PROGRAM HTVOL

The Determination of Tree Crown Volume by Layers

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ABSTRACT

A FORTRAN IV computer program calculates, from a few field measurements, the volume of tree crowns. This volume is in layers of a specified thickness of trees or large shrubs. Each tree is assigned one of 15 solid forms, formed by using one of five side shapes (a circle, an ellipse, a neiloid, a triangle, or a parabolalike shape), and one of three bottom shapes (a circle, an ellipse, or a triangle).

A test of the accuracy of this technique shows that it produces estimates within acceptable limits of error if the shape is carefully selected.

The program sorts these volume data by layer within species for each sample plot. Any number of plots can be run at one pass through the computer, and up to 100 species can be designated.

Keyword: Crown Volume

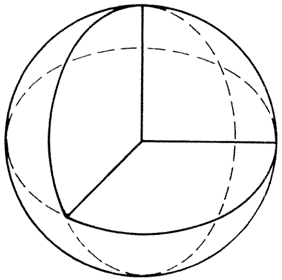
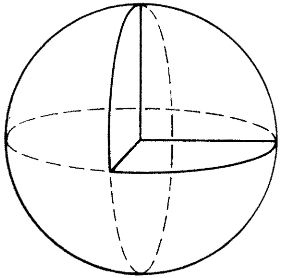
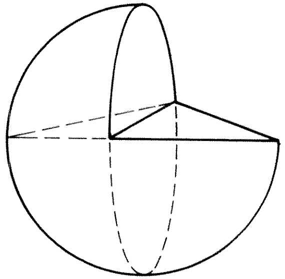
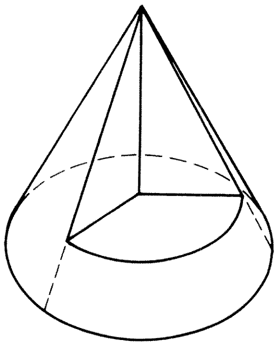
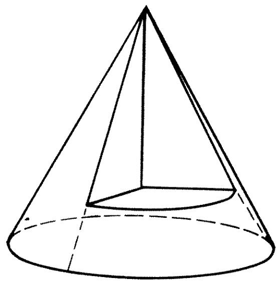
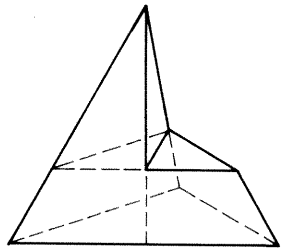
Estimates of the crown volume of trees and shrubs are often useful, particularly for understanding habitat associations of birds (*Sturman 1968, Thomas 1973*). Accurate measurement of these volumes is difficult, and estimating them by height classes requires difficult and tedious calculations. We have developed a computer program that calculates crown volume from a few simple field measurements and produces volume estimates for layers of any thickness. We use the term "crown volume" for the number of cubic feet of space within the crown. Bentley et al. (*1970*) used it with a similar meaning when they presented a technique for sampling low shrub vegetation by crown-volume classes.

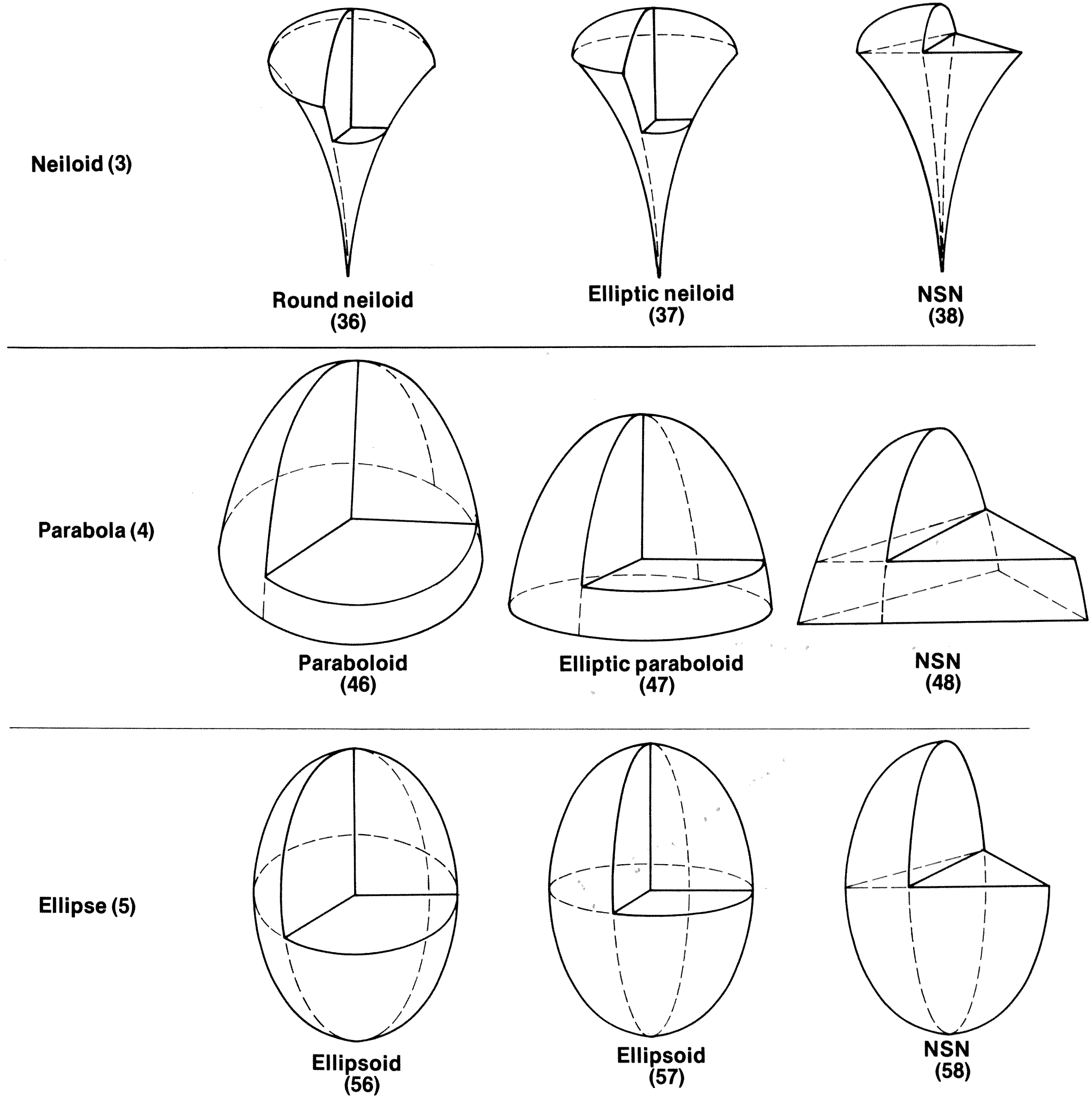
Earlier studies of bird habitat (*Sturman 1968*) refer only to the total crown volume, with the assumption that one or two general shapes (usually a circle or ellipse for the side shape and a circle for the bottom) are sufficient to determine this total. Others recognized that forest canopy layers affect bird distribution (*MacArthur and MacArthur 1961, MacArthur et al. 1962, MacArthur 1964*), but considered only ground vegetation, understory, and overstory, and did not attempt to quantify the volumes of the layers.

Bird habitat studies are not the only use for crown volume. Studies of crown fuels for evaluation of potential fire behavior (*Sando and Wick 1972*) or classical studies of crown development for site evaluation or thinning (competition studies) also would require some expression of this crown volume.

Program HTVOL (*H*eigh*t* *Vol*ume) calculates the gross volume occupied by the crown, including stems, branches, leaves, and the air between them. To do so it assumes that each tree or large shrub fits one of 15 geometric shapes. The program incorporates a density variable to supplement the estimate of crown volume. The program can calculate volumes for up to 100 layers, but each layer must be of the same thickness for a single program run. Subroutines can calculate volumes of other layers or combinations merely by adding volumes. The volumes are cal-

Figure 1. Shapes used in HTVOL program (with identifying numbers)

Profile shape	Plan Shape: Circle (6)	Plan Shape: Ellipse (7)	Plan Shape: Triangle (8)
Circle (1)	 Sphere (16)	 Ellipsoid (17)	 NSN* (18)
Triangle (2)	 Cone (26)	 Elliptic cone (27)	 Pyramid (28)



*NSN = *No Specific Name*

culated and presented by plot. There is no limit to the number of plots in any one run, nor to the number of trees in a plot.

TREE SHAPE

No one geometric form can be used to describe the crown shape of all species. But if a side view or profile shape and a bottom or plan shape are estimated, the resultant geometric solid can be used to describe volume. HTVOL uses five profile and three plan shapes to describe 15 geometric solid forms. The shapes are shown in figure 1, each with its name and an identification number. These identification numbers are used in the field (and in the program) to describe the shape.

The shapes are not evenly distributed among trees in any region of the country. A sample of the relative distribution from a suburban bird habitat study in Amherst, Massachusetts (*Thomas 1973*), illustrates this for 2,700 trees (table 1).

FIELD MEASUREMENTS

The following variables were measured or estimated for each tree:

1. Species—Species are identified with a four letter code. We used a species code using the first two letters of the genus and species. Example: Acer rubrum = ACRU; Salix species = SASP.
2. Crown profile class—assigned shape 1 to 5.
3. Crown plan class—assigned shape 6 to 8.
4. Total height—measured to nearest foot.
5. Bole height—average height to the live crown.
6. Plan radius—for a circle the radius R, for an ellipse plan the large (RL) and small (RS) radii. For a triangle the height (L) and base (W).
7. Diameter—dbh to nearest inch.
8. Density class—rated 1 to 5 according to the density of the crown. 1 = very dense; 5 = very sparse.

Diameter and density class are the only two variables not used in the calculations. They were useful for photographic interpretation and description of the kind of volume, and are included in all data formats of the program.

TEST OF ACCURACY OF PREDICTED VOLUMES

We estimated the error of our assumption that trees conform to one of 15 shapes for a sample of 49 trees. The selection was purposive, to try to include all the shapes.

The test was conducted in three stages:

1. For each selected tree, we measured total height, height to crown, and plan radius or radii in the field.
2. Two black and white photographs were taken of each tree at right angles to one another. A 10-foot range pole (graduated at one foot intervals) was placed against the tree.
3. In the office, each negative was projected on 1 inch x 1 inch graph paper and adjusted to a convenient scale (1 inch = 5 feet or 1 inch = 2 feet). The crown was sliced into layers at 5-foot height intervals, and the radii of each slice were recorded.

We calculated the true volume of each tree from the office measurements and compared it with the volume calculated by the HTVOL program.

Two points should be considered if this technique is applied to any study:

Table 1. ***Frequency distribution of tree shapes in a bird habitat study in Amherst, Massachusetts***

Profile shape	Plan shape: Circle		Ellipse		Triangle		Total	
	No.	%	*No.*	%	*No.*	%	*No.*	%
Circle	314	11.5	49	1.8	103	3.8	466	17.1
Triangle	296	10.9	44	1.6	85	3.1	425	15.6
Neiloid	432	15.8	126	4.6	400	14.7	958	35.1
Parabola	195	7.1	26	1.0	81	3.0	302	11.1
Ellipse	438	16.0	38	1.4	101	3.7	577	21.1
Total	1,675	61.3	283	10.4	770	28.3	2,728	100.0

1. Although a conifer may generally appear to be conical in profile, no tree tested had a true triangular shape—all were rather parabolic, i.e., the sides curved outward. The difference in volume estimates between the two shapes, real and assigned, was significant. This should be tested on the true firs of the West or other conical-crown types.

2. Most elm trees followed a neiloid profile only if the crown shape started at the base of the tree, i.e., the height to the live crown (bole height) is assumed to be 0 feet. Therefore, it is likely that any tree that looks like a neiloid should be given a bole height of 0. Again, this assumption should be tested before field work is started.

Table 2 is a summary of observed differences in volume for each solid and by side and bottom shapes. These differences were calculated by comparing the volumes obtained from the photographs with those obtained from field measurements used by the HTVOL program. Their frequency distribution is shown in table 3.

Table 2. *Differences between estimated and actual crown volume for trees of each solid shape by profile, plan shape, and height class*

Category	Number of trees	Percent difference
Shape no.		
16	3	− 3.56
36	4	−19.00
37	2	− .04
38	1	− 1.96
46	12	− 7.06
56	21	+ 5.32
57	1	+25.50
58	5	+ 1.00
Profile shape		
1	3	− 3.56
3	7	−14.09
4	12	− 7.06
5	27	+ 7.31
Plan shape		
6	40	− 4.30
7	2	+12.18
8	7	+ .21
Height class		
0-19	8	− 1.12
20-39	24	− .64
40-59	24	− 1.14
60+	6	− 5.12

Table 3. *Frequency distribution of individual tree differences between true and estimated volume*

Difference range	Positive differences	Negative differences	Total
Percent	*Number*	*Number*	*Number*
0-5	4	11	15
6-10	1	7	8
11-15	8	2	10
16-20	4	3	7
21-25	2	1	3
26-30	4	0	4
31-35	0	0	0
36-41	1	1	2
Total	24	25	49

Several general statements can be made about the results:

1. Although we tried to find samples of all the shapes, only 8 of the 15 were represented. Shapes 17, 18, 26, 27, 28, 47, and 48 (fig. 1) were not observed. This does not mean they do not exist, but it does mean that the conditions under which trees grow along roadsides in the Town of Amherst do not produce many trees that are conical in side view or ellipsoidal or triangular in plan. This is to be expected. Trees in open or semiopen locations tend to have circular crown bases. In tight or closed stands, triangular and elliptical plan shapes would be more prevalent.

2. The exact profile shape must be considered carefully. In the original choice in the field, conical profile shapes (Profile Shape 2) were assigned to conifers from 20 to 60 feet in height. After we looked at the photographs and took measurements in the office, we changed the profile shape to parabolic (Profile Shape 4) because it more closely represented the shape of the test trees.

3. The least accurate profile shape is the neiloid (Profile Shape 3), originally thought to fit the elm tree best. In our small test sample, the errors of volume exceeded our 10 percent limit. But when the profile was changed in the office to an ellipse or a parabola, the errors dropped to 8 percent, which was within our acceptable range.

4. Differences between estimated and actual volume for individual trees ranged from −40.9 to +38.2 percent. But the estimates for 33 trees were within 15 percent of the true volume. With sufficient numbers of trees—say 20 or more for

any one shape—the differences would average between 5 and 10 percent—a deviation we would call acceptable. The overall difference for the 49 trees we tested was 2.25 percent, well within our acceptable limits.

5. An analysis of the differences *by layer* (estimated volume minus true volume) showed differences ranging from ± 1 percent to more than 150 percent, but clustered around ± 20 percent. The typical profile of differences by layer showed that at the bottom of a crown the estimate tended to be low, while at the top it tended to be high. Only profile 3 showed a different pattern. For all samples of profile shape 3, the negative differences were in the middle layers. This is significant and caution is urged in selecting this profile shape because trees assigned neiloid shapes tend to be wider in the crown than the regular geometric profile shape allows.

6. To aid in selecting the best shape, a clear plastic templet with all the shapes etched on it could be used in the field. The tree would be viewed through the templet, held at the proper distance from the eye to place the tree entirely within the etched shape outline. This would also help choose the radius that would best insure a representative volume calculation by the single shape technique.

NOTE: Copies of Program HTVOL may be obtained from the authors at the Northeastern Forest Experiment Station, Hilton House, University of Massachusetts, Amherst, Mass. 01002.

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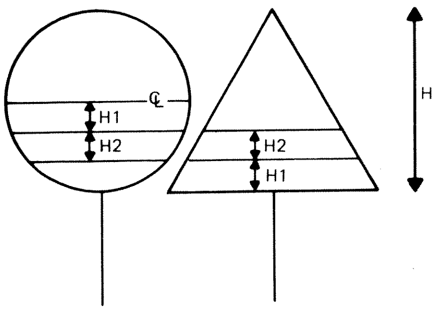
APPENDIX
FORMULAS FOR FRUSTRUMS OF SHAPES

The following formulas are used in HTVOL to compute the volume of a layer:

Definitions

- π = Pi = 3.141593
- R = Radius of a circle for bottom shape 6
- RS = Radius of small side for bottom shape 7
- RL = Radius of long side for bottom shape 7
- W = Width of triangle for bottom shape 8
- L = Length of triangle for bottom shape 8
- H1 = Distance from base to lower plane of frustrum. The base is the bottom of the crown for shapes triangle, neiloid, and parabola-like, and the base is the mid-diameter for an ellipse or circle.
- H2 = The thickness of a frustrum
- HC = Height of crown—the total height of the solid.



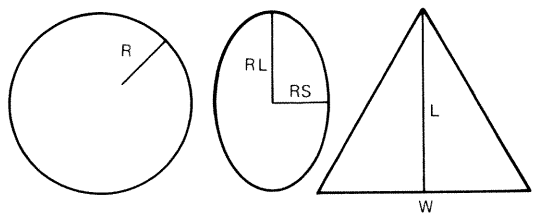


Figure No.	*Frustrum*	*Solid**
16	$\frac{\pi H2}{3}\left[3R^2-3H1^2-3H1H2-H2^2\right]$	$\frac{4}{3}\pi R^3$ 4.18879 R^3
17	$\frac{\pi H2RS}{3R}\left[3R^2-3H1^2-3H1H2-H2^2\right]$ where R = RL	$\frac{4}{3}\pi RSR^2$ 4.18879 $RSRL^2$
18	$\frac{2H2}{3}\left[3R^2-3H1^2-3H1H2-H2^2\right]$ $\left\{R=\frac{W}{2}\text{ or }\frac{L}{2}\text{ Since }W=L\right\}$	$\frac{8R^3}{3}$ 2.667 R^3
or	$\frac{2H2RS}{3R}\left[3R^2-3H1^2-3H1H2-H2^2\right]$ { RS= $\frac{W}{2}$ or $\frac{L}{2}$ whichever is smaller R= $\frac{W}{2}$ or $\frac{L}{2}$ whichever is= $\frac{HC}{2}$ since $W \neq L$ }	8 $\frac{RSR^2}{3}$ 2.667 RL^2RS

*Solid is calculated by setting $H2=\frac{HC}{2}=R, H1=0$

Figure No.	*Frustrum*	*Solid*

26

$$\frac{\pi R^2 H2}{3}\left[\left(1-\frac{H1}{HC}\right)^2+\left(1-\frac{(H1+H2)}{HC}\right)^2+\left(1-\frac{H1}{HC}\right)\left(1-\frac{(H1+H2)}{HC}\right)\right]$$

Solid: $\frac{\pi R^2 HC}{3}$, $1.04719\ HCR^2$

27

$$\frac{\pi RLRSH2}{3}\left[\left(1-\frac{H1}{HC}\right)^2+\left(1-\frac{(H1+H2)}{HC}\right)^2+\left(1-\frac{H1}{HC}\right)\left(1-\frac{(H1+H2)}{HC}\right)\right]$$

Solid: $\frac{\pi RLRSHC}{3}$, $1.04719\ HCRSRL$

28

$$\frac{LWH2}{6}\left[\left(1-\frac{H1}{HC}\right)^2+\left(1-\frac{(H1+H2)}{HC}\right)^2+\left(1-\frac{H1}{HC}\right)\left(1-\frac{(H1+H2)}{HC}\right)\right]$$

Solid: $\frac{LWHC}{6}$, $.1667\ LWRC$

36

$$\frac{\pi H2}{6}\left[\left(R-\sqrt{R^2\left(\frac{HC^1-H1}{HC^1}\right)}\right)^2+4\left(R-\sqrt{R^2\left(\frac{HC^1-H1-\frac{H2}{2}}{HC^1}\right)}\right)^2+\left(R-\sqrt{R^2\left(\frac{HC^1-H1-H2}{HC^1}\right)}\right)^2\right]$$

Solid: $\frac{\pi HCR^2}{6}\ 1.7573$, $.9201512\ HCR^2$

37

$$\frac{\pi H2}{6}\left[\left(RL-\sqrt{RL^2\left(\frac{HC^1-H1}{HC^1}\right)}\right)\left(RS-\sqrt{RS^2\left(\frac{HC^1-H1}{HC^1}\right)}\right)+\right.$$

$$4\left(RL-\sqrt{RL^2\left(\frac{HC^1-H1-\frac{H2}{2}}{HC^1}\right)}\right)\left(RS-\sqrt{RS^2\left(\frac{HC^1-H1-\frac{H2}{2}}{HC^1}\right)}\right)+$$

$$\left.\left(RL-\sqrt{RL^2\left(\frac{HC^1-H1-H2}{HC^1}\right)}\right)\left(RS-\sqrt{RS^2\left(\frac{HC^1-H1-H2}{HC^1}\right)}\right)\right]$$

Solid: $\frac{\pi HCRSRL}{6}\ 1.7573$, $.9201512\ HCR^2$

38

$$\frac{H2}{3}\left[\text{Same as 37}\right]\quad \text{If } W > L,\ RL = \frac{W}{2}\ RS = \frac{L}{2};\quad \text{If } W < L,\ RS = \frac{W}{2}\ RL = \frac{L}{2}$$

Solid: $.3827\ HCRSRL$

Figure No.	*Frustrum*	*Solid**
46	$\frac{\pi H2R^2}{2HC}\left[2HC-2H1-H2\right]$	$\frac{\pi R^2HC}{2}$ 1.57079 R^2HC
47	$\frac{\pi H2RSRL}{2HC}\left[2HC-2H1-H2\right]$	$\frac{\pi RSRLHC}{2}$ 1.57079 RSRLHC
48	$\frac{H2WL}{4HC}\left[2HC-2H1-H2\right]$	$\frac{HCLW}{4}$ .25 HCLW
56	$\frac{\pi H2R^2}{3HC^2}\left[3HC^2-12H1^2-12H1H2-4H2^2\right]$	$\frac{2\pi HCR^2}{3}$ 2.09439 R^2HC
57	$\frac{\pi H2RSRL}{3HC^2}\left[3HC^2-12H1^2-12H1H2-4H2^2\right]$	$\frac{2\pi HCRSRL}{3}$ 2.09439 RSRLHC
58	$\frac{H2WL}{6HC^2}\left[3HC^2-12H1^2-12H1H2-4H2^2\right]$	$\frac{2HCWL}{3}$ .6667 HCWL

* Solid is calculated by setting H2 = HC H1 = 0 for "4"

H2 = Hc/2 H1 = 0 for "5"