How much can natural resource inventory benefit from finer resolution auxiliary data?

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ABSTRACT

For remote sensing-assisted natural resource inventories, the effects of spatial resolution in the form of pixel size and the effects of subpixel information on estimates of population parameters were evaluated by comparing results obtained using Landsat 8 and RapidEye auxiliary imagery. The study area was in Burkina Faso, and the response variable of interest was firewood volume (m\textsuperscript{3}/ha). A sample consisting of 160 field plots was selected from the population following a two-stage sampling design. Models were fit using weighted least squares; the population mean, \( \mu \), and the variance of the estimator of the population mean, \( V(\hat{\mu}) \), were estimated using two inferential frameworks, model-based and model-assisted, and compared. For each framework, \( V(\hat{\mu}) \) was estimated both analytically and empirically. Empirical variances were estimated using bootstrapping that accounted for the two-stage sampling. The primary results were twofold. First, for the effects of spatial resolution and subpixel information, four conclusions are relevant: (1) finer spatial resolution imagery indeed contributed to greater precision for estimators of population parameters, but despite the finer spatial resolution of RapidEye, the increase was only marginal, on the order of 10% for model-based variance estimators and 36% for model-assisted variance estimators; (2) subpixel information on texture was marginally beneficial for inference of large area population parameters; (3) RapidEye did not offer enough of an improvement to justify its cost relative to the free Landsat 8 imagery; and (4) for a given plot size, candidate remote sensing auxiliary datasets are more cost-effective when their spatial resolutions are similar to the plot size than with much finer alternatives. Second, for the comparison between estimators, three conclusions are relevant: (1) sampling distribution for the model-based \( V(\hat{\mu}) \) was more concentrated and smaller on the order of 42% to 59% than that for the model-assisted \( \hat{V}(\hat{\mu}) \), suggesting superior consistency and efficiency of model-based inference to model-assisted inference; (2) bootstrapping is an effective alternative to analytical variance estimators; and (3) prediction accuracy expressed by RMSE is useful for screening candidate models to be used for population inferences.

1. Introduction

The state of ecosystems across Africa including vegetation trends and land cover is still little known, but is particularly important in understanding land degradation processes, predicting changes in climate and improving land management (Vågen et al., 2015). Baseline inventories of ecosystem properties may allow for a proper assessment of landscape performance and prediction of change over time. In West Africa, the increasing consumption of fuelwood has been considered a cause of forest degradation and deforestation in the region which, in return, is likely to make energy sources scarcer, more expensive as well as cause a deterioration in ecosystems and greater vulnerability to climate change (Arevalo, 2016; Papillon et al., 2006; Puentes-Rodriguez et al., 2017). Energy in Sub-Saharan Africa is significantly more expensive than in other parts of the world, and approximately 80% of all residents depend on fuelwood as their main energy source, mostly firewood and charcoal with a combustion efficiency less than 30% (UNDP, 2010). Burkina Faso has a very large annual population increment and small community development indices, is ranked among the most vulnerable countries to climate change, and faces energy crises for which fuelwood can be expected to play a crucial role (Arevalo, 2016). With the launch of the Reducing Emissions from Deforestation...
and Forest Degradation (REDD+) program in 2010, the government of Burkina Faso elaborated strategies for environmental protection, forestry and climate change adaptation and mitigation, and an accompanying investment plan for 2008–2018 that relies on recurrent inventories providing accurate and reliable information about the status of ecosystems (MEDD, 2012).

Traditionally, large-scale inventory programs use field surveys based on probabilistic sampling designs that support estimators with sufficient precision (Tomppo et al., 2010). This design-based inference is free from assumptions regarding the structure of the population, because it is based on the distribution of all possible estimates permissible under the strict terms of the sampling design (Cochran, 1977). However, because the desired properties of design-based estimators rely on sufficiently large sample sizes, they are apt to become unaffordable and relatively less cost-efficient; conversely, reduced sample sizes risk to satisfaction precision criteria. This dilemma would hinder developing countries from implementing repetitive inventories to regularly update the status of ecosystems and natural resources as is required today for understanding land degradation processes, predicting changes in climate and improving land management (MEDD, 2012).

Remote sensing-assisted Inventories have become increasingly popular. In particular, remotely sensed auxiliary data that are correlated with attributes of interest facilitate use of model-based inference (Gregoire, 1998). Model-based inference relies on a model as the basis for constructing inferences in forms such as confidence intervals for the population parameters (Cassel et al., 1977). The finite population is regarded as a realization of a random process called a superpopulation, and every finite population is seen as a sample of the infinite superpopulation (Särndal, 1978). The superpopulation is defined by the superpopulation model wherein the remotely sensed auxiliary information enters as independent variables. Properties of a model-based population parameter estimator (or predictor considering that the population parameters are random) are deduced conditionally with respect to the observed sample and the stipulated model, not the sampling design. A major concern with model-based inference is the potential for serious bias in the population parameter estimator if the presumed or stipulated model is misspecified (Hansen et al., 1983; Royall and Herson, 1973; Valliant et al., 2000). As an alternative to model-based estimators, model-assisted estimators also use models and auxiliary information to improve estimator precision (Baffetta et al., 2009). However, inference validity is contingent on probabilistic samples, so model-assisted estimators are still design-based.

The finest possible data recorded by pixels are constrained by the spatial resolution of a spectral remote sensing sensor. Each pixel represents a measured surface reflectance value, and the detail discernible in imagery depends on the spatial resolution of the spectral sensor. For a homogeneous feature to be correctly predicted, its size usually should not to be smaller than the pixel size, or the prediction is
apt to become poor because the average brightness of all features in that pixel will be recorded. For remote sensing applications, each population unit typically consists of a pixel or a contiguous group of pixels. Finer resolution provides more detailed data about each population unit size because it is similar to the size of the 0.1-ha field sample plots, and equals the size of a 6 × 6 block of the RapidEye pixels. Therefore, the selection of sample plot in the continuum of the study area can be approximately assimilated to the selection of pixel from a discrete population of N pixels. As should be evident, the RapidEye data provide much finer data for each population unit than do the Landsat 8 data. All analyses conform to three underlying assumptions: (1) a finite circular primary sampling units with radii of 564 m were randomly selected, and in the second stage, 10 circular plots with radii of 17.84 m (0.1 ha) were randomly selected from within each primary sampling unit. Plot centers were geo-referenced with Global Navigation Satellite System receivers with a real-time accuracy of 60 cm supported by free corrections of Satellite-Based Augmentation Systems based on European Geostationary Navigation Overlay Service. The mean distance between plot centers was 218.2 m. The two-tailed hypothesis test based on Moran’s I (p, value = 0.31 at α = 0.05) accepted a random pattern that averts problems associated with spatial autocorrelation among plot observations.

Plot-level firewood volume (m³/ha) was estimated from fallen or standing deadwood and living trees by selecting the woody material that was not rotten and was usable as fuelwood, and estimating the totals per hectare by aggregation. Because specific allometric models for particular tree species volume were not available, a general model for dry climates presented by Chave et al. (2005) was applied. For this study, the propagation of error did not take the uncertainty of the allometric model predictions into account. Altogether 54 tree species were observed of which the most common were Anogeissus leiocarpa and Vitellaria paradoxa. Most hardwood species except Parkia biglobosa and Steculia setigera were included as suitable for firewood; these included Anogeissus leiocarpa, Burkea africana, Crossopteryx febrifuga, Detarium microcarpum, Pterocarpus erinaceus, Terminalia avicennoides and Vitellaria paradoxa. Descriptive statistics for the plot measurements are shown in Table 1.

2. Materials

2.1. Study area

The study area of size 10,889 ha is in the rural commune of Kou in south-eastern Burkina Faso (11°45′N, 1°57′W) (Fig. 1, left). The topography is a plain with low relief and mean elevation of 350 m above sea level. The soil has a sandy clay texture and consists mainly of plinthosols with a subsurface accumulation of plinthite with small nutrient content (Jonsson et al., 1999). The mean annual precipitation is 790 mm/year, and the mean annual temperature 28 °C. The climate is semi-arid and bimodal (Peel et al., 2007). The monsoonal rainy season lasts seven months from April to October and accounts for 80% of the annual precipitation received, whereas the dry season covers the rest of a year (Nicholson, 2009).

The vegetation is characterized as dry savanna with a sparse tree coverage, and the natural savanna dynamics have been increasingly disturbed by agricultural land use, causing the current fragmented landscape with embedded farm areas (Fischer et al., 2011). The long-lasting drought during 1966–2000 severely affected local livelihoods and caused a tremendous loss in species diversity, also leaving introduced crop tree species in an unsuited environment (Nicholson, 2001; Wezel and Lykke, 2006). Agroforestry parklands are typical in this region where multifunctional tree species such as Parkia biglobosa and Vitellaria paradoxa are sustained (Basset and Crumney, 2003). Most inhabitants depend subsistence agriculture and livestock farming that strongly depend on goods provided by trees including timber, firewood, medicinal plants and animal fodder, with the firewood constituting an estimated share of 90% on the total energy supply (Beleem et al., 2007; Brännlund et al., 2009). Therefore, natural renewable resources play an extraordinary role for the sustainability of livelihood and the development of economy.

2.2. Field data

In this study, land cover types were ignored, and the entire study area was considered a spatially continuum population. Field data were collected during the dry season between late November 2013 and early February 2014. A two-stage sampling design proposed by the Land Degradation Surveillance Framework (Vågen et al., 2015) (Fig. 1, right) was used to select 160 sample plots. In the first stage, 16 equal-sized

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tree density (stems/ha)</td>
<td>10</td>
<td>1935</td>
<td>494</td>
<td>401</td>
</tr>
<tr>
<td>Mean diameter (cm)</td>
<td>6.4</td>
<td>40</td>
<td>15.2</td>
<td>8.5</td>
</tr>
<tr>
<td>Basal area (m²/ha)</td>
<td>0.2</td>
<td>16.1</td>
<td>5.6</td>
<td>3.6</td>
</tr>
<tr>
<td>Firewood volume (m³/ha)</td>
<td>0</td>
<td>29.1</td>
<td>6.6</td>
<td>6.2</td>
</tr>
</tbody>
</table>
population consisting of \( N \) units in the form of square, 0.09-ha cells, (2) a sample of \( n \) population units in the form of the 160 sample plots, and (3) availability of auxiliary data in the form of Landsat 8 and RapidEye variables for both sample and non-sample population units.

3.1. Model-based inference

For modeling purposes, a sample comprised of observations of both the dependent and independent variables is collected for estimating the model parameters, \( \beta \), where \( \beta = \{ \beta_1, \cdots, \beta_p \} \) is a vector of \( p \) parameters. A model, \( y_i = f(x_i \beta) + \epsilon_i \), is assumed to adequately represent the underlying relationship between the dependent and independent variables, was used to estimate the superpopulation model where \( i \) indexes plots or population units; \( y_i \) is the plot-level dependent variable; \( x_i \) is a vector of independent variables; \( \beta \) are parameter estimates, \( \hat{\beta} \), \( \epsilon_i \) is a random residual term. Because remotely sensed data have the advantage of providing wall-to-wall values, assuming that the model is correctly specified, a straightforward model-unbiased estimator of \( \mu \) is,

\[
\hat{\mu}_{mb} = \frac{1}{N} \sum_{i=1}^{N} f(x_i \hat{\beta}),
\]

which is simply the mean of the model predictions for all population units, where the subscript \( mb \) denotes a model-based estimator. For this study, we focus on \( \hat{\beta} \), and note that the total can be readily estimated from the mean as \( \hat{\tau} = \hat{\mu}_{mb} \).

An estimator of the variance of \( \hat{\mu}_{mb} \) is,

\[
\hat{\Lambda}(\hat{\mu})_{mb,an} = \sum_{j=1}^{p} \sum_{k=1}^{p} \text{Cov}(\hat{\beta}_j, \hat{\beta}_k) \left( \frac{\partial f}{\partial \beta_j} \right) \left( \frac{\partial f}{\partial \beta_k} \right),
\]

where the subscript \( an \) denotes this analytical estimator which is distinguished from an empirical estimator reported in Section 3.3; \( \text{Cov}(\hat{\beta}_j, \hat{\beta}_k) \) or \( \text{Cov}(\hat{\mu}) \) for short, is the estimated covariance between the \( j \)-th and \( k \)-th parameter estimates \( \hat{\beta}_j \) and \( \hat{\beta}_k \); and \( \frac{\partial f}{\partial \beta_j} \) is a vector, each with probability \( 1/N \). From Theorem 10.2 in Cochran (1977, p.278) where the residuals \( (y_i - f(x_i \hat{\beta})) \) are based on the sample; and \( \epsilon_i \) is a random residual term. Because remotely sensed data have the advantage of providing wall-to-wall values, assuming that the model is correctly specified, a straightforward model-unbiased estimator of \( \mu \) is,

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For Landsat 8 and RapidEye, their respective \( \hat{\mu}_{mb} \) and \( \hat{\Lambda}(\hat{\mu})_{mb,an} \) were calculated. The independent variables were selected respectively from Landsat 8 and RapidEye variables and served as proxies for the dependent variable. The dependent variable for each population unit can then be predicted using a fitted model. Methods for calculating remote sensors variables are reported in Section 3.4, and modeling details are reported in Section 3.5.

3.2. Model-assisted estimators

For the perspective of two-stage sampling, the population of \( N \) elements consists of \( C \) primary sampling units (PSUs), and each PSU consists of \( S \) second-stage sampling units (SSUs) in element or plot form, so the population size \( N = C \cdot S = 109 \cdot 1110 \). A simple random sample of \( s \) PSUs was selected at stage-1, and at stage-2 the other simple random sample of \( s \) elements was drawn from within each selected PSU, so the sample size \( n = c \cdot s = 16 \cdot 10 \). However, because the Landsat 8 pixels were assimilated to field sample plots that simplifies in plot size from the original 0.1-ha to 0.09-ha, an expansion factor may be used for rescaling the estimators as proven and instructed in the Appendix A.

Like model-based estimators, model-assisted estimators use a model constructed with auxiliary data to improve inferences, but they rely on probability samples for validity. The same model used for model-based inference was also used for model-assisted estimators. The model-assisted estimators for two-stage sampling arising out of modeling at the element level were introduced by Särndal et al. (1992, p.323):
to take the variability between PSUs and the variability within each PSU into account. With this pseudo-replication approach, the variability between replicate estimates accounts correctly for the between and within components without having to estimate them separately. A resample based on this scheme is drawn from the original sample in three steps: first, recalculate the number of PSUs (c*) and the number of elements within a PSU (s*) required for resampling: 

\[ c^* = \frac{k_1}{k_2}(c - 1) \]

and 

\[ s^* = \frac{k_1}{k_2}(s - 1), \]

where \( k_1 = C/c \) and \( k_2 = S/s \); second, randomly select \( c^* \) PSUs with replacement; and third, randomly select \( s^* \) elements within each selected PSU with replacement. The original sample was thereby resampled 30,000 times.

The important issue is not the number of resamples but rather whether the estimates stabilized. For this study, 30,000 repetitions were deemed sufficient for both inferential frameworks because the estimates changed little already after 10,000 repetitions. The first term on the right side of Eq. (3) does not necessarily equal \( \tilde{\sigma}_\mu \) in bootstrapping, because the stochastic process mimicked by a pseudo-replication approach differs for generating a resample of model calibration data. For each resample, the model parameters were estimated, and the resample-specific \( \tilde{\mu}_{\mu b} \) and \( \tilde{\mu}_{\mu a b} \) were calculated and recorded. The empirical bootstrap variance was calculated as,

\[
\tilde{\sigma}^* = \overline{\tilde{\sigma}^2} = \frac{1}{B - 1} \sum_{b=1}^{B} (\tilde{\mu}^*_b - \tilde{\mu}^*)^2 ,
\]

where the subscript \( b \) denotes an empirical estimator using bootstrapping; \( \tilde{\mu}^*_b = \frac{1}{s_b} \sum_{i=1}^{s_b} \tilde{\mu}_{i b}^* \) is the bootstrap estimate from the \( b \)-th resample, \( b = 1, \ldots, B \), and \( B = 30,000 \). Depending on Eqs. (1) or (3) adopted in Eq. (5), \( \tilde{\sigma}^* (\tilde{\mu}_{\mu b} b) \) or \( \tilde{\sigma}^* (\tilde{\mu}_{\mu a b}) \) was respectively used for notation.

An estimate of the bias of \( \tilde{\mu} \) was computed to assess the deviation as

\[
\tilde{B} \tilde{\sigma}^* (\tilde{\mu}) = \overline{\tilde{\mu}^2} - \tilde{\mu} ,
\]

where \( \tilde{\mu} \) is based on the original sample of 160 plots. Note that \( \tilde{B} \tilde{\sigma}^* (\tilde{\mu}) \) pertains only to resampling issues but not to issues related to model misspecification. Depending on Eqs. (1) or (3) adopted in Eq. (6), \( \tilde{B} \tilde{\sigma}^* (\tilde{\mu}_{\mu b} b) \) or \( \tilde{B} \tilde{\sigma}^* (\tilde{\mu}_{\mu a b}) \) was respectively used for notation.

### 3.4. Remote sensing variables

The coordinates of plot centers were used for the registration between sample plots and pixels. Although there is no assurance that any particular plot is completely contained within the pixel containing the plot center, any detrimental effects should be captured in sums of squared deviations between observations and predictions. Remote sensing variables are auxiliary data used as independent variables in modeling. These variables were calculated from the original spectral bands, the first principal component of the original spectral bands (PCA), the textural features of the first principal component, spectral indices and textural features for the respective spectral indices. The spectral indices considered here are described in Table 2. Most of the indices listed have been shown to contribute to statistically significantly improving the quality of fit of stem volume or aboveground biomass models in environmental conditions similar to those for the current study (Hou et al., 2011; Hou et al., 2013; Hou et al., 2017; Wu et al., 2013). Textural features were calculated for each first principal component and its respective spectral indices, and these form the first and second-order statistical features, including the mean, variance, homogeneity, contrast, dissimilarity, entropy, angular second moment and correlation, with equations available in Haralick et al. (1973) and Gonzalez (2008).

The variables of RapidEye were calculated at the spatial resolution of 5-m by 5-m, and then downscaled to 30-m by 30-m through averaging to stay consistent with Landsat 8. The textures were calculated for RapidEye only with a window size of 6 × 6, an offset distance of 1 (averaged over all directions), and a 64 grey level quantization resulting from the tradeoff between noise reduction and spatial detail preservation. In this sense, RapidEye permits texture metrics within each Landsat 8 pixel. Thus, this enables a specific comparison of model fit and inferences when using Landsat 8 with no texture to model fit and inferences when using RapidEye texture metrics within the Landsat pixel. Further, a comparison between using and not using texture metrics for RapidEye indicates their extent of utility for making inferences.

### 3.5. Modeling

Formulation of regression models and selection of independent variables can be difficult when the number of independent variables is large. Further, forward, backward, and stepwise variable selection techniques do not work well when correlations among independent variables are large (Harrell, 2001). For the Landsat 8 model (Model-1) and the two RapidEye models (without texture metrics, Model-2; and with texture metrics, Model-3), independent variables were selected parsimoniously using the “bootstrap stepAIC” procedure (Rizopoulos, 2009) which integrates bootstrapping to assess the variability in model selection when using a stepwise algorithm based on the Akaike information criterion (Akaike, 1974).

A linear form \( y = X\beta + \epsilon \) was selected for all models where \( y \) is the dependent variable; \( X \) is the design matrix of independent variables; \( \beta \) is the vector of model parameters; and \( \epsilon \) is the vector of random errors assumed to be normally distributed with \( E(\epsilon) = 0 \) and \( E (\epsilon^2) = \Phi \), a positive definite matrix. To accommodate heteroscedasticity, the model parameters were estimated using weighted least squares (WLS) for which \( \hat{\beta} = (X^T W X)^{-1} X^T W y \), and \( W = \Phi^{-1} \) is a diagonal matrix with \( w_i = 1/\alpha^2 \) (Carroll and Ruppert, 1988). The diagonal elements of \( W \) were estimated using a 4-step procedure introduced in McRoberts et al. (2016). First, the pairs \( (y_i, \hat{y}_i) \) were ordered with respect to \( \hat{y}_i \); second, the pairs were aggregated into groups of size 20 each; third, for each group, \( g \), the mean of the predictions, \( \overline{\hat{y}}_g \), and the variance, \( \hat{\sigma}_g^2 \), of the residuals, \( \hat{e}_g = y_g - \hat{y}_g \), were calculated; and fourth, the relationship between \( \hat{\sigma}_g^2 \) and \( \overline{\hat{y}}_g \) was modeled as \( \hat{\sigma}_g^2 = \lambda \overline{\hat{y}}_g + \hat{\sigma}_g^2 \), where \( \lambda \) is the parameter to be estimated (Fig. 2).

The following heteroscedasticity consistent estimator, \( \tilde{\text{Cov}}(\hat{\beta}) \) required in Eq. (2), introduced by MacKinnon and White (1985), was used,

\[
\tilde{\text{Cov}}(\hat{\beta}) = (X^T W X)^{-1} X^T W \text{diag} \left[ \frac{\hat{e}_g^2}{(1 - h_g)} \right]^{-1} X W (X^T W X)^{-1} ,
\]

where \( h_g = x_g^T W X x_g / \hat{y}_g^T W \hat{y}_g \). Dividing \( \hat{e}_g^2 (1 - h_g) \) inflates \( \hat{e}_g^2 \) so that the over-influence of observations with large variances is adjusted (Hou et al., 2017).

The degree to which measures of goodness of fit of a model or prediction accuracy correspond to the precision of estimators of the population parameter was assessed. Particularly for \( \tilde{\mu}_{\mu b} \), this estimator

### Table 2

Summary of the spectral indices. NIR, near infrared band; R, red band; B, blue band; SWIR, short-wave infrared band.

<table>
<thead>
<tr>
<th>Spectral indices</th>
<th>Formula</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enhanced vegetation index (EVI)</td>
<td>2.5(NIR-R)/(NIR + 6R - 7.5B + 1)</td>
<td>Huete et al. (2002)</td>
</tr>
<tr>
<td>Generalized Difference Vegetation Index (GNDVI)</td>
<td>(NIR2 − R2)/ (NIR2 + R2)</td>
<td>Wu et al. (2013)</td>
</tr>
<tr>
<td>Normalized Difference Vegetation Index (NDVI)</td>
<td>(NIR-R)/(NIR + R)</td>
<td>Rouse et al. (1973)</td>
</tr>
<tr>
<td>Normalized Difference Water Index (NDWI)</td>
<td>(NIR-SWIR2)/(NIR + SWIR)</td>
<td>Go (1996)</td>
</tr>
<tr>
<td>Specific Leaf Area Vegetation Index (SLAVI)</td>
<td>NIR/R + SWIR2</td>
<td>Lymburner et al. (2000)</td>
</tr>
<tr>
<td>Simple Ratio (SR)</td>
<td>NIR/R</td>
<td>Birth and McVey (1968)</td>
</tr>
</tbody>
</table>

* Not available for RapidEye in the absence of SWIR and SWIR2 bands.
is unbiased only if the model is correctly specified. However, whether the model is or is not correctly specified is usually unknown and about all that can be determined is whether the model does or does not exhibit systematic lack of fit to a particular set of sample data. The goodness of fit was assessed with Kvålseth’s coefficient of determination (Kvålseth, 1985; Willett and Singer, 1988) which measures the proportion of the variation in weighted-\( y \)- that can be accounted for by weighted-\( X \) as,

\[
R_{\text{adj}}^2 = 1 - \left( \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2} \right),
\]

where \( y_i = W^{-1/2}y \) and \( X_i = W^{-1/2}X \). The numerator of the second term in Eq. (8) is the sum of squares of the weighted residuals, and the denominator is the sum of squares of the weighted-\( y \)-values about their mean. The prediction accuracy was assessed with RMSE where \( \text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2} \), \( \text{RMSE}_m = \frac{\text{RMSE}}{\bar{y}} \times 100 \), \( n \) is the number of sample plots; \( \bar{y}_i \) is the field measured firewood volume; \( \hat{y}_i \) is the predicted firewood volume; and \( \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \).

4. Results and discussion

4.1. Constructed models

The three models developed using auxiliary data from Landsat 8 (Model-1) and from RapidEye without using texture metrics (Model-2) or with using texture metrics (Model-3), are summarized in Table 3.

![Predicted heteroscedastic residual variance](image)

Fig. 2. Predicted heteroscedastic residual variance.

Studentized residual graphs and graphs of observations versus predictions are reported in Fig. 3. Minimal numbers of independent variables were selected for each model because of the parsimony criterion, which also facilitates simplicity and comparability between models. Of particular interest, Model-1 attained prediction accuracy similar to Model-2 and Model-3 which were only slightly superior; the goodness of fit of Model-3 was slightly less than that of Model-2, ascribed to weighting; texture metrics contribute to improving the prediction accuracy, but only to a small extent.

4.2. Comparison between inferences

Because of the different assumptions underlying model-assisted and model-based estimators, rigorous comparisons are not always possible. In this study, this comparison was feasible because the constructed models were applied to all population units as indicated by Eqs. (1) and (3). The primary difference between the variance estimators is rooted in the fact that \( \hat{V}(\hat{\mu})_{\text{hub,as}} \) capitalizes on the variability of predictions for all population units (Eq. (2)), whereas \( \hat{V}(\hat{\mu})_{\text{hub,ms}} \) capitalizes on the variability of residuals for sample units (Eq. (4)).

Model-based and model-assisted estimates with their bootstrap counterparts based on RapidEye and Landsat 8 are reported in Table 4, with convergence graphs depicted in Fig. 4. Interestingly, estimates in Table 4 obtained with Model-1 were nearly as good as their counterparts obtained with Model-2 or Model-3, regardless of which estimation approach was used. As far as this study is concerned, this suggests equivalent or comparable effectiveness of these remotely sensed auxiliary datasets used for the inference. Estimates, \( \hat{\mu}_{\text{hub}} \) and \( \hat{\mu}_{\text{ms}} \), were deemed trustworthy in that they were similar to each other and to the estimate of the mean calculated using only the field observations, 6.6 m\(^3\)/ha (Table 1).

While \( \hat{V}(\hat{\mu})_{\text{hub,as}} \) and \( \hat{V}(\hat{\mu})_{\text{hub,as}} \) based on the original sample were close to their empirical counterparts of \( \hat{V}(\hat{\mu})_{\text{hub,as}} \) and \( \hat{V}(\hat{\mu})_{\text{hub,as}} \), model-based inference outperformed model-assisted inference in estimating \( \hat{V}(\hat{\mu}) \) with differences on the order of 42% to 59% that finally leads to shorter confidence intervals. This also suggests superior efficiency through the capitalization on the variability of predictions instead of residuals of sample units. For either inferential framework, both analytical and empirical variance estimators are credible, with the latter being conservative by providing slightly larger variance estimates. \( \hat{B}_{\text{bias}}(\hat{\mu})_{\text{hub,as}} \) or \( \hat{B}_{\text{bias}}(\hat{\mu})_{\text{hub,as}} \) did not appear to be a concern as their estimates were nearly zero. Consistent with conclusions in Hou et al. (2017), prediction accuracies rather than the goodness of fit (Table 3) corresponded closely to variance estimates (Table 4) so that RMSE is useful for screening models for both inferences with an underlying cause associated with the prediction errors.

The convergence of variance and bias estimates spanning the range of various bootstrap iterations was at about the same speed for Landsat 8 and RapidEye (Fig. 4). Both model-based and model-assisted variance estimates were generally less for Model-2 or Model-3 than for Model-1, suggesting again positive effects of a finer spatial resolution on estimation. Compared with model-assisted estimators, model-based inference produced smaller and concentrated variance estimates, and thus was deemed more consistent and efficient.

Empirical variance estimators are valid alternatives for analytical variance estimators for both inferential frameworks. Hou et al. (2017) reported recently that empirical variance estimators based on bootstrapping were also found effective even when temporally external auxiliary datasets were used for model-based inference, while analytical variance estimators failed. Bootstrapping is always an option because it is generally universally applicable, both for parametric models with or without knowing the covariance matrix and for nonparametric imputations. It also provides a bootstrap estimate of bias that is otherwise difficult to assess.

Table 3

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE (m(^3)/ha)</th>
<th>RMSE(_m), ( \text{RMSE}_m )</th>
<th>( R_{\text{adj}}^2 )</th>
<th>Independent variable</th>
<th>Estimate</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model-1</td>
<td>4.56</td>
<td>57.65</td>
<td>0.60</td>
<td>(Intercept)</td>
<td>−4.87</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>EVI</td>
<td>11.07</td>
<td>0.71</td>
<td></td>
<td>1.83</td>
<td>0.11</td>
</tr>
<tr>
<td>Model-2</td>
<td>4.55</td>
<td>57.48</td>
<td>0.66</td>
<td>(Intercept)</td>
<td>−4.97</td>
<td>4.53</td>
</tr>
<tr>
<td></td>
<td>PCA</td>
<td>1.88</td>
<td>0.15</td>
<td></td>
<td>91.66</td>
<td>32.74</td>
</tr>
<tr>
<td>Model-3</td>
<td>4.30</td>
<td>54.36</td>
<td>0.63</td>
<td>(Intercept)</td>
<td>−0.17</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>Textural mean of</td>
<td></td>
<td></td>
<td>SR</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Textural variance</td>
<td></td>
<td></td>
<td>of SR</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All parameter estimates were statistically significantly different from zero at \( \alpha = 0.01 \).
4.3. Effects of subpixel information and spatial resolution on inference

Model-3 using RapidEye data permits texture metrics within each Landsat 8 pixel based on which Model-1 was constructed. This specific comparison of model fit and inferences when using Landsat 8 with no texture (Model-1) to model fit and inferences when using RapidEye texture metrics within the Landsat 8 pixel (Model-3) facilitated assessment of the contribution of subpixel information for both forms of inference. Although texture metrics slightly contributed to the prediction accuracy of Model-3 (Table 3), model-based and model-assisted variance estimates based on this model did not categorically outperform those based on Model-1 (Table 4).

On the contrary, differences between $\hat{\mu}_{mb}$ and $\hat{\mu}_{mb}^{an}$ and between $\hat{\mu}_{ma}$ and $\hat{\mu}_{ma}^{an}$ were much larger than those of Model-1 or Model-2, indicating uncertainty increased by including these textures. Generally, the contribution of textures at subpixel level appeared to be marginally beneficial when it comes to population level inferences. This finding is analogous to conclusions from studies of the effects of subplot uncertainties such as individual tree volume model predictions (McRoberts et al., 2015) or tree measurement errors (Chave et al., 2004) on large area inferences.

For remote sensing-assisted natural resource inventories, the effects of spatial resolution in the form of pixel size on estimates of population parameters were evaluated by comparing results for RapidEye and Landsat 8 data. The Landsat 8 pixel was about the same size as the plot, whereas the RapidEye pixel was much smaller than the plot, merely 1/36 the size of a Landsat 8 pixel. The question then became whether the estimators of the population parameter were more precise when using the finer spatial resolution RapidEye data than when using the coarser

![Figure 3](image-url)  
**Fig. 3.** Studentized residual graph and the graph of observations versus predictions showing that these residuals are homogeneous, and no model exhibits meaningful systematic lack of fit to the data.

![Figure 4](image-url)  
**Fig. 4.** Convergence graphs of model-based and model-assisted estimates.
resolution Landsat 8 data.

Our results revealed that the finer spatial resolution may contribute to greater precision in the estimators, but despite the finer spatial resolution of RapidEye, the increase was only marginal, on the order of 10% for model-based variance estimators and 36% for model-assisted variance estimators (Table 4). An underlying reason may be that the correlation between the response variable of interest and the independent variables, key to model prediction accuracy, is not necessarily significantly increased through adopting finer resolution auxiliary data. This result is distinguished from the effects of using a finer spatial resolution on detection of object-based targets for which a coarser resolution is not always effective (Hou et al., 2013). Considering the minimal improvement to model performance when using RapidEye imagery, as well as a higher cost, smaller spatial extent, and shorter temporal record of RapidEye imagery, Landsat 8 appears to be a more cost-effective source of imagery for this application. Reducing the cost of data acquisition without compromising estimator precision has been a major concern of natural resource inventories. Assuming precision is the primary priority, relative to mapping increased heterogeneity, for which RapidEye may still provide an advantage, then our finding in favor of Landsat 8 could help to reduce costs for larger inventories requiring many acquisitions of auxiliary data.

Decisions regarding plot size should accommodate the inferential framework. For simple random sampling estimators that rely entirely on field observations, smaller plot sizes were reported to help increase estimator precision under a given constraint on sampling intensity or cost (Hou et al., 2015). However, for model-based and model-assisted estimators that rely on the correlation between the response variable of interest and auxiliary information, plot size affects the amount of correlated information obtainable from remote sensing data, and further manifests itself through the increases or decreases in estimator precision. Naesset et al. (2015) reported a clear increased trend in estimator precision by increasing the plot size by evaluating its effects on model-assisted estimation of aboveground biomass change using multi-temporal interferometric synthetic aperture radar and airborne laser scanning data. Further, with larger plots the problem of spatially registering field plots to remotely sensed data decreases (Saarela et al., 2016).

For a fixed area population, increasing the plot size or population unit size automatically decreases population size, N, which is often used in model-based or model-assisted estimators. In an extreme situation where N = 2, it is likely that the variance estimate for N = 2 would be smaller than variance estimates for larger N. This small variance is attributed to an extraordinarily large plot size for a finite study area, a condition that is not practicable for natural resource management and planning. Also, excessively large plots will comprise several different land-use categories while in practice the interest is often related to reports broken down by land-use category. Therefore, three corollaries that could be used as general guidelines are drawn: (1) decisions on proper plot size are recommended to take the spatial scale into account that would relate to management planning goals and homogeneity of survey targets; (2) a working model developed with a specific plot size may not necessarily work well for other plot sizes; and (3) when referring to an external model for making inferences, an option is to maintain the plot size used for constructing the external model. However, for a given plot size that is not adjustable, particularly for completed field campaigns, we argue that candidate remote sensing auxiliary datasets are more cost-effective when the spatial resolution is similar to the plot size than much finer alternatives.

5. Conclusions

For remote sensing-assisted natural resource inventories, the effects of spatial resolution in the form of pixel size and the effects of subpixel information on estimates of population parameters were assessed by comparing results obtained using RapidEye and Landsat 8 auxiliary imagery. The validity of analyses was ensured through the application and comparison between model-based inference and model-assisted estimators with variance estimated analytically and empirically.

As far as this study is concerned, first, four conclusions are relevant for the effects of spatial resolution and subpixel information: (1) finer spatial resolution imagery indeed contributes to greater precision for estimators of population parameter, but despite the finer spatial resolution of RapidEye, the increase was only marginal, on the order of 10% for model-based variance estimators and 36% for model-assisted variance estimators; (2) subpixel information on texture is marginally beneficial when it comes to inferences for large area populations; (3) cost-effectiveness is more favorable for the free Landsat 8 imagery than RapidEye imagery; and (4) for a given plot size, candidate remote sensing auxiliary datasets are more cost-effective when their spatial resolutions are similar to plot sizes than with much finer alternatives. Second, for the comparison between estimators, three conclusions are relevant: (1) sampling distribution for the model-based $\hat{V}(\hat{\mu})$ was more concentrated and smaller on the order of 42% to 59% than model-assisted variance estimates, suggesting superior consistency and efficiency of model-based inference to model-assisted inference; (2) bootstrapping is an effective alternative to analytical variance estimators; and (3) prediction accuracy expressed by RMSE is useful for screening candidate models to be used for population inferences. Third, three corollaries are drawn: (1) decisions on proper plot size require considerations about the spatial scale that would relate to management planning goals and homogeneity of survey targets; (2) a working model developed with a specific plot size may not necessarily work well for other plot sizes; and (3) when using an external model for making inferences, an option is to maintain the plot size used for constructing the external model.

Although the population size was relatively small, restricted to a typical region in Burkina Faso, the analytical approach is suitable for various biophysical attributes beyond the firewood, a renewable natural resource. Results of this study can contribute to discussions on land-use dynamics and designing of a regional monitoring system in dry savanna, particularly for Burkina Faso.

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Appendix A. Appendix

PROOF: the estimators before and after the plot size assimilation (1000 m² vs. 900 m²) are convertible by an expansion factor. Define,

$$Pop.\ area = A \text{ (in m}^2)$$

$$SSU.\ area_{Ori} = 1000 \text{ (in m}^2, \text{the plot size of the original second-stage sampling unit is 1000 m}^2);$$

$$SSU.\ area_{8} = 900 \text{ (in m}^2, \text{the plot size of the Landsat 8 second-stage sampling unit is 900 m}^2);$$

$$\text{Pop. size}_{Conv} = \frac{A}{SSU.\ area_{Ori}} = \frac{A}{1000} \text{ (the population size for the original plot size);}$$

$$\text{Pop. size}_{8} = \frac{A}{SSU.\ area_{8}} = \frac{A}{900}$$

38
Define Eq. (a.1) by Eq. (a.2), we get the ratio between SSU size and SSU size

\[
\frac{\text{SSU size}}{\text{SSU size}_{\text{Ori}}} = \frac{\text{Pop. size}_{\text{SSU size}}}{\text{Pop. size}_{\text{SSU size}_{\text{Ori}}}} = \frac{\frac{\text{PSU size}}{8}}{\frac{\text{PSU size}_{\text{SSU size}}}{900}} = 10 \times \frac{9}{0.9} = 10.
\]

Proof 2: the estimators before and after the plot size assimilation are linked by the expansion factor.

\[
\hat{\mu}_{\text{Ori}} = \hat{\mu}_{\text{PSU size}} = \frac{\text{Pop. size}_{\text{SSU size}}}{\text{Pop. size}_{\text{SSU size}_{\text{Ori}}}} = \frac{\frac{\text{PSU size}}{8}}{\frac{\text{PSU size}_{\text{SSU size}}}{900}} = 10 \times \frac{9}{0.9} = \frac{\hat{V}(\mu_{\text{PSU size}})}{\hat{V}(\mu_{\text{PSU size}}_{\text{SSU size}_{\text{Ori}}})} = \frac{\hat{V}(\mu_{\text{PSU size}})}{\hat{V}(\mu_{\text{PSU size}})}.
\]

Because \(\hat{\mu}_{\text{PSU size}}, \hat{\mu}_{\text{PSU size}_{\text{SSU size}_{\text{Ori}}}}\) and \(\hat{V}(\mu_{\text{PSU size}})\) Expansion factor are known, \(\hat{\mu}_{\text{Ori}}\) and \(\hat{V}(\mu_{\text{PSU size}_{\text{Ori}}})\) can be obtained accordingly. Please note that inference made for a population are per SSU, although the PSUs can be converted using the expansion factor applied to PSUs level estimators. This proof is generalized by defining SSU area and PSUs size as other constants. In this case, the conversion back to \(\hat{\mu}_{\text{PSU size}}, \hat{V}(\mu_{\text{PSU size}})\) was redundant, because the main topic studied (the effects of spatial resolution and subpixel data on inference) can be analyzed at the scale of \(\hat{\mu}_{\text{PSU size}}\) and \(\hat{V}(\mu_{\text{PSU size}})\) to stay consistent with the model-based estimators for a comparison.