Technical Note

Simulating Double-Peak Hydrographs from Single Storms over Mixed-Use Watersheds

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Abstract: Two-peak hydrographs after a single rain event are observed in watersheds and storms with distinct volumes contributing as fast and slow runoff. The authors developed a hydrograph model able to quantify these separate runoff volumes to help in estimation of runoff processes and residence times used by watershed managers. The model uses parallel application of two advection-diffusion equations and calibrates the model's fast and slow time parameters as well as a coefficient representing the relative size of the smaller hydrograph peak. The model provides an accurate representation of hydrograph timing, volume, peak, points of inflection, and recession rate, and its parameters represent physical processes of advection and diffusion and relate to watershed scale. The authors calibrated the model to match observed two-peak hydrographs with high efficiency on a watershed with distinct urban and rural land cover, and another watershed with distinct fast runoff from saturated areas. The Nash–Sutcliffe efficiency (NSE) of the simulated discharge was 0.93 for the urban watershed and 0.92 for the rural watershed. For the urban watershed, the simulated slow runoff volume was 89.6% of total runoff, and the fast runoff volume was 10.4% of total runoff; and for the rural watershed, the simulated slow runoff volume was 93.1% of total runoff, and the fast runoff volume was 6.9% of total runoff. This parsimonious two-peak hydrograph model can help researchers investigate how different storms and land cover types partition fast and slow flow and impact rainfall-runoff dynamics. **DOI:** 10.1061/(ASCE)HE.1943-5584.0001225.

Author keywords: Hydrograph model; Fast flow; Slow flow; Advection-diffusion equation.

Introduction

Antecedent conditions on a watershed influence runoff response and analysis of the runoff hydrograph can help inform how the watershed partitioned the precipitation into runoff, including what volumes had relatively long or short travel times. Accurate simulation of the hydrograph using watershed and precipitation inputs is important to many areas of resource management, including inference of surface and subsurface flows from fast and slow runoff times. The general shape of a hydrograph for a natural watershed is characterized by a fast rising limb, one peak flow value, and a relatively slow falling limb. Two-peak hydrographs are also observed; however, they have a relatively fast peak followed by a relative slow peak. The two distinct peaks reveal the existence of a fast runoff source and a slow runoff source contributing to the total runoff. For some watersheds, the fast runoff may be overland flow, while the slow runoff may be subsurface flow, and this can reflect mixed land, soil, and saturation properties of watersheds.

Spatially distributed hydrogeological models, such as MIKE SHE (Refshaard et al. 1995), SWMM (Huber and Dickinson 1992), and HEC-RAS (USACE 2008), can simulate multipeak hydrographs by identifying the sources area of fast flow/slow flow and assigning different flow parameters. However, these models contain many (10s–100s) parameters and include complex simulations, requiring considerable time dedicated to model parameterizing as well as significant uncertainties regarding equifinality relative to a unique set of parameters accurately characterizing the runoff response (Beven 2006). Therefore, there is a need for a parsimonious and efficient model to simulate two-peak hydrographs and analyze the partitioning of fast flow and slow flow. In this technical note, the authors propose the parallel application of the advection-diffusion hydrograph model (Yang and Endreny 2013) to simulate two-peak hydrographs and estimate the volume contributions of the two runoff sources.

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Methodology

Yang and Endreny (2013) developed the advection-diffusion hydrograph model with two time parameters $\alpha(t)$ and $\beta(t)$ relating to watershed scale x (L), flow celerity c (L/t) and flow diffusivity D (L^2/t)

$$Q_{\text{norm}} = \frac{Q}{Q_{\max}} = \left(\frac{t}{t_{\max}}\right)^{-3/2} e^{-\beta(1/t - 1/t_{\max})} e^{-1/\alpha(t - t_{\max})} \qquad (1)$$

where Q_{norm} = normalized runoff; Q = volume runoff per time (L^3/t); $\alpha(t)$ is defined as $\alpha = (4D/c^2)$; $\beta(t)$ is defined as $\beta = (x^2/4D)$; $t_{\max}$ is time to runoff peak defined as

$$t_{\max} = -\frac{3}{4}\alpha + \frac{3}{4}\alpha\sqrt{1 + \frac{16}{9}\frac{\beta}{\alpha}}$$

and $Q_{\max}$ is runoff peak defined as

$$Q_{\max} = \frac{P_{\text{eff}} \times A}{\sqrt{\pi}} \sqrt{\beta} \times t_{\max}^{-3/2} e^{-\beta/t_{\max}} e^{-t_{\max}/\alpha} e^{2\sqrt{\frac{\beta}{\alpha}}}$$

in which P_{eff} = effective precipitation; and A = area receiving P_{eff}.

Earlier research (Yang and Endreny 2013) has demonstrated the two-parameter advection-diffusion hydrograph model can simulate single-peak hydrographs efficiently when all flows contributing to the hydrograph have similar celerity c and diffusivity D. This two-parameter model is an extension of the one-parameter model of Criss and Winston (2008a) that represented subsurface flow diffusion and the rainfall-triggered pressure wave that displaces preexisting *old* pore water. The one-parameter subsurface flow diffusion model had a single time parameter and was shown to outperform alternative models on simulating the observed hydrograph properties of time to peak, runoff volume, rising and falling limb inflection points, and recession rate (Criss and Winston 2008b). The two-parameter model provides additional control of the shape of the rising limb and recession limb, or be configured to reduce to the one-parameter model. In cases with two-peak hydrographs, parallel application of the two-parameter hydrograph model can represent the two volumes of flow with distinct flow celerity and diffusivity.

The partitioning of slow runoff and fast runoff is determined by setting the relative peak amplitude coefficient of slow runoff, denoted as C, where the complimentary peak amplitude of fast runoff is $(1 - C)$, in which $0 \leq C \leq 1$. The total runoff Q for a parallel application is therefore

$$Q = C \times Q_1 + (1 - C) \times Q_2 \qquad (2)$$

in which Q = dimensionless total runoff; and Q_1 and Q_2 = normalized slow runoff and fast runoff defined by Eq. (1). It is important to note that Q is not the normalized value because the coefficient C and $(1 - C)$ lower the normalized peaks (value of 1) of slow runoff and fast runoff, and the peaks of Q_1 and Q_2 occur at different times.

Two conceptual examples of the parallel advection-diffusion hydrograph model in Eq. (2) are presented (Fig. 1). In the first example, the slow runoff parameters $\alpha_1 = 1$ and $\beta_1 = 10$ and the fast runoff parameters $\alpha_2 = 0.1$ and $\beta_2 = 1$ allow separated peaks in the combined hydrograph [Fig. 1(a)]. In the second example, the slow runoff parameters $\alpha_1 = 1$ and $\beta_1 = 10$ and the fast runoff parameters $\alpha_2 = 0.5$ and $\beta_2 = 5$ allow a merged peak [Fig. 1(b)]. Both illustrative hydrographs shown in Figs. 1(a and b) can be found in observed natural hydrographs.

The volume percentage of slow runoff Vol_{p1} can be calculated by

$$\mathrm{Vol}_{p1} = \frac{\int_0^{+\infty} C \times Q_1 dt}{\int_0^{+\infty} [C \times Q_1 + (1 - C) \times Q_2] dt} \qquad (3)$$

By integrating Eq. (3) with time, one can get

$$\mathrm{Vol}_{p1} = \frac{C \times \sqrt{\pi/\beta_1} \times t_{\max 1}^{3/2} \exp(\beta_1/t_{\max 1} + t_{\max 1}/\alpha_1 - 2\sqrt{\beta_1/\alpha_1})}{C \times \sqrt{\pi/\beta_1} \times t_{\max 1}^{3/2} \exp(\beta_1/t_{\max 1} + t_{\max 1}/\alpha_1 - 2\sqrt{\beta_1/\alpha_1}) + (1-C) \times \sqrt{\pi/\beta_2} \times t_{\max 2}^{3/2} \exp(\beta_2/t_{\max 2} + t_{\max 2}/\alpha_2 - 2\sqrt{\beta_2/\alpha_2})} \qquad (4)$$

in which $t_{\max 1}$ is the peak time of slow runoff and $t_{\max 2}$ is the peak time of fast runoff.

Applications

Application 1: Onondaga Creek Watershed (USGS 04240010) with an outlet at Spencer Street in Syracuse, New York

Onondaga Creek watershed of New York is an urbanized watershed with 229 km^2 in rural land cover and 56 km^2 in urban land cover concentrated in the lower watershed near the watershed outlet (Fig. 2). Two-peak hydrographs are commonly generated after rainstorms, and electrical conductivity monitoring in Onondaga Creek suggests each peak has a separate water source, with the first a rapid runoff from the urban area and the second slower runoff from the nonurban areas. In this study, the authors simulated the runoff event of October 17–21, 2013 with rainfall maximum intensity of 1 mm/5 min, observed at a gauge on the SUNY ESF Syracuse, NY campus. The hydrograph was recorded at 15-min intervals at the USGS gauge station 04240010. Stream electronic conductivity data were also collected at the outlet as supplementary data to characterize the two different runoff sources. The time constants α and β for fast runoff and slow runoff, the relative peak of slow runoff C, and the scale parameter were calibrated using the parameter estimation (PEST) software (Doherty 2001) to minimize the R^2. The authors' dual-application hydrograph model simulated the observed runoff with a high Nash–Sutcliffe efficiency (NSE) (Nash and Sutcliffe 1970) of 0.93 (Fig. 3). The simulated slow runoff time constant α_1 was 128.0 h and β_1 was 23.2 h; the fast runoff constant α_2 was 2.6 h and β_2 was 1.2 h, with a C value of 0.24 representing the relative peak amplitude of slow runoff. The simulation period used for this watershed provides insight on the impact of fitting a single set of time constants for two precipitation events during the same calibration. The figure shows how the model overestimated the presence of a two-peak hydrograph in the second, lower intensity precipitation event at hour 17:00. The model parameters are sensitive to the intensity of the precipitation, and the first intense precipitation event weighted the calibration for higher flow celerity and diffusivity parameters, resulting in lower time constants. These lower time constants performed relatively poorly for the second, lower intensity precipitation event. The calibration was for the entire hydrograph, and to get the best fit using the R^2, the calibrated parameters will emphasize fitting the higher peak from the first precipitation event. The peak time for fast runoff was 0.68 h, which is about 1/20 of the slow runoff peak time of 14.4 h. According to Eq. (4), the simulated slow runoff volume was 89.6% of total runoff, and the fast runoff volume was 10.4% of total runoff.

Application 2: Williams Creek near Peerless Park, Missouri (USGS 07019090)

Williams Creek watershed of Missouri has an area of 19.7 km^2 and is predominantly in forested land cover (Fig. 4); one-peak hydrographs are generally observed for this watershed. However, in extreme rainfall events the watershed has significant area in rapid runoff, perhaps as saturation excess and infiltration excess runoff, and two-peak hydrographs are observed. In this case, the first peak would be surface runoff and the second peak would be a slower subsurface runoff source. The two-peak hydrograph response was observed following the April 9, 2001, 12 mm/h rainstorm on

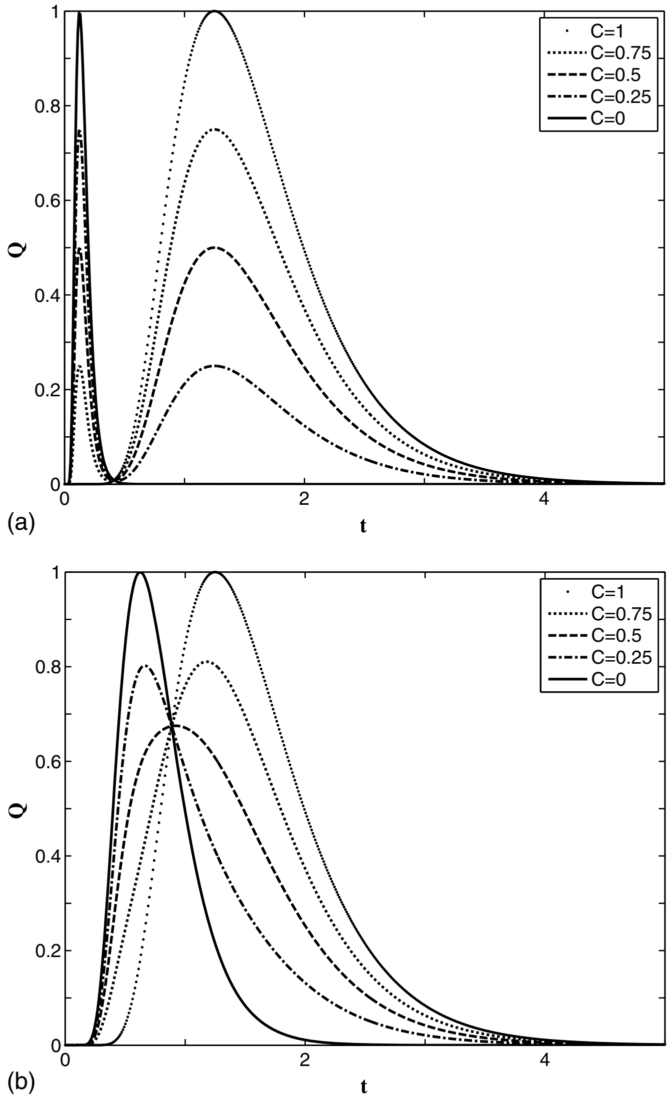

Fig. 1. Illustration hydrographs for different relative peak C of slow flow: (a) hydrographs for fast runoff parameters $\alpha_1 = 0.1$ and $\beta_1 = 1$ and slow runoff parameters $\alpha_2 = 1$ and $\beta_2 = 10$; (b) hydrographs for fast runoff parameters $\alpha_1 = 0.5$ and $\beta_1 = 5$ and slow runoff parameters $\alpha_2 = 1$ and $\beta_2 = 10$

Williams Creek, as recorded by the National Oceanic and Atmospheric Administration station at Cahokia/St. Louis (WBAN: 725314 99999) with a temporal resolution of 1 h, and the hydrograph recorded at USGS gauge 0709090. The parallel advection-diffusion hydrograph model was applied for this rainfall event, and using PEST the authors fit time constants α_1, β_1, α_2, and β_2, and C, the relative peak amplitude of slow runoff. The parallel hydrograph model Eq. (2) simulated the observed hydrograph with a high NSE of 0.92 (Fig. 5). The simulated slow runoff time constant α_1 was 17.2 h and β_1 was 194.1 h; the fast runoff time constant α_2 was 0.1 h and β_2 was 103.5 h; and the C coefficient of slow runoff to high runoff peak was 0.236. The peak time for fast flow is 3.1 h, which was about 1/4 of the slow runoff peak time of 11.1 h. According to Eq. (4), the simulated slow runoff volume was 93.1% of total runoff, and the fast runoff volume was 6.9% of total runoff. For both the New York and Missouri watersheds, the slow runoff volume dominated the total runoff hydrograph volume, but the fast runoff was distinguished by its separate peak in the hydrograph.

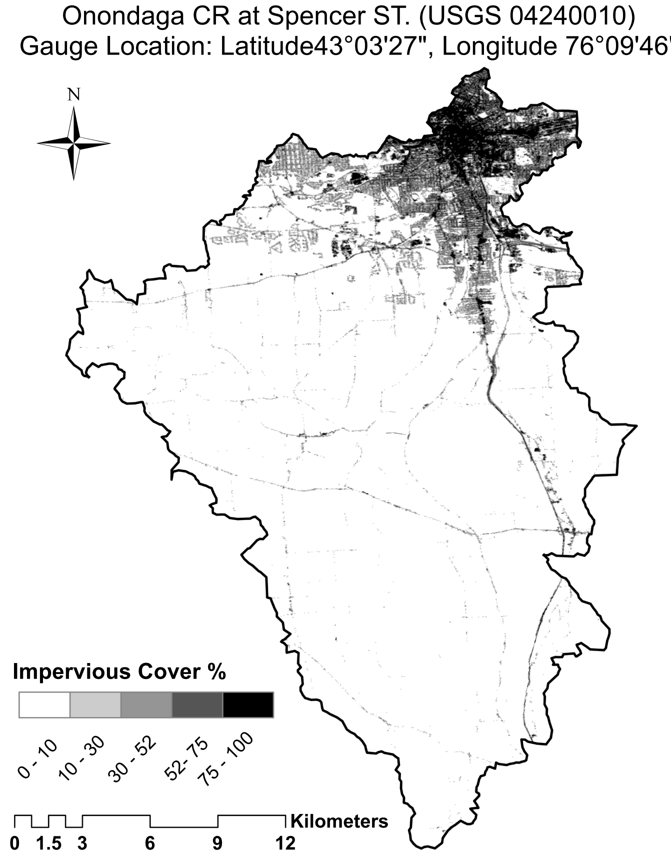

Fig. 2. Boundary and NLCD 2001 impervious cover percentages for Watershed of Onondaga Creek at Spencer Street, Syracuse, New York (data from Homer et al. 2007)

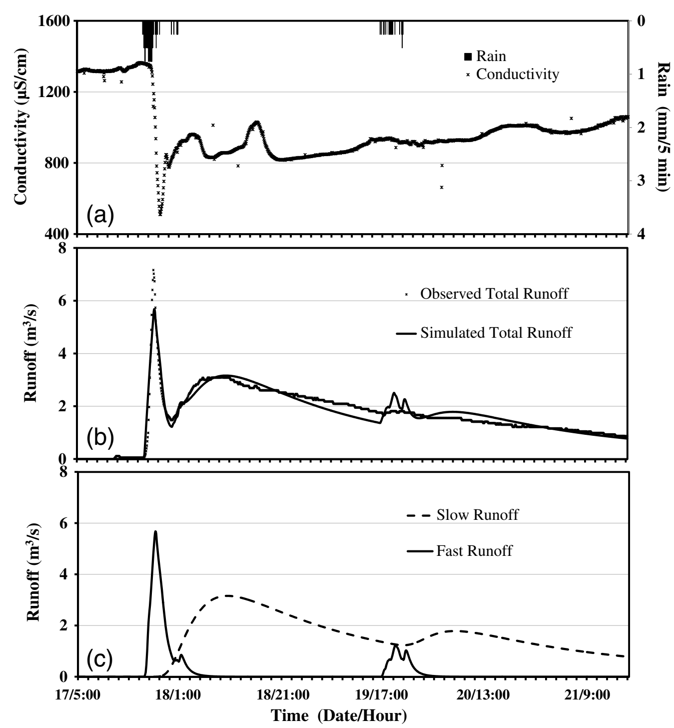

Fig. 3. (a) Observed rain and electronic conductivity at the outlet of Onondaga Creek at Spencer Street; (b) observed and simulated total runoff; (c) simulated slow runoff and fast runoff

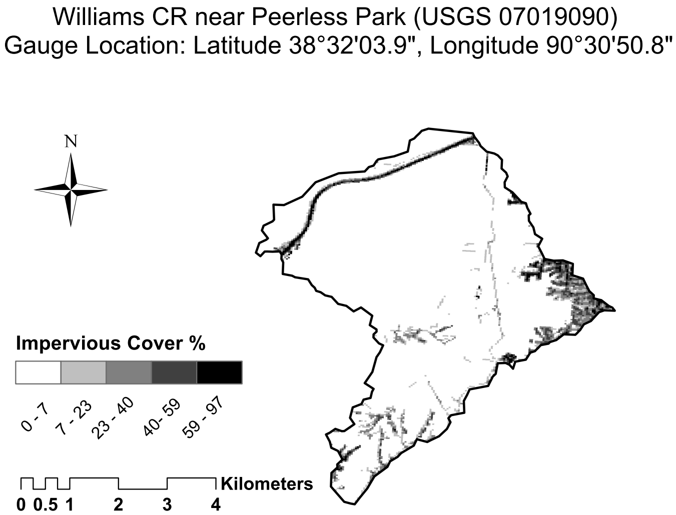

Fig. 4. Boundary and NLCD 2001 impervious cover percentages for Watershed of Williams Creek near Peerless Park, Missouri (data from Homer et al. 2007)

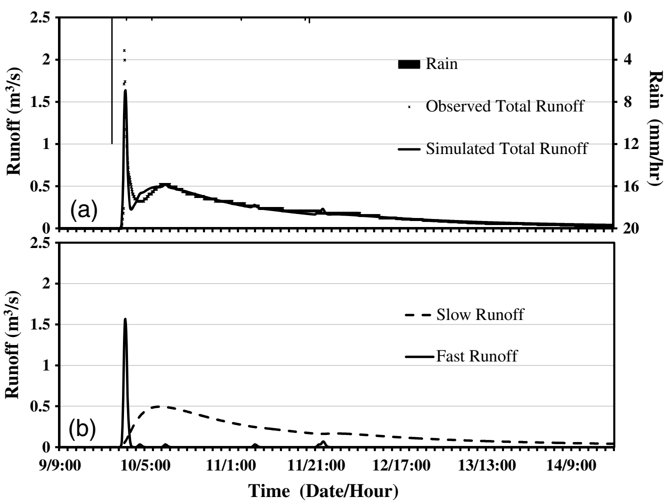

Fig. 5. (a) Observed rain and observed and simulated hydrographs for Williams Creek near Peerless Park, Missouri; (b) simulated slow runoff and fast runoff

Discussion

Two-peak hydrographs generated by single pulse precipitation events are a phenomenon that impacts water resources management by sending to receiving waters runoff contributions from different sources, with different volumes and residence times. Earlier studies by Criss and Winston (2008b) and Yang and Endreny (2013) have shown that the accurate representation of hydrograph timing, volume, peak, points of inflection, and recession rate is best achieved by routing equations based on diffusion and advection-diffusion theory. These hydrograph models have the additional benefit of using parameters representing physical processes of celerity and diffusion and calibrating with as few as two time parameters relating directly to watershed scale, flow celerity, and diffusivity. As such, the advection-diffusion hydrograph model is a natural candidate to simulate two-peak hydrographs caused by flows with distinct celerity and diffusivity.

The parallel advection-diffusion hydrograph model has the potential to be applied to separate subsurface and surface flows by assuming surface flow is fast flow and subsurface flow is slow flow. Various graphical and empirical techniques were developed to separate subsurface and surface flows in hydrographs to better understand watershed controls on runoff and predict runoff travel times, pollutant loads, and flood risk (Rinaldo et al. 2011; Smith and Ward 1998). Early techniques in hydrograph separation include recession curve analysis (McNamara et al. 1997; Wittenberg and Sivapalan 1999), and more physically based techniques involve separating hydrographs into source components using naturally occurring tracers (Hooper and Shoemaker 1986; Wels et al. 1991). However, none of the techniques can give an absolute estimation of the contribution of surface flow and subsurface flow (Joerin et al. 2002) because these two components are related and interact, such as subsurface flow changes to surface flow on saturated areas or surface flow changes to subsurface flow on unsaturated areas. As presented in the application on Williams Creek near Peerless Park, Missouri, by assuming the surface flow is fast flow and subsurface flow is slow flow, the parallel model provides a simple method to give rough estimates of the partitioning of surface flow and subsurface flow.

Conclusions

In this technical note, the authors developed a method to estimate the mixture of fast and slow runoff in two-peak hydrographs generated by a single precipitation event. The method involves combining two advection-diffusion hydrograph models, one for fast runoff and one for slow runoff, into a single equation that is efficiently calibrated by fitting time parameters based on celerity and diffusivity terms, as well as a coefficient representing the relative size of the smaller peak in the hydrograph. The model is designed for historical event simulations, allowing for analysis of the relative contribution of each runoff component; the model parameterization is highly uncertain in forecast event simulations because partitioning between the time parameters for fast and slow runoff and the runoff partitioning coefficient is dependent on antecedent conditions. Applications of the parallel model demonstrated its ability to achieve high Nash–Sutcliffe efficiencies in simulating observed two-peak hydrograph events in a rural-urban watershed and a rural watershed with variable saturation areas. The simulation results indicate that although slow runoff volume dominated the total runoff volume, the fast runoff was distinguished by its separate peak in the hydrograph due to the short flow time, and the peak amplitude formed by the fast runoff is significant higher than that formed by slow runoff; this information is important for flood control and water resource management. The application of the parallel two-peak hydrograph model to simulate fast runoff and slow runoff helps watershed managers assess how green infrastructure and stormwater interventions influence the volume of runoff in fast and slow flow, and the timing, peak, and recession characteristics of urban flood hydrographs.

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