Maximum Stand-Density Index of 40 Western Hemlock–Sitka Spruce Stands in Southeast Alaska

Nathan J. Poage, David D. Marshall, and Michael H. McClellan

Reineke’s (1933) maximum stand-density index (SDI_{max}) was determined for 40 unthinned, fully stocked, even-aged, hemlock-spruce stands in southeast Alaska. A nonlinear model was used to identify objectively the linear portions of the stands’ self-thinning trajectories for analysis. The objectives of the present study were (1) to use the modeled slopes and intercepts to determine the mean SDI_{max} of the stands and (2) to identify stand and site factors associated with the variability observed in SDI_{max}. The mean SDI_{max} of the 40 hemlock-spruce stands in southeast Alaska was 619. Individually, none of the stand or site factors examined accounted for >30% of the variability observed in SDI_{max}, when all 40 stands were analyzed together. Although the spruce proportion of total stand basal area of most stands increased over time and the hemlock proportion of total stand basal area of most stands decreased over time, SDI_{max} was not related to species proportion or changes in species proportion over time.

Keywords: stand-density index, self-thinning, Tsuga heterophylla, Picea sitchensis, Alaska, Pacific Northwest

Mixed-species stands of western hemlock (Tsuga heterophylla (Raf.) Sarg.) and Sitka spruce (Picea sitchensis (Bong.) Carr.) regenerate naturally after regeneration harvesting as well as heavy thinning in southeast Alaska (Ruth and Harris 1979, Deal and Farr 1994, Zasada and Packee 1994, Deal and Tappeiner 2002, Deal et al. 2002). Interest in actively managing these young-growth stands for a variety of objectives is growing (Harris and Farr 1974, McClellan et al. 2000, Peterson and Monsrud 2002, Wipfli et al. 2002). Reineke’s (1933) stand-density index (SDI), which describes stand density in terms of equivalent number of 10-in.-diameter trees per acre (TPA), is used in many growth models used by forest managers to project future forest development under different management scenarios (Long 1985, Dixon et al. 1992, Farnden 1996). Maximum SDI (SDI_{max}) is used to establish upper limits to stocking and to indicate the onset of mortality when developing density management regimes. Accurate estimates of SDI_{max} for young-growth hemlock-spruce stands are, therefore, critical if forest managers in southeast Alaska are to make realistic projections of forest development under different management scenarios.

In developing the concept of SDI, Reineke (1933) observed an exponential relationship between the number of TPA and the quadratic mean diameter (\(D_q\), the diameter of a tree of mean basal area) for even-aged stands undergoing self-thinning, as shown in Equation 1:

\[
TPA = c_0 \times D_q^c
\]  

For an even-aged stand for which multiple observations of TPA and \(D_q\) over time are available, plotting TPA and \(D_q\) on log-log axes typically results in a size-density trajectory that approaches and then curves to follow an angled line of reasonably constant slope. This linear (i.e., self-thinning) portion of the size-density trajectory represents the upper limit of stocking for the stand (Reineke 1933, Drew and Flewelling 1977) and can be modeled by Equation 2, which is derived by taking natural logarithms (ln or log.) of both sides of Equation 1, as shown in Equation 2:

\[
\ln(TPA) = \ln(c_0) + c_1 \times \ln(D_q)
\]  

Reineke (1933) observed that \(c_1\), the slope of the linear relationship described by Equation 2, was \(-1.605\) for most of the species he investigated. The intercept in Equation 2 \(\ln(c_0)\) was described by Reineke (1933) as a constant that varied by species. Because SDI defines stand density in terms of equivalent number of 10-in.-diameter TPA, SDI can be derived by setting \(c_1 = -1.605\) in Equation 2, as shown in Equation 3 (modified from Sterba [1981]):

\[
SDI_{max} = \frac{c_0 \times 10^{-1.605}}{c_0 \times D_q^{-1.605}} = \frac{10^{-1.605}}{D_q^{-1.605}} = \left(\frac{10}{D_q}\right)^{-1.605}
\]  

Equation 3 can be simplified further, as shown in Equation 4:

\[
SDI = TPA \times \left(\frac{10}{D_q}\right)^{-1.605}
\]  

Equation 4 can be used to calculate the SDI of a stand at any point in time for which TPA and \(D_q\) are known. A more difficult problem is to determine the maximum SDI (SDI_{max}) of a stand. The difficulty in determining SDI_{max} lies in

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WEST. J. APPL. FOR. 22(2) 2007 99
objective identifying the linear portion of the size-density trajectory (Weller 1987). The potential bias associated with ocularly selecting points to represent the linear portion of the size-density trajectory may be avoided by using the nonlinear model of Hann et al. (2003). This nonlinear model simultaneously fits both the linear (i.e., self-thinning) and curvilinear (i.e., onset of self-thinning) portions of each plot’s size-density trajectory, as shown in Equation 5:

\[
\ln(D_y) = \beta_0 + \beta_1 \times \ln(\text{TPA}) - \left(\frac{(\beta_0 \times \beta_2)^2}{\beta_0 + \beta_1 \times \ln(D_{\text{initial}})} - \ln(D_{\text{initial}})\right) \times e^{(\beta_1 \times \ln(\text{TPA}) - \ln(\text{TPA}))}
\]

(5)

The first part of Equation 5 \(\ln(D_y) = \beta_0 + \beta_1 \times \ln(\text{TPA})\) describes the inverse of the linear relationship between TPA and \(D_y\) originally described by Reineke (1933) in Equation 2 (i.e., \(\ln(\text{TPA}) = \ln(e_0) + e_1 \times \ln(D_y)\)). Consequently, \(\beta_0 = -(\ln(e_0))/e_1\) and \(\beta_1 = 1/e_1\). The second part of Equation 5, which models the curvilinear portion of the size-density trajectory (i.e., the onset of self-thinning), contains the additional coefficients \(\beta_2\), \(\beta_3\), \(D_{\text{initial}}\), and \(\text{TPA}_{\text{initial}}\). The \(D_{\text{initial}}\) and \(\text{TPA}_{\text{initial}}\) refer to \(D_y\) and TPA at the first measurement made on the plot.

If Equation 5 can be fit to a stand’s self-thinning trajectory and the slope coefficient \(\beta_1\) does not differ significantly from \(1/e_1 = 1/\ln(\text{TPA})\), SDI\(_{\text{max}}\) can be calculated using the intercept coefficient \(\beta_0\), as shown in Equation 6 (modified from Hann and Wang [1990]):

\[
\text{SDI}_{\text{max}} = e^{-1.605 \times (\ln(10) - \beta_0)}
\]

(6)

We determined the mean SDI\(_{\text{max}}\) for 40 unthinned, fully stocked, even-aged, hemlock-spruce stands in southeast Alaska. Our first objective was to use the nonlinear model of Hann et al. (2003) to identify objectively and model the linear (i.e., self-thinning) portions of the stands’ size-density trajectories. We then used the modeled slopes and intercepts to determine the mean SDI\(_{\text{max}}\) of the stands. Our second objective was to identify stand and site factors associated with the variability observed in SDI\(_{\text{max}}\). Of particular interest was whether SDI\(_{\text{max}}\) varied with the species proportions in these mixed-species stands.

Methods

We analyzed live tree data from 40 unthinned, permanent study plots maintained by the USDA Forest Service in southeast Alaska (Figure 1). The plots were located in the \(P. \text{sitchensis}–T. \text{heterophylla}\) forest type (Franklin and Halpern 2000). All plots were undergoing self-thinning and had \(>95\%\) of total stand basal area comprised of hemlock and spruce. The stands containing the plots were between 13 and 94 years old at the time of plot establishment. The north-to-south distribution of the plots was approximately 250 mi. Most plots were located close to the coastline, with 39 of 40 plots at elevations of \(\leq 500\) ft. Summaries of current stand and site characteristics are presented in Table 1. See DeMars (2000) for a more detailed description of the study area.

Plot sizes ranged from 0.036 to 0.600 ac: 17 plots were 0.036 ac, 14 plots were 0.200 ac, and 9 plots were 0.250–0.600 ac. The 0.036-ac plots—used in exceptionally dense young stands (generally greater than 1,500 TPA)—were sampled using nine 0.004-ac circular subplots spaced at 23.3 ft intervals on a \(3 \times 3\) grid. Species and dbh (dbh, the diameter at 4.5 ft) were recorded for each live tree with a dbh of \(\geq 0.5\) in. and taller than 4.5 ft within each plot. All trees measured at plot establishment were tagged. The diameters of all live tagged trees were remeasured 7–15 times since plot establishment, generally at 2- to 4-year intervals. Before analysis, \(D_y\) (quadratic mean diameter) and TPA were calculated for each of the 7–15 measurements made on each plot. See DeMars (2000) for a more detailed description of the plot design and sampling scheme.

To determine the mean SDI\(_{\text{max}}\) for the 40 plots (objective 1), we first constructed a size-density trajectory for each plot by plotting \(\ln(D_y)\) over \(\ln(\text{TPA})\) for the 7–15 measurements available for each plot. We fit each plot’s size-density trajectory individually using the nonlinear model of Hann et al. (2003; Equation 5). For plots where the nonlinear model would not converge because of the highly linear nature of the data, we fit each plot’s size-density trajectory individually using the linear model described previously as the first part of Equation 5 and shown here as Equation 7:

\[
\ln(D_y) = \beta_0 + \beta_1 \times \ln(\text{TPA})
\]

(7)

The SAS System for Windows, version 8.02 (SAS 2001), was used for all statistical analyses. Linear models were fit individually for 28 plots using the REG Procedure in SAS. Nonlinear models were fit individually for 12 plots in SAS using the NLIN Procedure with Marquardt’s computational method.

We used the modeled linear and nonlinear asymptotic slopes and intercepts of the self-thinning trajectories to determine the mean and maximum SDI\(_{\text{max}}\) of the stands (objective 1). We began by using t-tests to compare the fitted slope (\(\beta_1\)) of each plot with a slope of \(1/\ln(\text{TPA})\) and a slope of \(1/\ln(\text{TPA})\) indicated no change in total stand basal area). Preliminary results indicated no significant difference between the mean slope for all plots and a slope of \(1/\ln(\text{TPA})\). Consequently, we reran the best-fit model for each plot with a fixed slope of \(\beta_1 = 1/\ln(\text{TPA})\), estimated \(\beta_0\) (the intercept), and then used \(\beta_0\) in Equation 6 to estimate SDI\(_{\text{max}}\). As a methodological check, we ocularly (i.e., visually) identified points on the linear portion of each plot’s size-density trajectory, used
linear regression to calculate the slope of the line fit through the ocular points, and then compared the ocular and model slopes.

We used correlation analysis and linear regression to identify stand and site factors associated with the variability observed in $SD_{\text{max}}$ (objective 2). The stand factors examined as explanatory variables were total stand basal area ($BA_T$, in $\text{ft}^2/\text{ac}$), spruce proportion of total stand basal area ($P_{\text{spruce}}$), total stand age, Farr’s (1984) 50-year site index ($SI_{50}$), and the skewness of the dbh$^{1.5}$ distribution, a measure of stand uneven agedness and species mixture proposed by Sterba and Monserud (1993). The proportion of hemlock was not included as a stand factor because the proportions of spruce and hemlock were inversely related. The spatial variability of $BA_T$ was used as an additional explanatory variable for the 0.036-ac plots (which were sampled using nine, uniformly spaced, 0.004-ac subplots). The spatial variability of $BA_T$ was estimated for each 0.036-ac plot by using the individual basal areas of the nine 0.004-ac subplots to calculate the coefficient of variation of $BA_T$ ($cv_{BA_T}$) for the full 0.036-ac plot. The most recent measurements were used to calculate the foregoing stand factors. The site factors considered for each plot were plot slope (%), elevation (ft), aspect (azimuth, in degrees), latitude (decimal degrees north), and longitude (decimal degrees west). The combined effect of plot slope and aspect was modeled by multiplying the plot slope by (1) the sine of the aspect and (2) the cosine of the aspect (Stage 1976).

Backward stepwise linear regression was used to model $SD_{\text{max}}$ as a function of the stand and site factors for all 40 plots. Preliminary results indicated that the mean $SD_{\text{max}}$ was higher for the 0.036-ac plots, which were used in exceptionally dense young stands, than for larger plots (Table 2). To examine possible correlations between $SD_{\text{max}}$ age, and plot size, significant linear models for all 40 plots were refit separately for (1) plots = 0.036 ac and (2) plots > 0.036 ac.

**Results**

The size-density trajectories for the 40 hemlock-spruce stands investigated in southeast Alaska all exhibited highly linear, self-thinning portions of constant slope (Figure 2). The linear model (Equation 7) was the best-fit model for 28 of 40 stands and the nonlinear model (Equation 5) of Hann et al. (2003) was the best-fit model for the other 12 stands. The mean model slope for the relationship between $\ln(D_s)$ and $\ln(TPA)$ 1/$-1.657$ was not significantly different from a slope of 1/$-1.605$ (Table 2). This enabled us to use the intercepts ($\beta_0$) of the self-thinning trajectories in Equation 6 to calculate a mean $SD_{\text{max}}$ of 619 for the 40 hemlock-spruce stands (objective 1; Table 2). The $SD_{\text{max}}$ values observed for individual stands ranged from 442 to 839. The mean $SD_{\text{max}}$ of plots = 0.036 ac (670) was larger than that of plots > 0.036 ac (582). All stands with $SD_{\text{max}}$ > 700 were sampled using 0.036-ac plots, although only 53% (9 of 17) of the stands sampled using 0.036-ac plots had $SD_{\text{max}}$ > 700 and the lowest $SD_{\text{max}}$ (442) observed for all 40 plots was for a 0.036-ac plot.

The model slopes for individual stands ranged from 1$/-2.192$ to 1$/-1.310$. The t-tests of individual regression slope parameters indicated that 55 and 25% of the stands had model slopes different from 1$/-1.605$ at the $P < 0.05$ and $P < 0.01$ levels, respectively. Ninety percent (36 of 40) of the stands had model slopes different from 1$/-2.000$ at the $P < 0.05$ level and 85% of the stands had model slopes different from 1$/-2.000$ at the $P < 0.01$ level. Of the stands that differed from 1$/-2.000$ at the $P < 0.05$ level (36 of 40), all but one had model slopes < 1$/-2.000$. Model slopes < 1$/-2.000$ indicated increasing $BA_T$. The mean ocular and model

Table 1. Stand and site characteristics of the 40 western hemlock–Sitka spruce study stands in southeast Alaska.

<table>
<thead>
<tr>
<th>Variable (units)</th>
<th>Mean</th>
<th>SD</th>
<th>n</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (yr)</td>
<td>72.45</td>
<td>28.89</td>
<td>40</td>
<td>31.10</td>
<td>143.20</td>
</tr>
<tr>
<td>TPA</td>
<td>721.39</td>
<td>645.52</td>
<td>40</td>
<td>176.00</td>
<td>2,861.10</td>
</tr>
<tr>
<td>$D_s$ (in.)</td>
<td>11.59</td>
<td>4.39</td>
<td>40</td>
<td>3.50</td>
<td>21.30</td>
</tr>
<tr>
<td>$BA_T$ (ft$^2$/ac)</td>
<td>344.02</td>
<td>71.57</td>
<td>40</td>
<td>161.00</td>
<td>443.70</td>
</tr>
<tr>
<td>Spruce proportion</td>
<td>0.43</td>
<td>0.28</td>
<td>40</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>Hemlock proportion</td>
<td>0.57</td>
<td>0.28</td>
<td>40</td>
<td>0.00</td>
<td>0.97</td>
</tr>
<tr>
<td>$SI_{50}$ (ft)</td>
<td>96.63</td>
<td>12.96</td>
<td>40</td>
<td>61.00</td>
<td>120.00</td>
</tr>
<tr>
<td>Slope (%)</td>
<td>11.40</td>
<td>12.69</td>
<td>40</td>
<td>0.00</td>
<td>40.00</td>
</tr>
<tr>
<td>Elevation (ft)</td>
<td>211.63</td>
<td>246.15</td>
<td>40</td>
<td>10.00</td>
<td>1,500.00</td>
</tr>
<tr>
<td>Aspect (degrees)</td>
<td>203.12</td>
<td>78.53</td>
<td>25</td>
<td>20.00</td>
<td>360.00</td>
</tr>
<tr>
<td>Latitude (decimal degrees north)</td>
<td>56.03</td>
<td>0.94</td>
<td>40</td>
<td>54.49</td>
<td>58.32</td>
</tr>
<tr>
<td>Longitude (decimal degrees west)</td>
<td>133.24</td>
<td>0.94</td>
<td>40</td>
<td>131.08</td>
<td>135.33</td>
</tr>
</tbody>
</table>

Age, TPA, $D_s$, basal area, and species proportions are from the most recent measurements for individual stands. Farr (1984) was used for the $SI_{50}$. Aspects for plots with slope = 0% were considered "missing" data and were not included in the analyses; the aspect for the single north-facing plot was assigned an azimuth of 360° N.

SD, standard deviation; n, number of nonmissing values.

Figure 2. Size-density trajectories of the 40 unthinned, even-aged, western hemlock–Sitka spruce study stands in southeast Alaska. Stand density is shown on the x-axis as ln(TPA), where "ln" denotes the natural logarithm; or ln$_e$. Mean tree size is shown on the y-axis as ln($D_s$) (in.; the diameter of the tree of mean basal area). The thin solid lines show the size-density trajectories of individual stands. The heavy dashed line shows the theoretical upper limit to stocking for a stand with an $SD_{\text{max}}$ of 619, the mean $SD_{\text{max}}$ calculated for all 40 stands. The heavy dashed line passes through the point (TPA = 619 TPA; $D_s$ = 10 in.) and has a slope equal to 1/$-1.605$, the inverse of Reineke’s (1933) original slope of 1/$-1.605$ (note that Reineke [1933] plotted $D_s$ on the x-axis and TPA on the y-axis in his original SDI study). The y-axis of Figure 2 is identical to the y-axis of Figure 3.
slopes did not differ significantly based on paired t-tests ($P > 0.05$), indicating that we were able to ocularly identify the asymptotic portions of the self-thinning trajectories.

The mean model intercept for the relationship between $\ln(D_i)$ and $\ln(\text{TPA})$ was 6.1539 (Table 2). The model intercepts for individual stands ranged from 5.1260 to 7.2227. Similar to Johnson (2000, available online at www.cof.orst.edu/cof/fr/research/organon/pubs/WII_RR1.pdf; last accessed July 7, 2006), we found that model intercept was negatively and linearly associated with model slope ($r^2 = 0.9420$, MSE = 0.0101, and $P < 0.0001$).

Several of the stand and site factors we examined were significantly associated with the variability observed in $\text{SDI}_{max}$ (objective 2; see Table 3 for significant models). $\text{SDI}_{max}$ was significantly related to the most recently measured $\text{BA}_T$ (+) and age (−) when both were included as explanatory variables in the same multiple linear regression model for all 40 plots. (No significant linear interaction was observed between $\text{BA}_T$ and age in this multiple linear regression model.) When this same multiple linear model was refit separately for (1) plots $= 0.036$ ac and (2) plots $> 0.036$ ac, significant and positive relationships were found between $\text{SDI}_{max}$ and $\text{BA}_T$ for both sets of plots, but the associations between $\text{SDI}_{max}$ and age were either weak or not significant. Simple linear regressions of $\text{SDI}_{max}$ on $\text{BA}_T$ for (1) all plots, (2) plots $= 0.036$ ac, and (3) plots $> 0.036$ ac indicated that $\text{SDI}_{max}$ was significantly and positively related to $\text{BA}_T$. In contrast, simple linear regressions indicated that $\text{SDI}_{max}$ was not related to age for (1) all plots, (2) plots $= 0.036$ ac, or (3) plots $> 0.036$ ac (Table 3).

$\text{SDI}_{max}$ was not significantly related to the spruce proportion of total stand basal area ($P_{\text{spruce}}$). Although the $P_{\text{spruce}}$ of most stands increased as $D_i$ increased over time (Figure 3), $\text{SDI}$ did not increase as with increasing $D_i$ (Figure 4). $\text{SDI}_{max}$ was not significantly related to the skewness of the dbh distribution or the SI (SI). For the stands sampled using 0.036-ac plots, $\text{SDI}_{max}$ decreased with increasing spatial variability of $\text{BA}_T$ (as approximated by $\text{cBA}_T$, the coefficient of variation of $\text{BA}_T$ between 0.004-ac subplots). Backward stepwise linear regression of $\text{SDI}_{max}$ on the site factors indicated that suggested $\text{SDI}_{max}$ was weakly and positively associated with longitude but not with the other site factors.

### Discussion

The mean $\text{SDI}_{max}$ of 619 of the 40 hemlock-spruce stands we investigated in southeast Alaska (objective 1) was comparable with the mean $\text{SDI}_{max}$ estimates of 590 and 610 reported by Hann et al. (2003) and Johnson (2000, available online at www.cof.orst.edu/cof/fr/research/organon/pubs/WII_RR1.pdf; last accessed July 7, 2006), respectively, for stands of western hemlock in British Columbia, Washington, and Oregon. The mean $\text{SDI}_{max}$ of 619 we report was higher than estimates derived from the hemlock-spruce yield tables of Taylor (1934; mean $\text{SDI}_{max} = 540$) and Meyer (1937; mean $\text{SDI}_{max} = 519$) for southeast Alaska. The mean $\text{SDI}_{max}$ values derived from Taylor (1934) and Meyer (1937) for hemlock-spruce stands in southeast Alaska were within 1 SD of the mean $\text{SDI}_{max}$ we report. The mean $\text{SDI}_{max}$ of 619 the 40 hemlock-spruce stands was approximately three-quarters of the maximum $\text{SDI}_{max}$ (798) reported by Long (1985) for stands of western hemlock. However, the maximum $\text{SDI}_{max}$ (839) of the 40 hemlock-spruce stands we investigated was higher than Long’s (1985) maximum $\text{SDI}_{max}$ of 798. The mean $\text{SDI}_{max}$ of 619 we report was similar to the adjusted default $\text{SDI}_{max}$ of 638 used by FVS-SEAPROG (Dixon et al. 1992), the primary vegetation-growth simulator used in southeast Alaska.

The stand and site factors examined were not strongly associated with the variability observed in $\text{SDI}_{max}$ when all 40 stands were analyzed together. These results support Reineke’s (1933) long-standing assertion that $\text{SDI}_{max}$ is relatively independent of site quality and age. Interestingly, the combination of $\text{BA}_T$ and age accounted for $> 50\%$ of the variability observed in $\text{SDI}_{max}$ for the analysis of all 40 stands; both $\text{BA}_T$ and age were highly significant in this analysis (Table 3). Subsequent analyses on stands sampled using (1) plots $= 0.036$ ac and (2) plots $> 0.036$ ac clearly indicated that age was a nonsignificant or (at best) poor predictor of the variability observed in $\text{SDI}_{max}$ (in contrast to $\text{BA}_T$). When analyzed separately for the (1) plots $= 0.036$ ac and (2) plots $> 0.036$ ac, $< 1\%$ of the variability observed in $\text{SDI}_{max}$ was accounted for by age. Thus, the significance of age when combined with $\text{BA}_T$ in the analysis of all 40 stands reflects the fact that the stands with the highest $\text{SDI}_{max}$ values (i.e., $\text{SDI}_{max} > 700$) all had ages less than the median age of 67 years for all 40 stands and all were sampled with 0.036-ac plots.

We initially were concerned by the finding that values of $\text{SDI}_{max} > 700$ were observed only for stands sampled using the smallest plots (0.036 ac). However, the study design called for using 0.036-ac plots to sample exceptionally dense stands, which not only had many TPA but also were generally young. Therefore, it is not surprising that the largest $\text{SDI}_{max}$ values were observed for stands sampled using 0.036-ac plots. In addition, only one-half of the stands sampled using 0.036-ac plots had values of $\text{SDI}_{max} > 700$, and the remainder of the stands sampled using 0.036-ac plots spanned the $\text{SDI}_{max}$ range observed in the stands sampled using larger plots. It should be further noted that $\text{SDI}_{max}$ values derived from relatively small plots may lead to a change-of-scale bias when used to model the development of larger area, more heterogeneous stands (Monserud 2002).

The spatial variability of $\text{BA}_T$ (i.e., $\text{cBA}_T$) accounted for 28% of the variation observed in $\text{SDI}_{max}$. Although 28% is not a particularly large fraction of the variability observed in $\text{SDI}_{max}$, $\text{cBA}_T$ explained

### Table 2. Mean $\text{SDI}_{max}$, model slope, and model intercept of the 40 western hemlock–Sitka spruce study stands in southeast Alaska.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SE</th>
<th>n</th>
<th>Confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{SDI}_{max}$ (all plots)</td>
<td>619.249</td>
<td>16.132</td>
<td>40</td>
<td>575.566 – 662.932</td>
</tr>
<tr>
<td>$\text{SDI}_{max}$ (plots $= 0.036$ ac)</td>
<td>669.767</td>
<td>30.091</td>
<td>17</td>
<td>581.878 – 757.656</td>
</tr>
<tr>
<td>$\text{SDI}_{max}$ (plots $&gt; 0.036$ ac)</td>
<td>581.910</td>
<td>12.901</td>
<td>23</td>
<td>545.546 – 618.274</td>
</tr>
<tr>
<td>Slope</td>
<td>1/1.657</td>
<td>1/104.167</td>
<td>40</td>
<td>1/1.731 – 1/1.588</td>
</tr>
<tr>
<td>Intercept</td>
<td>6.514</td>
<td>0.065</td>
<td>40</td>
<td>5.978 – 6.330</td>
</tr>
</tbody>
</table>

Mean $\text{SDI}_{max}$ is also shown for (1) plots $= 0.036$ ac and (2) plots $> 0.036$ ac. Mean model slope is shown in fractional rather than decimal form.

SE, standard error; $n$, number of nonmissing values; confidence interval, lower and upper limits of 99% confidence interval of mean.
### Table 3. Significant associations between model SDI\textsubscript{max} and explanatory stand and site factors for the 40 western hemlock–Sitka spruce study stands in southeast Alaska.

<table>
<thead>
<tr>
<th>Regression model</th>
<th>SDI\textsubscript{max} (all plots)</th>
<th>SDI\textsubscript{max} (plots = 0.036 ac)</th>
<th>SDI\textsubscript{max} (plots &gt; 0.036 ac)</th>
<th>SDI\textsubscript{max} (all plots)</th>
<th>SDI\textsubscript{max} (plots = 0.036 ac)</th>
<th>SDI\textsubscript{max} (plots &gt; 0.036 ac)</th>
<th>SDI\textsubscript{max} (all plots)</th>
<th>SDI\textsubscript{max} (plots = 0.036 ac)</th>
<th>SDI\textsubscript{max} (plots &gt; 0.036 ac)</th>
</tr>
</thead>
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<tr>
<td>\text{r}^2</td>
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<td>MSE</td>
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<tr>
<td>P-value</td>
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</tr>
</tbody>
</table>

Standard errors of the estimated parameter coefficients are shown in parentheses below the estimates. For all but the last two models, models are shown for (1) all plots, (2) plots = 0.036 ac, and (3) plots > 0.036 ac.

MSE, mean square error; SDI\textsubscript{max}, model maximum stand-density index; age, total stand age (yr); \text{BA}_T, total stand basal area (ft\textsuperscript{2}/ac); \text{cvBA}_T, coefficient of variation of \text{BA}_T (a measure of spatial variability of \text{BA}_T); Lon, longitude (decimal degrees west).

Degree of significance for parameter estimates indicated by superscripts on standard errors: n.s., not significant; \text{P} > 0.05, \text{P} \leq 0.05, **\text{P} < 0.01, ***\text{P} < 0.001, ****\text{P} < 0.0001.

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**Figure 3.** Changes in Sitka spruce proportion of total stand basal area with changes in mean tree size for the 40 unthinned, even-aged, western hemlock–Sitka spruce study stands in southeast Alaska. Sitka spruce proportion of total stand basal area is shown on the \textit{x}-axis. Mean tree size is shown on the \textit{y}-axis as ln(\text{D}_0) [in.; the diameter of the tree of mean basal area], where “ln” denotes the natural logarithm or \text{log}_e. The thin solid lines show the trajectories of individual stands. The large dots identify the most recent measurements for individual stands. The \textit{x}-axis of Figure 3 is identical to the \textit{x}-axis of Figure 4, and the \textit{y}-axis of Figure 3 is identical to the \textit{y}-axis of Figure 2.

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**Figure 4.** Changes in Sitka spruce proportion of total stand basal area with changes in Reineke’s (1933) SDI for the 40 unthinned, even-aged, western hemlock–Sitka spruce study stands in southeast Alaska. Sitka spruce proportion of total stand basal area is shown on the \textit{x}-axis. SDI is shown on the \textit{y}-axis. The thin solid lines show the trajectories of individual stands. The large dots identify the most recent measurements for individual stands. The \textit{x}-axis of Figure 4 is identical to the \textit{x}-axis of Figure 3.

as much of the variability found in SDI\textsubscript{max} as any of the other site and stand factors. The spatial distribution of trees in an even-aged stand can influence SDI\textsubscript{max} and primarily is a consequence of the initial stand conditions. Random differences in the preestablishment populations of shrubs and advance regeneration, microsite availability, seed availability, seedbed condition (possibly related to logging), and herbivory all contribute to the variability in initial conditions that may influence stand development for, in some cases, centuries. Given that all the plots in this study were installed in established stands, however, we were unable to control and account for the effects of initial stand conditions on SDI\textsubscript{max}. Partial or minor disturbances such as windstorms also can lead to local variations in stocking. However, the 40 stands did not appear to have experienced significant postestablishment levels of disturbance.

A key finding of this study was that most of the mixed-species hemlock-spruce stands we investigated exhibited an increase in the spruce proportion of total stand basal area (\textit{P}_{spruce}) over time (Figure 3). Interestingly, SDI\textsubscript{max} was not related to species proportion or changes in species proportion over time. If single-species stands are stands in which a single species accounts for >0.80 of total stand
basal area (Puettmann et al. 1992), our data indicate that SDI$_{max}$ does not differ significantly between single-species hemlock stands, mixed-species hemlock-spruce stands, and single-species spruce stands in southeast Alaska (Figure 4). Although the largest SDI values observed were in the stand with the highest proportion of spruce ($P_{\text{spruce}} > 0.95$ for all measurements), the SDI values of this single-species spruce stand decreased over time and eventually fell below the SDI values of a mixed-species hemlock-spruce stand with $P_{\text{spruce}} < 0.50$.

Literature Cited


