

# RATIO EQUATIONS FOR LOBLOLLY PINE TREES

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**Abstract**—The conversion factors (CFs) or expansion factors (EFs) are often used to convert volume to green or dry weight, or from one component biomass to estimate total biomass or other component biomass. These factors might be inferred from the previously developed biomass and volume equations with or without destructive sampling data. However, how the factors are related to tree size such as DBH, height or tree volume had not been examined. Using the tape and biomass measurement data of about 2000 destructively sampled loblolly pine trees, we developed several nonlinear equations to relate ratios between stem green/dry weights and stem volume to DBH and height, or tree volume. We also developed tree fractional biomass component equations with the Dirichlet regression and logratio regression approaches. These two approaches guarantee all component proportions sum to 1, and have almost the same performance. The ratios are functions of tree size and can be better estimated by DBH and HT than by stem volume.

The conversion factors (CFs) and expansion factors (EFs) are commonly used to convert tree volume to green and dry weights or from one component or total biomass to other components. These factors are usually derived from previously developed biomass and/or volume equations. How the factors are related to tree size had not been formally tested. Traditionally, separate tree fractional biomass component equations were developed, but this approach cannot hold the constraint that all component proportions sum to one. Using loblolly pine expanded datasets and new modeling approaches, in the study we developed a series of ratio equations for: (1) ratio of stem total green weight to total outside volume, (2) ratio of stem-wood dry weight to total outside volume, (3) proportions of stem-wood and bark in stem biomass, and (4) proportions of stem-wood, bark, branch, and foliage components in total tree aboveground biomass. We also compared two new modeling approaches.

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## DATA AND METHODS

Data are from (1) 1280 trees with taper measurements and green weight of cut-bolts, from which tree stem outside volume and stem green weight with bark were calculated; (2) 274 trees with taper measurements, green weights of disks, and dry weight without bark, from which tree stem outside volume, stem green weight with bark, and dry weight of stem without bark were calculated; (3) 481 new destructively sampled trees with taper measurements, green weights of cut-bolts and branch with foliage, green weights of disks, subsampled branch with foliage, dry weights of disk wood, bark, branch and foliage. For the new sampled trees, stem green weight, dry weight of stem without

bark (stem-wood), dry weight of bark, dry weight of branches, and dry weight of foliage were calculated for each tree. Stem volume for each tree was obtained using Bailey's (1995) over-lapping-bolts method.

For the ratio of stem weight to volume, the ratio equations were fitted in a system of stem volume, stem green or dry weight, and ratio equations with NSUR approach following Zhao et al. (2015) 4-step fitting strategies. The ratio equations were also directly fitted to stem volume using the OLS.

The Dirichlet regression Model (DRM) and log-ratio regression OLS Model (LGRM) approaches were used to model stem biomass composition and total aboveground biomass composition.

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## RESULTS

The ratio of stem green weight to volume ( $R_{gw/v}$ ) and ratio of stem dry weight to volume ( $R_{dw/v}$ ) can be estimated by tree diameter at breast height (DBH, cm) and total height (HT, m) or stem outside volume (VOL, m<sup>3</sup>):

$$R_{gw/v} = 547.648993DBH^{0.026656} HT^{0.42614}, \text{ or } R_{gw/v} = 950.0VOL^{0.0504}$$

$$R_{dw/v} = 152.455601DBH^{0.040590} HT^{0.259236} \quad R_{dw/v} = 432.447VOL^{0.10692} e^{-0.08677VOL}$$

Proportions of stem-wood and bark in stem biomass can be estimated with the following equations:

$$P_{stemwood} = \frac{87.8561DBH^{-0.3584} HT^{0.8903}}{87.8561DBH^{-0.3584} HT^{0.8903} + 149.6651DBH^{-0.4311}}, \text{ or}$$

$$P_{stemwood} = \frac{1}{1 + 1.60724DBH^{-0.06283} HT^{0.88325}}, \text{ or}$$

$$P_{stemwood} = \frac{544.7898V^{0.32036} e^{-0.28403V}}{33.824 + 544.7898V^{0.32036} e^{-0.28403V}}$$

$$P_{stembark} = 1 - P_{stemwood}$$

Biomass allocation in total tree aboveground biomass can be estimated by DBH and HT using either DRM or LGRM:

$$a_0 = 7.829 + 1.5964DBH^{-1.2763} HT^{2.9237} + 2.6327DBH^{-1.3144} HT^{2.0156} + 1.6966HT^{0.9634}$$

$$P_{wood} = 1.5964DBH^{-1.2763} HT^{2.9237} / a_0$$

$$P_{bark} = 2.6327DBH^{-1.3144} HT^{2.0156} / a_0$$

$$P_{branch} = 1.6966HT^{0.9634} / a_0$$

$$P_{foliage} = 7.829 / a_0$$

, or

$$c_0 = 1 + 1.60724DBH^{-0.06283} HT^{0.88325} + DBH^{1.28554} HT^{-1.95407} + 4.66716DBH^{1.28151} HT^{-2.93341}$$

$$P_{wood} = 1 / c_0$$

$$P_{bark} = 1.60724DBH^{-0.06283} HT^{0.88325} / c_0$$

$$P_{branch} = DBH^{1.28554} HT^{-1.95407} / c_0$$

$$P_{foliage} = 4.66716DBH^{1.28151} HT^{-2.93341} / c_0$$

## DISCUSSION AND CONCLUSIONS

Both the Dirichlet regression and log-ratio regression approaches can be used to model biomass allocation, with almost the same performance. The approaches guarantee all components sum to one. The models based on DBH and HT perform better than the model based on stem volume only.

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