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Computation of times of sunrise, sunset, and twilight in or near mountainous terrain

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An electronic calculator with trigonometric functions can be used to compute times of sunrise, sunset, or twilight, or time of desired illumination at any location in mountainous terrain. The method is more convenient and versatile, and less cumbersome than using tables. Latitude, longitude, elevation, day of the year (1 to 366), and slope to the horizon at the azimuth of the sun at sunrise and sunset are necessary for the computations. The effects of elevation of the observation point and blocking of the sun's radiation by terrain surrounding the observation point are included in the computations.

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Times of sunrise and sunset at specific mountainous locations often are important influences on forestry operations. The change of heating of slopes and terrain at sunrise and sunset affects temperature, air density, and wind. The times of the changes in heating are related to the times of reversal of slope and valley flows, surfacing of strong winds aloft, and the penetration inland of the sea breeze. The times when these meteorological reactions occur must be known if we are to predict fire behavior, smoke dispersion and trajectory, fallout patterns of airborne seeding and spraying, and prescribed burn results. Knowledge of times of different levels of illumination, such as the beginning and ending of twilight, is necessary for scheduling operations or recreational endeavors that require natural light.

The times of sunrise, sunset, and twilight at any particular location depend on such factors as latitude, longitude, time of year, elevation, and heights of the surrounding terrain. Use of the tables (such as *The Air Almanac*[1]) to determine times is inconvenient because each table is applicable to only one location. Different tables are needed for each location and corrections must then be made to the tables to account for elevation and for terrain characteristics. Additional tables are needed to obtain times of desired illumination. The availability of computers, especially small computers with trigonometric functions, has made it more convenient and usually more accurate to determine mathematically times of sunrise and sunset or times when enough sunlight is available for a particular job. Times in rugged terrain at various locations, at different elevations, and in various terrain can be computed. This note describes a simple, useful method by which the required times are calculated in eight steps from nine known or measured variables.

PROCEDURE

Times of sunrise and sunset, or times of desired illumination, at a specific location in mountainous terrain, can be determined by obtaining the information listed in steps 1 through 9 in the checksheet (*fig. 1*) and performing the calculations in steps 10 through 17. The equations can be solved with a hand-held electronic calculator with trigonometric functions, or more easily, with a programmable calculator with at least six registers, four of which may be stack registers. The checksheet may be reproduced and the copies used to record the intermediate values obtained on a nonprogrammable calculator.

An example of calculations of times of sunrise and sunset at Riverside, California, is included in *figure 1*. The observed time of sunrise for this day and location was 0647 PST and the observed time of sunset, 1648 PST. The Box Springs Mountains to the east of Riverside create a slope to the horizon of 7.5°, which on this date delayed sunrise by approximately 40 minutes. The Santa Ana Mountains to the west caused sunset to be 10 minutes earlier than it would have been in a nonmountainous area. If the site had been at sea level instead of at 306 m, sunrise would have been about 3 minutes later and sunset 3 minutes earlier.

ACCURACY

Calculations are most accurate in midlatitudes from 30° to 40° North. When elevation and height of surrounding terrain are not considered, as in the tables of *The Air Almanac*,[1] calculations are generally within 1 minute of the table value but vary to 3 minutes near the vernal equinox. These errors occur because of the simplified relationships used in the calculations. Other inaccuracies occur due to variations of refraction because of varying atmospheric temperature lapse rates. Some error may occur in mountainous areas if the azimuth of the sun at sunrise and sunset is approximated (see equations (6) and (7) below) and not obtained by observation. In comparisons of computed times with observed times of sunrise and sunset in mountainous areas between 30° North and 40° North, differences of 4 to 5 minutes were common. Illumination due to scattering and indirect radiation may vary because the amount of indirect radiation received at a point depends on the size and distance of the land mass or clouds that obstruct the direct radiation. The errors due to these variables are difficult to quantify. They tend to be large in high latitudes where atmospheric temperature lapse rates can vary greatly and where the sun rises and sets obliquely over the mountains.

BACKGROUND AND DEVELOPMENT

The equations shown in the checksheet (*fig. 1*) are given in order of use in a typical calculation. The more detailed discussion here follows the order of development of the procedure. Additional background and explanation are given in the discussion that follows.

The time of sunrise (when the upper limb of the sun appears on the horizon), the beginning of morning twilight, or the beginning of required illumination can be estimated by the relationship

$$t_{sr} = (h_r + 180)/15 - \Delta t - Q \qquad (1)$$

and the time of sunset (when the upper limb of the sun can last be seen on the horizon), the end of evening twilight, or the end of required illumination by

$$t_{ss} = (h_s + 180)/15 - \Delta t - Q \qquad (2)$$

where h_r is the hour angle of the sun (the angle from the meridian of the observer, negative to the east in degrees) in the morning, h_s is the hour angle in the evening, Δt is the factor that corrects time to Local Standard Time, and Q is the equation of time.

The hour angle of the sun in the morning is estimated as

$$h_r = -\arccos\{[-\sin(0.8 + R - S_r) - \sin\phi \sin d]/(\cos\phi \cos d)\} \qquad (3)$$

and the hour angle in the evening as

$$h_s = \arccos\{[-\sin(0.8 + R - S_s) - \sin\phi \sin d]/(\cos\phi \cos d)\} \qquad (4)$$

where R is the angle between the sun's rays tangent to the earth at sea level and tangent at the elevation of the site; S_r is the slope to the horizon at the azimuth of the sun at sunrise, and S_s is the slope at sunset; ϕ is the latitude of the observer; and d is the declination of the sun.

Corrections for refraction of the radiation by the atmosphere and the semidiameter of the sun are included in the factor 0.8. Refraction varies with variation of the vertical density lapse rate along the path of the solar radiation through the atmosphere, so some variation of times of sunset and sunrise will occur.

The factor

$$R = \arccos[(6.37 \cdot 10^6)/6.37 \cdot 10^6 + 1.32\,E)] \qquad (5)$$

where E is the elevation (meters) of the site, corrects the hour angle at sunrise and sunset to account for variations of the depression angle that are due to elevation of the observation point. The constant 1.32 is applied to fit variation of the depression angle that

LOCATION (Univ. California, Riverside) DATE (Nov. 1, 1976)

1. Day of the year (from 1 to 366). $D =$ (306)

2. Elevation of the site (meters). $E =$ (320 m)

3. Latitude of site (degrees and decimal degrees). $\phi =$ (33.95°N)

4. Longitude of site (degrees and decimal degrees). $L =$ (117.25°W)

5. Difference Local Standard Time and Greenwich Mean Time (hours). $H =$ (+8 hr)

6. Azimuth of sun at sunrise (degrees from north) $A_r =$ (110°)

   $A_r = 90 + 31 \cos(0.986\, D + 7.9)$

7. Azimuth of sun at sunset (degrees from north) $A_s =$ (250°)

   $A_s = 270 - 31 \cos(0.986\, D + 7.9)$

8. Slope to horizon at azimuth of sun at sunrise (degrees). $S_r =$ (7.5°)

9. Slope to horizon at azimuth of sun at sunset (degrees). $S_s =$ (2.0°)

10. Correction for elevation of site (degrees)

    a. For sunrise and sunset

    $R = \arccos[(6.37 \cdot 10^6)/(6.37 \cdot 10^6 + 1.32\, E)]$ $R =$ (-0.66°)

    b. For civil twilight, $R = 5.2$

    c. For nautical twilight, $R = 11.2$

11. Declination of the sun (degrees) $d =$ (-14.58°)

    $d = 23.45 \sin[0.973\,(D - 81.5)]$

12. Hour angle[1] of sun at sunrise (degrees from meridian)

    $h_r = -\arccos\{[-\sin(0.8 + R - S_r) - \sin\phi \sin d]/(\cos\phi \cos d)\} - 90$ $h_r =$ (-72.17°)

13. Hour angle[1] of sun at sunset (degrees from meridian)

    $h_s = \arccos\{[-\sin(0.8 + R - S_s) - \sin\phi \sin d]/(\cos\phi \cos d)\} + 90$ $h_s =$ (79.23°)

14. Correction to obtain Local Standard Time (decimal hours) $\Delta t =$ (0.18 hr)

    $\Delta t = H - L/15$

15. Equation of time, Q (in decimal hours) $Q =$ (0.26 hr)

    $Q = [0.7 \sin(-0.986\, D) + \sin(-1.97\, D - 15.78)]/6$

16. Time[2] of sunrise (hours and decimal hours) $t_{sr} =$ (0645 P.s.t., 6.75 hr)

    $t_{sr} = (h_r + 180)/15 - \Delta t - Q$

17. Time[2] of sunset (hours and decimal hours) $t_{ss} =$ (1650 P.s.t., 16.84 hr)

    $t_{ss} = (h_s + 180)/15 - \Delta t - Q$

[1]If $S_s = S_r$, then $h_s = -h_r$. Times of twilight or times of desired solar illumination are determined by using the same procedures and equations listed in steps 12 through 17. The value of R, from step 10, differentiates the resulting time.

[2]For daylight saving time, add 1 hour. To obtain minutes, multiply decimal portion of answers 16 and 17 by 60.

Figure 1—This checksheet shows the steps used for computation of times of sunrise and sunset in uneven terrain. A sample computation (in parentheses) is included. Time of twilight can also be computed by the method shown.

is due to variation of refraction caused by the different path lengths of the sun's rays through the atmosphere.

For calculation of the time of the beginning of morning civil twilight and the time of ending of evening civil twilight, R = 5.2; for nautical twilight, R = 11.2. The time of the beginning of morning civil twilight and the time of ending of evening civil twilight are defined as the times (in good conditions and in the absence of other illumination) when the degree of illumination is such that the brightest stars are just visible and terrestrial objects can be easily distinguished. This occurs when the sun is 6° below the horizon. Morning nautical twilight begins and evening nautical twilight ends when the sun is 12° below the horizon. The degree of illumination of the beginning of morning nautical twilight and end of evening nautical twilight (in good conditions and in the absence of other illumination) is such that the general outlines of ground objects are visible—although the horizon is probably indistinct—all detailed operations have become impossible, and all navigation stars can be seen. These are arbitrary levels of illumination and other depressions can be determined to correspond to different levels necessary for a particular purpose or job.

To calculate times of greater or less illumination smaller depression angles can be used, or for times of less illumination greater depression angles can be employed. Depression angles for required levels of illumination are more variable in mountainous areas than on level terrain because the sunlight is often scattered by land masses. Some adjustment of the term R may be necessary to simulate the depression angle for the necessary illumination. The same R value can be used for different times of the year and for a large area.

The adjustment often necessary in mountainous terrain, when the solar radiation is blocked by land masses, is made using the slope to the horizon along the azimuth, which is either directly observed or approximated. The azimuth of the sun at sunrise can be approximated by

$$A_r = 90 + 31 \cos(0.986\,D + 7.9) \qquad (6)$$

where D is the day of the year. The azimuth at sunset can be approximated by

$$A_s = 270 - 31 \cos(0.986\,D + 7.9) \qquad (7)$$

Equations (6) and (7) provide good approximations of the correct azimuth angles from 30° to 45° North but are not accurate in other latitudes.

The slope to the horizon along the azimuth where the sun's rays are last blocked in the morning is S_r; where the sun's rays are last blocked in the evening it is S_s. These slopes can be obtained from a topographic map or by surveying the site.

The sun's declination d can be estimated from

$$d = 23.45 \sin[0.973\,(D - 81.5)] \qquad (8)$$

The declination of the sun is its angular distance north (+) or south (-) of the celestial equator.

The two factors, Δt and Q, are needed in equations (1) and (2) to equate solar time and clock time. The time at the observer's longitude is changed to Local Standard Time by the factor

$$\Delta t = H - L/15 \qquad (9)$$

where H is the time difference between Greenwich Mean Time and Local Standard Time, and L is the longitude of the site. (For example, the time difference H between Greenwich Mean Time and Local Standard Time in the Pacific Standard Time Zone is +8 hours.)

The equation of time Q compensates for the difference between clock time and solar time caused by the variation in the time of complete rotation of the earth relative to the sun. It may be approximated by the relationship

$$Q = [0.7 \sin(-0.986\,D) + \sin(-1.97\,D - 15.78)]/6 \qquad (10)$$

This equation is accurate to about ±2 minutes.

NOTE

[1] United States Naval Observatory. 1975. *The air almanac.* Washington, D.C., 734 p.

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