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Optimal uneven-aged stocking guides: an application to spruce-fir stands in New England

Jeffrey H. Gove[1]* and Mark J. Ducey[2]

[1]*USDA Forest Service, Northern Research Station, 271 Mast Road, Durham, NH 03824, USA*
[2]*Department of Natural Resources, University of New Hampshire, 114 James Hall, Durham, NH 03824, USA*

*Corresponding author. Tel.: +603 8687667; E-mail: jgove@fs.fed.us

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Management guides for uneven-aged forest stands periodically need to be revisited and updated based on new information and methods. The current silvicultural guide for uneven-aged spruce-fir management in Maine and the northeast (Frank, R.M. and Bjorkbom, J.C. 1973 A silvicultural guide for spruce-fir in the northeast. General Technical Report NE-6, Forest Service. U.S. Department of Agriculture) presents two options for each of three different cutting cycles, and all guides are based on a negative exponential distribution for the number of trees by diameter class. While this guide claims to be optimal, it will be demonstrated that it is not necessarily optimal in any commonly accepted sense and is overly restrictive in its adherence to the negative exponential model of stand structure. We review a methodology that allows the objective determination of optimal stocking guides for uneven-aged stands that can be tailored to the individual stand and provides consideration for a number of alternative objectives and cutting cycles. These guides do not necessarily conform to the strict negative exponential distribution. They also suggest residual basal areas somewhat lower than those recommended by the current spruce-fir silvicultural guide for the northeastern USA.

Introduction

A common problem in uneven-aged stand management is the determination of residual stocking levels after a cutting cycle. Existing management guides, especially those that have been around for more than a few decades, are normally based on limited experimental treatments, often supplemented with the expertise of the authors accrued over years of silvicultural experience in the forest type under consideration. Many of these older guides are still in use, even though it may be difficult to trace exactly how the final stocking structures were derived, or why they were determined to be 'optimal' in some sense to begin with. Often, these guides were based on the negative exponential or *q* (Meyer, 1952; Kerr, 2014) model, because it was commonly accepted to be the natural form for uneven-aged stocking prescriptions. However, it is now well known that a *q* distribution can be an overly constraining residual target stocking model and that using it in no way imparts any sense of optimality in the resulting guide. Therefore, updating such guides using more advanced and flexible methods seems to be a reasonable undertaking.

As an example, for more than 35 years, the standard silvicultural guide for managing spruce-fir forests in the northeastern USA has been that of Frank and Bjorkbom (1973). These authors present two sets of target diameter distributions based on *q* with differing cutting cycles. The first, corresponding to the maximization of spruce-fir pulpwood, presents a distribution based on a *q* value of 1.5 for diameter classes that are 2.54-cm (1-inch) wide. The first guide has many small trees in the sapling- to pole-size classes, with a maximum stand diameter at breast height (DBH) of 39.37 cm (15.5 inches). The second set of guides encompasses three broad objectives of multiple products for the maximization of sawtimber, veneer and non-timber values for spruce-fir and possibly other species within the stand. This guide has a *q* of 1.3, carrying more large trees up to a DBH of 49.53 cm (19.5 inches). Frank and Bjorkbom (1973) (hereafter F&B) state that there was little research available on stocking and structural goals for uneven-aged spruce-fir stands at the time when they published their guide, and it can reasonably be inferred from their narration that, realizing the deficiency in knowledge at the time, their guides should be taken more as rough goals than strict rules.

This last point is quite important, because while F&B present their guides as optimal, their reasons for choosing the specific BDq (residual basal area per acre, maximum stand diameter and *q* value) combinations are never addressed. An optimal stocking guide is one that maximizes (or minimizes) some quantity under certain constraints. For example, the biological growth capacity inherent in the stand is one such constraint: a stand cannot be forced to grow beyond the physical means of the site given its inherent structure. F&B speak in terms of guides that will produce the maximum sawtimber and pulpwood production for a given cutting cycle. This is a statement of optimality – to maximize production of some quantity. Clearly F&B were motivated by optimal management of uneven-aged spruce-fir, though they lacked a formal justification for their use of these target structures in the

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sense of being mathematically optimal. However, as mentioned earlier, we must remember that what F&B wrote was a complete, thoughtful and forward-looking synthesis of what was known at the time. Furthermore, computational tools for exploring optimality that we now take for granted were not available to them and indeed were only just about to be introduced to the forestry literature.

Shortly after F&B published their guide, a landmark paper was published on the optimal management of uneven-aged stands by Adams and Ek (1974). These authors present an objective method for deducing the target stand structure in uneven-aged stands that meets the definition of optimality. Aside from the quantitative representation of the target objective in terms of the quantity to be maximized (e.g. volume growth) and the problem constraints (e.g. some target level of basal area stocking), the only requirement for their method is a simple stand-level growth model. More complex regional stand growth models (e.g. FIBER Solomon *et al.* (1995)) can also be utilized for this exercise. However, such models are often overly complex for this type of application, and the crux of the model can be surrounded with program code that applies bounds to the model from published research or expert opinion in such a way as to introduce 'discontinuities' in the model predictions when it is embedded within an optimization scheme. If we envision the search for a best or optimal stand structure as akin to searching for the top of a hill, the discontinuities are akin to cliffs and crevasses. These discontinuities can lead to problems within an optimization framework, necessitating the use of computationally more costly direct search algorithms for the problem solution (Haight and Monserud, 1990). Alternative optimization methods based on stochastic search techniques are now more popularly available and could be helpful in ameliorating such problems. However, rather than optimizing a regional growth model, in this paper we use a very simple growth model along the lines suggested by Adams and Ek (1974) and demonstrate how it can be used to tailor the target structural guide to the stand – or group of similar stands – in question. This approach requires only some permanent plot information, or the use of remeasurement data from similar stands in public data sources such as those taken by many national forest inventory (NFI) efforts like the Forest Inventory and Analysis Database in the USA (Anonymous, 2010). Whatever the source of the growth data, using either annual remeasurements or periodic remeasurements that coincide with the cutting cycle length will facilitate the optimization procedure.

As previously noted, the BDq approach in general may be overly restrictive when developing optimal target structures. There are at least two reasons for this. First, many studies have shown that uneven-aged stands, or those stands in transition to uneven-aged, often follow more of a rotated-sigmoid distribution, though the underlying mechanism determining their genesis, and even its interpretation, is still evidently an open question. Goff and West (1975) were the first to postulate that rotated-sigmoid distributions, rather than reverse J-shaped distributions, were the characteristic equilibrium distribution form in old-growth stands. Lorimer and Frelich (1984) and Leak (1996) both observed that rotated-sigmoid structures were disturbance-related. Subsequently, Leak (2002) used simulation to show that both rotated-sigmoid and reverse J-shaped distributions could be realized from different growth and mortality schedules depending on initial stand conditions. In another simulation study, Hansen and Nyland (1987) showed how rotated-sigmoid structures would result in pure sugar maple (*Acer saccharum* Marsh.) stands under uneven-aged management. Similarly, optimization studies have also found rotated-sigmoid structures for uneven-aged northern hardwood stands (Adams and Ek, 1974; Gove and Fairweather, 1992). Both the simulation approaches and the optimization approaches lead to sustainable structures. Finally, in a recent study of northern hardwoods under conversion from even- to uneven-aged management, Gove *et al.* (2008) found that the majority of stand structures were clearly rotated-sigmoid twenty-five years after initial treatment.

A second reason to question the efficacy of the BDq approach is that it may, in certain stands, restrict the structure to a distribution that is not optimal for its growth characteristics at the time. In the realm of optimal stand structures, we must find the maximum of a function – the objective function – that has a (hopefully) concave shape defined over the decision variables. When constraints are added to an optimization problem, the global unconstrained maximum can no longer be realized because the constraints act as a barrier, making the global maximum infeasible. Instead, one must find the maximum within the restrictions of the constraints. The BDq approach imposes the restriction that the optimal diameter distribution must conform to a negative exponential or *q* distribution. This constraint eliminates other structures that are potentially better in terms of the objective function, including rotated-sigmoid or other distributional forms. This was pointed out originally by Bare and Opalach (1988) and is easily verifiable with models such as the one presented here.

The objectives of this study are threefold. First, we review a method based on the optimization model of Adams and Ek (1974) that will provide target stand structures in a particular spruce-fir stand that clearly maximize some quantifiable criterion. Second, we show how these structures change in an intuitive manner depending upon which criterion is being optimized. That is, the target structures for maximizing carbon stocking may be quite different from those found by maximizing a financial objective. Indeed, Gove (1998) found that not only were the solutions to these two objectives quite different, but that they were at odds with each other. In addition, the structures derived in that study showed that adding constraints that would preserve horizontal (diameter distribution) and vertical (foliage-height distribution) diversity to that of some antecedent stand condition imposed large penalties on the final solutions. Lastly, it will be shown (but will quickly become quite apparent without much discussion) that the optimal structures when contrasted to the F&B guide provide dramatically different solutions to management of uneven-aged spruce-fir stands in New England. The key result, however, is that optimal stocking guides are indeed verifiably optimal, unlike the older guides. The methods presented here can be used on any stand where growth equations are available or can be estimated. The guides so developed are tailored to the individual stand or set of stands that the growth equations apply to and provide an objective method for determining target structures for uneven-aged management.

Methods

One of the objectives of this study is to compare the optimal structures derived from the model given below to multi-product structures in the F&B guide (their Table 4). Their structures are presented in 2.54-cm (1 inch) diameter classes. However, in terms of practical application, 5.08-cm (2 inch) or larger DBH classes become more reasonable for marking in the field; indeed, Leak and Gottsacker (1985) recommend only three classes for marking, but here we take the middle ground noting

that one can always collapse into larger classes if need be. Therefore, we first convert the F&B structure for the 5-year cutting cycle to a 5.08-cm guide as shown in Table 1, using the methods outlined in Ducey and Gove (2014). It is important to note that the smallest diameter class in this guide and each of the optimal guides presented later include small trees between 1.27 and 2.54 cm.

The stand used for demonstrating the methodology is from the Penobscot experimental forest (PEF), located in the townships of Bradley and Eddington, Maine, USA. Two compartments, or stands (9 and 16) on the PEF, have been managed by intensive single-tree selection system since 1954 and 1957, respectively. The cutting cycle for these compartments is 5 years with measurements taken pre- and post-harvest and a BDq goal of 22.96 m^2 ha^{-1}, 48.26 cm maximum DBH and 2.54 cm q value of 1.4. The species composition is 85–90 per cent softwoods and 10–15 percent hardwoods. Softwood species on these compartments include eastern hemlock (*Tsuga canadensis* (L.) Carr.), red spruce (*Picea rubens* Sarg.), white spruce (*P. glauca* (Moench) Voss), balsam fir (*Abies balsamea* (L.) Mill.), northern white cedar (*Thuja occidentalis* L.) and eastern white pine (*Pinus strobus* L.). The most common hardwood species are red maple (*A. rubrum* L.) and paper birch (*Betula papyrifera* Marsh.), with small numbers of other northern hardwood species scattered throughout. Ten- and twenty-year cutting cycles are also under compartment management at the PEF. A detailed description of these and other compartments on the PEF is found in Brissette (1996) and Sendak *et al.* (2003). In what follows we restrict our attention to Compartment 16. This stand may contain more mixedwood (hemlock and hardwoods) than a pure spruce-fir stand might allow (though the amount varies over time), and several authors (e.g. Brissette, 1996; Sendak *et al.*, 2003) have adopted the term 'Acadian' forest type (Halliday, 1937, Braun, 1950, p. 422, Rowe, 1972) to distinguish it from pure spruce-fir. Older studies on the PEF refer to the type as spruce-fir (Frank and Blum, 1978), or spruce-fir-hemlock (Solomon and Frank, 1982), indicating that the F&B guide would be applicable to such stands.

Growth model

A simple growth model was developed by Gove (1998) using the 1977–81 cutting cycle in Compartment 16, a stand of ~6.6 ha in area. The data used for model development were from 20 remeasured concentric growth plots within the stand. All trees 11.68 cm DBH and larger were measured on 0.081-ha (1/5th acre) circular plots whereas all trees between 1.27 and 11.67 cm DBH were measured on 0.020-ha (1/20th acre) circular plots. In general, the simple growth accounting for the stand in terms of numbers of stems moving from one DBH class to the next during the growth period from $t-1$ to t (i.e. some 5-year cutting cycle) is given by (Ek, 1974, Adams and Ek, 1974):

$$\begin{aligned} N_j(t) &= N_j(t-1) + I(t) - U_j(t) - M_j(t) && \text{for } j = 1 \\ N_j(t) &= N_j(t-1) + U_{j-1}(t) - U_j(t) - M_j(t) && \text{for } j = 2, \ldots, n_d \\ N_{n_d+1}(t) &= U_{n_d}(t), && \text{for the last diameter class} \end{aligned} \tag{1}$$

Table 1 Trees per hectare and net change (in TPH) for the F&B guide and Compartment 16

DBH class (cm)	TPH		Net Δ TPH[a]		Value ($)[b]	Cords[c]	Board feet[d]
	F&B	CMPT 16	F&B	CMPT 16	Per tree	Per tree	Per tree
5.08[e]	915.9	1403.6	−50.9	284.8	0.00	0.00	0.0
10.16	396.4	268.1	11.3	42.7	0.17	0.01	0.0
15.24	231.0	140.4	6.4	7.4	0.60	0.04	0.0
20.32	134.6	79.1	6.5	5.7	1.32	0.08	0.0
25.40	78.4	59.3	5.6	1.7	2.43	0.15	0.0
30.48	45.7	57.3	4.5	−1.0	5.63	0.24	89.0
35.56	26.6	40.8	3.4	2.7	8.62	0.34	129.0
40.64	15.5	9.9	2.6	8.8	11.86	0.44	168.0
45.72	9.0	0.5	1.9	4.8	15.57	0.59	214.0
50.80	0.0	0.0	4.7	0.6	20.94	0.71	283.0
Total TPH	1853.2	2058.9	−4.0	358.3	–	–	–
BPH	27.1	22.7	–	–	–	–	–
VGS ($)	1386.1	1177.4	–	–	–	–	–
VG ($)	240.2	234.0	–	–	–	–	–
LEV ($)	−516.6	−330.3	–	–	–	–	–
BFS	12 047.3	12 128.1	–	–	–	–	–
BFG	2994.6	2952.5	–	–	–	–	–
BAG	1.9	3.4	–	–	–	–	–
q	1.7	2.1	–	–	–	–	–

[a]Net change in number of trees per hectare, where growth is calculated from Compartment 16 equations.
[b]Prorated based on average per cent sawtimber per tree.
[c]From PEF local cubic foot volume table assuming 2.407 m^3 (85 ft^3) per cord.
[d]From PEF local board foot volume table.
[e]Includes trees from 1.27 to 2.54 cm.
BPH = basal area ha^{-1}, VGS = value of growing stock, VG = value growth, LEV = land expectation value, BFS = board foot stocking, BFG = board foot growth, BAG = basal area growth (m^2 ha^{-1}) and q is the diminution quotient (Meyer, 1952).

where

n_d is the number of DBH classes with $j = 1, \ldots, n_d = 9$ in this study (i.e. $j = 1$ specifies the 5.08-cm (two-inch) DBH class, $j = 2$ is the 10.16-cm (four-inch) class and so on).

$N_j(t)$ is the number of trees per hectare in DBH class $j = 1, \ldots, n_d + 1$ at time t (i.e. the end of the 5-year growth period; $t-1$ refers to the beginning of the period).

$I(t)$ is ingrowth: the number of trees per hectare growing into the 5.08-cm-diameter class during the growth period.

$U_j(t)$ is upgrowth: the number of trees per hectare growing from diameter class j to $j+1$ during the growth period.

$M_j(t)$ is mortality: the number of trees per hectare dying in diameter class j during the growth period.

For example, the first equation states that the number of trees in the smallest DBH class at the end of the 5-year cutting cycle is equal to the number at the start of the period plus the number of ingrowth trees, less the number of trees growing into the next higher DBH class and mortality.

The actual growth model fitted to the compartment 16 data and used in the dynamic accounting in equation set (1) is given as (Gove, 1998):

$$I(t) = -8014.8877 + \frac{9292.3869}{\bar{D}_q(t-1)} + \frac{35051.324}{1.471552 \log_e(\mathrm{BPH}(t-1))}$$

$$U_j(t) = 1.281562 N_j(t-1)^{0.725770} \times \frac{1.6272473 D_j^{0.477678}}{\bar{D}_q(t-1)} \qquad j = 1, \ldots, n_d$$

$$M_j(t) = 0.44282764 N_j(t-1)^{1.90044} \times \exp(-1.83953 D_j) \qquad j = 1, \ldots, n_d$$

where $\bar{D}_q$ and BPH are the quadratic mean stand diameter (cm) and basal area ($m^2\ ha^{-1}$), respectively, for all trees larger than 1.27 cm (0.5 inches), and D_j is the midpoint DBH for class j. It is worth re-emphasizing that this growth model is specifically for Compartment 16 during the 1977–81 cutting cycle, and as such, it has a limited applicability beyond the conditions present in this stand. Therefore, we caution that the model may not extrapolate well to other stands. In particular, the ingrowth equation as parameterized can predict negative numbers of trees for larger $\bar{D}_q$ and BPH, which is specifically accounted for in the optimization model as the solution algorithm searches over the ranges of decision variables for the optimal solution. Alternative formulations for this simple accounting growth model can be found, e.g. in Ek (1974) and Adams and Ek (1974), which can serve as a good starting point for other stand-based models.

Optimization model

The growth accounting system (1) is used in conjunction with some quantitative objective function for a given diameter distribution within an optimization setting. In this sense, the values of the diameter distribution (decision variables) are varied by the optimization algorithm to find the distribution that provides the maximum value of the objective. This can be formally stated as a nonlinear programming model, as first formulated by Adams and Ek (1974):

$$\underset{N_1, N_2, \ldots, N_{n_d}}{\text{Maximize}} \quad F(N_1, N_2, \ldots, N_{n_d}) \tag{2}$$

$$\text{St}: \quad N_j(t) - N_j(t-1) \geq 0 \qquad j = 1, \ldots, n_d + 1 \tag{3}$$

$$N_j \geq 0 \qquad j = 1, \ldots, n_d \tag{4}$$

$$\mathrm{BPH} = B \qquad \text{(optional)} \tag{5}$$

The above formulation is interpreted as follows. In equation (2), the objective is to maximize some function F, which takes its value based on the diameter distribution subject to three sets of constraints. The objective functions that we will consider here are presented below. Constraint equations (3) simply state that the growth must be positive in each DBH class in terms of number of trees. This set of constraints are called the 'sustainability constraints' because if the growth were to be negative in any diameter class, the distribution would not be sustained over time given the current growth dynamics. These are coupled with the following constraints, (6), which prohibit negative numbers of trees in each DBH class. The final constraint, (5), allows us to set some level of basal area per hectare, B, that the optimal guide must meet if so desired. This is the only optional constraint, and we will see how relaxing it will improve solutions.

We consider three different objectives. The first is a financial objective that produces what Adams (1976) referred to as 'investment efficient' guides (when optimized at different levels of value stocking), by maximizing the land expectation value (LEV), *viz.*

$$\mathrm{LEV} = \frac{\mathrm{VG}}{((1+a)^{n_c} - 1)} - \mathrm{VGS} \tag{6}$$

where

$$\mathrm{VG} = \sum_{j=1}^{n_d+1} V_j N_j(t) - \mathrm{VGS}$$

$$\mathrm{VGS} = \sum_{j=1}^{n_d} V_j N_j(t-1)$$

additionally $a = 5$ per cent is the alternative rate of return, $n_c = 5$ is the cutting cycle length in years and VG is the 5-year value growth determined using the tree values (V_j) in Table 1 and is the value of growing stock.

The last two objectives are board foot stocking (BFS) and periodic board foot growth (BFG). Note that the board foot content of a tree is defined as a prediction of the yield of rough, green lumber if the tree were to be sawn into uniform products; a board foot is 0.00236 m^3 of lumber (Husch *et al.*, 2003, p. 201). Both the stocking and growth are calculated in the usual way from the tree list and net change information (e.g. Table 1) using local volume tables from the PEF. Many other objective function values could have been chosen; these might include basal area growth, biomass or carbon stocking, cords or cubic metre volume and the like. Alternatively, as mentioned earlier, some of these objectives could also be couched as constraints depending on the goals of the optimization. For example, one might want to carry a given minimum level of carbon or preserve the amount of carbon in the stand compared with some benchmark stand. In this case, the carbon would be another simple inequality constraint. The two board foot objectives were chosen specifically to be in accordance with F&B's multi-product guide and idea of maximizing the production of high-value timber products. The LEV objective provides a financial alternative that, as we will show, does not necessarily conform well to the same goal given the tree values used here. Maximizing BFS is very close to maximizing value of growing stock (VGS) with the tree values given in Table 1 because these values are so heavily weighted towards the larger diameter trees that contain the most board foot volume. Therefore, BFS should be considered a very good objective to matching that of F&B.

It should be noted that there are other approaches to the stand-level optimization problem that are often considered to be more generally applicable because they take the transition strategies to some equilibrium or steady-state structure into account over fixed or infinite time horizons (cutting cycles). The majority of these methods are some adaptation or extension of either fixed endpoint (FE) or equilibrium endpoint (EE) problems (e.g. Getz and Haight, 1989, Chapters 3 and 5). For example, the steady-state investment-efficient solution can be viewed as one choice for the terminal state in fixed-endpoint problems (Haight and Getz, 1987; Getz and Haight, 1989, p. 269). We adopt the simpler model in equations (2–5) for two reasons. First, the optimality theory that underlies both the FE and EE methods is based on discounted economic returns over (normally) more than one cutting cycle, establishing transition regimes that can often

span many decades. In contrast, our goal is to provide stocking guides that are applicable to the current cutting cycle. Second, our desire is to determine a stand structure that is reasonable for the growth dynamics and markets associated with the current cutting cycle that is close in spirit to the F&B guide, where the maximization of volume stocking or growth are our main objectives. The LEV objective is included in the analysis to elucidate trade-offs in stand structure that can result from a net present value form of objective, as will become clear in the next section. A recent review of the literature on this subject can be found in Hyytiäinen and Haight (2012).

Results

The methods of the previous section were used to look at two different scenarios, each encompassing three solutions corresponding to the three objectives BFG, BFS and LEV. In the first scenario, constraint (5) is applied, fixing the level of basal area per hectare in the stand to that of the F&B guide. For the 5.08-cm guide, this is 27.1 m^2 ha^{-1} as shown in Table 1. In the second scenario, we relax this constraint and find the solution that also yields the natural basal area that is inherent in the stand given the growth equations for the particular objective. In what follows, we contrast the optimal solutions for each scenario and against the F&B guide.

Comparison to the F&B guide

First, however, note the differences between the F&B guide and Compartment 16 as shown in Table 1. The former carries almost 5 m^2 ha^{-1} of basal area more than the latter. While Compartment 16 has 50 per cent more trees in the 5.08-cm class, the F&B guide carries higher numbers of trees in all other DBH classes except the small sawtimber, contributing to the higher basal area stocking. These differences are a consequence of the q values for each stand, though the q for Compartment 16 was fitted by least squares and is therefore an average for the stand. The overall VG and VGS are relatively comparable between the two stands; however, the latter is higher in the F&B guide because of the larger trees that are present in the guide. What may not be apparent at this point is why both stands have a negative LEV; we will address the reasons for this in section Scenario I: basal area constrained. Finally, the F&B guide is not a sustainable target given the current growth of Compartment 16, due to the negative growth in the smallest DBH class. This illustrates one of the values of some stand-based growth information. If one were considering trying to increase the basal area of Compartment 16 while using the F&B guide, and if the growth dynamics remained similar (which they may not), the negative sustainability constraint in the 5.08-cm class warns that the ingrowth is not sufficient to balance upgrowth and mortality in this class. Eventually, the stand would have to be managed under some other target distribution because it is unsustainable. Seymour and Kenefic (1998) have also suggested that these recommended levels of basal area retention may not allow adequate recruitment and growth of trees in small size classes and may be difficult to sustain without modification. The small amount of negative growth in the 30.48-cm (12-inch) class of the Compartment 16 guide is inconsequential in the context of managing this stand. The stand is sustainable (or could be simply adjusted to be so) for all practical purposes given its current growth dynamics and therefore is being managed at a more reasonable target BDq.

Scenario I: basal area constrained

The results of the first scenario are presented in Table 2. This table shows the optimal diameter distributions as the solution to the maximization problem (2–5) given the growth model for Compartment 16, for each of the three objective functions. Figure 1 presents the target diameter distributions for each solution graphically. Note, however, that because of the large number of trees in the 5.08-cm class, only the 10.16-cm and larger classes are shown. The first point to notice is that the target structures are all sustainable, and all have a basal area equal to the F&B guide as determined by constraint (5). Because they are sustainable, each could be a reasonable target structure for Compartment 16 if we desired to manage the stand at higher basal area stocking level. However, that is where the similarities for the three solutions end. The BFS solution carries the lowest number of total TPH and attempts to equilibrate the DBH distribution for trees >12.7 cm by carrying almost as many trees in the large sawtimber classes as in the small sawtimber classes. This is in stark difference to the other two solutions. In addition, this solution is the only one with negative LEV as in the F&B guide. The reason for this, and the answer to the question posed earlier, is found by a simple rearrangement of equation (6) as follows: $\text{LEV} \propto \text{VG} - \text{HC}$, where $\text{HC} = ((1+a)^{n_c} - 1) \times \text{VGS}$ are holding costs (Adams, 1976; Martin, 1982). In this form, we can see that the holding costs add a penalty to the objective function such that if they are larger than the five-year value growth, then the LEV will be negative. The holding costs are directly associated

Table 2 Optimal guides for PEF Compartment 16 scenario I: BPH constrained to the F&B guide stocking level

DBH class (cm)	TPH			Net Δ TPH[a]		
	BFG	BFS	LEV	BFG	BFS	LEV
5.08[b]	2751.7	1835.9	4133.7	0.0	0.0	0.0
10.16	353.0	607.2	1472.7	99.3	35.3	84.5
15.24	163.0	55.6	66.2	13.9	46.0	142.1
20.32	134.9	46.0	54.7	0.0	0.0	0.0
25.40	116.5	39.7	47.3	0.0	0.0	0.0
30.48	23.1	35.2	6.6	20.8	0.0	15.4
35.56	10.4	31.8	2.6	4.2	0.0	2.5
40.64	9.5	29.1	2.3	0.0	0.0	0.0
45.72	8.8	26.9	2.2	0.0	0.0	0.0
50.80	0.0	0.0	0.0	6.4	12.5	2.9
Total TPH	3570.9	2707.4	5788.2	144.6	93.9	247.4
BPH	27.1	27.1	27.1	–	–	–
VGS ($)	1088.6	1531.1	597.8	–	–	–
VG ($)	312.6	296.0	269.2	–	–	–
LEV ($)	43.0	−459.8	376.4	–	–	–
BFS	6879.8	17 897.1	1773.0	–	–	–
BFG	4205.8	3545.8	2520.9	–	–	–
BAG	1.6	1.2	2.5	–	–	–

[a]Net change in number of trees per hectare.
[b]Includes trees from 1.27 to 2.54 cm.
BPH = basal area ha^{-1}, VGS = value of growing stock, VG = value growth, LEV = land expectation value, BFS = board foot stocking, BFG = board foot growth, BAG = basal area growth (m^2 ha^{-1}).

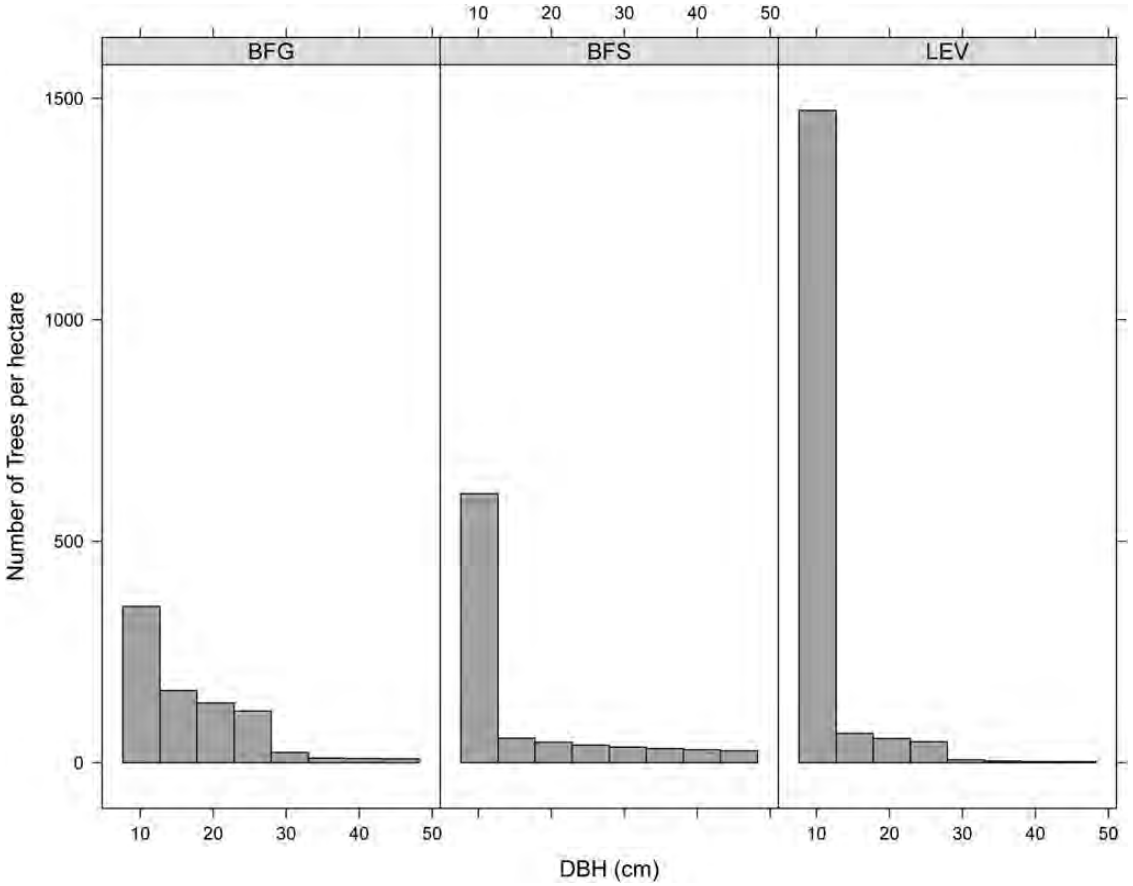

Figure 1 Scenario I diameter distributions for each objective when BPH is constrained to that of the F&B guide (5.08-cm class not shown; see Table 2 for this value).

with the discounted value of trees – specifically large trees. The more large trees in the DBH distribution, the greater the penalty the holding costs impose on LEV. Both the F&B guide and the optimal BFS solution seek to carry more large trees in the stand (a natural goal when seeking to maximize board foot volume), which drives the LEV negative; this is also true of Compartment 16 (Table 1) for this particular cutting cycle.

The optimal LEV solution in Table 2 is at the other end of the spectrum from the optimal BFS solution. Notice that this solution carries twice the number of overall TPH as the BFS solution and shifts as many trees as possible into the smallest diameter classes. The result is only 1773 board feet per hectare, less than one tenth that in the BFS solution. This shift, in the sense of being polar opposite solutions, is due to the fact that when we seek to maximize LEV, mathematically what we are doing is in essence minimizing the holding costs (through the VGS). As noted earlier, to do this, simply get rid of all the retained value in the stand, i.e. the high-volume, high-value larger diameter trees. The optimal BFG solution is somewhat of a compromise between the two, not because of any economic constraints, but because the small-to-medium sawtimber trees are accruing the highest growth in board foot volume. The resulting stand has a large number of trees in the 5.08-cm class, but from there, the distribution is putting trees into position to attain BFG when they reach the sawtimber size classes. This solution is trivially positive in LEV and, as seen, has the highest overall BFG of the three solutions as it should, while carrying a more moderate amount of BFS than the optimal BFS solution. The net change shows that we could harvest ~25 trees ha^{-1} in the 30.48- to 35.56-cm classes in addition to 6.4 trees 50.8 cm in diameter every five years. The optimal BFS solution forgoes the harvest of small sawtimber, instead setting the harvest of 13 trees in the 50.8-cm class each cycle. Note again from our growth accounting equations (1) that all trees that grow into the 50.8-cm class are destined to be cut at the end of each cycle. The LEV solution has the lowest VG and VGS of all three solutions but still has some VG to cut at the end of the period.

Scenario II: basal area unconstrained

The two compartments under selection system with five-year cutting cycles at the PEF show a remarkably stable residual target of ~23 m^2 ha^{-1}, with pre-harvest basal area reaching ~28 m^2 ha^{-1} on average over 40 years of management (Sendak *et al.*, 2003). In the second scenario, we therefore recognize that the 27 m^2 ha^{-1} residual basal area suggested by the F&B guide (5.08-cm version used here) may be too optimistic for this stand. Therefore, the basal area constraint (5) was dropped in the optimization model and the three optimizations repeated with basal area allowed to take its natural level based on the underlying

biology of the system. The results of the three solutions for this scenario are shown in Table 3 and Figure 2. The first thing to notice about all three solutions is that, with regard to their individual objective function values (BFG, BFS and LEV), the solutions provided in Table 3 are indeed all higher than those presented in Table 2. Again, this is due to lifting a constraint on the objective function surface, which allows us to venture to a higher solution. The second general observation is that these three solutions do indeed settle to a lower overall basal area per hectare than that recommended by the F&B guide. The basal areas are all within ~3 m^2 ha^{-1} of the current residual stand in Compartment 16 and therefore would be reasonable target structures for this stand – more so than those solutions in Table 2.

Several other trends in the second scenario are similar to those in the first. For example, in the optimal BFS solution, we find that again, the solution has tried to equilibrate the diameter distribution such that it has become more even than scenario one. On the other extreme, the LEV solution has again been more successful at packing more small trees into the stand at the expense of larger, more valuable stems. While the basal area per hectare is 4 m^2 ha^{-1} higher in the BFS solution as compared with the LEV solution, the latter has six times the number of TPH. The VGS is slightly higher for the two board foot objectives in the unconstrained solutions, whereas that for LEV has decreased by half, reflecting the movement of trees into the smaller classes. This latter adjustment for LEV is again in keeping with our previous observation that maximizing LEV is akin to minimizing holding costs, and the solution does this by decreasing the value of standing growing stock in the target distribution. Likewise, the VG that one is able to cut during the cycle has changed in each solution: BFG is higher, but BFS is lower, whereas the LEV solution remains about the same (the increase in the optimum value coming at the expense of VGS). Finally, the BFG solution is again somewhere in between the other two for the second scenario. This solution again produces the highest level of value growth for the three solutions (the quantity Adams and Ek (1974) used as an objective), by cutting trees in the small- to mid-sawtimber sizes where growth is fastest in the sawtimber class. Other comparisons can certainly be made between the two scenarios, the original target structure and the F&B guide and are left to the reader to explore further.

Table 3 Optimal guides for PEF Compartment 16 scenario II: BPH not constrained to the F&B guide stocking level

DBH class (cm)	TPH			Net ΔTPH[a]		
	BFG	BFS	LEV	BFG	BFS	LEV
5.08[b]	3944.6	853.2	7369.4	0.0	0.0	0.0
10.16	141.4	88.2	62.5	187.4	28.1	471.8
15.24	108.3	67.6	41.8	0.0	0.0	2.0
20.32	89.6	55.9	34.6	0.0	0.0	0.0
25.40	77.4	48.3	29.9	0.0	0.0	0.0
30.48	31.4	42.8	7.5	11.5	0.0	11.7
35.56	15.9	38.7	3.3	5.2	0.0	3.1
40.64	14.6	35.4	3.1	0.0	0.0	0.0
45.72	13.5	32.8	2.8	0.0	0.0	0.0
50.80	0.0	0.0	0.0	9.9	10.3	4.7
Total TPH	4436.7	1262.8	7554.8	214.0	38.4	493.4
BPH	25.9	24.9	20.6	–	–	–
VGS ($)	1092.3	1751.4	304.9	–	–	–
VG ($)	349.1	220.3	272.3	–	–	–
LEV ($)	171.2	−954.1	680.6	–	–	–
BFS	10 185.5	21 764.4	2213.3	–	–	–
BFG	4500.7	2912.6	2770.1	–	–	–
BAG	1.9	0.6	3.4	–	–	–

[a]Net change in number of trees per hectare.
[b]Includes trees from 1.27 to 2.54 cm.
BPH = basal area ha^{-1}, VGS = value of growing stock, VG = value growth, LEV = land expectation value, BFS = board foot stocking, BFG = board foot growth, BAG = basal area growth (m^2 ha^{-1}).

Discussion

The optimization approach to uneven-aged management provides an alternative way to specify the target distribution when strict adherence to a *q* distribution may not be tenable or desired. In the solutions, we have demonstrated that the optimal guides do not conform to a *q* distribution, but rather, have distributions that can be far from what would be considered 'balanced' in terms of the *q* model. It is interesting to note at this point that actually the F&B guide did not do too bad of a job at attaining its ostensible goal of maximizing high-value product yield. Note in Table 1 that the F&B growing stock value (VGS) is higher than what is being maintained in Compartment 16 during the growth period in this analysis, although one could logically argue that this is due to the higher residual basal area stocking in the F&B guide. Comparing the F&B guide with the optimal distributions from the first scenario (Table 2), we see that only the BFS objective exceeded the F&B guide with regard to VGS. So, for a guide carrying the same level of basal area stocking, with the restrictions of a *q* model, the F&B guide is certainly not a failure – in fact these authors did quite well. One could indeed constrain the solutions in the first scenario even further by imposing a negative exponential distribution restriction on the solution and see just how close the F&B guide would be to that solution if we chose to maximize VGS rather than BFS. In the set of solutions in the second scenario, we performed experiments along these lines; for example, simply by maximizing BFS as a surrogate of growing stock value, we find that the lower basal area solution in Table 3 has substantially higher VGS than any of the other distributions presented, including the F&B guide and the Compartment 16 stand distribution. Such experiments offer strong commendation of the optimization approach with regard to its inherent flexibility.

One point that we have not addressed is that of the distribution of stems that require harvesting in the optimal guides. It was noted earlier that the F&B guide could not be applied to the Compartment 16 stand as long as the growth dynamics stayed at the same levels as the current period analysed because the distribution is not sustainable (one would have to plant trees to make up for the negative net change in the 5.08-cm class). In contrast, the actual residual stand structure for Compartment 16 shows a reasonable net change where cutting would be required to some level in almost every diameter class, with the most in the small trees. The optimal guides, however, show that only select diameter classes have accruals that will require harvests. Often, a few trees are

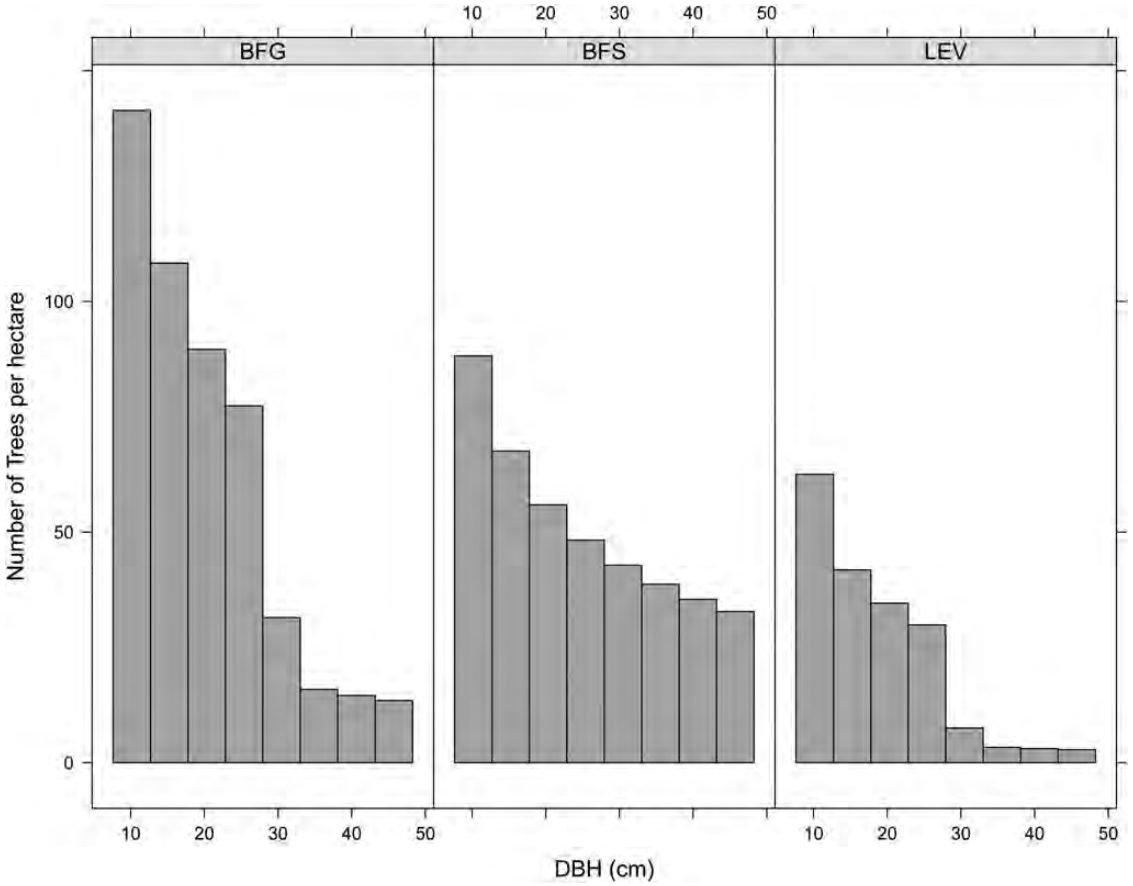

Figure 2 Scenario II diameter distributions for each objective when BPH is left unconstrained (5.08-cm class not shown; see Table 3 for this value).

harvested from the mid-to-larger diameter classes, with many trees requiring cutting in the 10.16-cm class and none in the 5.08-cm class. The lack of cutting in the 5.08-cm class speaks to the balance that the class is in with respect to ingrowth. But since this is a managed stand, the ingrowth equations already reflect mortality due to average harvesting damage, as do the upgrowth and mortality equations. Thus, there may be a need under these guides to do a small amount of thinning in some of the lower classes to make up the difference. However, the optimization approach could also be used to fine-tune the optimal DBH distribution in such a way that we balanced out some of the accumulation of growth in the 10.16-cm class by adding a few more constraints. Again, this would modify the final objective and optimal distribution, but such constraints are admissible within the inherent limits of the stands biological growth capacity to attain them. In addition, one must consider the impact of the optimal distributions on the residual species mix within the stand with regeneration of more tolerant species favoured in those solutions that carry more large trees (e.g. BFS), while favouring intolerant species in the LEV solutions that carry fewer trees over 25 cm.

According to Sendak *et al*. (2003), the original study plan for Compartment 16 called for a residual basal area of 26.2 m^2 ha^{-1}, though the initial stand basal area prior to treatment was closer to 27.5 m^2 ha^{-1} (this is an average of both compartments for this treatment, see Sendak *et al*. (2003), Table 6). The pre-harvest inventories given by these authors also showed that the stands grew to somewhat above the 27.5 m^2 ha^{-1} mark by the end of the 5-year cutting cycle. Taking these facts into consideration, the higher basal areas of the first scenario may not be unreasonable for this stand. Species mix, stocking and a changing diameter distribution will all work to change the growth dynamics for the stand by the time it was managed at that target level. At that point, new growth equations would be required and another round of optimizations would need to be undertaken to find the new target structure. However, given the species mix and growth rates of Compartment 16 for the time period used in this study, the solutions in the second scenario are more appropriate.

The cycle of continuous monitoring in the form of updating the growth equations at the end of a cutting cycle, re-optimization and fine-tuning of target structures for the next cycle does have some commendable points. It does not tie future management into a strict regime of applying the same treatment repeatedly, when we know that both endogenous and exogenous factors will contribute to change things like species mix, growth rates, markets, ownership, to name only a few, over a period as brief as a 5-year cutting cycle. These factors may affect not only our constraints on the target structure, but indeed even the target objective to which we manage over time. Relatedly, the target structure must be attainable from the current stand diameter distribution, not only in terms of basal area stocking, but in terms of having enough trees present in each diameter class to attain the target goal. Adams and Ek (1974) presented a second optimization model that showed how

to determine the optimal transition strategy over a period of cutting cycles. In addition, Haight *et al*. (1985) provided alternative methods for the transition cutting schedule. In general, these approaches take more than one cutting cycle and therefore may not be appropriate for the cutting cycle update scheme just discussed. However, extensions to these methods can be made that will facilitate the single cycle scheme.

The LEV objective was included in the optimal scenarios because one might associate maximization of value, as in the F&B guide with regard to high-value products, with a discounted set of investment returns. As mentioned earlier, Adams (1976) terms these guides investment-efficient, because they yield the maximum value growth over the cutting cycle for initial *value* stocking levels, but not necessarily physical stocking levels. The use of LEV conflicts with the update or adaptive approach given earlier because it assumes that once a stand is in the target structure, it remains in that equilibrium condition for all future cutting cycles. However, because management conditions change based on factors such as those mentioned earlier, tree values will change with markets and so will the maximum LEV level – simply, the equilibrium interpretation may be somewhat unrealistic in practice. It should also be noted that where a different set of tree values used with less value for larger trees, the optimal LEV solution would be more in line with (less penalty than) the other solutions. An example where using less disparate tree values over all DBH classes led to LEV solutions that were closer to other objectives can be found in Adams (1976).

The optimal structures derived here are guides in a double meaning of the word. Obviously, they are each a target structural guide for Compartment 16 on the PEF. In addition, they should be looked at as being somewhat flexible since it is impossible to attain partial numbers of trees as dictated in the solutions. Gove and Fairweather (1992) showed how simple stand variability will change the growth equations within the stand, had the growth plots been established at alternate locations. This in turn affects the optimization model solution. They presented a method for determining a sort of confidence interval on the optimal distribution, just like on a stand inventory estimate. Practically speaking, there is, therefore, a range of optimal distributions that are close to what we presented here, when we consider things like sampling variability in the model generation and solution. Departure from the exact optimal solution might lead to slight negative values in net change when viewed strictly from the current growth equations, but the point is there is enough uncertainty in both the growth equations and tree values to allow some leeway when implementing the guides. That one reason is why the slight negative growth in the 30.48-cm class of the Compartment 16 distribution is not of practical concern for the management of this stand. Indeed, uncertainty in growth equations and tree values provide compelling arguments for the iterative updating, re-optimization and fine-tuning approach suggested earlier even if we were to expect management objectives to remain relatively stable. Given this stochastic leeway then, it is fairly clear that only a slight modification of the current Compartment 16 distribution (Table 1) at harvest for one or two cutting cycles would allow conversion to the unconstrained BFS guide, if that were the objective of management. This would be accomplished by cutting the excess trees in the 30.48-cm and lower classes, while allowing the stock to build in the 35.56- to 45.72-cm classes. Conversion to the other optimal guides, if desired, would be more problematic and likely take several cutting cycles, again advocating for the adaptive scheme.

Summary and conclusions

We have presented a review and application of a well-tested methodology that provides a reasonable alternative to the standard BDq approach to the management of spruce-fir stands in the northeast under the selection system. The method used here requires more information than one might normally be able to provide for an individual stand in the form of a set of growth equations. But as we mentioned, there are alternatives to establishing growth plots within every stand such as setting up a network of plots in similar stands (which many larger landowners already have) or using NFI plots from similar stands. We consider these latter two options more reasonable than perhaps optimizing a more general growth model, but that is possible as well with constraints used to match the site condition within such models.

The optimal distributions presented for Compartment 16 at the PEF point to two conclusions. First, a lower basal area stocking level than the F&B guide is appropriate, commensurate with the level at which the stand is currently managed (in the 1977–81 growing cycle). Second, the optimal distributions depart from the traditional BDq model while showing improvements in every stand parameter discussed over both the current target guide for the stand and that of the F&B guide.

Many other possible objectives are available for use in the optimization approach; essentially any quantity that is a function of the decision variables (i.e. the diameter distribution) can be used in the objective or the constraints. For example, horizontal structural diversity is a function of the diameter distribution and can be either maximized or maintained when viewed as an object or constraints, respectively (Gove *et al*., 1994; Gove *et al*., 1995). And some constraints can be viewed loosely as fixed alternative objectives, like the physical basal area stocking level in that used in the first set of scenarios. With the current speed of personal computers and the availability of good nonlinear optimizers, solutions to the basic optimization problem can be found in microseconds. This allows the easy exploration of any number of alternative objectives that might be considered reasonable for management as part of the normal cycle of updating management plans.

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Conflict of interest statement

None declared.

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