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Merchantable Sawlog and Bole-Length Equations for the Northeastern United States

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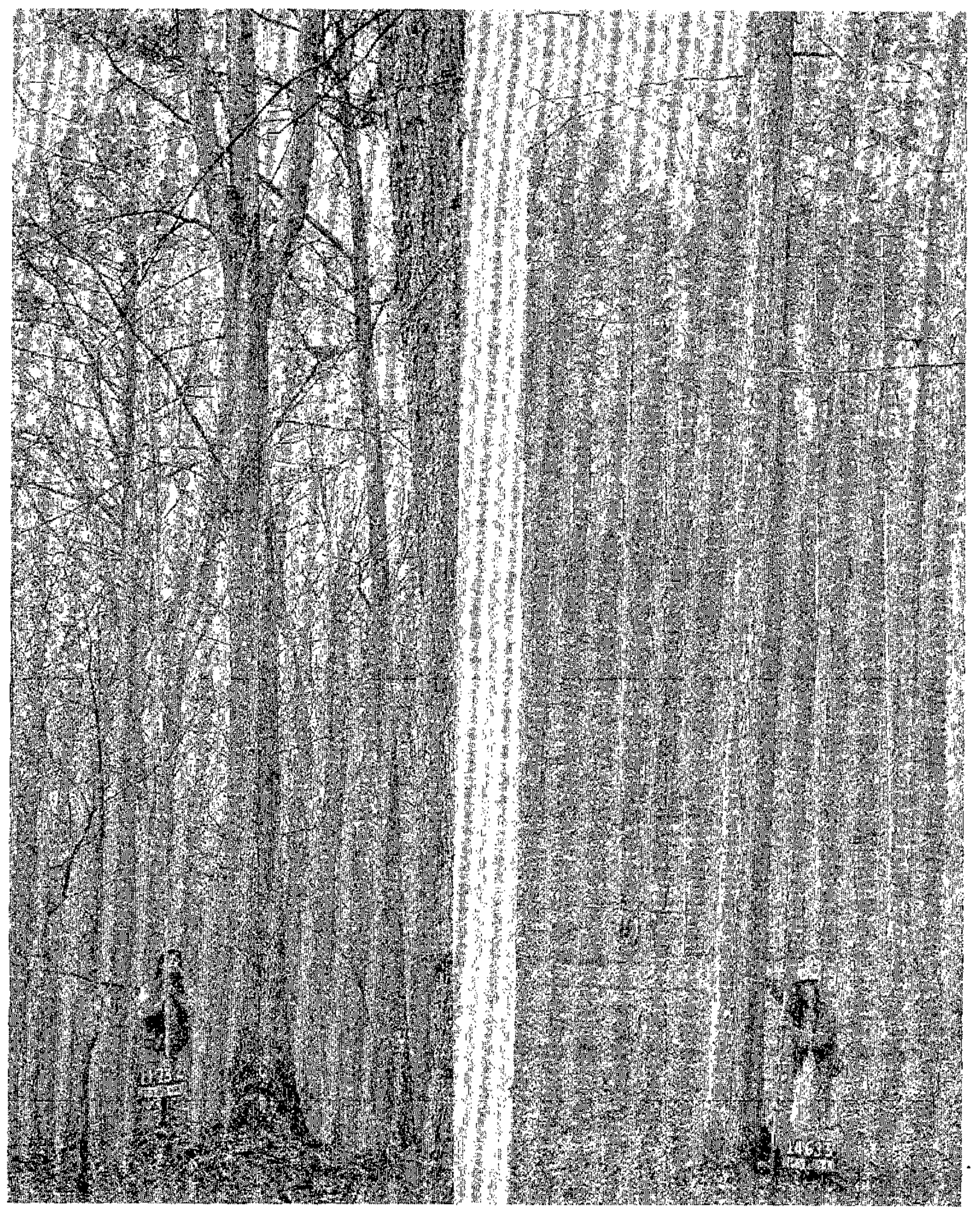

Abstract

Describes development of merchantable-height equations in terms of sawlog and bole length for important tree species of the Northeastern United States. A modified Richards growth model is used to develop species-specific coefficients for 25 common species or species groups. Variability in merchantable sawlogs or bole length is accounted for by species, d.b.h., and site index. Predicted merchantable height when used in conjunction with published cubic-foot-volume or board-foot-volume equations provide realistic estimates of stand volume for average stand conditions in the Northeastern United States. Regression coefficients for merchantable bole length and sawlogs have been incorporated into NE-TWIGS.

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COVER PHOTO.—Wide variation of merchantable sawlog length between two trees of same species, same size.

Introduction

Stem diameter and merchantable bole height are two key measurements needed for reliable estimates of tree volume, either in terms of cubic or board measure. Stand volume, of course, is an important stand attribute because it allows the manager to place an economic value on the timber, especially when one volume is known by species and size class. It is important to have accurate estimates of current volume or value of a stand, particularily at the time it is to be harvested or sold. However, even more important to the manager choosing from among several possible silvicultural options is the relative change in volume or value over time. If information can be developed that shows the relative change in volume or value as a result of applying each of several silvicultural treatments, then the manager has a very objective, rational basis for selecting the appropriate option.

To help managers develop this type of information on stand volume changes, researchers have created various growth-and-yield prediction models that simulate how trees and stands respond to different silvicultural treatments over time. Most of the programs that simulate individual-tree growth utilize regression equations that predict how much a tree of a given species grows in dimensions such as d.b.h. or height in relation to various tree and stand characteristics such as age, stocking, competitive status, site quality, and so on. This paper describes how the generalized merchantable-height equations used in the individual-tree growth-and-yield model NE-TWIGS (Hilt and Teck 1989) were developed.

Data

Data for this study were collected by the Forest Inventory and Analysis (FIA) unit of the Northeastern Forest Experiment Station as part of the periodic survey of forested lands. They are a part of the data set used in developing the NE-TWIGS growth-and-yield model described by Teck (1990). Data were collected from more than 4,400 1/5-acre permanent plots during the 1960's, 70's, and 80's and included measurements from 14 Northeastern states. These survey plots covered a wide range of age, site, and stocking conditions. Basal area per acre ranged from 30 to 255 ft^2/acre. Site index (base age 50 years) was measured on each plot for the dominant species and ranged from 30 to 90 feet. For other species on the plot, we used conversion equations to assign the appropriate site index to each tree, depending on species.[1] Quadratic-mean stand diameter ranged from 5 to 13 inches, which would indicate a wide range in the apparent age of size-class distribution of the sampled stands.

Table 1 lists the 25 species groups used in this study. Although the NE-TWIGS program provides growth estimates for 28 separate species or species groups, there were insufficient height data to develop merchantable-height equations for red pine and loblolly pine so they were combined with other pines as indicated in Table 2. Similarily, we combined paper birch and other birches with yellow birch because of insufficient height data. Our total sample consisted of more than 70,000 trees measured for bole length and about 30,000 trees measured for sawlog length (Table 2).

Sawlog length and bole length as used here are defined by FIA in their data collection procedures as follows:

Sawlog length is the measurement in feet from a 1-foot stump to the first of:

a. minimum-top-sawlog diameter (9-inch-diameter outside bark (d.o.b.) for hardwoods and 7-inch d.o.b. for softwoods). Minimum length of sawlog is 8 feet.

b. the point above which no log, whether or not merchantable, can be produced because of excessive limbs, forks, or crooks.

c. on trees broken off, the point of the break.

Bole length is the measurement in feet from a 1-foot stump to the first of:

a. a 4-inch-top d.o.b. Minimum length is 5 feet.

b. the point where the central stem terminates by branching before reaching a 4-inch-top d.o.b. However, bole length can extend up through major subdivisions of the central stem. When this occurs, the measurement should follow the largest (in diameter) of the divisions.

c. on trees broken off, the point of the break.

Variation in merchantable bole and sawlog height is great between individual trees even within a given diameter class. Figure 1 indicates the extent of this natural variation in sawlog length for chestnut oak. Among the factors contributing to this large variation in the merchantable height of trees within stands and between stands are species, site quality, and stand density. Other natural factors such as ice, wind, or other damage and human-caused disturbances such as type of harvest cutting can lead to great variation in merchantable tree height for a tree within a given species and d.b.h. class. A major source of variation in reporting merchantable heights is the difficulty in measuring them. The range in merchantable height is probably much larger than it would be in a specific stand because survey plots represent all types of stand conditions. Some plots represent severely high-graded stands where only the low-quality, forked, and cull trees remain, while some plots represent the very best stand conditions where the stand has been silviculturally treated to leave only the tallest, straightest, and highest quality stems. Table 2 shows the mean, maximum, and standard deviation for bole length and sawlog height of each species. The minimum-bole and sawlog lengths are not shown, but in most cases these were as defined, that is, an 8-foot minimum for sawlogs and a 5-foot minimum for bole length.

[1]Teck, R. M.; Fuller, L. G.; Hilt, D. E. 1988. Untitled report, on file with authors.

Table 1.—List of species, by species group, that were modeled

Species group	Species
American beech	American beech
Balsam fir	Balsam fir
Black cherry	Black cherry
Black oak	Black oak
Chestnut oak	Chestnut oak
	Swamp chestnut oak
Eastern hemlock	Eastern hemlock
Hickory	Hickory spp.
Noncommercial hardwoods	Boxelder
	Striped maple
	Serviceberry
	Gray birch
	American hornbeam
	Eastern redbud
	Flowering dogwood
	Hawthorne
	Apple
	Ironwood
	Pin cherry
Northern red oak	Northern red oak
Northern white-cedar	Atlantic white-cedar
	Northern white-cedar
	Eastern redcedar
Other hardwoods	American buckeye
	Yellow buckeye
	Hackberry
	Persimmon
	Honey locust
	American holly
	Butternut
	Black walnut
	Water tupelo
	Blackgum
	Sourwood
	Sycamore
	Black locust
	Black willow
	Sassafras
	Basswood
	American elm
	Slippery elm
Other pines	Loblolly pine
	Pitch pine
	Scotch pine
	Shortleaf pine
	Red pine
Quaking aspen	Balsam poplar
	Bigtooth aspen
	Quaking aspen
	Cottonwood
Red maple	Red maple
Red spruce	Black spruce
	Norwary spruce
	Red spruce
Scarlet oak	Scarlet oak
	Southern red oak
	Swamp red oak
	Shingle oak
	Water oak
	Pin oak
	Willow oak
Sugar maple	Sugar maple
Tamarack	Tamarack
Virginia pine	Virginia pine
White ash	White ash
	Black ash
	Green ash
	Blue ash
White oak	White oak
	Swamp white oak
	Bur oak
	Chinkapin oak
	Post oak
White pine	White pine
White spruce	White spruce
Yellow birch	Birch species
Yellow-poplar	Yellow-poplar
	Sweetgum
	Cucumbertree

Table 2.—Descriptive statistics of sample trees, by species

Species group	Sawtimber trees: Sample size	D.b.h. Mean	D.b.h. maximum	D.b.h. SD	Sawlog length Mean	Sawlog length maximum	Sawlog length SD	Pulpwood trees: Sample size	D.b.h. Mean	D.b.h. maximum	D.b.h. SD	Bole length Mean	Bole length maximum	Bole length SD
American beech	1,201	14.8	35.6	3.73	25.3	54.0	8.45	3,001	10.0	35.6	4.67	28.2	76.0	11.83
Balsam fir	1,403	10.5	19.9	1.49	27.7	64.0	8.60	5,635	7.5	19.9	2.07	28.3	76.0	11.07
Black cherry	447	14.0	32.8	2.73	26.1	56.0	9.16	1,324	10.3	32.8	4.20	29.6	70.0	12.92
Black oak	656	15.4	43.6	4.09	30.5	60.0	9.37	1,411	12.1	39.7	4.96	37.3	85.0	13.09
Chestnut oak	622	14.6	32.7	3.41	29.3	60.0	9.73	1,543	11.6	32.7	4.58	33.2	76.0	13.67
Eastern hemlock	2,474	12.3	31.3	3.04	26.1	80.0	10.34	3,747	9.8	30.9	3.70	28.8	80.0	12.78
Hickory	469	13.9	29.7	2.91	30.2	58.9	9.88	1,706	9.5	29.7	3.61	31.8	8.0	13.93
Loblolly pine	a				a			a				a		
Noncommercial	–	–	–	–	–	–	–	1,099	6.7	21.1	1.98	18.7	52.0	7.92
N. red oak	1,526	15.1	56.5	4.14	29.8	68.0	9.18	2,966	11.8	62.1	5.29	35.2	88.0	13.00
N. white-cedar	1,325	11.6	22.9	2.38	20.4	48.0	6.46	2,696	11.8	22.9	3.01	23.9	70.0	8.90
Other hardwoods	887	14.4	33.0	3.17	27.9	62.0	10.93	3,035	9.8	33.0	4.06	29.0	78.0	14.14
Other pines	1,196	11.8	25.0	2.45	34.2	66.0	12.34	1,979	10.8	25.0	3.31	38.7	80.0	15.95
Paper birch	b				b			b				b		
Quaking aspen	513	13.1	36.0	2.40	29.6	56.0	8.23	1,768	8.8	36.0	3.07	32.6	70.0	11.36
Red maple	2,759	14.2	35.7	3.49	25.7	64.0	8.86	9,349	9.2	35.7	4.24	29.6	80.0	11.70
Red pine	a				a			a				a		
Red spruce	2,331	11.5	26.4	2.33	29.0	72.0	9.76	5,518	8.7	26.4	2.94	31.8	84.0	12.15
Scarlet oak	581	14.7	34.2	3.54	30.6	60.0	9.29	1,403	11.4	34.2	4.52	36.4	78.0	13.56
Sugar maple	2,159	15.0	40.1	3.99	27.9	67.0	9.13	4,778	10.4	40.1	4.72	32.6	90.0	12.72
Tamarack	121	11.3	16.4	1.78	30.1	55.0	8.31	190	8.5	16.1	2.81	31.8	68.0	12.74
Virginia pine	610	11.2	18.5	1.78	33.0	66.0	10.24	1,237	9.7	18.5	2.85	41.2	84.0	13.61
White ash	694	14.4	37.0	3.36	30.6	66.0	10.70	1,826	10.1	37.0	4.30	32.3	89.0	14.23
White oak	991	15.1	40.3	4.18	28.8	65.0	10.10	2,817	11.1	40.3	4.86	32.3	84.0	13.69
White pine	2,562	13.6	43.0	4.25	32.3	86.0	12.70	3,741	11.5	23.3	5.15	37.5	70.0	15.75
White spruce	275	11.7	23.3	2.49	29.4	58.0	9.43	564	8.9	23.3	3.23	31.7	70.0	13.05
Yellow birch	1,667	14.1	36.4	3.20	25.2	56.0	8.00	5,609	9.3	36.4	3.78	29.4	78.0	10.80
Yellow-poplar	936	16.3	40.0	4.64	39.6	78.0	11.93	1,775	12.7	40.0	5.74	42.9	98.0	17.36

[a]Red pine and loblolly pine included in other pines due to insufficient data.
[b]All birches included in yellow birch due to insufficient data.

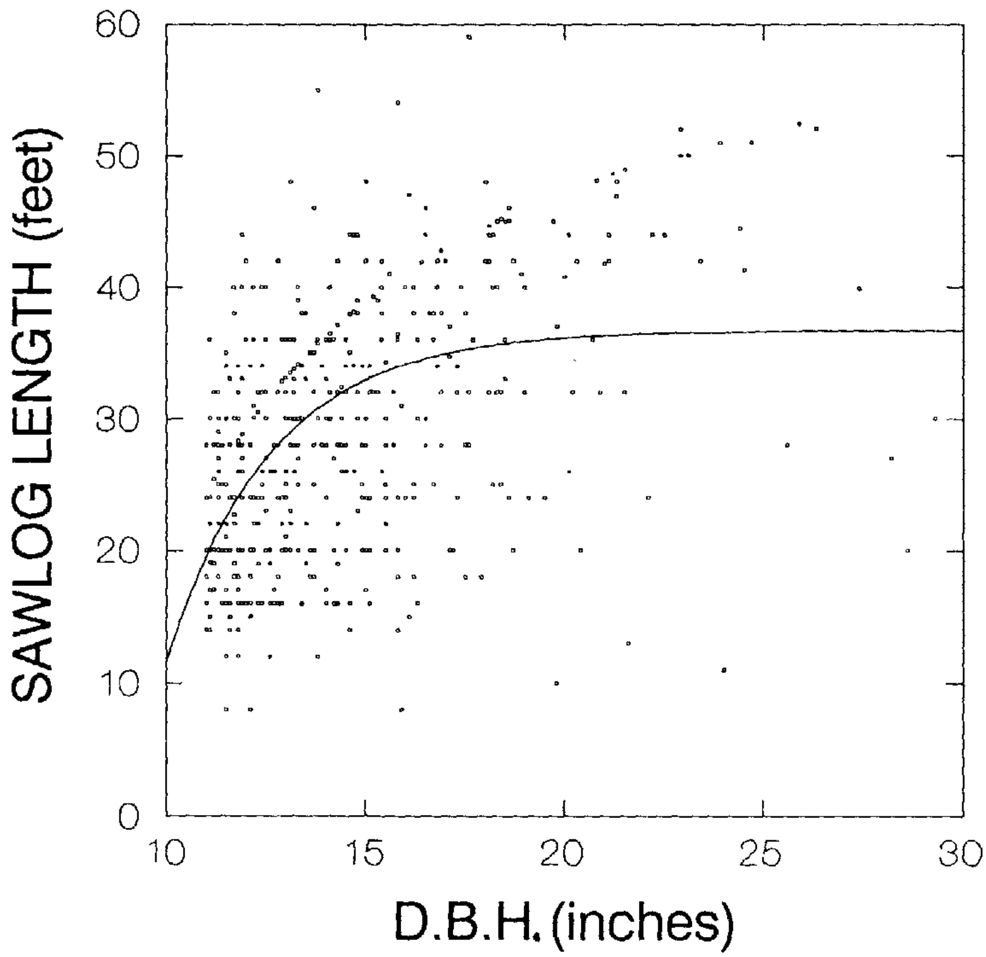

Figure 1.—Chestnut oak sawlog lengths for all sites. The solid line represents the fitted equation with site index set at 60.

Methods

The Richards growth model (Richards 1959) frequently has been modified and used to fit total height of site-index trees as a function of site and stand age (Lundgren and Dolid 1970; Beck 1971). Ek (1971) obtained an excellent fit for white spruce using the expression:

$$H = b_1 S^{b_2}(1 - EXP(b_3 A))^{b_4 S^{b_5}} \qquad (1)$$

where:

H = total height of dominant and codominant trees in feet
S = site index (height at age 50)
A = age in years
b's = constant parameters of the model.

This model, referred to as Model 1, also has given excellent results when used to fit data read directly from site-index curves for many other species (Payandeh 1974; Hilt and Dale 1982).

One of the problems we encountered when using Model 1 was the use of stand age as a predictor variable because many of the timber stands in the Northeastern United States contain trees of several age classes. Because of uneven-aged stand conditions or the wide variation in tree ages within a stand, it is almost impossible to determine stand age with any degree of precision; hence, stand age was not used as a predictor variable in the NE-TWIGS growth-and-yield modeling effort. So, for the merchantable-height models, tree diameter was substituted for age. Upon plotting our merchantable-height data, it did not appear to be doubly asymptotic in form as implied by Model 1, therefore, we tried the simplified form

$$H = b_1 S^{b_2}(1 - EXP(b_3(D - TD))) \qquad (2)$$

where:

H = the merchantable height, either sawlog or bole length
D = diameter at breast height (d.b.h.) outside bark at a point 4.5 feet above the ground, in inches
TD = the minimum top diameter outside bark, as stated in the definitions above; that is, 9 inches for hardwood sawlogs, 7 inches for softwood sawlogs, and 4 inches for pulpwood for all species.

For each species group we used the merchantable-bole length or sawlog data to fit Model 2, using the nonlinear procedures provided by the Statistical Analysis System (SAS) (SAS Institute 1982).

Results

The nonlinear regression coefficients computed for Model 2 are given for each species in Table 3 for bole length and sawlog length. The species groups are listed as they appear in the NE-TWIGS growth-and-yield simulator, so where the data were pooled for some species, the same set of coefficients is listed more than once.

Table 3.—*Coefficients for predicting sawlog and bole length for northeastern species*

Species	Sawlog length			Index[a]	Bole length			Index
group	b_1[b]	b_2	b_3	of fit	b_1	b_2	b_3	of fit
American beech	15.530	0.167	–0.405	0.207	16.430	0.212	–0.328	0.468
Balsam fir	10.366	0.322	–0.467	0.254	17.394	0.252	–0.326	0.621
Black cherry	11.837	0.219	–0.337	0.096	8.301	0.369	–0.310	0.388
Black oak	16.636	0.201	–0.328	0.327	13.901	0.291	–0.283	0.439
Chestnut oak	10.405	0.308	–0.379	0.242	10.573	0.338	–0.301	0.345
Eastern hemlock	33.152	0.057	–0.216	0.376	49.866	0.000	–0.167	0.608
Hickory	39.321	0.000	–0.360	0.273	44.770	0.036	–0.209	0.613
Loblolly pine	5.199	0.530	–0.321	c	2.085	0.791	–0.224	c
Noncommercial	–	–	–		26.129	0.000	–0.493	0.259
N. red oak	25.095	0.086	–0.403	0.232	34.608	0.062	–0.294	0.456
N. white-cedar	24.385	0.024	–0.376	0.253	32.794	0.024	–0.259	0.586
Other hardwoods	35.152	0.000	–0.369	0.179	48.125	0.007	–0.206	0.513
Other pines	5.199	0.530	–0.321	0.490	2.085	0.791	–0.224	0.644
Paper birch	20.458	0.061	–0.882	c	18.922	0.176	–0.400	c
Quaking aspen	10.736	0.290	–0.546	0.207	11.015	0.338	–0.371	0.567
Red maple	14.197	0.190	–0.473	0.143	22.319	0.149	–0.342	0.494
Red pine	5.199	0.530	–0.321	c	2.085	0.791	–0.224	c
Red spruce	17.995	0.203	–0.352	0.310	24.180	0.186	–0.280	0.629
Scarlet oak	22.206	0.132	–0.370	0.284	27.724	0.141	–0.239	0.572
Sugar maple	25.696	0.064	–0.418	0.202	21.237	0.182	–0.294	0.517
Tamarack	37.572	0.000	–0.431	0.250	23.357	0.194	–0.286	0.694
Virginia pine	10.278	0.334	–0.436	0.287	12.054	0.351	–0.350	0.464
White ash	13.321	0.238	–0.468	0.158	26.321	0.135	–0.268	0.497
White oak	11.050	0.283	–0.387	0.221	19.406	0.206	–0.247	0.492
White pine	39.900	0.031	–0.234	0.360	33.035	0.138	–0.173	0.668
White spruce	32.532	0.056	–0.327	0.327	25.540	0.175	–0.258	0.660
Yellow birch	22.540	0.050	–0.673	0.064	18.922	0.176	–0.400	0.438
Yellow-poplar	50.572	0.000	–0.276	0.414	65.206	0.000	–0.160	0.681

[a]Index of fit = $\frac{\text{(corrected total sums of squares – residual sums of squares)}}{\text{(corrected total sums of squares)}}$.

[b]Model form: $Length_i = b_1 SI^{b_2}(1 - EXP(b_3(DBH - TD)))$, where i = sawlog, or bole.
SI = site index, DBH = outside bark diameter at 4.5 feet, TD = the appropriate top diameter.

[c]Indicates species that were combined due to insufficient data. The combinations are:

1. white spruce, red spruce
2. red pine, loblolly pine, other pines
3. yellow birch, paper birch.

The indices of fit, commonly known as the calculated R^2, are appreciably higher for the bole-length equations than are those for sawlogs. This is partially because bole length is more closely related to total height, which depends on site and age or size class. Also, a better fit was expected because, by definition, the merchantable bole length can extend up through major forks or subdivisions of the central stem, whereas major branches, forks, or crooks limit sawlog length. In general, our model performed somewhat better for the coniferous species than for most hardwoods for both sawlog and bole-length predictions. This was not unexpected since conifers generally are more excurrent in form than hardwoods. Tulip-poplar and hickory are exceptions since they usually have an undivided stem, and this characteristic is evident in the higher index of fit for these two hardwoods. This compares favorably with most conifers.

Although these merchantable-height equations fail to explain a large portion of the variation, they do provide useful predictions for many purposes. They are consistent with well-known trends that show merchantabe height increasing as d.b.h. and site index increase (Fig. 2). When used in growth models such as NE-TWIGS, along with diameter growth and volume equations, the manager can obtain estimates of the relative change in volume or value for a wide range of possible silvicultural treatments. This relative change in volume from the present to some future date, with and without the proposed silvicultural treatment, provides substantive support for the appropriate decision. Users are cautioned, however, that our merchantable-height equations probably should not be used to estimate present volume of a specific stand, especially if the volume estimate is the basis of a sale or a logging contract. In such cases we

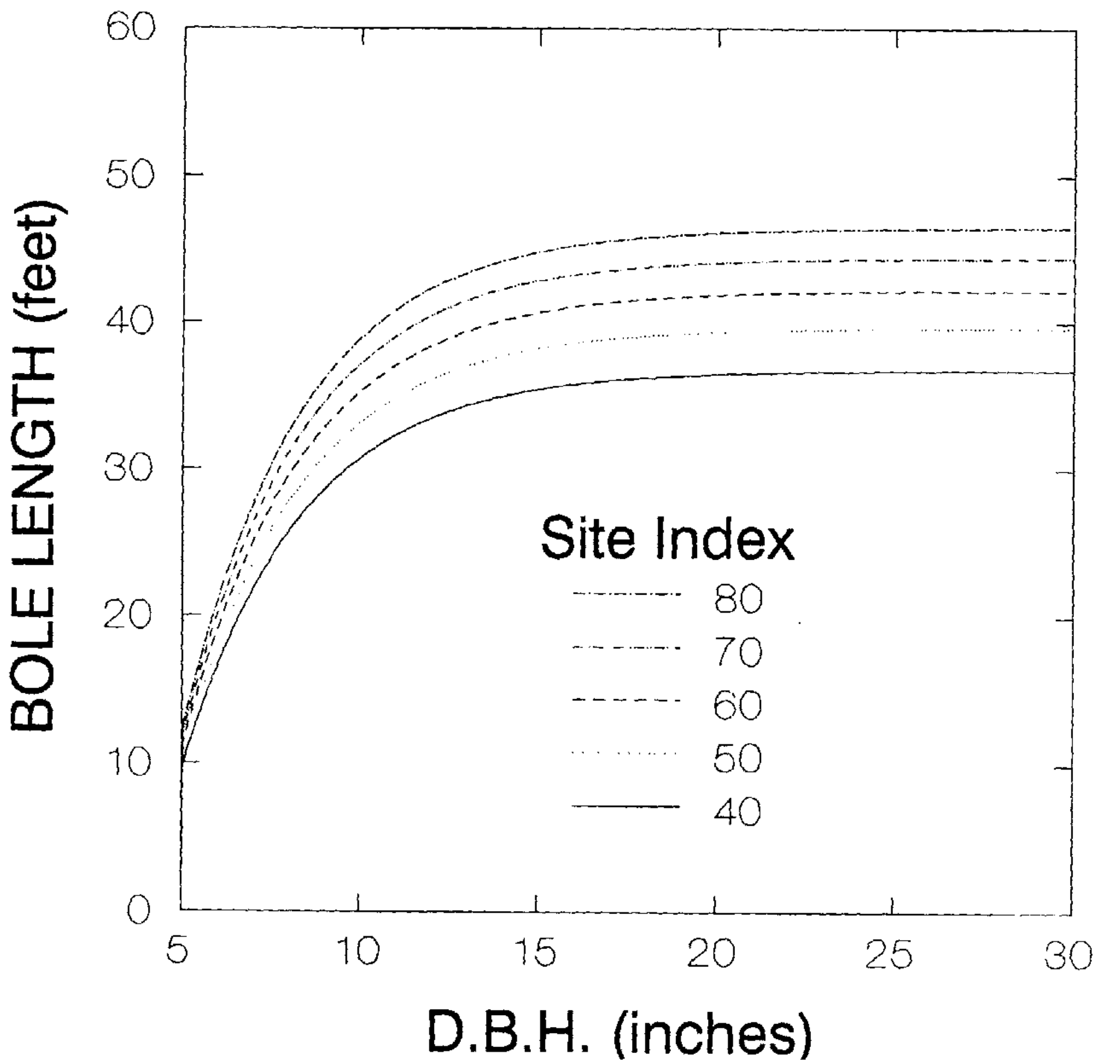

Figure 2.—Form of the bole-length equation fitted with specified site indices for chestnut oak.

would recommend traditional inventory procedures that entail measuring every tree or a sample of trees for stem diameter and estimating or measuring stem height to either a fork or some minimum top diameter that restricts merchantability. Timber cruise software is available that uses these measurements to determine present volumes and values (Yaussy and Brisbin 1990). Remember, the merchantable-height equations presented here are regional averages and, as such, they may not apply to a specific stand since individual timber stands may vary widely from location to location, as noted earlier.

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A modified Richards growth model is used to develop species-specific coefficients for equations estimating the merchantable sawlog and bole lengths of trees from 25 species groups common to the Northeastern United States. These regression coefficients have been incorporated into the growth-and-yield simulation software, NE-TWIGS.

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