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An assessment of uncertainty in volume estimates for stands reconstructed from tree stump information

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Assessment of pre-harvest stand conditions after unplanned tree removals often requires reconstruction of the stand based on stump information. Prediction of diameter at breast height (d.b.h.) from stump measurements is a common practice because d.b.h. is usually a necessary precursor for estimating diameter distributions and predicting tree volume. Although not a widespread exercise, tree volumes are sometimes predicted directly from stump dimensions. Regardless of the approach taken, statistical models are invariably used in some manner and the model predictions are erroneously assumed to be without error. In this study, several methods for tree volume prediction arising from stump information were evaluated for the contribution of model-related uncertainty to the error in population estimates of total volume. When the entire population was enumerated, the model-related uncertainty was 1–2 per cent of the estimate depending on the volume estimation method. Sampling approaches based on individual stumps and 0.042 ha plots were evaluated, where the total uncertainty due to both model and sampling error was considerably larger when using the plot-based method. Generally, the smallest amount of error was present when predicting d.b.h. and then estimating tree volume from d.b.h. The uncertainty was largest for estimation of tree volume directly from stump dimensions when sampling proportions were ~0.35 or smaller; otherwise, the largest uncertainty resulted from prediction of d.b.h. and merchantable height which were both used as predictor variables in the volume model.

Introduction

From a global perspective, a non-trivial proportion of the wood supply originates from unauthorized timber harvest. While the offence is most prominent in tropical forests, it occurs to some extent ubiquitously (World Bank, 2006). There are numerous economic, environmental and social consequences that result from this practice (Brack, 2005); however, from a landowner economic perspective, considerable wood volume (and value) may have been removed from the site. Regardless of whether the owner is a public or private entity, an economic loss was incurred. Unfortunately, it is often the case that the loss is difficult to quantify and/or the ability to obtain restitution is limited (Mortimer *et al.*, 2005). A difficulty often encountered in quantifying the loss is appropriately defining the population to be sampled. Assuming this potential difficulty has been overcome and recouping the financial loss seems likely, it is usually necessary to reconstruct the pre-harvest stand to estimate the timber value (Chhetri and Fowler, 1996; Wiant and Brooks, 2007).

Stand reconstruction efforts often involve determination of tree attributes from the remaining stumps in order to satisfy one or more information needs (Özçelík *et al.*, 2010). The most common practice is to use statistical models to predict diameter at breast height (d.b.h.) from stump dimensions (Westfall, 2010; Pond and Froese, 2014). The d.b.h. predictions can then be used to create diameter distributions and serve as predictors in models that predict other tree attributes such as height and volume (Muukkonen, 2007; Temesgen *et al.*, 2007). Alternatively, direct prediction of tree volume from stump information can be undertaken; although implementation of this approach is hindered by the limited species and geographic scope of published models (Bylin, 1982; Parresol, 1998; Corral-Rivas *et al.*, 2007; Özçelík *et al.*, 2010; Aigbe *et al.*, 2012). Regardless of the methods used, the resultant tree volume predictions are usually incorrectly treated as observations without error. In cases where the stumps are completely enumerated, the estimated total volume would presumably be viewed as a known quantity without uncertainty; whereas if a sample of stumps is obtained, the uncertainty in the estimated stand total would only include sampling variability (Gregoire and Valentine, 2008). Thus, the amount of uncertainty in the stand-level volume is either unknown (improperly considered to be zero) or underestimated.

To credibly evaluate the amount of uncertainty in the estimated population volume (and the subsequent valuation) obtained using stump information, the variability due to the underlying statistical models needs to be accounted for. In this context, model-related uncertainty specifically refers to uncertainty in the parameter estimates and the residual variance (McRoberts *et al.*, 2016). In this study, the contributions to stand volume uncertainty from various statistical models used to estimate tree volume from stump dimensions were assessed. The objectives were to: (1) compare model-related uncertainty amongst various models used in the prediction of tree volume and (2) evaluate the relative contributions of sampling error and model-related uncertainty for estimates of stand volume and associated economic value over a range of sample intensities.

Methods

Data

Two independent data sources were used in this study. The first dataset resulted from a region-wide tree taper study in the Northeastern U.S. (Westfall and Scott, 2010); a subset of which was used to model the relationship between d.b.h. and stump dimensions in Westfall (2010; Table 1). These data are hereafter referred to as the NE data and are used to fit various statistical models that facilitate stand reconstruction and individual-tree volume prediction. The second dataset was comprised of tree taper measurements taken on the Allegheny, Green Mountain and White Mountain National Forests in the Northeastern U.S. (hereafter NFS data). In the NFS data, usually a single measure of height and diameter below d.b.h. was taken near 0.3 m in height and these observations were considered to be stump measurements of a harvested stand. Given the emphasis in this study on merchantable tree volume, trees having d.b.h. of 25 cm inches and larger were retained for analysis. For consistency, the same species groupings used in the NE data were applied to the NFS data. The resultant NFS data included 898 trees across 15 species groups (Table 1).

Analysis

Three likely approaches to reconstructing the total volume in the NFS data were considered:

(1) Stump dimensions were used to predict tree d.b.h., tree volume was predicted from d.b.h.
(2) Stump dimensions were used to predict tree d.b.h., merchantable height was predicted from d.b.h., tree volume was predicted using d.b.h. and merchantable height.
(3) Tree volume was predicted directly from stump dimensions (diameter and height).

As is common in stand reconstruction exercises, multiple statistical models were used and regression analyses were conducted independently for each species group (Corral-Rivas *et al.*, 2007). The NE data were used to fit all the models listed below using maximum likelihood methods (SAS, 2009). When modelling tree size/volume attributes, heteroscedasticity is often encountered in the residual variance. Thus, estimating the magnitude of the residual variance was also addressed in the model fitting process. The model describing the relationship between d.b.h. and stump dimensions was (Westfall, 2010):

$$\text{d.b.h.} = d \times (1.37/h)^{\beta_1} + \beta_2(1.37 - h) + \varepsilon, \tag{1}$$

$$\varepsilon \sim N(0, \beta_3 d^{\beta_4}), \tag{2}$$

where: d.b.h. = diameter at breast height of 1.37 m (cm),
d = stump diameter (cm),
h = stump height (m),
$\beta_1 - \beta_4$ = estimated parameters,
ε = random error.

To estimate merchantable height at a 10.2 cm top limit, a Chapman–Richards (Richards, 1959) formulation was used:

$$HT_m = \theta_1(1 - \exp(\theta_2\ \text{d.b.h.}))^{\theta_3} + \varepsilon, \tag{3}$$

$$\varepsilon \sim N(0, HT_m^{\theta_4}), \tag{4}$$

where: HT_m = merchantable height (m),
$\theta_1 - \theta_4$ = estimated parameters,
ε = random error.

Table 1 Summary of stump diameter and d.b.h. for 898 trees in the NFS data by species group

Group	# trees	Stump diameter (cm)				d.b.h. (cm)			
		Min.	Mean	Max.	Std. dev.	Min.	Mean	Max.	Std. dev.
1	52	29.0	45.4	78.2	10.9	25.4	39.4	64.5	9.1
2	42	28.7	45.9	70.9	12.2	25.7	36.3	51.8	7.6
3	9	28.2	36.9	46.0	5.9	25.9	31.2	38.9	4.3
4	44	27.7	46.4	86.9	14.3	25.4	39.0	76.2	10.7
7	97	26.2	42.3	67.3	9.7	25.4	36.2	55.6	7.3
8	29	29.5	62.3	93.7	16.8	26.7	53.1	76.7	13.4
9	78	29.7	48.0	84.6	13.5	26.2	39.7	67.3	9.3
10	60	29.2	57.0	91.4	14.4	25.7	51.0	81.0	12.8
11	144	29.0	44.5	75.4	10.6	25.4	35.6	53.6	7.2
12	65	26.4	40.2	63.2	9.0	25.4	35.2	52.3	7.4
13	24	32.5	53.0	86.4	13.2	27.7	43.1	67.1	9.8
14	99	29.2	51.8	118.9	16.3	25.4	41.1	81.0	10.9
15	20	29.0	42.8	61.7	10.2	26.4	36.7	51.8	7.8
17	48	27.9	49.1	76.7	10.1	26.7	39.5	62.0	8.2
18	87	29.2	47.9	92.5	12.7	26.2	41.5	81.5	10.7

The model used to predict merchantable tree volume using d.b.h. as the only predictor variable was:

$$V_d = \delta_1 + \delta_2 \text{ d.b.h.}^2 + \varepsilon, \tag{5}$$

$$\varepsilon \sim N(0, \text{d.b.h.}^{\delta_3}), \tag{6}$$

where: V_d = tree volume (m^3) based on d.b.h.,
$\delta_1 - \delta_3$ = estimated parameters,
ε = random error.

The model used to predict merchantable tree volume using d.b.h. and merchantable height as predictor variables was:

$$V_{dh} = \lambda_1 + \lambda_2(\text{d.b.h.}^2 HT_m)^{\lambda_3} + \varepsilon, \tag{7}$$

$$\varepsilon \sim N(0, \text{d.b.h.}^{\lambda_4 HT_m}), \tag{8}$$

where: V_{dh} = tree volume (m^3) based on d.b.h. and merchantable height,
$\lambda_1 - \lambda_4$ = estimated parameters,
ε = random error.

The model used to predict merchantable tree volume using stump information as predictor variables was:

$$V_{st} = \gamma_1 d^{\gamma_2} h^{\gamma_3} + \varepsilon, \tag{9}$$

$$\varepsilon \sim N(0, \gamma_4 d^{\gamma_5}), \tag{10}$$

where: V_{st} = tree volume (m^3) based on stump dimensions,
$\gamma_1 - \gamma_5$ = estimated parameters,
ε = random error.

Goodness-of-fit statistics for the models presented above were assessed via the root mean squared error (RMSE) and the concordance correlation (r_c; Vonesh *et al.*, 1996). Due to the various attributes being predicted, the formulations are given generically:

$$\text{RMSE} = \sqrt{\frac{\sum (y - \hat{y})^2}{n}}, \tag{11}$$

$$r_c = 1 - \frac{\sum (y - \hat{y})^2}{\sum (y - \bar{y})^2 + \sum (\hat{y} - \bar{\hat{y}})(y - \hat{y}) + n(\bar{y} - \bar{\hat{y}})}, \tag{12}$$

where $\hat{y}$ is the model prediction, $\bar{\hat{y}}$ is the mean model prediction, y is the observed value, $\bar{y}$ is the mean observed value and n is the number of observations. The r_c statistic spans the interval between −1 and +1, with $r_c = 1$ indicating a perfect fit to the data.

To evaluate the uncertainty associated with each model, both the variability due to the estimated parameters and the residual variance needed to be accounted for (McRoberts and Westfall, 2014). The methods were identical for all the models presented; the volume models (5) and (6) will be used to describe the process. To account for both the uncertainty in the parameter estimates as well as the covariance among parameter estimates, 5000 simulation replications were performed where a bootstrap sample (Efron and Gong, 1983) was drawn and the model fitted to sample data. Thus, 5000 potential sets of parameter estimates were produced (Figure 1). To create a predicted volume for each tree, one of the 5000 parameter sets was randomly chosen. The uncertainty due to the residual variance was then incorporated by selecting a random value (z) from a $N(0,1)$ distribution, multiplying by the square root of the residual variance, and adding this value to the original prediction. Thus, the predicted tree volume is obtained from $\hat{V}_d = \hat{\delta}_1 + \hat{\delta}_2 \text{ d.b.h.}^2 + z\sqrt{\text{d.b.h.}^{\hat{\delta}_3}}$.

The model-related uncertainty applied to the NFS data was assessed via a Monte Carlo simulation. For each of 5000 iterations, d.b.h. is predicted for each tree using (1) and (2). Using this predicted d.b.h., the merchantable height is calculated from (3) and (4). The three tree volume estimates, V_d, V_{dh} and V_{st}, are then calculated using the appropriate predictor variables for each model. Analysis of the simulation results was performed by evaluating the total estimated volumes and the variation among the totals over the 5000 iterations:

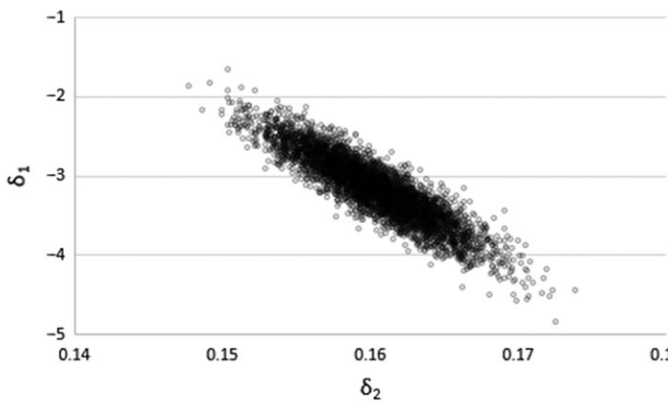

Figure 1 Range and covariance of estimated parameters from model (5) over 5000 bootstrap samples.

$$V^T_{r(\bullet)} = \sum_{i=1}^{898} V_{i(\bullet)}, \tag{13}$$

$$\bar{V}^T_{(\bullet)} = \frac{\sum_{r=1}^{5000} V^T_{r(\bullet)}}{5000}, \tag{14}$$

$$\text{SE}(\bar{V}^T_{(\bullet)}) = \sqrt{\frac{\sum_{r=1}^{5000}\left(V^T_{r(\bullet)} - \bar{V}^T_{(\bullet)}\right)^2}{(5000 - 1)}}, \tag{15}$$

where $V_{i(\bullet)}$ is the estimated volume for tree i where (•) indicates the model for V_d, V_{dh} or V_{st}, $V^T_{r(\bullet)}$ is the estimated total volume for replication r (r = 1, … ,5000), $\bar{V}^T_{(\bullet)}$ is the average total volume overall replications and SE($\bar{V}^T_{(\bullet)}$) is the standard error of the average total volume overall replications. The specification of (15) is the usual estimator for the standard deviation of the sample mean, however, in this simulation context it refers to the distribution of the population estimates and thus is equivalent to the standard error of the estimate.

In the above estimation, the 898 trees in the NFS data are considered to be the population and the population is completely enumerated. However, from a practical perspective, it may not be feasible to measure the entire population and a sample may be taken instead. If a sample is taken, there would also exist additional uncertainty due to sampling variability. To assess the implications of taking a sample, sampling proportions over the range of 0.05, 0.10, … , 0.50 were evaluated using 5000 simulation replications. As the total volume in the population is still the attribute of interest, Equation (13) is rewritten more generally as

$$V^T_{r(\bullet)} = \frac{N}{n}\sum_{i=1}^{n} V_{i(\bullet)}, \tag{16}$$

where n = sample size and N = number of population elements. The uncertainty is still assessed as described by (15); however, the $V^T_{r(\bullet)}$ will also include variation due to different samples being chosen at each replication. The finite population correction (Köhl *et al.*, 2006) is implicit in the selection of samples among the simulation replications.

Two potential sampling paradigms were evaluated: (1) sampling of individual stumps and (2) plot-based sampling where all stumps within each plot were measured. A key difference between these alternatives is that individual stumps would be the primary sampling unit (PSU) under design #1; whereas the plot would represent the PSU under design #2. To implement design #1, 5000 simulation replications were conducted where stumps were randomly selected to attain sample sizes appropriate for the sampling proportion and the statistics were calculated as shown above (16), (14) and (15), in respective order.

Implementation of design #2 was more complex as an area component was needed. The NFS data had no specified area to expand the plot-level mean to a population total. Given the desirability for consistency in estimated totals between the sampling options, the area used in the analysis was approximated at 4.6 ha. Using a selected plot size of 0.042 ha, the total number of plots in the population was set at N = 110. To conduct the sampling simulation, each tree was randomly assigned to one of the 110 plots. For a given sampling proportion, the appropriate number (n) of plots was randomly selected at each iteration and tree volumes were summed to the plot level. The mean over all sampled plots was then calculated and expanded to the population total for each iteration. These steps were repeated 5000 times and the overall statistics calculated accordingly.

To provide a coarse approximation of the uncertainty in terms of value, tree cubic metre volume was converted to board-foot volume (International ¼″ Rule) using data collected in the state of Pennsylvania from 2010 to 2015 by the Forest Inventory and Analysis programme of the U.S. Forest Service (U.S. Forest Service, 2013). These data suggested a 158.9:1 ratio of board feet to cubic metres for both softwood and hardwood species (Miles, 2016). Note that the cubic volume measure is based on a merchantable top diameter of 10.2 cm, whereas the board-foot volume only occurs in the sawlog portion of the bole (to a 17.8 cm top diameter (softwoods) or 22.9 cm diameter (hardwoods); Woudenberg *et al.*, 2010). This conversion facilitated the use of current pricing guides to develop an estimated valuation of the reconstructed stand. Based on recent stumpage prices in Pennsylvania (Timber Market Report, 2016), the approximate average value of 1000 board feet (MBF) was about US $100 for softwoods; US$250 per MBF for hardwoods. These prices were used to assign a stumpage value to each tree. The analytical procedure mimicked that described above (13)–(16) for estimates of volume. For notational purposes, the average total value and the standard error of average total value are signified as $\bar{\tilde{S}}^T_{(\bullet)}$ and $\mathrm{SE}(\bar{\tilde{S}}^T_{(\bullet)})$, respectively. Ninety-five per cent confidence intervals were constructed for the valuation via $\bar{\tilde{S}}^T_{(\bullet)} \pm 2 \times \mathrm{SE}(\bar{\tilde{S}}^T_{(\bullet)})$.

Results

Model fit as assessed by the r_c statistic indicated values greater than 0.93 for prediction of d.b.h. from stump diameter (1) and all three tree volume models (5), (7) and (9); with the exception of species/groups 3 and 6 for volume prediction from stump information (9) where the r_c values were near 0.8 (Table 2). Models for prediction of merchantable height (3) exhibited poorer fits to the data with r_c ranging in value from 0.79 to 0.94. Based on the determinations of McBride (2005), $r_c < 0.90$ is indicative of poor agreement between the observations and predictions, whereas $0.90 \le r_c < 0.95$ is considered moderate agreement and $0.95 \le r_c$ is judged to be substantial agreement. Thus, most of the results for models (1), (5), (7) and (9) suggest moderate to substantial agreement between observed and predicted values; however, the agreement was generally less for merchantable height models.

The RMSE statistics indicated average prediction errors were generally in the range of 1.5–2.5 cm for the d.b.h. prediction model (1) (Table 2). Due to varying stand conditions and their effects on tree taper, prediction of merchantable height (3) using d.b.h. as the sole predictor variable results in an average RMSE of 2.6 m across all 18 groups (range 1.5–3.3 m). These results are consistent with previously published studies in the region (Wharton, 1984; Peng *et al.*, 2001; Westfall, 2010). Prediction of tree volume using d.b.h. only (5) resulted in RMSE of approximately 0.1–0.2 m^3; however, the addition of merchantable

Table 2 Concordance correlation (r_c) and RMSE by regression model and species group

Group	# trees	Model (1)		Model (3)		Model (5)		Model (7)		Model (9)	
		r_c	RMSE (cm)	r_c	RMSE (m)	r_c	RMSE (m^3)	r_c	RMSE (m^3)	r_c	RMSE (m^3)
1	123	>0.995	1.50	0.84	2.98	0.99	0.19	>0.999	0.025	0.98	0.23
2	123	0.99	1.79	0.92	1.96	0.98	0.08	>0.999	0.005	0.95	0.15
3	127	0.98	1.98	0.84	2.35	0.97	0.09	>0.999	0.005	0.82	0.24
4	127	0.99	2.33	0.87	2.71	0.98	0.23	>0.999	0.017	0.95	0.35
5	136	0.99	1.34	0.82	2.70	0.96	0.11	>0.999	0.007	0.97	0.11
6	112	0.98	3.14	0.94	1.45	0.99	0.07	>0.999	0.008	0.77	0.43
7	127	0.99	2.15	0.79	2.89	0.98	0.21	>0.999	0.019	0.97	0.23
8	114	0.99	1.75	0.91	3.05	0.99	0.21	>0.999	0.013	0.97	0.29
9	51	0.99	2.10	0.92	2.33	0.99	0.18	>0.999	0.016	0.95	0.43
10	102	0.99	1.82	0.84	2.78	0.98	0.16	>0.999	0.016	0.97	0.22
11	125	0.99	1.92	0.90	1.83	0.99	0.10	>0.999	0.012	0.94	0.23
12	131	0.99	2.36	0.85	3.05	0.98	0.25	>0.999	0.012	0.97	0.31
13	98	0.99	1.85	0.86	3.34	0.98	0.24	>0.999	0.053	0.98	0.26
14	153	0.99	2.44	0.93	2.28	0.99	0.18	>0.999	0.014	0.97	0.35
15	116	0.97	4.29	0.84	3.07	0.99	0.23	>0.999	0.020	0.94	0.50
16	131	0.99	1.52	0.94	2.09	0.99	0.10	>0.999	0.014	0.93	0.38
17	126	0.99	2.38	0.90	2.44	0.99	0.18	>0.999	0.009	0.97	0.31
18	106	0.99	2.58	0.86	2.77	0.98	0.20	>0.999	0.007	0.95	0.33

height as a predictor variable (7) substantially reduced RMSE to less than 0.02 m^3 for most species groups. The RMSE spanned approximately 0.1–0.5 m^3 (average 0.3 m^3) for tree volume predictions obtained directly from stump information (9). These values are slightly larger than those reported by Corral-Rivas *et al.* (2007), whose RMSE was generally in the range of 0.1–0.2 for species-specific models.

Generally, the simulations showed the estimated total volume was close to 1100 m^3 with an associated value of approximately US$40 000, depending on which volume method was used (Table 3). For the population of 898 stumps, the most precise estimates of volume/value were obtained from volume method #1 (stump dimensions were used to predict tree d.b.h., tree volume was predicted from d.b.h.). The next most reliable method was #3 (tree volume was predicted directly from stump dimensions). Method #2 (stump dimensions were used to predict tree d.b.h., merchantable height was predicted from d.b.h., tree volume was predicted using d.b.h. and merchantable height) incurred the most model-related uncertainty of the three methods tested. The empirical model-related uncertainty is assessed in the complete enumeration setting, where the results indicate the standard deviation is ~1.0, 1.9 and 1.2 per cent of the estimated value for V_d, V_{dh} and V_{st}, and their associated valuations, respectively. For V_d, model (1) contributed 0.68 per cent of the uncertainty; with model (5) providing the remaining 0.32 per cent. Model-related uncertainty for V_{dh} was 0.59 per cent for model (1), 1.23 per cent for model (3) and 0.08 per cent for model (7). As only model (9) was used for V_{st}, the entire 1.2 per cent is credited to that source.

When a sampling approach is necessary, additional uncertainty is incurred due to sampling error. As the sampling proportion (and associated sample size) decrease, the sampling error increases. For sampling design #1 (stumps), a 0.50 sampling proportion resulted in uncertainty increases to 2.8, 3.3 and 3.1 per cent of the estimate for V_d, V_{dh} and V_{st}, respectively. In comparison, uncertainty in V_d, V_{dh} and V_{st} increases to 4.3, 4.7 and 4.6 per cent when using a plot-based approach (sample design #2) for the same 0.50 sampling proportion. This pattern was evident throughout the results, suggesting that the variability among stumps was smaller than the variability among plots.

For both sampling methods, the relative differences in uncertainty among V_d, V_{dh} and V_{st} decrease with reduced sampling proportions. However, the magnitude of uncertainty increases substantially with smaller samples, where confidence interval half-widths exceed 20 per cent of the estimate for the smaller sampling intensities (Table 3). As expected, the total variability increased in a nonlinear fashion as sampling proportions decreased (Figure 2). A related phenomenon is that the relative contribution of the model-related uncertainty decreases as sampling error increases. The model contribution is ~10 per cent for V_d and V_{st} and nearly 16 per cent for V_{dh} at a 0.05 sampling proportion (Figure 3). At 0.50 sampling proportion, the influence of model-related uncertainty is considerably larger with V_d and V_{st} contributing ~40 per cent and V_{dh} providing nearly 60 per cent of the total uncertainty. Due to the plot-based approach (design #2) having larger sampling errors, the relative contributions of the model-related uncertainty to total error were slightly smaller than for design #1.

Discussion

Although the volume prediction model (7) for V_{dh} had the largest r_c and smallest RMSE of the three volume models studied, there seems to be no advantage of using V_{dh} over V_d. This is largely due to the substantial uncertainty the merchantable height prediction model (3) brings to the volume estimation process. Furthermore, variations in height tend to have minor effects on the predicted volume as these height differences occur in the upper stem where the amount of volume is relatively small. Similarly, although an extra model is involved, V_d provides more precise estimates of volume than V_{st}. It follows that the combined variability due to models (1) and (5) to estimate V_d must be less than the variability due to model (9) for V_{st}. This outcome underscores the importance of having well-fitting models at all stages of the volume prediction process.

The ranking of the V_{dh} and V_{st} methods depends on the sampling intensity. At sampling intensities of ~0.35 and larger, V_{st} provides less uncertainty than V_{dh} and the advantage of V_{st} increases with increase in sampling effort; albeit, the V_{dh} method is more precise for volume estimates when sampling intensities are less than 0.35 and the advantage of V_{dh} improves with decreasing sampling proportions. Presumably due to the influence of the HT_m variable in (7), V_{dh} is the most precise of the three volume estimation methods when only sampling error is considered. Thus, for sampling proportions near 0.35 and smaller, the smaller sampling error/larger model-related uncertainty of V_{dh} overcomes the larger sampling error/smaller model-related uncertainty of V_{st} to the extent that the overall uncertainty is less for V_{dh}. The implication of this outcome is that contributions from both the sampling and model-related uncertainty should be assessed at the planned level of sampling intensity to determine the relative performances of the three volume methods.

As the stand valuation was directly obtained from stand volume, the same relative magnitudes of uncertainty were found and the same factors influence the valuation results. When the entire population is measured, the model(s) variability creates little additional uncertainty in the valuation (Table 3). However, depending on the stand value and use of the uncertainty information, the valuation differences could have considerable practical significance. A possible assertion may be the value basis should be the upper 95 per cent confidence bound instead of the mean; a contrary appeal might suggest the lower confidence bound as the basis. Using an average difference of US$1500 between these two valuations, the valuation could vary by up to ~4 per cent. Considerably larger variations in valuation would be found where sampling proportions are small and sampling variance is large. In stands having high-value due to large area and/or desirable species, these differences could translate into substantial amounts of money. To avoid such potential negative impacts, methods that minimize stand valuation uncertainty should be sought.

Several factors influenced the results obtained in this study: (1) sample sizes of the model fitting data – larger samples will result in more precise parameter estimates, (2) model specification and fit – as model goodness-of-fit increases, smaller residual variance is obtained, (3) when using species/group-specific models – the overall uncertainty due to the models depends on the relative

Table 3 Estimates and uncertainty in total volume (m^3) and economic value (US$) for three types of volumes estimates (V_d, V_{dh} and V_{st}) and two sampling designs across various sampling proportions

PSU	Sampling proportion	$\bar{V}^T_{(d)}$	SE($\bar{V}^T_{(d)}$)	$\bar{\hat{S}}^T_{(d)}$	L95($\bar{\hat{S}}^T_{(d)}$)	U95($\bar{\hat{S}}^T_{(d)}$)	$\bar{V}^T_{(dh)}$	SE($\bar{V}^T_{(dh)}$)	($\bar{\hat{S}}^T_{(dh)}$)	L95($\bar{\hat{S}}^T_{(dh)}$)	U95($\bar{\hat{S}}^T_{(dh)}$)	$\bar{V}^T_{st}$	SE($\bar{V}^T_{(st)}$)	($\bar{\hat{S}}^T_{(st)}$)	L95($\bar{\hat{S}}^T_{(st)}$)	U95($\bar{\hat{S}}^T_{(st)}$)
Stump	0.05	1121	126	40 801	30 612	50 990	1069	127	38 928	28 843	49 013	1093	139	39 773	28 825	50 720
	0.10	1120	86	40 760	33 814	47 706	1068	89	38 940	31 876	46 003	1093	97	39 746	32 084	47 409
	0.15	1121	69	40 696	35 227	46 166	1069	72	38 904	33 267	44 540	1092	77	39 714	33 658	45 770
	0.20	1120	58	40 796	36 064	45 527	1069	61	38 928	34 109	43 746	1092	64	39 677	34 589	44 765
	0.25	1122	51	40 737	36 631	44 842	1068	54	38 912	34 571	43 253	1092	56	39 740	35 299	44 180
	0.30	1121	46	40 740	37 188	44 293	1069	48	38 920	35 024	42 816	1093	50	39 746	35 733	43 758
	0.35	1121	41	40 736	37 464	44 008	1070	45	38 929	35 431	42 426	1092	45	39 769	36 171	43 366
	0.40	1121	37	40 769	37 835	43 703	1069	41	38 920	35 654	42 187	1092	41	39 753	36 503	43 003
	0.45	1120	33	40 734	38 066	43 403	1070	38	38 910	35 913	41 907	1092	37	39 732	36 791	42 672
	0.50	1121	31	40 749	38 268	43 230	1069	35	38 921	36 163	41 678	1092	34	39 765	37 069	42 461
Plot	0.05	1125	210	40 935	25 486	56 384	1069	212	38 911	23 452	54 370	1093	217	39 758	23 727	55 789
	0.10	1121	148	40 786	29 991	51 581	1069	146	38 938	28 281	49 595	1089	148	39 638	28 703	50 574
	0.15	1119	114	40 680	32 331	49 030	1069	114	38 951	30 579	47 322	1091	118	39 709	30 995	48 423
	0.20	1123	98	40 830	33 626	48 034	1070	95	38 933	31 914	45 952	1092	101	39 754	32 329	47 180
	0.25	1120	85	40 706	34 423	46 989	1068	85	38 861	32 590	45 131	1092	86	39 745	33 433	46 056
	0.30	1120	74	40 715	35 270	46 161	1068	74	38 857	33 393	44 321	1091	76	39 729	34 092	45 365
	0.35	1121	66	40 771	35 947	45 595	1069	66	38 903	34 018	43 789	1093	68	39 805	34 772	44 838
	0.40	1121	59	40 759	36 424	45 095	1069	60	38 922	34 493	43 351	1092	61	39 734	35 284	44 184
	0.45	1121	53	40 767	36 867	44 667	1067	55	38 854	34 782	42 927	1090	55	39 692	35 646	43 737
	0.50	1120	49	40 735	37 149	44 321	1069	50	38 910	35 244	42 577	1092	50	39 749	36 060	43 437
	1.00	1121	11	40 757	39 900	41 615	1069	21	38 922	37 345	40 500	1092	13	39 748	38 745	40 752

L95 and U95 denote lower and upper 95 per cent confidence limits.

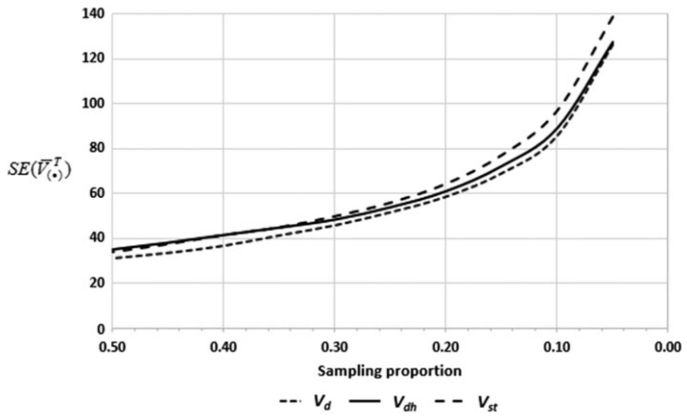

Figure 2 Total uncertainty when sampling stumps at various sampling intensities for three methods of volume estimation (V_d, V_{dh} and V_{st}).

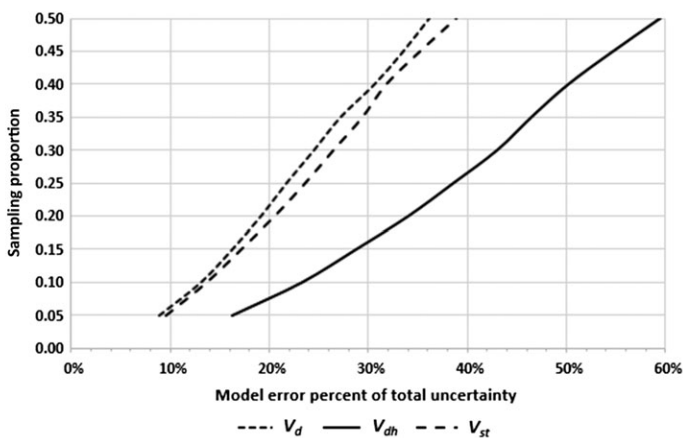

Figure 3 Model-related uncertainty percent of total variability in relation to sampling proportion for 3 methods of volume estimation (V_d, V_{dh} and V_{st}) when sampling stumps.

frequencies of species occurrence and the differences in model performance among species and (4) tree size distribution – model-related uncertainty usually increases as tree size increases. In regard to factors #1 and #2, McRoberts and Westfall (2014) found that when the proportion of variance explained by the volume models was at least 0.95 and the sample sizes were ~100 trees, the result was a negligible contribution of volume model-related uncertainty to overall error of population volume estimates. A similar conclusion would be drawn in this study, i.e. the model-related uncertainty would be inconsequential for very small sampling proportions (Figure 3). However, it is important to note that the results are highly dependent on the model formulations and resultant fits to the data. Other model formulations may perform better or worse than those used in this study. Similarly, the NE data used to fit the models were from a regional study and the variability reflects that spatial scale. Data collected at a different spatial scale (e.g. more locally) may produce different contributions of model error. Regardless of the contribution from the models, as sampling error decreases, the model-related uncertainty becomes increasingly influential in the overall variability. Small sampling errors may result from large sampling proportions, large sample sizes or estimation procedures that incorporate auxiliary information, e.g. stratified or model-assisted regression estimators (McRoberts and Westfall, 2016; McRoberts *et al.*, 2016). When the sampling error is small, foresters should consider both the parameter uncertainty and the goodness-of-fit for the model(s) being used when devising procedures to estimate the volume/value of reconstructed stands.

Factors #3 and #4 were not evaluated in this study, but the results suggest that increases in species abundance for species/groups having models of relatively poor fit/large RMSE would increase the uncertainty. For example, in the context of V_{st} (9), increased proportions of species associated with groups 6, 9 and 15 (RMSE > 0.40; Table 2) would increase the overall uncertainty; whereas less overall uncertainty would be realized with increased proportions of species from groups 2 and 5 (RMSE ≤ 0.15). Similarly, stands being comprised primarily of large trees would be subject to more uncertainty than stands with mostly small trees or a mixture of tree sizes.

In practice, a more rigorous assessment should be performed to accurately estimate stand value. Of considerable importance would be to apply species-level values, particularly when a stand is predominantly comprised of low-value or high-value species or special circumstances exist where, e.g. the large trees are of a desirable species and smaller trees are of lesser-valued species. Also, the cubic-foot to board-foot conversion would be more reliable if tree d.b.h. were taken into account. Such differentiation would make the valuation results more sensitive to species-mixes and tree size distributions. More accurate valuations may also be realized by matching the pricing units with the units of volume prediction, thus avoiding the use of approximate conversion factors such as cubic metres to board feet. Of course, the valuation undertaken here only includes the value of trees, not other potential losses such as aesthetic beauty, wildlife habitat or other environmental amenities.

This study did not evaluate uncertainty due to measurement variation. The measurement repeatability of some variables used in model construction (d.b.h., merchantable height) and the associated effects on tree volume estimates have been evaluated in other studies (Berger *et al.*, 2014; McRoberts and Westfall, 2016). While measurement of stump height and diameter is technically fairly straightforward, there likely exists greater measurement variability than might initially be expected. Also, development and subsequent application of models such as those that predict d.b.h. or volume from stump measurements often depend on accurate species identification; which is usually more difficult to assess for stumps in comparison to standing trees. However, no analysis of stump measurement repeatability is known at this time and may be an area for further study.

Conclusion

Stand reconstruction is necessary in all cases where pre-harvest tree size information was not obtained but the previous stand conditions need to be re-established. Whenever a stand is reconstructed using statistical models, an assessment of uncertainty in the predictions should be undertaken. In this study, regression analyses were conducted to obtain the parameter uncertainty and residual variability for the requisite models. Foresters using existing models should seek such information from the published papers to perform the uncertainty analyses. Although the results shown in this study indicated a relatively small contribution from model-related uncertainty, this outcome should not be assumed in all cases; particularly when using models based on small sample sizes and/or having poor fit to the data. When the number of stumps is too large to perform a complete enumeration, other important considerations are the sample design and the sampling proportion. Although the uncertainty due to the models can be small, the total uncertainty can be large due to the sampling variance.

The results of this study indicated that uncertainty due to models was the smallest when first estimating d.b.h. from the stump dimensions and then predicting tree volume from d.b.h. only. This is fortunate as the availability of models to predict d.b.h. from stump size is much greater than those developed to predict tree volume directly from stump measurements. Similarly, applicable tree volume models that use d.b.h. as the predictor variable are often easily procured. Thus, this method is recommended due to both error minimization and practicality of implementation. As the need to estimate losses from unauthorized timber harvest is nearly universal, the statistical framework described in this paper should have worldwide applicability to reconstructed stands. Conducting these types of analyses will provide an empirical basis to rigorously assess the reliability of the estimated pre-harvest volume.

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Conflict of interest statement

None declared.

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