual OC and ASN functions. The differences between the exact and actual test and related OC and ASN functions are due to (a) above, while the differences between the actual and observed OC and ASN functions are due to (b) above. The observed OC and ASN functions are used to approximate the OC and ASN functions of the exact test.

Comparison of New Procedure

To evaluate the properties and the possible applicability of the new test (TSTT), we compared it with the fixed sample size t-test (RAND) and Barnard's one-sided sequential t-test (BSTT) for the following problem:

$$H_0: E(X) = \mu = \mu_0$$

$$H_1: \mu > \mu_0$$

$$X \sim \text{NIID}(\mu, \sigma^2)$$

$$\sigma^2 \text{ unknown}$$

Our truncated test with $\alpha = 0.05$, $n_0 = 8$, and Pattern 3 probability boundary (*table 4, fig. 1*) yielded a Monte Carlo estimate of β of 0.057 at $\delta = 1.5$, so we compared it with these tests: (a) Barnard's test with nominal $\alpha = 0.05$, $\beta = 0.05$ at $\delta = 1.5$; and (b) fixed sample size t-test with nominal $\alpha = 0.05$ and $\beta = 0.039$ at $\delta = 1.5$ (sample size of 7).

The upper and lower decision boundaries of Barnard's tests were obtained with the aid of special tables (Davies 1967). The test statistic at each stage is

$$U_n = \frac{\sum_{i=1}^n (x_i - \mu_0)}{\sqrt{\sum_{i=1}^n (x_i - \mu_0)^2}}$$

The decision boundaries for Barnard's test are illustrated in *fig. 4*. Barnard's test is open in that the decision boundaries never meet.

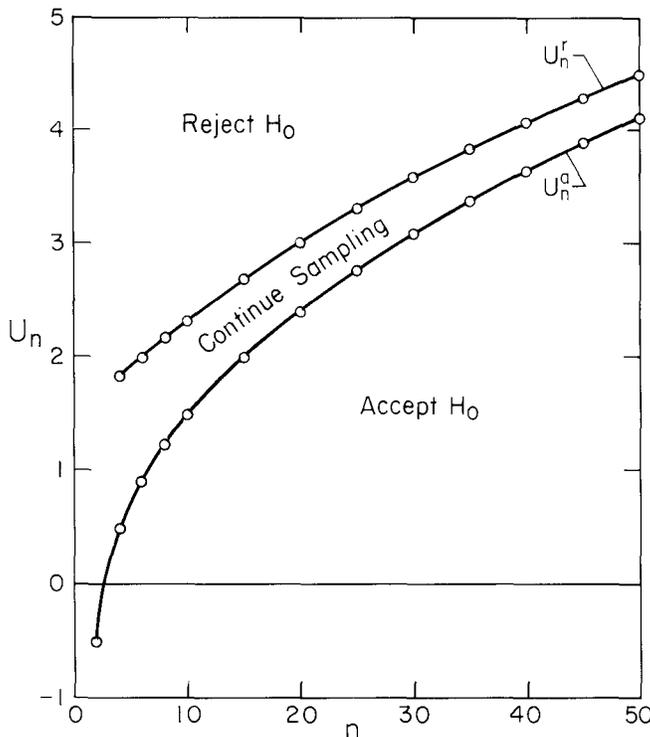


Figure 4—Upper rejection (U_n^r) and lower acceptance (U_n^a) boundaries of Barnard's one-sided sequential t-test ($\alpha = 0.05$ and $\beta = 0.05$ at $\delta = 1.5$). Encircled values are from Davies (1967, Table L•6).

Table 6—OC and ASN values as a function of the non-centrality parameter (δ) for a TSTT (Pattern 3, $S = 7$, $n_0 = 8$, $n_1 = 2$, $n_s = 1$ ($s > 1$) and $\alpha = 0.05$), a fixed sample t-test ($n = 7$), and Barnard's one-sided open test ($\alpha = 0.05$, $\beta = 0.05$ at $\delta = 1.5$).

δ	OC			ASN		
	Truncated	Fixed	Barnard	Truncated	Fixed	Barnard
-3.0	1.000	1.000	1.000	2.64	7.00	2.00
-2.0	1.000	1.000	1.000	2.83	7.00	2.01
-1.0	1.000	1.000	1.000	3.41	7.00	2.17
.0	.949	.952	.968	5.28	7.00	3.86
.5	.712	.655	.733	6.14	7.00	6.98
1.0	.308	.232	.188	5.95	7.00	8.04
1.5	.057	.041	.016	4.97	7.00	5.74
2.0	.002	.001	.001	4.29	7.00	4.68
3.0	.000	.000	.000	3.69	7.00	3.96
4.0	.000	.000	.000	3.48	7.00	3.75
5.0	.000	.000	.000	3.35	7.00	3.60
6.0	.000	.000	.000	3.21	7.00	3.46

The fixed sample t-test with a sample size of 7 is based on the test statistic

$$t_6 = (\bar{x} - \mu_0) / \sqrt{\sum_{i=1}^7 (x_i - \bar{x})^2 / 42}$$

which is compared with $t_{0.05,6} = 1.943$ in making

the appropriate decision for the conventional fixed sample size test.

Each of 12 normal distributions with δ varying from -3 to 6 were sampled a thousand times (table 6, fig. 5) with each of the three tests. Both OC and ASN functions are presented as functions of δ .

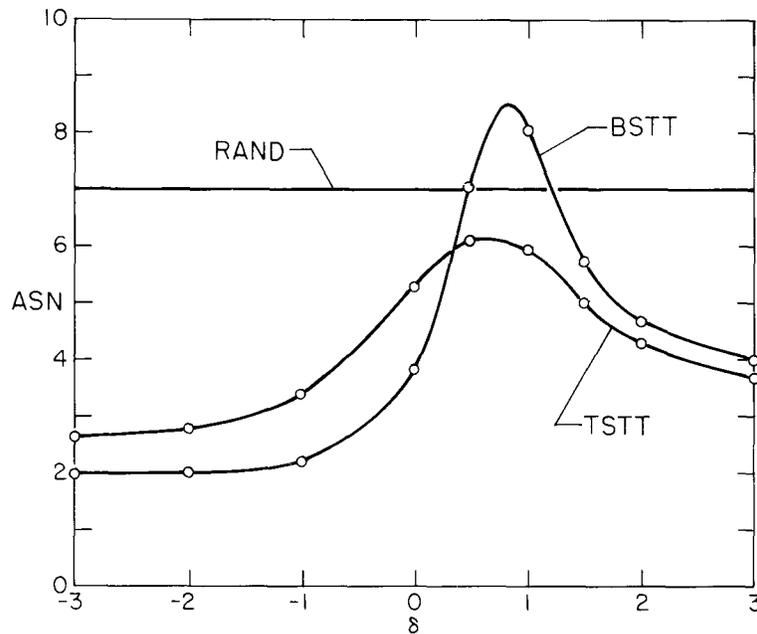


Figure 5—ASN as a function of the non-centrality parameter (δ) for TSTT (Pattern 3, $S = 7$, $n_0 = 8$, $n_1 = 2$, $n_s = 1$ ($s > 1$), and $\alpha = 0.05$), RAND ($n = 7$), and BSTT ($\alpha = 0.05$, $\beta = 0.05$ at $\delta = 1.5$). Computed points are encircled. Curves are fitted by eye.

Table 7—Observed probabilities of sample numbers (SN) as a function of the non-centrality parameter (δ) for TSTT (Pattern 3, $S = 7$, $n_1 = 2$, $n_s = 1$ ($s > 1$) and $\alpha = 0.05$) and BSTT ($\alpha = 0.05$, $\beta = 0.05$ at $\delta = 1.5$). Observed ASN and its standard error (s_{ASN}) are given.

SN	$\delta=-1.0$		$\delta=0.0$		$\delta=1.0$		$\delta=2.0$	
	Truncated	Barnard	Truncated	Barnard	Truncated	Barnard	Truncated	Barnard
2	0.1025	0.8665	0.0310	0.3590	0.0115	0.0465	0.0145	0.0030
3	.4835	.1030	.1050	.2295	.0485	.0565	.1515	.0775
4	.3345	.0225	.1900	.1355	.1500	.1720	.4265	.5075
5	.0695	.0055	.2390	.0830	.1630	.1455	.2750	.2550
6	.0100	.0015	.2295	.0600	.1710	.1070	.0945	.0755
7	0	.0010	.1475	.0415	.2180	.0900	.0300	.0405
8	0	0	.0580	.0290	.2380	.0510	.0080	.0210
9	0	0	0	.0225	0	.0495	0	.0095
10	0	0	0	.0105	0	.0435	0	.0065
11-20	0	0	0	.0280	0	.1930	0	.0040
21-30	0	0	0	.0015	0	.0375	0	0
31-40	0	0	0	0	0	.0065	0	0
41-50	0	0	0	0	0	.0015	0	0
\hat{ASN}	3.4010	2.1755	5.2055	4.0110	6.0395	8.3660	4.4055	4.6565
(s_{ASN})	.0180	.0114	.0331	.0625	.0279	.1320	.0234	.0306

The results of a comparison of the distribution of sample sizes for four normal distributions, each sampled 2000 times, for TSTT and BSTT are given in table 7. Given the absolute guarantee of a limiting sample size, TSTT compares favorably with RAND and BSTT.

The Monte Carlo estimates of α at $\delta = 0$ are quite close to the nominal α for TSTT and RAND and distinctly less than the nominal α for BSTT (table 6). The estimate of β at $\delta = 1.5$ is relatively close to the nominal β for RAND and distinctly less than the nominal β at $\delta = 1.5$ for BSTT. TSTT was chosen such that the Monte Carlo estimate of β at 1.5 was 0.057. The standard deviation ($s_{\hat{\alpha}}$) for the estimate of the OC point is 0.007 and indicates that β is quite close to the true unknown β at $\delta = 1.5$.

The Monte Carlo estimate of the ASN function for TSTT is uniformly lower than the ASN function for RAND and uniformly lower than the ASN function for BSTT for $\delta > 0.3$ (fig. 5).

The range of sample sizes is considerably larger for BSTT than for TSTT because of the open nature of the decision boundaries of BSTT (table 7). The sample size for TSTT can be no larger than $n_0=8$ for the above example. The standard errors of the Monte Carlo estimates of the ASN points (s_{ASN}) indicate that \hat{ASN} is quite close to ASN for both TSTT and BSTT. Fowler⁴ found this to be true for other examples of TSTT.

These results indicate the possible applicability of TSTT to many problems for which fixed sample size or other sequential t-tests have been used in the past.

APPLICATIONS IN FORESTRY

We know of no applications of sequential t-tests in the field of forestry. The application of sequential procedures has been largely limited to Wald's SPRT. We offer two examples where the SPRT has been used but where sequential t-tests may be more efficient.

Sequential procedures based on Wald's SPRT have been developed for (a) sampling of ribes populations

⁴ Fowler, G. W. *An investigation of some new sequential procedures for use in forest sampling*. 1969. (Unpublished Ph.D. thesis on file at University of California, Berkeley)

in the control of white pine blister rust (*Cronartium ribicola* Fisher) in California (Offord 1966); and (b) sampling of brood densities of the Engelmann spruce beetle (*Dendroctonus engelmanni* Hopk.) in standing trees to determine infestation trends (Knight 1960). In each case, preliminary field studies indicated that real-world distribution of the pertinent variable was best described by the negative binomial distribution.

In the ribes work, the problem is to test whether the average number of feet of live stem (FLS) per acre of *Ribes* spp. (native currants or gooseberries) on a forest area meets a certain standard (μ_0) after a private contractor has completed eradication work on that area. Ribes eradication is carried out in order to eliminate the alternate host (ribes plants) of white pine blister rust and thereby break the weakest link in the life cycle of this forest disease.

An SPRT is constructed to determine whether or not the contractor met the standard. This standard is set according to the blister rust hazard of the area. The null hypothesis $H_0: \mu \leq \mu_0$ FLS/plot is tested against the alternative hypothesis $H_1: \mu > \mu_0$ with α and β set at μ_0 and μ_1 respectively, and a "pooled" estimate of K (the non-test parameter of the negative binomial distribution) being determined. The sequential procedure is to compare the cumulative FLS/plot, at each stage of the test, with the appropriate decision boundaries.

In the beetle work, in late June in stands of Engelmann spruce, the problem is to ascertain whether or not immediate control work is needed. An SPRT is constructed to determine if brood densities are high or low. The null hypothesis $H_0: \mu = 4$ beetles per 6- by 6-inch bark sample (no control necessary) is tested against the alternative hypothesis $H_1: \mu = 5$ beetles per bark sample (treatment necessary) with α (set at μ_0) and β (set at μ_1) and a "pooled" estimate of K being determined. The sequential procedure is to compare the cumulative number of beetles per bark sample, at each stage of the test, with the appropriate decision boundaries.

It would seem that sequential t-procedures and, in particular, the truncated sequential t-test would compare quite favorably to the SPRT in each of the above two examples. The SPRT, besides being an open test, assumes prior knowledge of the underlying distribution and that the "pooled" estimate of K is approximately equal to the K of the population being sampled. In a preliminary study, Fowler⁵ found that the truncated sequential t-test compared favorably with the SPRT – even for distributions that were other than normal and highly irregular. These results plus the economic desirability of an upper limit to the number of observations in a given sample seem to indicate that the proposed test should be considered in certain forest sampling problems.

FIELD APPLICATION

To find out if the new procedure is operationally feasible in the field, we investigated the field procedure necessary to sample real-world forestry populations.

Since d_n is a rather difficult statistic to calculate in the field at each stage of the test, d_n can be transformed to U_n in which

$$U_n = n d_n \sqrt{n(n-1 + d_n^2)} = \frac{\sum_{i=1}^n (x_i - \mu_0)}{\sqrt{\sum_{i=1}^n (x_i - \mu_0)^2}}$$

This formulation is a monotone function of d_n and considerably easier to calculate. The OC and ASN functions that describe a particular test based on d_n will, of course, also describe the related test based on U_n . It is a simple procedure to convert the decision boundaries of a one-sided truncated sequential t-test from values of d_n to U_n .

When discrete distributions are sampled, the possibility of a series of observations starting with the first observation having the same value, say x^0 , arises.

Therefore, we have

$$\begin{aligned} U_n &= \frac{\sum_{i=1}^n (x^0 - \mu_0)}{\sqrt{\sum_{i=1}^n (x^0 - \mu_0)^2}} \\ &= n(x^0 - \mu_0) \sqrt{n(x^0 - \mu_0)^2} \end{aligned}$$

which reduces to \sqrt{n} or $-\sqrt{n}$ depending on whether $x^0 > \mu_0$ or $x^0 < \mu_0$. The statistic U_n is set at zero when $x^0 = \mu_0$. U_n is superior to d_n for this case since

$$\begin{aligned} d_n &= (\bar{x} - \mu_0) \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n(n-1)}} \\ &= (x^0 - \mu_0) \sqrt{\frac{(x^0 - x^0)^2}{(n-1)}} \end{aligned}$$

⁵ Fowler, G. W. *An investigation of some new sequential procedures for use in forest sampling*. 1969. (Unpublished Ph.D. thesis on file at University of California, Berkeley)

which reduces to $(x^0 - \mu_0)/0$. d_n can be set at zero when $x^0 = \mu_0$ but no meaningful value of d_n can be obtained when $x^0 \neq \mu_0$. In such cases, observations would have to be taken until the first observation with a different value occurred before starting the decision process. This would greatly affect the OC and ASN functions of the test.

The ribes example considered earlier will be used to illustrate the field application of the new test. Assume that the forest sampler has chosen the truncated test considered earlier, with $n_0 = 8$, $\alpha = 0.05$, and Pattern 3. Assume that this test yields the combination of OC and ASN that the forest sampler considers desirable for this problem with appropriate decision boundaries in terms of the statistic U_n (fig. 6).

Assume $H_0: \mu \leq 15$ FLS/acre is tested against $H_1: \mu > 15$. A Field Tabulation Sheet (fig. 7) can be used in the application of the proposed truncated test to a simulated ribes population using the statistic U_n . The sampling procedure is as follows:

1. Take two observations at random from the population, yielding observations 10 and 15 FLS for our example.
2. Subtract $\mu_0 = 15$ from $x_1 = 10$ and $x_2 = 15$, calculate $(x_1 - \mu_0)^2$ and $(x_2 - \mu_0)^2$ (can be accomplished with a pocket electronic calculator), and then calculate $\sum_{i=1}^2 (x_i - \mu_0)$ and $\sum_{i=1}^2 (x_i - \mu_0)^2$

3. Calculate $\sqrt{\sum_{i=1}^n (x_i - \mu_0)^2}$ (can be accomplished using a pocket electronic calculator), calculate

$$U_2 = \frac{\sum_{i=1}^2 (x_i - \mu_0)}{\sqrt{\sum_{i=1}^2 (x_i - \mu_0)^2}} \text{ on the Field}$$

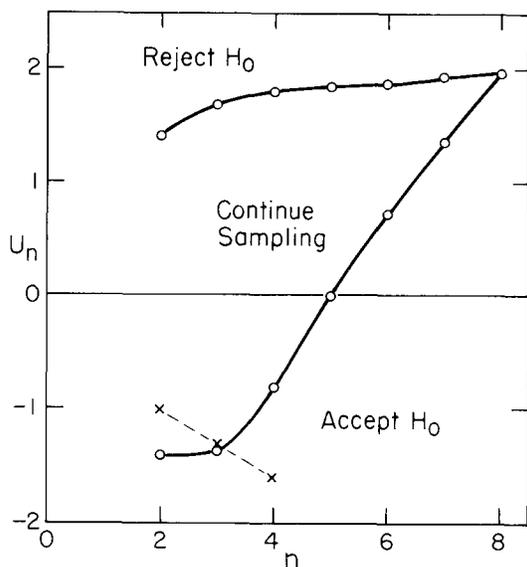


Figure 6—Acceptance and rejection boundaries in terms of U_n for a Pattern 3 test with $S = 7$, $n_0 = 8$, $n_1 = 2$, $n_s = 1$ ($s > 1$), and $\alpha = 0.05$. Computed points are encircled. Curves are fitted by eye. Dashed lines connecting points (x) illustrate a field application (see Fig. 7).

Tabulation Sheet (fig 7) and compare $U_2 = -1.00$ with U_2^a and U_2^r . Since $U_2^a < U_2 < U_2^r$, take another observation.

4. Repeat steps 2 and 3 for $n = 3$. This yields $U_3 = -1.27$, and since $U_3^a < U_3 < U_3^r$, take another observation.
5. Repeat steps 2 and 4 for $n = 4$. This yields $U_4 < U_4^a$, stop and accept H_0 .

The results of the above example can be followed graphically in fig. 6.

The field operation of the new procedure seems to be feasible and straightforward. The use of the transformation U_n , simplifies the operation considerably. Some sample calculations must be made in the field at each stage of the test, but with the aid of a small pocket-size electronic calculator, such calculations are not too difficult.

We suggest that researchers consider the use of the new procedure instead of the usual fixed sample size test for all sampling problems where acceptance or rejection of some standard (μ_0) is desired. The new procedure would be particularly applicable where observations are time-consuming, expensive, and/or destructive, as on the average, only 40% to 60% as many observations are needed as for equally reliable fixed sample-size procedures.

To choose a specific one-sided truncated sequential t-test, the researcher would have to choose meaningful values of α and β for the specific problem and decide on a truncation point. The sampling frame for the population to be sampled should be constructed so as to minimize the difficulty of taking a sequential random sample in the field. A procedure for supplying random numbers to the sampler one at a time should be implemented to insure independence of observations. Field personnel should be thoroughly trained in the operation and decision process and in the calculation procedure for the new test. Field tabulation sheets should be waterproof and bound in a field book with a water-proof hard cover.

FIELD TABULATION SHEET FOR A ONE-SIDED TRUNCATED SEQUENTIAL t-TEST					
(PAT 3): $n_0 = 8, \alpha = 0.05, \beta = 0.05 @ \delta \approx 1.5.$					
n	ACCEPT H_0	ACCEPTANCE BOUNDARY	U_n	REJECTION BOUNDARY	REJECT H_0
2		-1.4062	-1.00	1.4142	
3		-1.3699	-1.27	1.7053	
4	X	-0.8186	-1.61	1.8065	
5		0.0011		1.8540	
6		0.7432		1.8817	
7		1.3696		1.8898	
8		1.9778		1.9778	

n	x_i	μ_0	$x_i - \mu_0$	$\sum_1^n (x_i - \mu_0)$	$(x_i - \mu_0)^2$	$\sum_1^n (x_i - \mu_0)^2$	$\sqrt{\frac{\sum_1^n (x_i - \mu_0)^2}{n}}$	U_n
1	10	15	-5	-5	25	25		
2	15	15	0	-5	0	25	5.00	-1.00
3	0	15	-15	-20	225	250	15.81	-1.27
4	1	15	-14	-34	196	446	21.12	-1.61
5								
6								
7								
8								

COMMENTS OR CALCULATIONS:

Figure 7—Suggested field tabulation sheet for a one-sided truncated sequential t-test (Pattern 3, $S = 7, n_0 = 8, n_1 = 2, n_s = 1 (s > 1),$ and $\sigma = 0.05).$

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1974. **One-sided truncated sequential t-test: application to natural resource sampling.** USDA Forest Serv. Res. Paper PSW-100, 17 p., illus. Pacific Southwest Forest and Range Exp. Stn., Berkeley, Calif.

A new procedure for constructing one-sided truncated sequential t-tests and its application to natural resource sampling are described. Monte Carlo procedures were used to develop a series of one-sided truncated sequential t-tests and the associated approximations to the operating characteristic and average sample number functions. Different truncation points and decision boundary patterns were examined. The fixed sample size t-test and Barnard's open one-sided sequential t-test were compared with the new procedure. The upper one-sided test described can easily be modified to a lower one-sided test.

Oxford: 524.63–015.5.

Retrieval Terms: biometrics; sampling; sequential sampling; t-test.

Fowler, Gary W., and William G. O'Regan

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