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Application of Advection-Diffusion Routing Model to Flood Wave Propagation: A Case Study on Big Piney River, Missouri USA

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ABSTRACT: Flood wave propagation modeling is of critical importance to advancing water resources management and protecting human life and property. In this study, we investigated how the advection-diffusion routing model performed in flood wave propagation on a 16 km long downstream section of the Big Piney River, MO. Model performance was based on gaging station data at the upstream and downstream cross sections. We demonstrated with advection-diffusion theory that for small differences in watershed drainage area between the two river cross sections, inflow along the reach mainly contributes to the downstream hydrograph's rising limb and not to the falling limb. The downstream hydrograph's falling limb is primarily determined by the propagated flood wave originating at the upstream cross section. This research suggests the parameter for the advection-diffusion routing model can be calibrated by fitting the hydrograph falling limb. Application of the advection diffusion model to the flood wave of January 29, 2013 supports our theoretical finding that the propagated flood wave determines the downstream cross section falling limb, and the model has good performance in our test examples.
KEY WORDS: advection-diffusion equation, hydrograph, flood wave propagation, recession limb.

0 INTRODUCTION

Flood prediction is of critical importance given when human life and property are vulnerable to the destructive power of flooding (Middelmann-Fernandes, 2010; Younis et al., 2008; Burn, 1999). One approach to advance flood management is to better determine the physical controls on flood wave propagation, specifically the processes controlling flood volume, flood peak amplitude and time. This improved understanding of physical controls will support river management, dam design, flow regulation, and flood preparedness (Cao et al., 2011; Meire et al., 2010; Sakkas and Strelkoff, 1976).

Prior studies on flood wave propagation can be classified into experimental methods (Di Baldassarre and Montanari, 2009; Brakenridge et al., 2005) and theoretical methods (Milzow and Kinzelbach, 2010; Adamowski, 2008; Campolo et al., 1999). Experimental methods can be further classified into small scale laboratory experiments (Järvelä, 2002) and field observations (Biggin, 1996; Andrews, 1980). Small scale laboratory experiments are often implemented in flumes or designed channels with simplified and controlled boundary and initial conditions, including flow speed, volume and timing. These laboratory experiments have provided important conceptual insights on the physics of flood wave propagation, but they are not easily translated to predictions of real flood wave propagation due to the over simplified experimental conditions and the small scale. Field observations can address this issue of scale and realistic boundary and initial conditions provided, but too often insights are limited by measurement constraints.

Theoretical methods, as an alternative to experimental approaches, have been widely applied to investigate flood wave propagation. Theory is typically based on the principles of fluid mechanics, advanced simulation algorithms, and high speed computing. Physics-based river flood routing models have a wide spectrum of sophistication, from the simplest linear reservoir Muskingum method (Wurbs and James, 2002) to the full scale dynamic wave model (Saint-Venant, 1871). Although a full scale dynamic wave model can usually provide the most accurate simulations, simplified forms of the dynamic equations have been generally applied by considering the quality of given data, the availability of computational power, the fiscal constraints and the safety requirements. The dynamic wave model equations can be simplified with different levels of wave approximations, including the kinematic wave, non-inertia wave, gravity wave, and quasi-steady dynamic wave. Yen and Tsai (2001) demonstrated physically and mathematically that the advection-diffusion equation can be formulated from different levels of wave approximations of the dynamic

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wave model equations under the assumption that the wave celerity and diffusivity are step-wise constant. The advection-diffusion equation has been widely used to simulate the transport of a water wave along the runoff pathway such as a river or over land surface (Yang and Endreny, 2013; Kumar et al., 2010; Kirchner et al., 2001; Singh, 1995; Gillham et al., 1984).

Linking theoretical studies of flood wave propagation to experimental or field flow conditions is important to test theory with observation. In this study, we develop and test the advection-diffusion flood wave propagation theory using upstream and downstream field measurements of river discharge on the Big Piney River, MO. The water level data measured at the upstream and downstream cross sections of the river channel were utilized to calibrate and evaluate the model. In sections 1 and 2 below we introduce the model methodology and application; in sections 3 and 4 we present a discussion and conclusion.

1 METHODOLOGY

The generalized advection-diffusion equation defined by Yen and Tsai (2001) to describe the one-dimensional channel flow is

$$\frac{\partial q}{\partial \mathrm{t}}+c\frac{\partial q}{\partial x}=D\frac{\partial^2 q}{\partial x^2} \tag{1}$$

In Eq. (1), q [L/t] is the flow rate at a distance x [L] downstream from the point x=0 where the wave perturbation happens, c (L/t) is the kinematic wave celerity, and D [L^2/t] is the diffusivity which reflects the tendency of the water wave to disperse longitudinally as it travels downstream. Assuming c and D are constants, an analytical solution for Eq. (1) can be obtained under a unit delta perturbation (Yang and Endreny, 2013; Brutsaert, 2005)

$$q(x,t)=\frac{Px}{2\sqrt{\pi D}}t^{-\frac{3}{2}}\mathrm{e}^{-\frac{(x-ct)^2}{4Dt}} \tag{2}$$

Equation (2) is the response function of the channel under a unit trigger P [L] at the inlet of the channel. Criss and Winston (2008) also used delta function forcing of a transport equation in order to derive a theoretical hydrograph; Eq. (2) has the same functional form as their Eq. 4a, except that x has been replaced by (x–ct) in the exponent. Nevertheless, this distinction is important, as it allows the fundamental shape of the hydrograph to change downstream, which is a behavior seen in natural hydrographs along many rivers.

Equation (2) can be used as the Kernel function to simulate the output signal $g(t)$ at the outlet x=L under any time series triggers $f(t)$ at x=0

$$g(t)=\int_0^t f(\tau)q(L,t-\tau)\mathrm{d}\tau \tag{3}$$

Analytical representation of function $g(t)$ is seldom obtainable, thus a numerical calculation is generally employed, such as

$$g(n)=\sum_{m=1}^{n-1}f(m)q(n-m) \tag{4}$$

in which $g(n)$, $f(m)$ and $q(n–m)$ are discrete time series for output, input triggers and unit response (n and m are indexes for time steps).

In flood wave propagation analysis we consider how to obtain function $g(t)$ to remove inflows along the reach. For the upstream cross section boundary condition, we use the flood wave time series hydrograph as $f(t)$. If no lateral inflow or new precipitation were added to the river channel between the upstream and downstream cross sections, the hydrograph measured at the downstream cross section of the river channel should be the output flood wave $g(t)$. In this situation, which may approximate a dam break, the hydraulic properties of the flood wave, such as flow celerity and diffusivity, can be examined and obtained by the observed hydrographs at the two cross sections of the river channel. In the more common situation when lateral inflow and precipitation contribute water to the flood wave as it travels between cross sections, the downstream hydrograph is not predicted by the propagated flood wave function $g(t)$. For these conditions, our research question is how to extract the pure propagated flood wave $g(t)$ and investigate the flood propagation properties from the observed hydrographs at the upstream and downstream cross sections. We constrain this problem by considering: only lateral inflow, with no new precipitation, added to the flood wave between the two cross sections, and a small increase in watershed area between the upstream and downstream cross sections. Under this constrained condition, we postulate the downstream hydrograph recession limb can be represented by $g(t)$, and the hydraulic properties of the flood wave can be investigated by fitting the modeled recession limb to the observed hydrograph at the downstream cross section of the river channel. Figure 1 is a conceptual illustration for this assumption.

2 APPLICATION ON BIG PINEY RIVER

We tested the proposed model on the downstream section of the Big Piney River, MO (Fig. 2). Big Piney River is a tributary to the Gasconade River, which flows into the Missouri River, and then drains to the Mississippi River. Our study reach along the Big Piney River was defined using upstream and downstream USGS gauging stations, which monitored river water level and discharge (Fig. 2). The upstream cross section is at USGS gauge 06930000 (Station 1), near Big Piney, MO, and the downstream cross section is at USGS gauge 06930060 (Station 2), below Ft. Leonard Wood, MO. These USGS gauges recorded 15 minute time series water level data, available since 1999. The total drainage area for the upstream station is of 1 450 km^2 representing 94.4% of the total drainage area of the downstream watershed (1 536 km^2 at Station 2). The Big Piney watershed is part of the Ozark plateaus province, whose well developed karst features can be attributed to abundant rainfall, rugged topography and widespread units of soluble carbonate rocks (Criss et al., 2009; Vineyard and Feder, 1982).

We theorize that the discharge at the downstream cross section is generated by channel flow from Station 1 and the surface and subsurface flow from the sub-watershed area between the two stations. Surface and subsurface flow from the sub-watershed area between the two stations (1 536–1 450 km^2) are assumed to have a much lower peak amplitude and volume than the hydrographs at the upstream or downstream cross

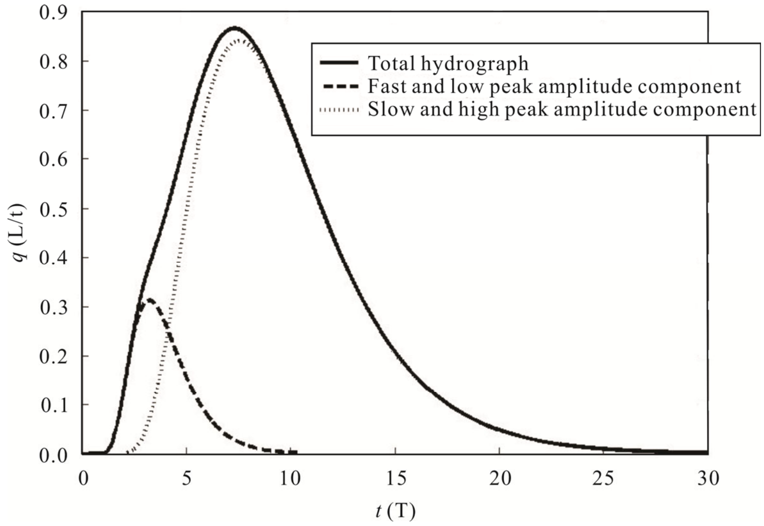

Figure 1. Conceptual illustration on the roles of the fast and slow components on determining the final hydrograph.

sections. We further theorize that the travel time for the hydrograph to arrive at cross sections can be approximated by Synder discharge lag time t_L (h) (Wurbs and James 2002): $t_L = C_t(LL_c)^{0.3}$, in which C_t is a constant ranging from 1.35 to 1.65, and L (km) is the length of the main stream from outlet to divide, and L_c (km) is the length of the main stream from outlet to a point nearest the watershed centroid. For the upstream watershed draining to Station 1, the lag time is approximately 27 h, while for the sub-watershed area between the two stations the lag time is approximately 5.5 h. Based on the ratio of the sub-watershed and Station 1 watershed lag times, we assume that the recession limb of the hydrograph measured at Station 2

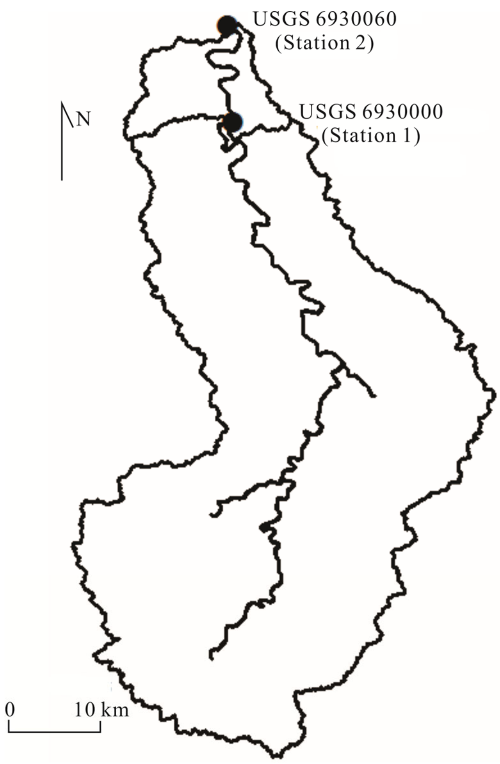

Figure 2. Watershed of Big Piney River. The river channel between Station 1 and Station 2 is our study object.

is primarily determined by the flow from Station 1.

The channel length from Station 1 to Station 2 is approximately 16 km, and the flow celerity c and diffusivity D were estimated as (Liu et al., 2003)

$$c = \frac{5}{3}v \tag{5}$$

$$D = \frac{vR}{2s} \tag{6}$$

in which v (L/t) is flow velocity; s is the slope; and R (L) is hydraulic radius. To estimate the average v in the channel between the two stations, we used Manning equation (Wurbs and James, 2002)

$$v = \frac{R^{2/3}s^{1/2}}{n} \tag{7}$$

in which n is the average river bed roughness, s is the average slope and R is the average hydraulic radius of the channel. We equated R to the average water level, assuming that channel width is much larger than the average water level.

To apply our model, we used the hydrograph measured at the upstream cross section as the input flood wave signal $f(t)$. To obtain $q(t)$ in Eq. (2) we substituted in c from Eq. (5), D from Eq. (6) and a channel length x=16 km. According to the digital elevation map (DEM) of the watershed, the elevation difference at the two gauge stations is about 15 m, and the channel length is about 16 km, so the average slope s is about 15/16 000=0.000 93. The water levels were only measured at the two gauge stations, and the water level measured at Station 1 was used to approximate the average R of the whole channel, then the only unknown parameter of the model is n.

Equation (4) was applied to simulate the flood wave $g(t)$, and parameter n for Eq. (7) was calibrated to match the recession limb of $g(t)$ and the recession limb of the measured hydrograph at the downstream cross section. We applied this model on the flood wave of January 29, 2013, which is a typical flood wave with moderate amplitude.

We obtained a manually calibrated parameter n of 0.22. The calibrated value of n is beyond the general range for natural river channels, because n also carries the uncertainty of R (we used water level at Station 1 to represent the average R for the whole channel). Therefore, the calibrated n is an effective parameter and does not represent the real roughness of the channel. The simulated hydrograph for the sub-watershed area between the two cross sections was derived using the difference of the $g(t)$ and $f(t)$ functions. This simulated sub-watershed hydrograph was similar

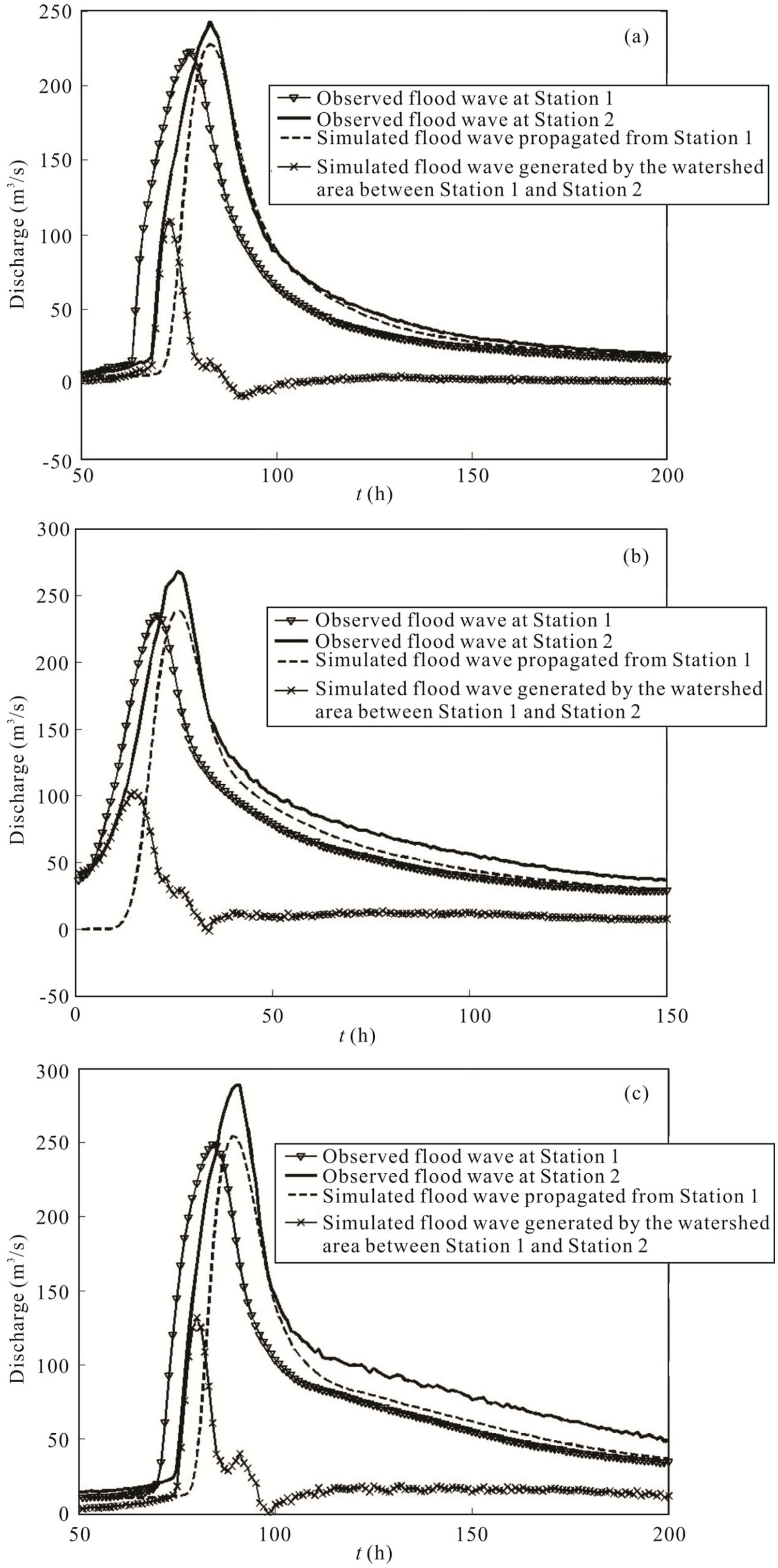

Figure 3. Observed and simulated flood waves. (a) flood wave on January 29, 2013 to February 4, 2013, of which the river bed roughness was calibrated; (b) flood wave on April 27, 2013 to May 3, 2013; (c) flood wave on November 15, 2009 to November 21, 2009.

in shape to the flood wave $f(t)$ at the upstream cross section, but with a smaller peak amplitude and smaller width (e.g., volume), and with anomalous negative discharge values. These negative values were caused when there were higher values of $g(t)$ than the observed hydrograph at the downstream cross section for some time steps. We accept these anomalies as a reasonable cost given the uncertainty of the measured hydrograph, the low amplitude of the mismatch, and the simplicity of the 1 parameter n advection-dispersion model.

After calibration, we applied the model with the same n value to two additional flood waves from April 27, 2013 and November 15, 2009, to validate the model. These two flood waves have similar peak amplitude as the one we used for model calibration. In both applications, the simulated hydrographs match our expectations, as illustrated by the conceptual plot of Fig. 1. The downstream recession limbs were dominated by the recession limbs of the upstream propagated flood waves, and not strongly influenced by the hydrographs generated by the sub-watershed area between the two cross sections (Figs. 3b and 3c). Our simulation shows that the travel time of the flood waves between the two cross sections, from Station 1 to Station 2, is about 6 hours and the peak amplitudes do not significantly change.

3 DISCUSSION

Hydrologists have recently been challenged by McDonnell and Beven (2014) to adopt an explicit and routine use of flow celerities and velocities in runoff routing model development and testing, arguing such an approach will improve our understanding of hydrological processes. Unfortunately, values for flow velocity and celerity of river discharge are not easy to measure or estimate. River discharge is the integration of both spatial and temporal information for the watershed, channel, and weather conditions. One path forward through this complexity is analysis of the discharge data combined with physics-based equations to extract important information about the river channel, watershed and weather conditions. This study demonstrates this novel approach by utilizing discharge data from upstream and downstream gauging stations to extract information on channel flood wave propagation is feasible. It has the potential to be applied as routine procedure to estimate the river channel roughness and thus determine the flow velocity, celerity and diffusivity.

4 CONCLUSION

In this study, we applied an advection-diffusion routing model based on a single calibrated parameter n to predict the flood wave at the downstream cross section of a 16 km reach of the Big Piney River, MO. The model used the observed discharge from the upstream cross section as a boundary condition. The only one parameter n was calibrated to best fit the observed hydrograph falling limbs, and the model performed well for two separate flood wave predictions. This paper presents an efficient method on studying flood wave propagation and estimating the flow velocity, celerity and diffusivity of flood waves.

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