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Origin of Sigmoid Diameter Distributions

William B. Leak

Abstract

Diameter distributions—numbers of trees over diameter at breast height (d.b.h.)—were simulated over 20-years using six diameter-growth schedules, six mortality trends, and three initial conditions. The purpose was to determine factors responsible for the short-term development of the arithmetic rotated sigmoid form of diameter distribution characterized by a plateau, near plateau, or bump in the mid-diameter d.b.h. classes. A distinct rotated sigmoid developed when the diameter-growth schedule dropped precipitously in the mid-diameter classes; this might reflect a decadent overstory and thrifty understory. When the initial diameter distribution was distinctly sigmoid, diameter-growth schedules characterized by slow growth in the small-diameter classes tended to maintain sigmoid characteristics. A parabolic growth trend with or without parabolic mortality, also produced moderate rotated sigmoid characteristics. Under most other growth or mortality schedules, the 20-year diameter distribution was J shaped. The results reinforce the concept that diameter distributions tend to maintain or return to the J shape particularly with respect to New England northern hardwoods.

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Introduction

F. DeLiocourt (1898) and H. Arthur Meyer (1952) were right! In both managed and undisturbed forests, there is a pervasive tendency for the diameter distribution—number of trees over diameter at breast height (d.b.h.)—to approximate the reverse J shape. Many other field studies have corroborated the utility of this general curve (e.g., Nyland 1998). Through simulation, I examined the effects of a variety of diameter-growth trends, mortality schedules, and initial conditions on the short-term development of the diameter distribution in New England northern hardwoods. The reverse J shape predominated in many (but not all) situations.

The primary purpose of this study was to determine the factors that might lead to another form of diameter distribution — the reverse or rotated sigmoid form. For this purpose, rotated sigmoid refers to a curve of number of trees over d.b.h. that has an obvious plateau, near plateau, or bump in the mid-d.b.h. range. Logarithmic rotated sigmoid is a curve with a plateau, near plateau, or bump when the log of tree numbers is plotted over d.b.h. A reverse J shape describes a curve of number of trees over d.b.h. that is steeply and steadily declining. Many distributions that appear J shaped plot as logarithmic rotated sigmoid. When a J-shape curve plots as a straight line on semi-log paper, the distribution is called the negative exponential; this curve has a constant quotient (Q) between numbers of trees in successive d.b.h. classes.

Goff and West (1975) suggested that the logarithmic rotated sigmoid was the natural steady-state distribution in small unmanaged hardwood-hemlock stands in Wisconsin, and that it might result from low mortality and rapid diameter growth of trees entering the main crown canopy; some of their stands also would appear as rotated sigmoid (without logarithmic transformation). Lorimer and Frelich (1984) suggested that logarithmic rotated sigmoids were natural steady-state distributions in unmanaged Michigan hardwoods. They based their conclusion on long-term simulations that included parabolic diameter-growth and mortality functions (rapid growth and low mortality in the mid sizes). However, the initial distributions in their two old-growth stands were rotated sigmoid (without logarithmic transformation). Schmelz and Lindsey (1965) believed that the logarithmic rotated sigmoid tendency was a temporary stage in the recovery of midwestern old-growth hardwoods from an earlier disturbance. Some of the economically optimal and sustainable distributions developed through simulations (Adams and Ek 1974; Gove and Fairweather 1992) have moderate rotated sigmoid properties that would be more pronounced with logarithmic transformation. A recent study in the Lake States (Goodburn and Lorimer 1999) showed that old-growth northern hardwood stands classified as balanced (based on areas of free-to-grow crowns) had logarithmic rotated sigmoid distributions, whereas negative exponential distributions were most common in selection stands classified as balanced. Several northern hardwood stands in New Hampshire developed logarithmic rotated sigmoid diameter distributions (though J shaped) following a variety of partial cuttings, whereas a nearby old-growth stand remained as a negative exponential distribution (Leak 1996); several stands that appeared J shaped exhibited a parabolic trend of log (number of trees) over d.b.h., which implies an increasing Q with increasing d.b.h., a form of distribution described some years ago (Leak 1964).

There are examples of intensively managed stands that developed rotated sigmoid (nonlogarithmic) distributions. On the Fernow Experimental Forest in West Virginia, a hardwood stand harvested four times over 24 years by removing trees throughout the d.b.h. range developed rotated sigmoid tendencies with a plateau in the mid-d.b.h. classes (Fig. 1). A comparable stand in which only sawlog-size trees were harvested developed a J-shape form (Trimble and Smith 1976). A 30-acre northern hardwood stand on the Bartlett Experimental Forest in New Hampshire developed a distinct bump in the mid-d.b.h. classes following three cuts over 48 years (Fig. 2).[1]

Thus, the literature conflicts somewhat as to the origin and occurrence of various forms of diameter distribution. It remains unclear whether sigmoid and other forms of diameter distribution are characteristic of old-growth, disturbed stands or intensively managed stands. In addition, there is little information on how a variety of growth and mortality schedules and initial conditions influence the development and maintenance of sigmoid vs. J-shape distributions. To further define the influential factors particularly as related to conditions in New England, simulations were conducted to determine the effects of six diameter-growth trends, six mortality schedules, and three initial diameter distributions on the development of diameter distributions over 20 years. A wide range of growth and mortality patterns was included to produce pronounced effects, and because field studies continue to show that growth and mortality trends are variable and unpredictable due to mixed species, suppression, site relationships, insect/disease effects, and natural damage (e.g., ice storms).

[1]Leak, W. B.; Sendak, P. E. Changes in species, grade, and structure over 48 years in a managed New England northern hardwood stand. In preparation.

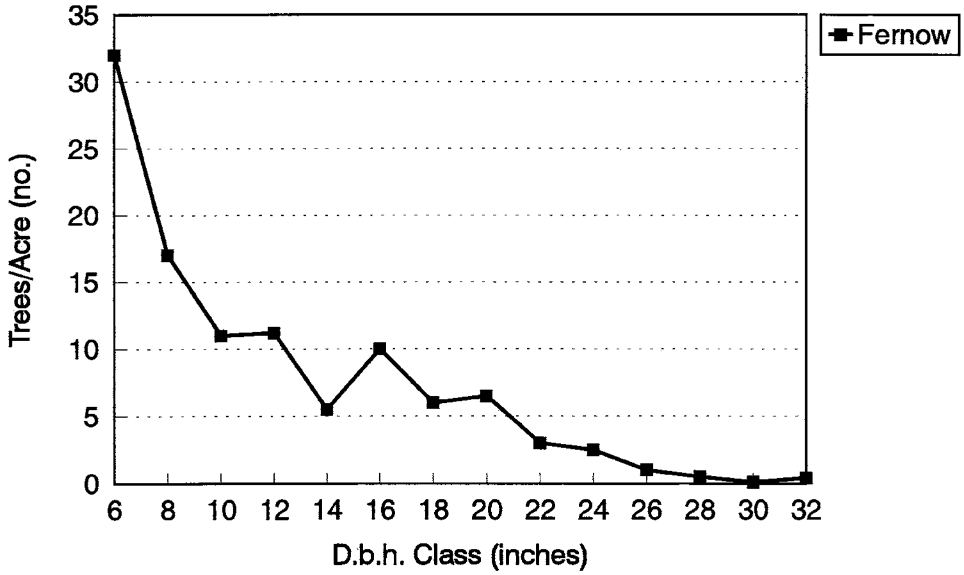

Figure 1.—Stand structure after 24 years of single-tree selection: a tract on the Fernow Experimental Forest, West Virginia, where harvesting was conducted throughout the range of d.b.h. classes (Trimble and Smith 1976).

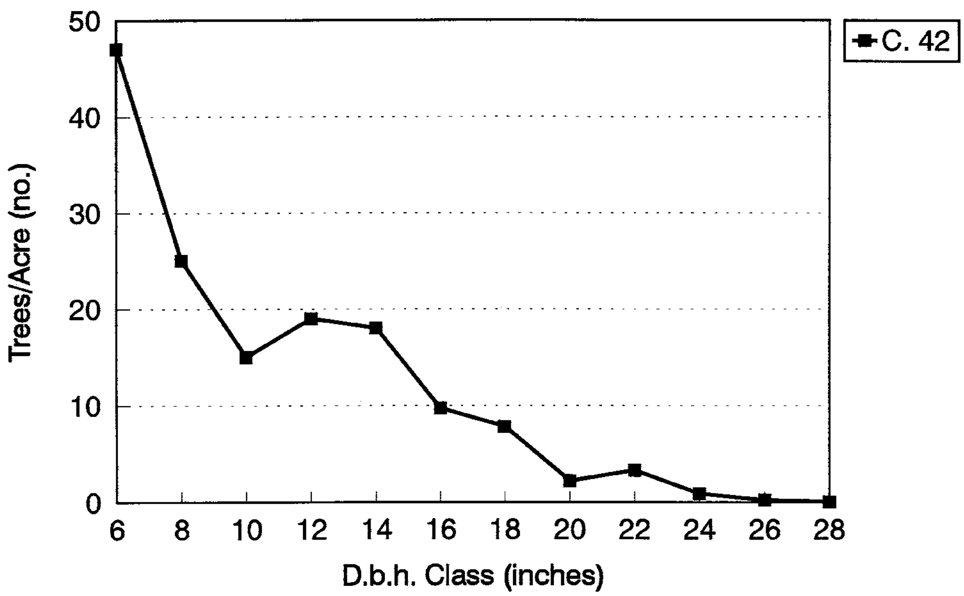

Figure 2.—Stand structure after 48 years of single-tree selection: a tract on the Bartlett Experimental Forest, New Hampshire.[1]

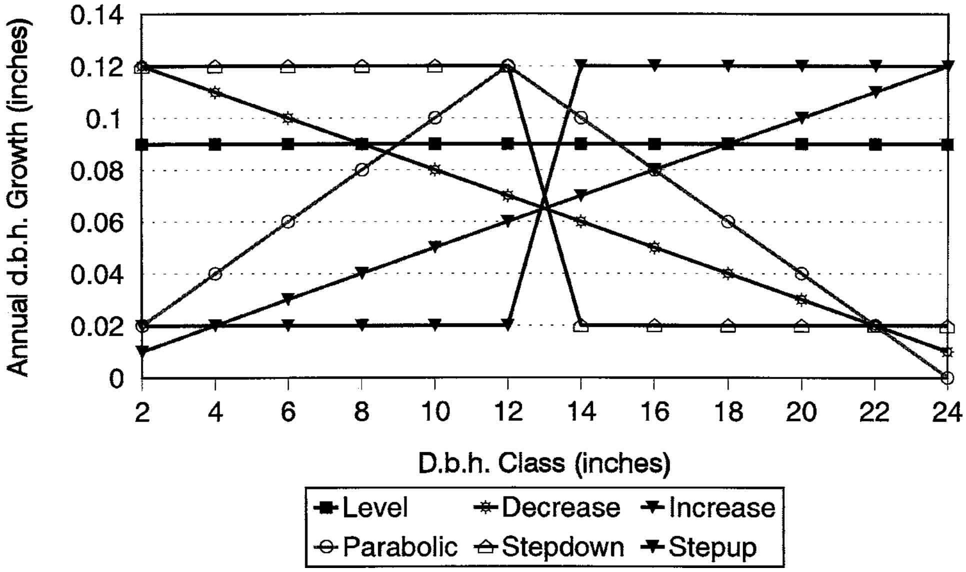

Figure 3.—Annual d.b.h. growth trends used as simulation input.

Methods

The growth and mortality data were bounded by the results of a comprehensive study of density and structure in a small sawtimber stand (somewhat even-aged with holdovers) on the Bartlett Experimental Forest (Solomon 1977) in New Hampshire. As a result, the findings are especially applicable to conditions in New England. Preliminary work with the simulation model as well as long-term field experiments (e.g., Leak 1996) indicated that diameter distributions in New England hardwoods are resistant to perturbations. Accordingly, the study was designed to examine extreme conditions and to detect visually distinct effects.

The six diameter-growth trends (Fig. 3) were:

Level.—Diameter growth was constant across all d.b.h. classes from 2 to 24 inches. This is fairly characteristic of the Solomon (1977) study in which poletimber and sawtimber grew at about the same rate. This trend tends to be typical of fairly good sites in New England (Leak 1983).

Decrease.—Diameter growth declined with d.b.h. This trend tends to be characteristic of mediocre sites in New England where initial diameter growth is high but falls off rapidly with increasing d.b.h. (Leak 1983).

Increase.—Diameter growth increased with d.b.h. (Leak and Graber 1976). This trend has been observed in older New England hardwood stands in which the overstory is too heavy or dense or the understory has been slow to respond — perhaps due to long periods of suppression.

In some instances, apparently where the largest overstory trees are low in vigor, the trend is:

Parabolic.— Growth is slow in the largest and smallest trees and greatest in the mid-d.b.h. classes. This trend is similar to that tested by Lorimer and Frelich (1984) and partially represents the theory proposed by Goff and West (1975) for the development of sigmoid distributions.

Stepdown.— Growth is rapid in the small classes and slow in the large ones. It is intended to represent an overstory of low-vigor or diseased trees (e.g., a beech bark disease infestation) coupled with a vigorous understory, possibly of a different species. For example, data from Solomon's (1977) study show that poletimber eastern hemlock (*Tsuga canadensis*) can grow two to six times as fast as beech (*Fagus grandifolia*) or sugar maple (*Acer saccharum*) poletimber even under fairly dense stockings of 80 to 100 ft^2 of basal area per acre.

Stepup.— Growth is slow in the small sizes and rapid in the large ones. This trend might be typical of a fast-growing intolerant/intermediate overstory with an understory of slower growing tolerants.

The six mortality schedules (percent mortality) were defined as for the growth trends (Fig. 4). Again, Solomon's (1977) data were used as bounds. The exception was the Parabolic trend in which the high mortality rates were in the large and small trees. This trend was similar to that tested by Lorimer and Frelich (1984). By contrast, Solomon (1977) found fairly constant mortality rates in both poletimber and sawtimber. Apart from these results, there are few available data with which to corroborate the various mortality schedules. However, the scenarios described under the growth trends apply generally to the mortality

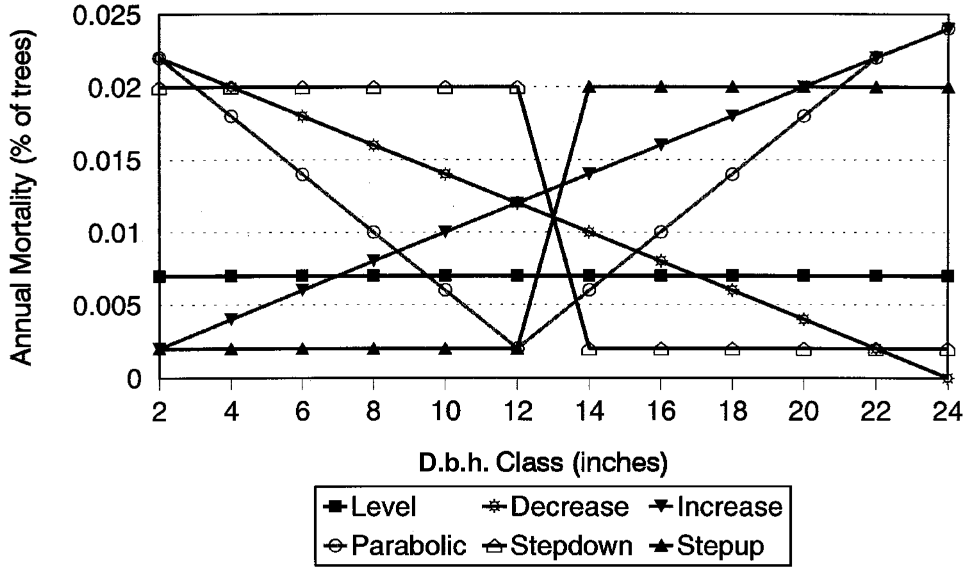

Figure 4.—Annual percent mortality schedules used as simulation input.

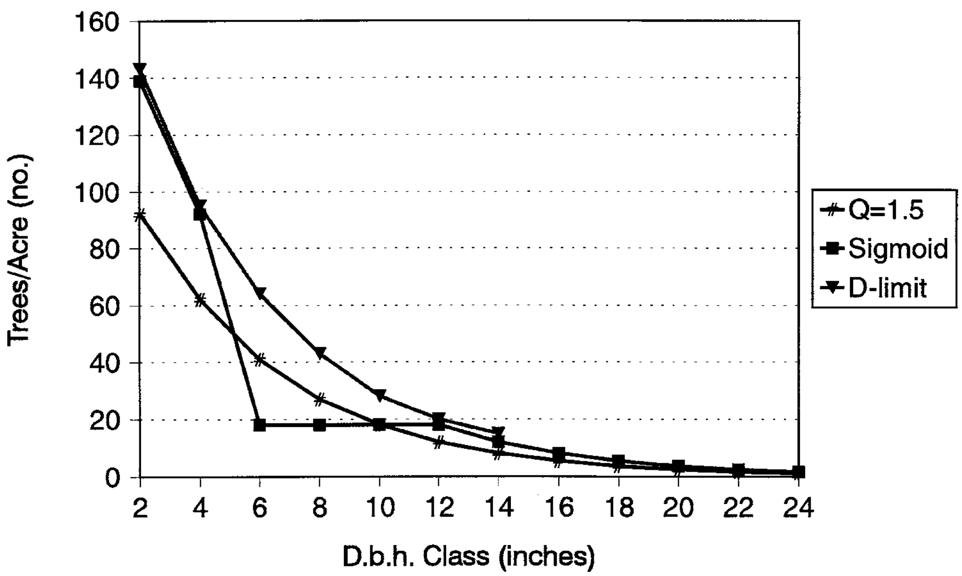

Figure 5.—Initial diameter distributions: negative exponential, Q = 1.5, 80 ft^2 of basal area per acre; sigmoid distribution, 79 ft^2 of basal area per acre; and diameter-limit, 80 ft^2 of basal area per acre.

schedules, if it is assumed that slow growth implies high mortality and vice versa.

The three initial conditions (Fig. 5) represented: (1) a typical residual stand in uneven-aged northern hardwoods: 80 ft^2 of basal area per acre (trees 2 inches and larger in d.b.h.), 72 ft^2 in trees 6 inches and larger, a Q of 1.5 and a maximum tree diameter of 24 inches; (2) an initial sigmoid distribution with a constant number of trees in the 6- through 12-inch classes, 79 ft^2 of basal area, and 68 ft^2 in trees 6 inches and larger in d.b.h. (this condition was designed to represent the situation where past disturbance or heavy marking in the small and large sizes had created sigmoid characteristics); and (3) a residual stand from a diameter-limit cut with 80 ft^2 per acre total, 69 ft^2 in trees 6 inches and larger in d.b.h. and no trees above 14 inches. Only the Level growth/mortality schedules were applied to this initial condition.

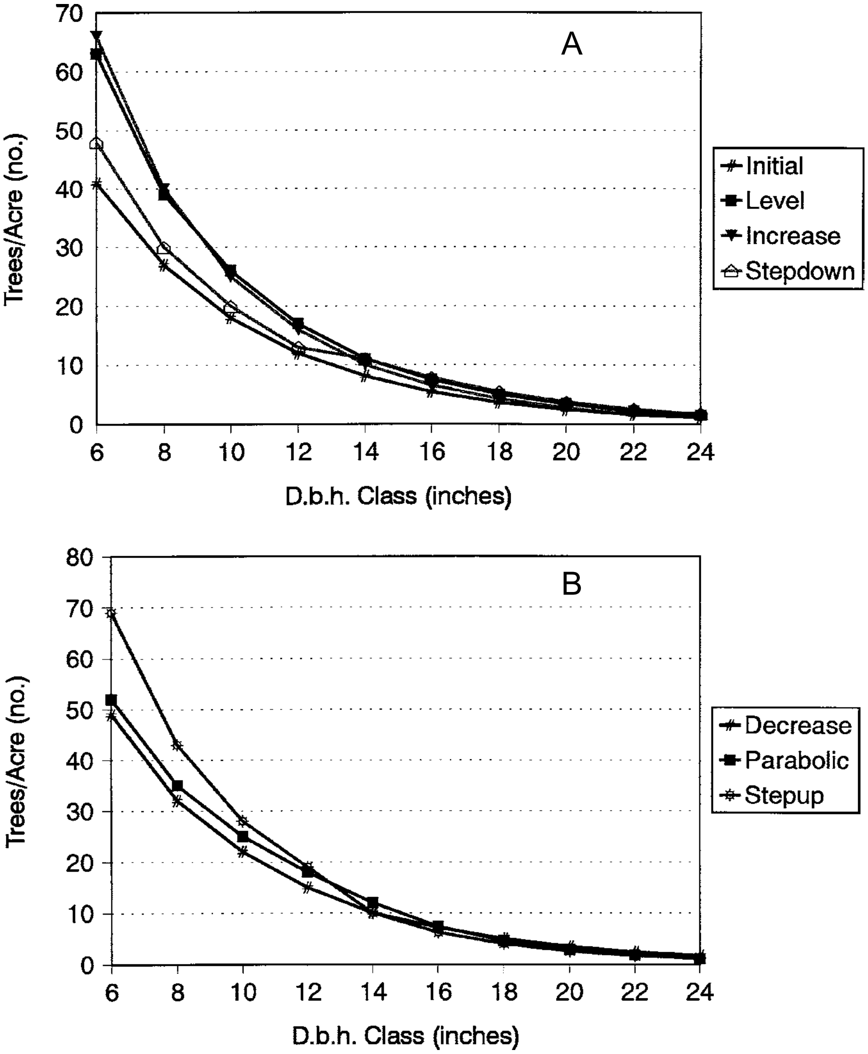

Figure 6.—Simulated diameter distributions after 20 years using (A) the Initial, Level, Increase, and Stepdown mortality schedules, and (B) the Decrease, Parabolic, and Stepup mortality schedules. All simulations using the Level growth trend and beginning with the negative exponential distribution.

The simulation model was a simple stand projection system written in Visual Fortran.[2] Beginning with an initial stand, percent mortality was applied by d.b.h. class. Trees moved into the next d.b.h. class based on the proportion between d.b.h. growth and d.b.h. class width. Ingrowth was determined by applying a constant proportion (0.10) to the tree numbers in the 2-inch class. The model was designed to incorporate the effects of stand density on d.b.h. growth, mortality, and ingrowth. However, for this study, the growth and mortality schedules in Figures 3 and 4 were applied over 20 years. This short projection period represents a fairly typical cutting cycle in northern hardwoods; using it avoids the inaccuracies inherent in long-term projections, especially those caused by regeneration cycles. The results are presented in untransformed numbers of trees in the 6-inch and larger d.b.h. class since the logarithmic form tends to exaggerate any sigmoid tendency, and because most management decisions and interpretations are based on untransformed distributions. The simulations are void of natural variability, so no statistical tests were applied. Interpretations of the results were based on visually distinct effects: the distribution was classified as a rotated sigmoid if it exhibited a plateau, near plateau, or incline in the mid-d.b.h. classes.

Results

When the mortality schedules (Fig. 4) were applied in combination with the Level growth schedule, most of the diameter distributions exhibited a J shape (or slightly sigmoid form) in 20 years regardless of the initial stand condition (Figs. 6-7). The exception was the Stepdown schedule which coupled with the sigmoid initial condition produced a mid-diameter plateau

[2]The use of trade, firm, or corporation names in this paper is for the information and convenience of the reader. Such use does not constitute an official endorsement or approval by the U.S. Department of Agriculture or Forest Service of any product or service to the exclusion of others that may be suitable.

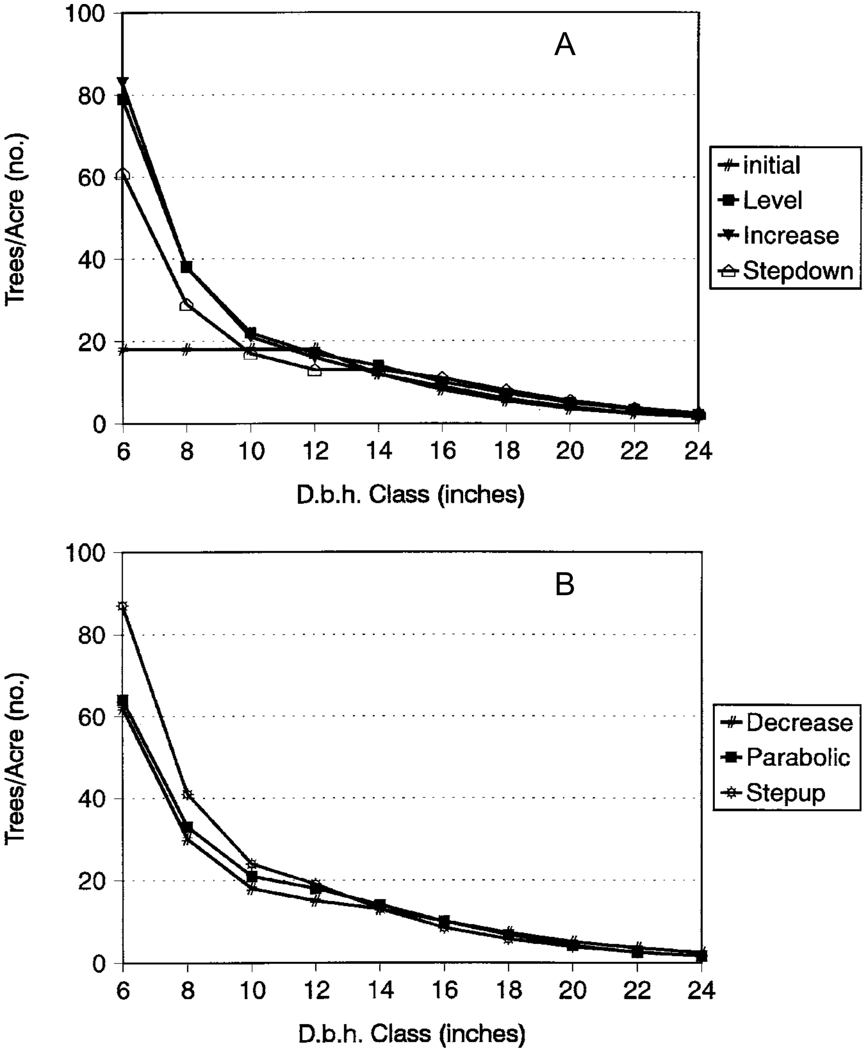

Figure 7.—Simulated diameter distributions after 20 years using (A) the Initial, Level, Increase, and Stepdown mortality schedules, and (B) the Decrease, Parabolic, and Stepup mortality schedules. All simulations using the Level growth trend and beginning with the sigmoid diameter distribution.

characteristic of the rotated sigmoid distribution. Decreasing trends in percent mortality (Stepdown, Decrease, and Parabolic schedules) resulted in curves that were less steep than those of other schedules. The sigmoid initial condition resulted in diameter distributions that are relatively steep in the lower d.b.h. classes (Fig. 7), that is, the Q between d.b.h. classes is larger in the smaller than in the larger classes. This is the type of diameter distribution recognized by Hansen and Nyland (1987), i.e., J shaped but with a larger Q representing the smaller d.b.h. classes.

When the growth schedules (Fig. 3) were applied with the Level mortality schedule, the diameter distributions remained essentially J shaped when the initial condition was J shaped (Fig. 8). There were three exceptions. First, the Stepdown growth schedule produced a well-defined sigmoid diameter distribution; this results when trees move into the mid-diameters faster than they move out. As mentioned previously, this general form of d.b.h. growth might be caused by a somewhat decadent overstory coupled with an understory influx of a more vigorous species. In fact, two long-term field studies in New Hampshire that exhibited sigmoid tendencies over time had a beech overstory with a developing hemlock understory (Leak 1996) or a hemlock understory with a bubble of midtolerant species.[1]

The Stepup growth schedule also produced a deviant from the J-shape distribution in the form of a deficiency in the 14-inch d.b.h. class (Fig. 8). Apparently, this was the result of trees moving out of this class faster than they move in. The Parabolic growth form also produced a slight plateau in the diameter distribution. This partially supports the results by Lorimer and Frelich (1984) as well as the suggestion of Goff and West (1975) that vigorous mid-diameter growth might produce a sigmoid distribution.

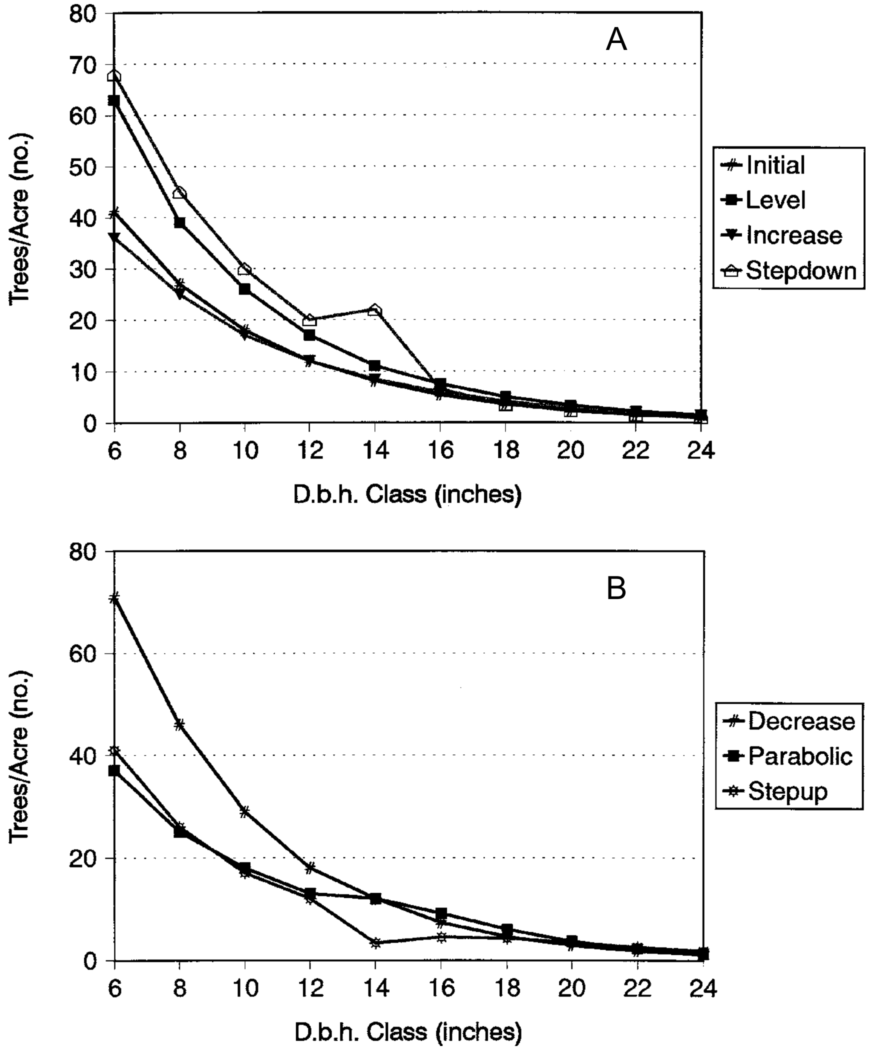

Figure 8.—Simulated diameter distributions after 20 years using (A) the Initial, Level, Increase, and Stepdown growth schedules, and (B) the Decrease, Parabolic, and Stepup growth schedules. All simulations using the Level mortality schedule and beginning with the negative exponential distribution.

When the initial distribution was sigmoid, schedules with slow d.b.h. growth in the small sizes (Parabolic, Stepup, Increase) responded slowly over the 20-year simulation period. This tended to maintain the sigmoid form (Fig. 9). The Stepdown trend produced a well-defined sigmoid distribution. Under the Decrease and Level growth schedules, the J-shape form essentially returned but with a steep (high Q) segment in the smaller d.b.h. classes.

Simulations beginning with a J-shape distribution using a mixture of decreasing growth and mortality or decreasing growth and increasing mortality maintained the J shape (Fig. 10). The Parabolic combination (rapid growth and high mortality in the mid-diameters) produced a slight sigmoid tendency in the 20-year distribution.

Beginning with the residual stand from the diameter-limit cut, the Level growth and mortality schedules resulted in a return to a fairly smooth J-shape (Fig. 11). This result agrees with earlier findings on the Bartlett Forest where diameter-limit cuts through the 10-inch or 13-inch d.b.h. classes developed J-shape curves, sometimes with moderate logarithmic sigmoid tendencies (Leak 1996).

Discussion

A simulation study cannot account for all of the biological and historical factors that affect diameter distributions, for example, historical cutting patterns, species mixtures, insect/disease infestations and regeneration cycles. However, several generalizations (backed by long-term field observations) can be made with respect to New England northern hardwoods.

First, for a variety of growth and mortality schedules, there is a tendency to maintain or revert to a J-shape diameter distribution, sometimes with steeper segments in the lower d.b.h. range (Hansen and Nyland 1987). Beginning with a J-shape initial distribution, a distinct

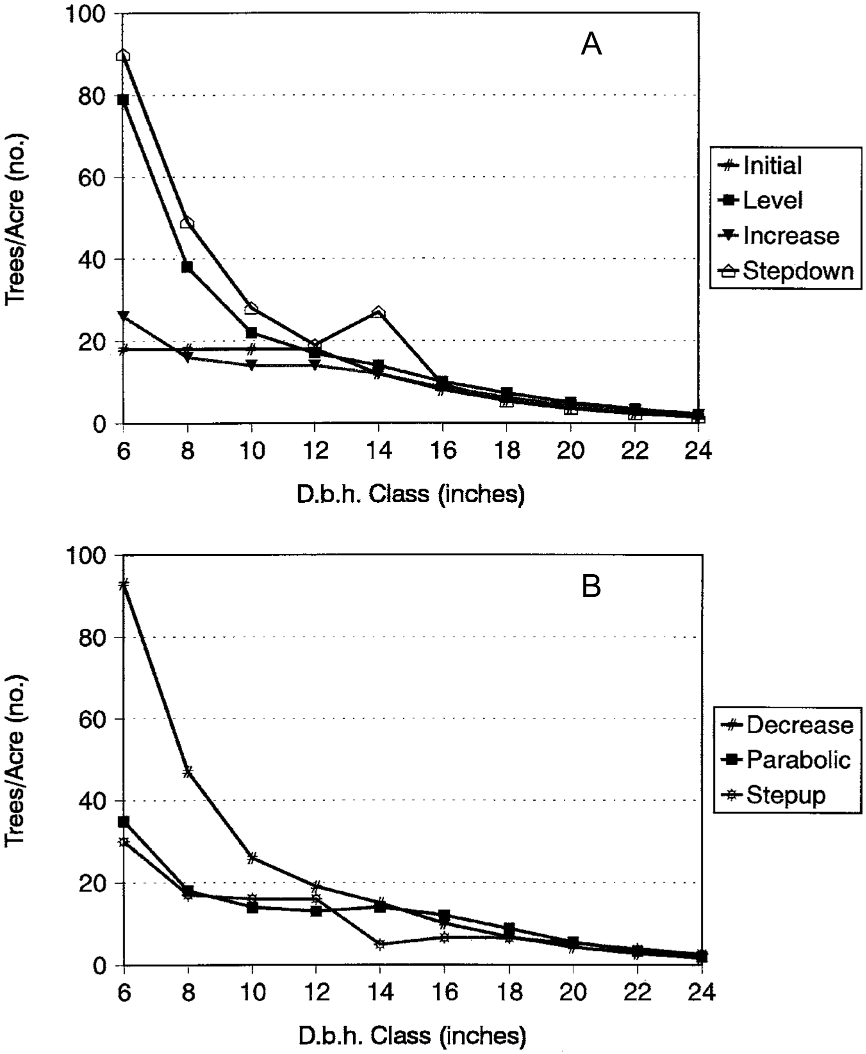

Figure 9.—Simulated diameter distributions after 20 years using (A) the Initial, Level, Increase, and Stepdown growth schedules, and (B) the Decrease, Parabolic, and Stepup growth schedules. All simulations using the Level mortality schedule and beginning with the sigmoid diameter distribution.

sigmoid distribution resulted from a Stepdown growth schedule, which is characterized by a sharp drop in d.b.h. growth rates in the mid-diameter classes. This form could represent a decadent overstory with a more vigorous understory of perhaps a different species, e.g., hemlock. A slightly sigmoid form was produced by a Parabolic diameter-growth schedule (rapid mid-diameter growth) and to a lesser degree by Parabolic diameter growth along with Parabolic mortality (low mid-diameter mortality), as suggested by Goff and West (1975) and Lorimer and Frelich (1984).

Beginning with a distinctly sigmoidal initial distribution, diameter-growth schedules with slow growth in the smaller d.b.h. classes tended to maintain sigmoid characteristics. Those with level d.b.h. growth or rapid growth in the smaller classes reverted to essentially a J-shape structure over the 20-year simulation period.

The form of the mortality schedule had less effect on the resulting shape of the diameter distribution than the growth schedule.

With regard to management practices, it would seem that a variety of marking/harvesting approaches can be used in northern hardwoods in New England without greatly affecting the development of a diameter distribution that resembles the J-shape or slightly sigmoid form. This observation agrees with long-term observations in New England (Leak 1996). These forms generally are accepted as sustainable and productive, i.e., they will support repeated harvesting over time. One exception that seems to significantly skew the distribution toward the sigmoid form is heavy removals in the smaller size classes, along with a low-vigor, slow-growing understory — possibly due to excessive stocking in the larger d.b.h. classes or long-term suppression of the smaller trees.

Are sigmoid distributions characteristic of old-growth stands, earlier disturbance, intensive management, or economically directed timber marking? The answer depends primarily on how these scenarios affect diameter-growth trends, initial stand conditions, and, to a lesser extent, mortality schedules.

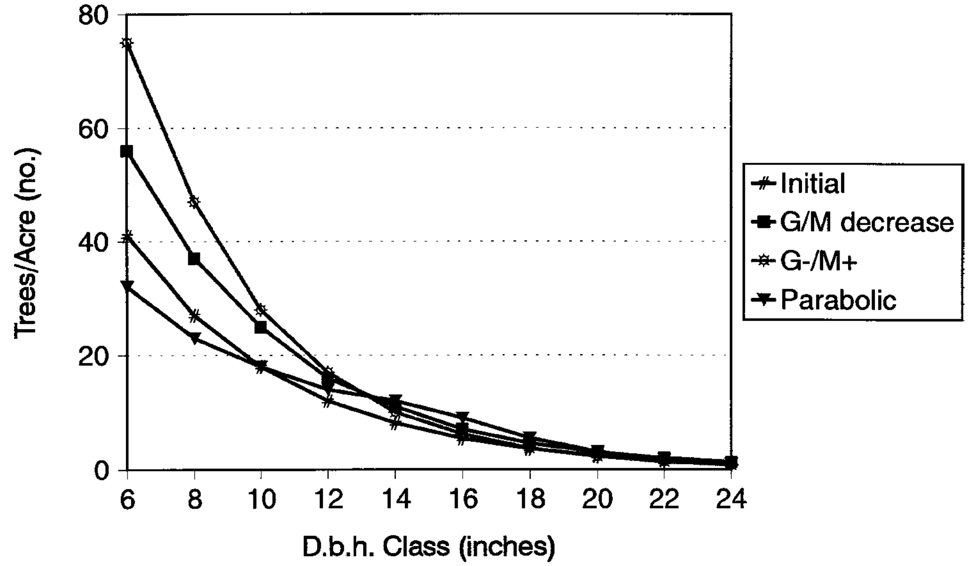

Figure 10.—Simulated diameter distributions after 20 years using growth and mortality combinations: growth Decrease/mortality Decrease and growth Decrease/mortality Increase (initial stand is negative exponential).

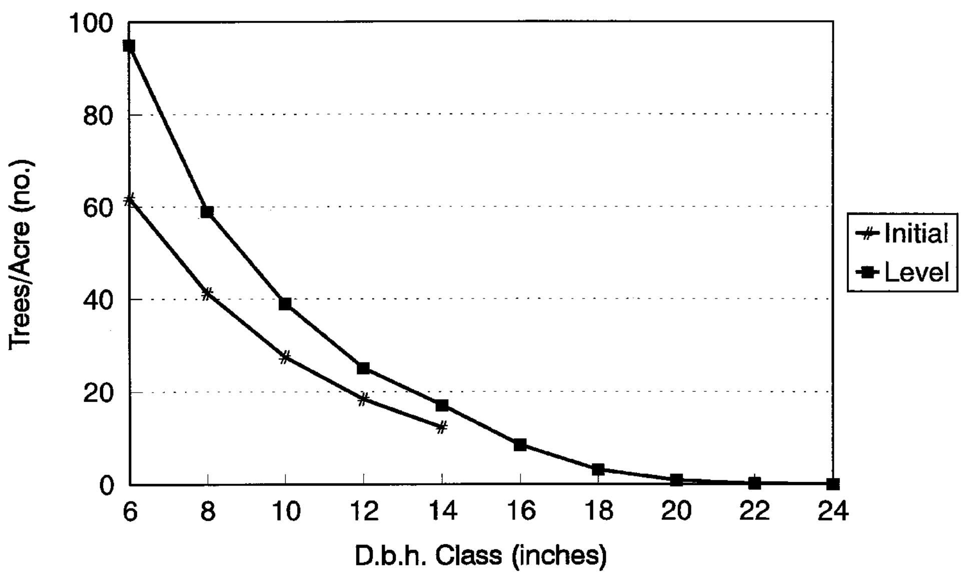

Figure 11.—Simulated diameter distribution after 20 years using Level growth and mortality (initial stand is a diameter limit).

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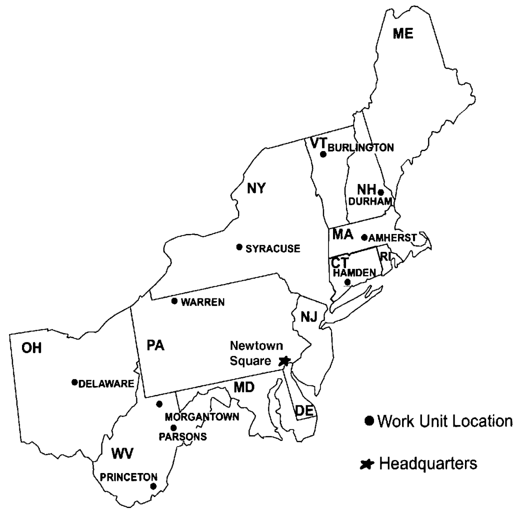

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