

United States
Department of
Agriculture

Forest Service



Southern
Research Station

Research Paper
SRS-39

Equations for Merchantable Volume for Subtropical Moist and Wet Forests of Puerto Rico

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November 2005

Southern Research Station
P.O. Box 2680
Asheville, NC 28802

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Abstract

In Puerto Rico, where locally grown woods are primarily used for furniture and crafts production, estimation of wood volume makes it possible to estimate the monetary value of one of the many commodities and services forests provide to society. In the Puerto Rican forest inventories of 1980 and 1990, workers calculated stem volume directly by applying geometric formulae to bole sections of merchantable trees. Field crews recorded several diameter and height measurements along the bole of each tree. If tree volume estimates were based on fewer tree measurements, this would significantly increase field crew productivity. For this reason, tree volume equations have been derived from Puerto Rican forest inventory data by directly calculating stem volume, then creating regression equations that estimate inside and outside bark merchantable stem volume from tree diameter at breast height and total height.

Keywords: Forest Inventory and Analysis, merchantable stem volume, Puerto Rico, tropical forest, volume equations.

Introduction

Estimates of wood volume are derived from forest inventory data. This is done by means of volume tables, geometric formulae, or regression equations that convert tree measurements such as diameter at breast height (d.b.h.) and total tree height to some measure of wood volume in the tree. Even in the absence of large-scale harvesting and forest-based industries, which is the case for Puerto Rico where locally grown woods are primarily used for furniture and crafts production (Kicliter 1997), estimation of wood volume makes it possible to estimate the monetary value of one of the many commodities and services that forests provide to society. Additionally, studies of forest resource sustainability and international assessments of forest resources, such as the Food and Agriculture Organization of the United Nations Forest Resource Assessments, require the reporting of growing-stock volumes (FAO 2001).

Volume equations have been developed for some Puerto Rican forests and species. Wadsworth (1949) developed a linear regression model that estimates total cubic foot volume from d.b.h. and total tree height for the species found in the subtropical wet forest life zone's tabonuco forest type (Ewel and Whitmore 1973). Francis (1988, 1989) produced linear

and nonlinear equations that estimate tree volumes from d.b.h. and merchantable bole length for plantation-grown oxborn bucida (*Bucida buceras*) and mahoe (*Hibiscus elatus*). Bauer and Gillespie (1990) produced linear volume equations using d.b.h. and total tree height for Honduras mahogany x West Indian mahogany (*Swietenia macrophylla* x *S. mahagoni*). Equations for noncommercial subtropical hardwoods are available from the U.S. Department of Agriculture Forest Service's National Information Management Systems (NIMS); these will be used to estimate tropical tree volumes for southern Florida, an area that has some tree species and genera in common with Puerto Rico. However, in Puerto Rico where an island-wide forest inventory has the potential to encounter over 700 tree species (Francis and Liogier 1991, Little and others 1974), additional resources must be used to produce tree volume estimates.

In Puerto Rican forest inventories conducted in 1980 (Birdsey and Weaver 1982) and 1990 (Franco and others 1997), workers calculated merchantable volumes directly by applying a geometric formula to bole sections of trees classified as growing stock. For a tree to be considered growing stock, one-third or more of the gross volume in its saw-log section must meet grade, soundness, and size requirements for commercial logs, or the tree must have the potential to meet these requirements if it is poletimber size with $12.5 \text{ cm} \leq \text{d.b.h.} < 27.5 \text{ cm}$. Inventory field crews took multiple diameter and height measurements along the bole of each tree so that these calculations could be made—a time-consuming process. If tree volume estimates could be based on fewer tree measurements, this would significantly increase the productivity of field crews in future forest inventories.

For this reason, we have derived tree volume equations from Puerto Rican forest inventory data by first directly calculating stem volumes using geometric formulae, and then fitting linear and nonlinear regression equations for estimating merchantable stem volume inside and outside bark from d.b.h. and total tree height measurements. Equations were fitted to data for all species combined, and then for species and genera represented by 25 or more growing-stock trees.

To determine which equations would produce the best estimates of merchantable volume from future Caribbean forest inventory data, we compared directly calculated volumes from a test dataset to estimates made using our models, estimates made using the Wadsworth (1949) method, and estimates made using the NIMS equations.

Methods

Study Area and Datasets

The forests surveyed in 1980 and 1990 by the U.S. Department of Agriculture Forest Service's Forest Inventory and Analysis Program in Puerto Rico were those considered to have the potential for commercial wood production (i.e., timberland). These surveys excluded forested lands unsuitable for commercial timber production, such as watersheds in which it is necessary to maintain forest cover, forests in areas with excessive rainfall or inoperably steep slopes, dry forests, and all public forests (Birdsey and Weaver 1982, 1987). The surveyed areas were in the subtropical rain, wet, and moist forest life zones (Ewel and Whitmore 1973). The 1990 forest inventory dataset we used in constructing volume equations

in this study consisted of data for 1,247 trees of 102 species with d.b.h. ≥ 12.5 cm after the removal of data for palms, tree ferns, and poorly formed cull trees that did not meet the standards for growing stock. The 1980 inventory dataset used to test the equations consisted of data for 591 trees of 76 species with d.b.h. ≥ 12.5 cm, again with data for cull trees, palms, and tree ferns removed. Figure 1 presents the diameter distribution of trees from the 1990 and 1980 forest inventory datasets by 10-cm d.b.h. classes. Table 1 shows maximum and minimum d.b.h. and heights for species or genera in the 1980 and 1990 datasets that were represented by more than 25 trees in both datasets combined.

The resource bulletins based on the 1980 and 1990 Puerto Rican forest inventories (Birdsey and Weaver 1982, Franco and others 1997) gave volume estimates (in m^3) that were calculated directly by applying the formula for the volume of a conic frustum to bole sections and summing these section volumes to obtain total merchantable volume. Total merchantable volume was defined as the inside bark portion of the tree's stem between a stump 30 cm tall and a 10-cm upper-stem diameter outside bark (d.o.b.), excluding wood volume in branches. The details of these direct volume calculations are given below.

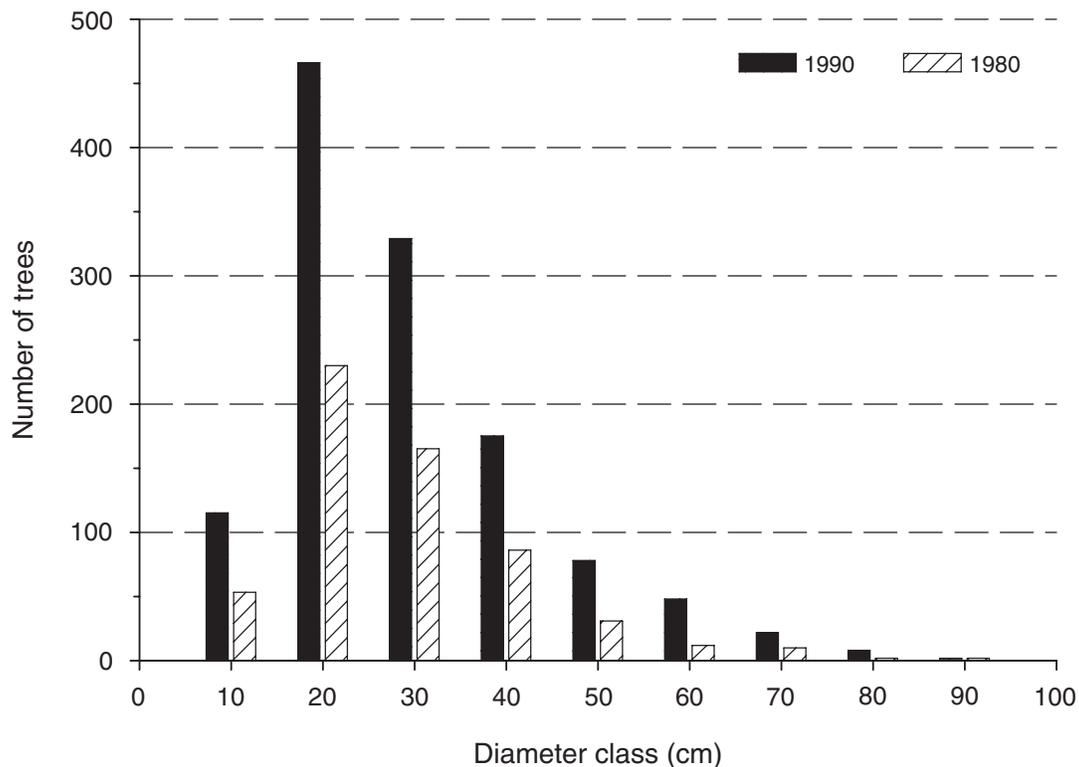


Figure 1—Distribution of trees by 10-cm diameter classes (d.b.h.) and year.

Table 1—Number of trees, mean, standard error, maximum, and minimum measurements for d.b.h. and height by species and year, Puerto Rico

| Species | Trees | D.b.h. | | | | Height | | | |
|-------------------------------|---------------|-----------|----------------|---------|---------|----------|----------------|---------|---------|
| | | Mean | Standard error | Maximum | Minimum | Mean | Standard error | Maximum | Minimum |
| | <i>number</i> | <i>cm</i> | | | | <i>m</i> | | | |
| <i>Andira inermis</i> | | | | | | | | | |
| 1980 | 33 | 22.1 | 1.3 | 41.6 | 12.5 | 12.9 | 0.6 | 21.0 | 7.0 |
| 1990 | 56 | 24.3 | 1.4 | 55.4 | 12.8 | 15.0 | 0.6 | 30.0 | 8.0 |
| <i>Cecropia schreberiana</i> | | | | | | | | | |
| 1980 | 28 | 30.3 | 1.9 | 53.8 | 17.5 | 16.1 | 0.7 | 23.0 | 6.0 |
| 1990 | 70 | 32.8 | 1.4 | 57.5 | 12.7 | 17.8 | 0.6 | 29.0 | 7.0 |
| <i>Cordia</i> spp. | | | | | | | | | |
| 1980 | 18 | 24.4 | 1.7 | 35.8 | 14.3 | 14.3 | 0.8 | 24.0 | 9.0 |
| 1990 | 30 | 23.6 | 1.3 | 42.3 | 13.5 | 14.7 | 0.9 | 25.0 | 3.0 |
| <i>Erythrina poeppigiana</i> | | | | | | | | | |
| 1980 | 15 | 29.7 | 2.1 | 49.9 | 20.2 | 13.1 | 1.0 | 18.0 | 5.0 |
| 1990 | 26 | 35.8 | 2.5 | 68.7 | 18.9 | 16.5 | 1.3 | 31.0 | 6.0 |
| <i>Guarea guidonia</i> | | | | | | | | | |
| 1980 | 94 | 31.9 | 1.3 | 62.8 | 12.7 | 14.7 | 0.3 | 21.0 | 8.0 |
| 1990 | 187 | 34.2 | 1.2 | 76.5 | 12.7 | 16.1 | 0.3 | 29.0 | 5.0 |
| <i>Inga laurina</i> | | | | | | | | | |
| 1980 | 25 | 27.0 | 1.5 | 48.5 | 15.5 | 16.2 | 0.8 | 23.0 | 9.0 |
| 1990 | 50 | 31.5 | 1.6 | 65.3 | 14.7 | 17.6 | 0.9 | 34.0 | 5.0 |
| <i>I. vera</i> | | | | | | | | | |
| 1980 | 70 | 26.9 | 1.1 | 51.3 | 12.7 | 14.1 | 0.4 | 22.0 | 8.0 |
| 1990 | 110 | 29.4 | 1.0 | 61.8 | 13.0 | 16.8 | 0.4 | 30.0 | 7.0 |
| <i>Mangifera indica</i> | | | | | | | | | |
| 1980 | 22 | 57.0 | 4.2 | 85.1 | 27.1 | 14.7 | 0.7 | 22.0 | 8.0 |
| 1990 | 35 | 57.1 | 3.4 | 87.9 | 17.1 | 17.5 | 0.7 | 27.0 | 10.0 |
| <i>Ocotea</i> spp. | | | | | | | | | |
| 1980 | 16 | 22.4 | 1.6 | 36.5 | 13.4 | 12.5 | 0.8 | 22.0 | 9.0 |
| 1990 | 33 | 24.0 | 1.4 | 44.5 | 13.1 | 12.4 | 0.6 | 19.0 | 4.0 |
| <i>Schefflera morototonii</i> | | | | | | | | | |
| 1980 | 22 | 27.8 | 3.4 | 65.3 | 13.6 | 15.9 | 0.9 | 23.0 | 9.0 |
| 1990 | 38 | 28.5 | 2.4 | 72.5 | 12.7 | 15.6 | 1.0 | 31.0 | 6.0 |
| <i>Spathodea campanulata</i> | | | | | | | | | |
| 1980 | 8 | 30.8 | 3.6 | 46.8 | 15.5 | 17.0 | 1.7 | 23.0 | 11.0 |
| 1990 | 121 | 25.8 | 1.0 | 68.5 | 12.8 | 14.2 | 0.4 | 28.0 | 3.0 |
| <i>Syzygium jambos</i> | | | | | | | | | |
| 1980 | 15 | 24.0 | 1.9 | 40.0 | 14.5 | 11.3 | 0.9 | 18.0 | 6.0 |
| 1990 | 39 | 22.1 | 1.2 | 43.4 | 13.5 | 11.6 | 0.6 | 22.0 | 6.0 |
| <i>Tabebuia heterophylla</i> | | | | | | | | | |
| 1980 | 15 | 21.1 | 2.1 | 37.1 | 13.1 | 13.8 | 0.7 | 18.0 | 8.0 |
| 1990 | 53 | 21.1 | 1.0 | 42.8 | 12.5 | 14.4 | 0.4 | 20.0 | 8.0 |
| Other ^a | | | | | | | | | |
| 1980 | 209 | 26.8 | 0.8 | 67.4 | 12.7 | 13.8 | 0.7 | 26.0 | 5.0 |
| 1990 | 399 | 28.2 | 0.6 | 76.5 | 12.6 | 15.4 | 0.3 | 37.0 | 5.0 |
| All species | | | | | | | | | |
| 1980 | 590 | 28.4 | 0.5 | 85.1 | 12.5 | 14.2 | 0.2 | 26.0 | 5.0 |
| 1990 | 1,247 | 29.4 | 0.4 | 87.9 | 12.5 | 15.5 | 0.1 | 37.0 | 3.0 |

D.b.h. = diameter at breast height.

^aSpecies for which a species- or genus-specific equation was not fitted.

Measurements and Calculations for Estimating Stem Volume

During the Puerto Rican forest inventories, the following measurements were taken on all trees with a d.b.h. ≥ 12.5 cm (fig. 2A):

- D.b.h. outside bark
- Double bark thickness (twice the bark thickness measured with a bark gauge, at breast height [1.37 m])
- Stump d.o.b. taken at 30 cm height
- Total bole length (length from a stump 30 cm tall to a 10-cm top d.o.b.)
- Pole top d.o.b. (if not 10 cm)
- Total height (H_T) from ground

For trees with d.b.h. ≥ 27.5 cm (sawtimber size), additional measurements were taken (fig. 2B):

- Saw-log length to d.o.b. of 22.5 cm
- Saw-log d.o.b. (if not 22.5 cm)

Stem volume was calculated by applying the formula for a conic frustum to sections of the bole.

For poletimber-size trees (fig. 2A), there were two bole sections:

1. Stump top to breast height
2. Breast height to pole top

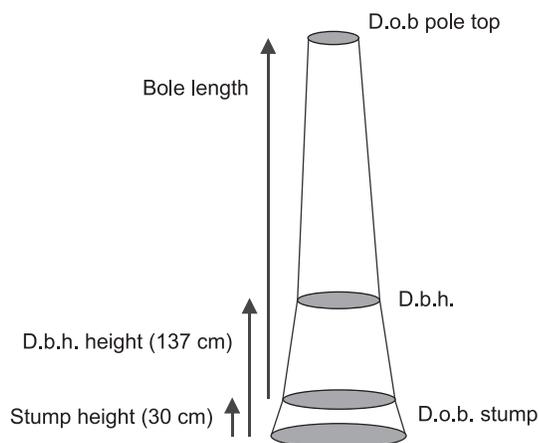
For sawtimber-size trees (fig. 2B), the sections were:

1. Stump top to breast height
2. Breast height to saw-log top
3. Saw-log top to pole top

Estimates of outside bark merchantable stem volume were based on outside bark diameters, and no deductions were made for bark thickness. To estimate inside bark merchantable stem volume, the diameters for all sections were converted from outside bark to inside bark. To deduct for bark thickness, bark ratio was first calculated using the following formula for hardwoods:

$$BR = \frac{D_{BH} - D_{BT}}{D_{BH}} \quad (1)$$

(A)



(B)

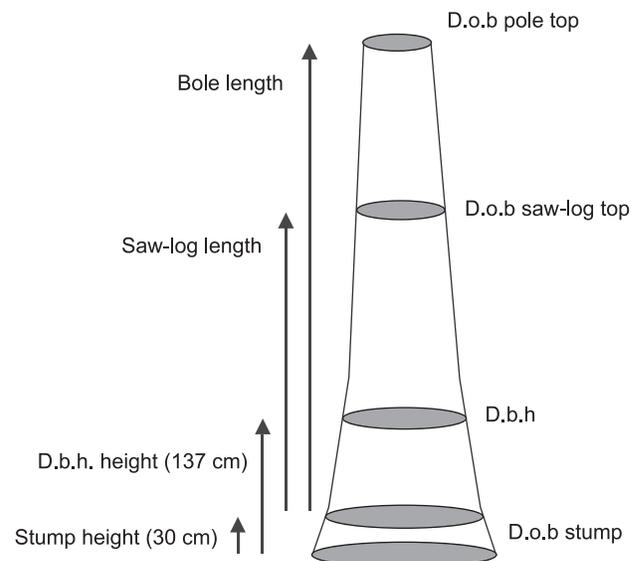


Figure 2—Measurements taken on (A) poletimber-size trees (minimum d.b.h. = 12.5 cm, minimum top d.o.b. = 10.0 cm) and (B) sawtimber-size trees (minimum d.b.h. = 27.5 cm, minimum saw-log top d.o.b. = 22.5 cm) in the 1980 and 1990 forest inventories of Puerto Rico.

where

BR = bark ratio

D_{BH} = d.b.h. outside bark

D_{BT} = double bark thickness

Then, diameters inside bark at stump, breast, saw-log top, and pole-top heights were calculated with the following formula for hardwoods:

$$D_{IB} = \frac{9 \cdot D_{OB} \cdot BR}{10 - (D_{OB}/D_{BH})} \quad (2)$$

where

D_{IB} = diameter inside bark

D_{OB} = diameter outside bark

Section lengths and inside and outside bark diameters were used in the formula for a conic frustum to calculate the wood volume of individual sections (example for inside bark section volume):

$$V_{SEC} = \{H_{SEC} \cdot [D_{IB1}^2 + (D_{IB1} \cdot D_{IB2}) + D_{IB2}^2] \cdot 0.00007854\} / 3 \quad (3)$$

where

V_{SEC} = section volume in m^3

H_{SEC} = section length in m

D_{IB1} = diameter in cm inside bark at one end of section

D_{IB2} = diameter in cm inside bark at other end of section

0.00007854 is a constant derived from the expression

$$D_i^2 \cdot [\pi / (4 \cdot 10,000)] \quad (\text{Husch and others 1993}) \quad (4)$$

where

D_i = section diameter in cm

The section volumes are then added together to obtain a total stem volume. Total stem volume is defined here as the inside or outside bark portion of the tree's stem between a 30-cm-tall stump and a 10-cm upper stem diameter. This volume estimate does not include branchwood volume. Stem volumes were calculated using this method for all trees in the 1990 forest inventory dataset that met minimum diameter requirements and were not abnormally formed.

Fitting of Regression Equations

Figure 3 presents directly calculated mean volumes for the 1990 and 1980 Puerto Rico forest survey datasets by 10-cm d.b.h. classes (there was only one tree in the 90-cm class so no standard error is presented for that class). First, equations were fitted to the 1990 inventory data for all species combined. These equations used d.b.h. and total tree height in different combinations, including the often used model $D_{BH}^2 H_T$. Mallows's C_p , PRESS, Variance Inflation Factor, and R^2 statistics (Stafford and Sabin 1997) were examined to determine which model fit best. Separate equations for poletimber and sawtimber-size stems were also tried, but this did not improve the fit appreciably. Once equations were fitted to the entire dataset, separate equations were fitted to individual species or genera represented by more than 25 individual trees.

The equations that fit the data best were those in which d.b.h. and total tree height were the independent variables and logarithmic transformations were applied to both the dependent and independent variables to stabilize variance. Two other models were also fitted to the data. An equation with only d.b.h. as an independent variable was fitted so that volume can be estimated when forest inventories do not include data on tree height, which is often hard to measure accurately in forest settings. An equation using only d.b.h. could also be used where tree crowns have been damaged by hurricanes, which is common in the Caribbean.

We also fit a nonlinear equation that incorporates a taper function based on the models mathematically described by Clutter (1980) and applied by Zarnoch and others (2003) to the same 1990 dataset (by species and genera) used for linear regression model fitting. This was done despite concerns that fewer upper stem measurements (only two—saw and pole-log top diameters) were available in the inventory datasets than are usually taken on each tree when developing a taper function (Cost 1978, Thomas and Parresol 1991, Van Deusen and others 1982). This results in more extrapolation from breast height to pole and saw-log heights, and a less than ideal representation of tree taper. The advantage of an equation with an incorporated taper function is the flexibility to change the upper stem diameter merchantability limit, currently set for this study at 10 cm d.o.b., if future needs change. We used the SAS NLIN (nonlinear regression) procedure (Cody and Smith 1997) with the Marquardt method as the fitting algorithm, and provided a starting value grid to avoid local minima (Freund and Littel 2000).

Bias introduced into the volume estimate by the logarithmic transformation was corrected for as suggested by Baskerville (1972) by multiplying estimated individual tree volume by $e^{0.5MSE}$. As the main focus of this analysis is on volume and

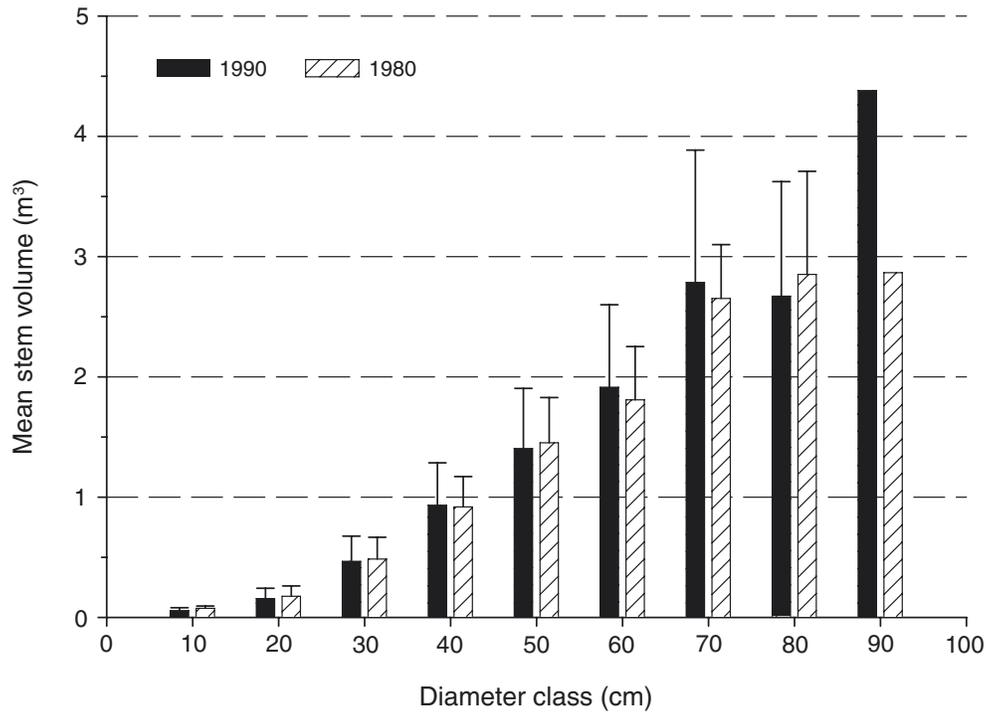


Figure 3—Mean stem volumes with standard errors directly calculated from multiple stem measurements, by 10-cm diameter classes (d.b.h.) and year. (Stem volume measurement is between a 30-cm tall stump and a 10-cm outside bark upper stem diameter, excluding wood volume in branches).

not the logarithm of the volume, the mean square error and R^2 were calculated using estimated volume and measured volume as follows (Kvålseth 1985):

$$R^2 = 1 - \frac{\sum (V_{meas} - V_{stem})^2}{\sum (V_{meas} - \bar{V}_{meas})^2} \quad (5)$$

where

V_{meas} = measured volume

V_{stem} = merchantable stem volume

The R^2 obtained when volume is used is slightly lower than that obtained when the logarithm of volume is used.

Volume Estimates and Comparisons

The 1980 Puerto Rico forest survey data were used as a test dataset. Stem volume was calculated directly using the forest inventory methods described above for all trees with a d.b.h. ≥ 12.5 cm in the 1980 forest survey (fig. 3). Stem volume was also estimated by inputting each tree's d.b.h. and total tree height values into the regression equations. For species without a species-specific volume equation, the general equations were used. For the nonlinear equation with incorporated

taper function, the upper stem diameter merchantability limit was set at 10 cm.

For the sake of comparison, volume was also estimated for each tree in the 1980 dataset using the Wadsworth (1949) volume equation for trees in the tabonuco forest type in the Caribbean National Forest for trees ranging in size from a d.b.h. of 5 to 125 cm and total tree height from 3.0 to 36.5 m. [Note that Wadsworth (1949) used English units, where stem volume is in cubic feet, d.b.h. is in inches, and total tree height is in feet. The volume estimates produced by Wadsworth's equation (equation 6) for this comparison were converted to metric units.]

$$\begin{aligned} \text{Log}(Y_s) = & 2.2020 \cdot \text{Log}(D_{BH}) + \\ & 0.4434 \cdot \text{Log}(H_T) - 1.8374 \end{aligned} \quad (6)$$

where

Y_s = stem volume

Volume was also estimated using the NIMS noncommercial subtropical hardwood equations that will be used to estimate subtropical tree volumes for southern Florida. NIMS has an equation for poletimber-size (d.b.h. < 27.5 cm) trees,

$$Y_{POLE} = -0.378830 + 0.001855 \cdot (D_{BH})^2 \cdot H_T \quad (7)$$

where

Y_{POLE} = poletimber volume

and another equation for sawtimber-size trees (d.b.h. ≥ 27.5 cm),

$$Y_{SAW} = 0.82350 + 0.001630 \cdot (D_{BH})^2 \cdot H_T \quad (8)$$

where

Y_{SAW} = sawtimber volume

Again, the volume estimates produced by these equations have been converted to metric units.

Pair-wise t-tests were used to determine whether the mean differences between the four volume estimates (log transformed linear d.b.h. and total tree height equation, log transformed d.b.h. alone equation, Wadsworth equation, and NIMS equations) and the directly calculated merchantable volume were greater than zero by chance alone (Cody and Smith 1997). Comparisons were made on all trees in the poletimber size class (12.5 \leq d.b.h. < 27.5 cm), sawtimber size class (d.b.h. ≥ 27.5 cm), and by 10-cm d.b.h. classes up to 70 cm. There were too few trees in the 80- and 90-cm d.b.h. classes to permit accurate testing.

Results

Model Building

For all species combined, the best fitting equation that used d.b.h. and total tree height was:

$$V_{stem} = e^{b_0 + b_1(\ln D_{BH}) + b_2(\ln H_T) + b_3(D_{BH})} \quad (9)$$

where

V_{stem} = merchantable stem volume in m^3

A second regression was fitted with only D_{BH} as the independent variable,

$$V_{stem} = e^{b_0 + b_1(\ln D_{BH}) + b_2(D_{BH})} \quad (10)$$

The nonlinear taper equation used had the form:

$$V_{stem} = b_o \cdot D_{BH}^{b_1} \cdot H_T^{b_2} - K \frac{D_{upper}^{b_3}}{D_{BH}^{(b_3-2)}} \cdot (H_T - 1.37) \quad (11)$$

where

D_{upper} = upper stem diameter merchantability limit in cm

$K = 0.00007854 \cdot (1 - 2/b_3)$

The taper function implied by this merchantable volume equation (equation 11) can be used to estimate upper stem diameter at any merchantable height, and height at any upper stem diameter as follows:

$$D_{upper} = D_{BH} \left(\frac{H_T - h}{H_T - 1.37} \right)^{\frac{1}{b_3-2}} \quad (12)$$

and

$$h = H_T - (H_T - 1.37) \left(\frac{D_{upper}}{D_{BH}} \right)^{b_3-2} \quad (13)$$

where

h = merchantable height in m

When volume was estimated with the nonlinear equation, the upper stem diameter merchantability limit was set at 10 cm.

Coefficients of equations for merchantable stem volume inside and outside bark for all species combined and 13 individual species or genus groupings are given in tables 2, 3, and 4 (table 2 for d.b.h. and total tree height equations, table 3 for d.b.h. only equations, and table 4 for nonlinear d.b.h. and total tree height equations). If one is using the species-specific equations to estimate a forest's volume, one should use the "other species" equation coefficients in tables 2, 3, and 4 to estimate volumes for species that don't have their own equations.

Equation Testing

Volume comparisons were made for all trees in the poletimber size class (12.5 cm \leq d.b.h. < 27.5 cm), sawtimber size class (d.b.h. ≥ 27.5 cm), and by 10-cm d.b.h. classes up to 70 cm. The results with standard errors and 95-percent confidence limits are presented in table 5. Note that the 10-cm d.b.h. class consists only of trees with d.b.h. ranging from 12.5 to 14.9 cm. Only comparisons up to the 70-cm d.b.h. class are presented in table 5 because the 80- and 90-cm classes had only two trees each.

The log-transformed linear equation with d.b.h., the nonlinear taper equation with d.b.h., and the log-transformed linear equation with d.b.h. and total tree height produced merchantable stem volume estimates that were very similar overall and were acceptable in accuracy when compared to the directly calculated volumes. Assessing model performance

Table 2—Model coefficients for estimating inside and outside bark merchantable stem volume from d.b.h. and total height (log transformed) by species, Puerto Rico, 1990^a

| Species | Trees number | b ₀ | b ₁ | b ₂ | b ₃ | R ² | MSE |
|-------------------------------|-----------------|----------------|----------------|----------------|----------------|----------------|---------|
| <i>Andira inermis</i> | 56 | | | | | | |
| Inside bark | | -10.73531 | 2.33133 | 0.77468 | -0.00748 | 0.89225 | 0.02102 |
| Outside bark | | -10.28954 | 2.28989 | 0.71767 | -0.00729 | 0.89010 | 0.02697 |
| <i>Cecropia schreberiana</i> | 70 | | | | | | |
| Inside bark | | -13.56336 | 3.55071 | 0.78797 | -0.04695 | 0.87448 | 0.04774 |
| Outside bark | | -13.14265 | 3.44680 | 0.78694 | -0.04487 | 0.87223 | 0.06064 |
| <i>Cordia</i> spp. | 30 | | | | | | |
| Inside bark | | -11.95066 | 3.06860 | 0.57666 | -0.02958 | 0.92034 | 0.00572 |
| Outside bark | | -11.74761 | 3.07771 | 0.58133 | -0.03390 | 0.92032 | 0.00681 |
| <i>Erythrina poeppigiana</i> | 26 | | | | | | |
| Inside bark | | -14.27432 | 3.63718 | 0.73037 | -0.03270 | 0.94862 | 0.06127 |
| Outside bark | | -14.50486 | 3.83840 | 0.69531 | -0.03801 | 0.95913 | 0.06134 |
| <i>Guarea guidonia</i> | 187 | | | | | | |
| Inside bark | | -12.56150 | 3.05691 | 0.74885 | -0.02617 | 0.89648 | 0.06431 |
| Outside bark | | -12.27119 | 3.01970 | 0.75533 | -0.02674 | 0.89667 | 0.07929 |
| <i>Inga laurina</i> | 50 | | | | | | |
| Inside bark | | -12.70218 | 2.83593 | 1.01689 | -0.02070 | 0.92292 | 0.01989 |
| Outside bark | | -13.00902 | 3.00540 | 1.02641 | -0.02627 | 0.93027 | 0.02186 |
| <i>I. vera</i> | 110 | | | | | | |
| Inside bark | | -10.83036 | 2.50263 | 0.73109 | -0.01560 | 0.91807 | 0.01670 |
| Outside bark | | -10.84585 | 2.55560 | 0.73265 | -0.01738 | 0.91968 | 0.01999 |
| <i>Mangifera indica</i> | 35 | | | | | | |
| Inside bark | | -10.16260 | 1.92058 | 1.08297 | -0.00294 | 0.73210 | 0.41411 |
| Outside bark | | -9.41269 | 1.82991 | 0.97767 | -0.00155 | 0.73835 | 0.48507 |
| <i>Ocotea</i> spp. | 33 | | | | | | |
| Inside bark | | -11.10688 | 2.19819 | 1.05004 | -0.00256 | 0.97535 | 0.00148 |
| Outside bark | | -10.95222 | 2.20138 | 1.04221 | -0.00322 | 0.97829 | 0.00163 |
| <i>Schefflera morototonii</i> | 38 | | | | | | |
| Inside bark | | -10.70757 | 2.16136 | 0.97939 | -0.00323 | 0.99144 | 0.01111 |
| Outside bark | | -10.12920 | 2.06841 | 0.89646 | -0.00022 | 0.99523 | 0.00808 |
| <i>Spathodea campanulata</i> | 121 | | | | | | |
| Inside bark | | -11.83160 | 2.70894 | 0.86993 | -0.01669 | 0.98328 | 0.00497 |
| Outside bark | | -11.14226 | 2.55388 | 0.83876 | -0.01385 | 0.98075 | 0.00759 |
| <i>Syzygium jambos</i> | 39 | | | | | | |
| Inside bark | | -10.54334 | 2.08250 | 0.89368 | 0.00008 | 0.83125 | 0.01450 |
| Outside bark | | -10.33813 | 2.02344 | 0.91053 | 0.00132 | 0.83866 | 0.01607 |
| <i>Tabebuia heterophylla</i> | 53 | | | | | | |
| Inside bark | | -11.57430 | 2.49205 | 0.89277 | -0.00703 | 0.95610 | 0.00222 |
| Outside bark | | -11.36222 | 2.48892 | 0.90955 | -0.01059 | 0.95513 | 0.00286 |
| Other ^b | 399 | | | | | | |
| Inside bark | | -12.13380 | 2.70518 | 0.95270 | -0.01870 | 0.92474 | 0.02722 |
| Outside bark | | -11.82546 | 2.64837 | 0.94063 | -0.01708 | 0.92436 | 0.03570 |
| All species | 1,247 | | | | | | |
| Inside bark | | -12.02348 | 2.75276 | 0.88961 | -0.02072 | 0.90827 | 0.04140 |
| Outside bark | | -11.68173 | 2.69449 | 0.87782 | -0.01975 | 0.90924 | 0.05188 |

d.b.h. = diameter at breast height.

MSE = mean square error.

^a $V_{stem} = e^{b_0 + b_1 (\ln D_{BH}) + b_2 (\ln H_T) + b_3 (D_{BH}^3)}$, where V_{stem} = merchantable stem volume in m³, D_{BH} = diameter at 1.37 m, and H_T = total tree height in m.

^b Species for which a species- or genus-specific equation was not fitted.

Table 3—Model coefficients for estimating inside and outside bark merchantable stem volume from d.b.h. (log transformed) by species, Puerto Rico, 1990^a

| Species | Trees <i>number</i> | b ₀ | b ₁ | b ₂ | R ² | MSE |
|-------------------------------|------------------------|----------------|----------------|----------------|----------------|---------|
| <i>Andira inermis</i> | 56 | | | | | |
| Inside bark | | -9.89365 | 2.74883 | -0.01023 | 0.78206 | 0.04251 |
| Outside bark | | -9.50640 | 2.67666 | -0.00983 | 0.78106 | 0.05373 |
| <i>Cecropia schreberiana</i> | 70 | | | | | |
| Inside bark | | -13.66618 | 4.36740 | -0.06043 | 0.78402 | 0.08234 |
| Outside bark | | -13.24115 | 4.26242 | -0.05833 | 0.78183 | 0.10355 |
| <i>Cordia</i> spp. | 30 | | | | | |
| Inside bark | | -13.26963 | 4.24623 | -0.06536 | 0.81290 | 0.01342 |
| Outside bark | | -13.07582 | 4.26489 | -0.06997 | 0.79597 | 0.01744 |
| <i>Erythrina poeppigiana</i> | 26 | | | | | |
| Inside bark | | -15.87797 | 4.82072 | -0.04694 | 0.84485 | 0.18501 |
| Outside bark | | -16.02184 | 4.96512 | -0.05157 | 0.86773 | 0.19853 |
| <i>Guarea guidonia</i> | 187 | | | | | |
| Inside bark | | -11.37162 | 3.29002 | -0.02417 | 0.86666 | 0.08283 |
| Outside bark | | -11.06816 | 3.25483 | -0.02471 | 0.86490 | 0.10367 |
| <i>Inga laurina</i> | 50 | | | | | |
| Inside bark | | -14.76853 | 4.70259 | -0.06525 | 0.75718 | 0.06266 |
| Outside bark | | -15.09003 | 4.88954 | -0.07125 | 0.76266 | 0.07440 |
| <i>I. vera</i> | 110 | | | | | |
| Inside bark | | -9.95936 | 2.89215 | -0.01964 | 0.81276 | 0.03817 |
| Outside bark | | -9.97137 | 2.94596 | -0.02143 | 0.82018 | 0.04475 |
| <i>Mangifera indica</i> | 35 | | | | | |
| Inside bark | | -10.49736 | 3.07171 | -0.02132 | 0.58428 | 0.64259 |
| Outside bark | | -9.69613 | 2.86912 | -0.01815 | 0.60684 | 0.72887 |
| <i>Ocotea</i> spp. | 33 | | | | | |
| Inside bark | | -9.53696 | 2.54212 | -0.00418 | 0.78678 | 0.01282 |
| Outside bark | | -9.39237 | 2.54274 | -0.00482 | 0.78423 | 0.01618 |
| <i>Schefflera morototonii</i> | 38 | | | | | |
| Inside bark | | -10.75646 | 3.08376 | -0.01346 | 0.93960 | 0.07838 |
| Outside bark | | -10.15879 | 2.91271 | -0.00958 | 0.94095 | 0.09998 |
| <i>Spathodea campanulata</i> | 121 | | | | | |
| Inside bark | | -11.08799 | 3.19974 | -0.01805 | 0.91919 | 0.02400 |
| Outside bark | | -10.42197 | 3.02710 | -0.01516 | 0.91732 | 0.03259 |
| <i>Syzygium jambos</i> | 39 | | | | | |
| Inside bark | | -9.65948 | 2.53683 | -0.00496 | 0.60120 | 0.03428 |
| Outside bark | | -9.43608 | 2.48633 | -0.00381 | 0.60566 | 0.03927 |
| <i>Tabebuia heterophylla</i> | 53 | | | | | |
| Inside bark | | -11.75282 | 3.58078 | -0.04111 | 0.93698 | 0.00318 |
| Outside bark | | -11.54393 | 3.59811 | -0.04531 | 0.93484 | 0.00415 |
| Other ^b | 399 | | | | | |
| Inside bark | | -11.25341 | 3.23528 | -0.02031 | 0.82853 | 0.06202 |
| Outside bark | | -10.95213 | 3.17175 | -0.01866 | 0.83424 | 0.07823 |
| All species | 1,247 | | | | | |
| Inside bark | | -11.41282 | 3.35947 | -0.02733 | 0.81503 | 0.08347 |
| Outside bark | | -11.07369 | 3.29316 | -0.02627 | 0.81746 | 0.10434 |

d.b.h. = diameter at breast height.

MSE = mean square error.

^a $V_{stem} = e^{b_0 + b_1(\ln D_{BH}) + b_2(D_{BH})}$, where V_{stem} = merchantable stem volume in m³ and D_{BH} = diameter at 1.37 m.

^b Species for which a species- or genus-specific equation was not fitted.

Table 4—Model coefficients for estimating inside and outside bark merchantable stem volume from d.b.h. and total height in a nonlinear regression taper equation by species, Puerto Rico, 1990^a

| Species | Trees <i>number</i> | b ₀ | b ₁ | b ₂ | b ₃ | SSE |
|-------------------------------|------------------------|----------------|----------------|----------------|----------------|-------------------------|
| <i>Andira inermis</i> | 56 | | | | | |
| Inside bark | | 0.00004 | 1.81263 | 1.11977 | 3.35135 | 2.52271 |
| Outside bark | | 0.00002 | 1.90521 | 1.21305 | 3.24424 | 4.17579 |
| <i>Cecropia schreberiana</i> | 70 | | | | | |
| Inside bark | | 0.00010 | 1.81935 | 0.83918 | 3.29290 | 7.92211 |
| Outside bark | | 0.00007 | 1.95023 | 0.83456 | 3.26138 | 11.48275 |
| <i>Cordia</i> spp. | 30 | | | | | |
| Inside bark | | 0.00006 | 2.23743 | 0.50421 | 3.46169 | 0.29389 |
| Outside bark | | 0.00003 | 2.52160 | 0.44382 | 3.19032 | 1.13561 |
| <i>Erythrina poeppigiana</i> | 26 | | | | | |
| Inside bark | | 0.00002 | 1.98406 | 1.12400 | 3.51253 | 1.91242 |
| Outside bark | | 0.00005 | 1.97705 | 0.93928 | 3.44792 | 2.28506 |
| <i>Guarea guidonia</i> | 187 | | | | | |
| Inside bark | | 0.00008 | 1.92664 | 0.70748 | 2.96476 | 25.98568 |
| Outside bark | | 0.00010 | 1.88734 | 0.73166 | 2.96413 | 33.09052 |
| <i>Inga laurina</i> | 50 | | | | | |
| Inside bark | | 0.00002 | 2.02317 | 1.15868 | 3.29652 | 3.05359 |
| Outside bark | | 0.00001 | 2.04250 | 1.22339 | 3.25192 | 4.16100 |
| <i>I. vera</i> | 110 | | | | | |
| Inside bark | | 0.00008 | 1.85341 | 0.87317 | 3.27694 | 6.42406 |
| Outside bark | | 0.00007 | 1.92919 | 0.84339 | 3.22199 | 9.87309 |
| <i>Mangifera indica</i> | 35 | | | | | |
| Inside bark | | 0.00009 | 1.56418 | 1.19588 | 2.93596 | 25.90205 |
| Outside bark | | 0.00015 | 1.54875 | 1.09195 | 2.91972 | 28.84107 |
| <i>Ocotea</i> spp. | 33 | | | | | |
| Inside bark | | 0.00003 | 1.93042 | 1.15826 | 3.57446 | 0.19879 |
| Outside bark | | 0.00001 | 2.13112 | 1.26462 | 3.37728 | 0.71280 |
| <i>Schefflera morototonii</i> | 38 | | | | | |
| Inside bark | | 0.00008 | 1.94121 | 0.78641 | 3.71168 | 1.63683 |
| Outside bark | | 0.00006 | 2.03835 | 0.80043 | 3.54624 | 2.77364 |
| <i>Spathodea campanulata</i> | 121 | | | | | |
| Inside bark | | 0.00006 | 1.86027 | 0.96010 | 3.69325 | 3.43735 |
| Outside bark | | 0.00005 | 1.94002 | 1.00073 | 3.54513 | 6.77271 |
| <i>Syzygium jambos</i> | 39 | | | | | |
| Inside bark | | 0.00002 | 2.09996 | 1.14408 | 3.42865 | 0.67420 |
| Outside bark | | 0.00001 | 2.43478 | 1.05685 | 3.22420 | 1.29064 |
| <i>Tabebuia heterophylla</i> | 53 | | | | | |
| Inside bark | | 0.00004 | 2.08728 | 0.77360 | 3.52155 | 0.31078 |
| Outside bark | | 0.00002 | 2.35746 | 0.82028 | 3.19140 | 1.60281 |
| Other ^b | 399 | | | | | |
| Inside bark | | 0.00011 | 1.79592 | 0.80149 | 3.13759 | 35.14657 |
| Outside bark | | 0.00008 | 1.92245 | 0.78102 | 3.13227 | 47.89941 |
| All species | 1,247 | | | | | |
| Inside bark | | 0.00009 | 1.76072 | 0.95231 | 3.18682 | 4.29 × 10 ²⁰ |
| Outside bark | | 0.00011 | 1.75178 | 0.93862 | 3.20789 | 5.18 × 10 ²⁰ |

d.b.h. = diameter at breast height.

SSE = sum of square error.

^a $V_{stem} = (b_0 \cdot D_{BH}^{b_1} \cdot H_T^{b_2}) - K \frac{D_{upper}^{b_3}}{D_{BH}^{(b_3-2)}} \cdot (H_T - 1.37)$, where V_{stem} = merchantable stem volume in m³, D_{BH} = diameter at 1.37 m (in cm),

H_T = total tree height in m, D_{upper} = upper stem diameter merchantability limit in cm, and $K = 0.00007854 \cdot [1 - (2 / b_3)]$.

^b Species for which a species- or genus-specific equation was not fitted.

Table 5—Mean volume difference between volume equation estimate and directly calculated merchantable stem volume, standard error of the mean, confidence limits, and pair-wise t-test result by size class, diameter class, and equation type, Puerto Rico, 1990

| Class | Trees <i>number</i> | Mean difference | Standard error | Confidence limits | | Pr > t |
|--|------------------------|--------------------|-------------------|---------------------|---------------------|----------|
| | | | | Lower 95 percent | Upper 95 percent | |
| ----- <i>m</i> ³ ----- | | | | | | |
| Size class | | | | | | |
| Sawtimber (d.b.h. ≥ 27.5 cm) | 330 | | | | | |
| D.b.h. / H_T equation ^a | | 0.0081 | 0.0027 | 0.0028 | 0.0134 | 0.0027 |
| D.b.h. equation ^b | | 0.0159 | 0.0033 | 0.0095 | 0.0224 | < 0.0001 |
| Nonlinear taper equation ^c | | 0.0034 | 0.0029 | -0.0022 | 0.0091 | 0.2333 |
| NIMS equation ^d | | -0.0164 | 0.0029 | -0.0222 | -0.0106 | < 0.0001 |
| Wadsworth equation ^e | | 0.0606 | 0.0028 | 0.0550 | 0.0661 | < 0.0001 |
| Poletimber (12.5 cm ≤ d.b.h. < 27.5 cm) | 261 | | | | | |
| D.b.h. / H_T equation ^a | | 0.0450 | 0.0141 | 0.0172 | 0.0727 | 0.0016 |
| D.b.h. equation ^b | | 0.1143 | 0.0192 | 0.0765 | 0.1520 | < 0.0001 |
| Nonlinear taper equation ^c | | -0.0685 | 0.0124 | -0.0929 | -0.0442 | < 0.0001 |
| NIMS equation ^d | | -0.1112 | 0.0146 | -0.1400 | -0.0824 | < 0.0001 |
| Wadsworth equation ^e | | 0.3287 | 0.0326 | 0.2646 | 0.3928 | < 0.0001 |
| All stems | 591 | | | | | |
| D.b.h. / H_T equation ^a | | 0.0244 | 0.0064 | 0.0117 | 0.0370 | 0.0002 |
| D.b.h. equation ^b | | 0.0594 | 0.0089 | 0.0419 | 0.0768 | < 0.0001 |
| Nonlinear taper equation ^c | | -0.0284 | 0.0059 | -0.0399 | -0.0168 | < 0.0001 |
| NIMS equation ^d | | -0.0583 | 0.0069 | -0.0719 | -0.0447 | < 0.0001 |
| Wadsworth equation ^e | | 0.1790 | 0.0155 | 0.1486 | 0.2093 | < 0.0001 |
| Diameter class | | | | | | |
| 10 cm (5.0 cm ≤ d.b.h. ≤ 14.9 cm) ^f | 53 | | | | | |
| D.b.h. / H_T equation ^a | | 0.0002 | 0.0020 | -0.0037 | 0.0042 | 0.9020 |
| D.b.h. equation ^b | | 0.0045 | 0.0024 | -0.0003 | 0.0093 | 0.0658 |
| Nonlinear taper equation ^c | | 0.0031 | 0.0030 | -0.0029 | 0.0092 | 0.3015 |
| NIMS equation ^d | | -0.0102 | 0.0018 | -0.0139 | -0.0065 | < 0.0001 |
| Wadsworth equation ^e | | 0.0292 | 0.0018 | 0.0256 | 0.0329 | < 0.0001 |
| 20 cm (15.0 cm ≤ d.b.h. ≤ 24.9 cm) | 230 | | | | | |
| D.b.h. / H_T equation ^a | | 0.0066 | 0.0033 | 0.0001 | 0.0131 | 0.0451 |
| D.b.h. equation ^b | | 0.0135 | 0.0037 | 0.0061 | 0.0208 | 0.0004 |
| Nonlinear taper equation ^c | | 0.0031 | 0.0035 | -0.0038 | 0.0099 | 0.3801 |
| NIMS equation ^d | | -0.0148 | 0.0037 | -0.0221 | -0.0074 | < 0.0001 |
| Wadsworth equation ^e | | 0.0592 | 0.0031 | 0.0530 | 0.0653 | < 0.0001 |
| 30 cm (25.0 cm ≤ d.b.h. ≤ 34.9 cm) | 165 | | | | | |
| D.b.h. / H_T equation ^a | | 0.0119 | 0.0074 | -0.0027 | 0.0265 | 0.1100 |
| D.b.h. equation ^b | | 0.0423 | 0.0097 | 0.0231 | 0.0614 | < 0.0001 |
| Nonlinear taper equation ^c | | -0.0268 | 0.0081 | -0.0427 | -0.0109 | 0.0011 |
| NIMS equation ^d | | -0.0744 | 0.0081 | -0.0904 | -0.0584 | < 0.0001 |
| Wadsworth equation ^e | | 0.1143 | 0.0084 | 0.0978 | 0.1308 | < 0.0001 |

continued

Table 5—Mean volume difference between volume equation estimate and directly calculated merchantable stem volume, standard error of the mean, confidence limits, and pair-wise t-test result by size class, diameter class, and equation type, Puerto Rico, 1990 (continued)

| Class | Trees <i>number</i> | Mean difference | Standard error | Confidence limits | | Pr > t |
|---------------------------------------|------------------------|--------------------|-------------------|---------------------|---------------------|----------|
| | | | | Lower 95 percent | Upper 95 percent | |
| Diameter class (continued) | | | | | | |
| 40 cm (35.0 cm ≤ d.b.h. ≤ 44.9 cm) | 86 | | | | | |
| D.b.h./ H_T equation ^a | | 0.0154 | 0.0182 | -0.0208 | 0.0516 | 0.3994 |
| D.b.h. equation ^b | | 0.0588 | 0.0242 | 0.0106 | 0.1069 | 0.0173 |
| Nonlinear taper equation ^c | | -0.0913 | 0.0193 | -0.1297 | -0.0529 | < 0.0001 |
| NIMS equation ^d | | -0.1654 | 0.0182 | -0.2016 | -0.1292 | < 0.0001 |
| Wadsworth equation ^e | | 0.2031 | 0.0194 | 0.1645 | 0.2417 | < 0.0001 |
| 50 cm (45.0 cm ≤ d.b.h. ≤ 54.9 cm) | 31 | | | | | |
| D.b.h./ H_T equation ^a | | 0.0786 | 0.0465 | -0.0163 | 0.1735 | 0.1012 |
| D.b.h. equation ^b | | 0.1588 | 0.0589 | 0.0384 | 0.2792 | 0.0114 |
| Nonlinear taper equation ^c | | -0.0736 | 0.0400 | -0.1552 | 0.0081 | 0.0756 |
| NIMS equation ^d | | -0.1853 | 0.0428 | -0.2726 | -0.0980 | 0.0002 |
| Wadsworth equation ^e | | 0.4722 | 0.0410 | 0.3885 | 0.5560 | < 0.0001 |
| 60 cm (55.0 cm ≤ d.b.h. ≤ 64.9 cm) | 12 | | | | | |
| D.b.h./ H_T equation ^a | | 0.2318 | 0.1297 | -0.0536 | 0.5172 | 0.1013 |
| D.b.h. equation ^b | | 0.5007 | 0.1395 | 0.1937 | 0.8077 | 0.0042 |
| Nonlinear taper equation ^c | | -0.0425 | 0.1117 | -0.2884 | 0.2034 | 0.7108 |
| NIMS equation ^d | | 0.0360 | 0.1322 | -0.2549 | 0.3270 | 0.7904 |
| Wadsworth equation ^e | | 1.1151 | 0.1288 | 0.8315 | 1.3986 | < 0.0001 |
| 70 cm (65.0 cm ≤ d.b.h. ≤ 74.9 cm) | 10 | | | | | |
| D.b.h./ H_T equation ^a | | 0.1383 | 0.1508 | -0.2028 | 0.4794 | 0.3830 |
| D.b.h. equation ^b | | 0.4316 | 0.2238 | -0.0747 | 0.9378 | 0.0859 |
| Nonlinear taper equation ^c | | -0.2042 | 0.1552 | -0.5552 | 0.1468 | 0.2207 |
| NIMS equation ^d | | -0.1715 | 0.1521 | -0.5155 | 0.1726 | 0.2888 |
| Wadsworth equation ^e | | 1.3288 | 0.2165 | 0.8391 | 1.8185 | 0.0002 |
| Total | 587 | | | | | |

Pr > |t| = the probability that the mean difference between estimated and calculated volumes is greater than zero by chance alone.

^a $V_{stem} = e^{b_0 + b_1 (\ln D_{BH}) + b_2 (\ln H_T) + b_3 (D_{BH})}$, where V_{stem} = merchantable stem volume in m³, D_{BH} = diameter at 1.37 m, and H_T = total tree height in m.

^b $V_{stem} = e^{b_0 + b_1 (\ln D_{BH}) + b_2 (D_{BH})}$.

^c $V_{stem} = (b_0 \cdot D_{BH}^{b_1} \cdot H_T^{b_2}) - K \frac{D_{upper}^{b_3}}{D_{BH}^{(b_3-2)}} \cdot (H_T - 1.37)$, where V_{stem} = merchantable stem volume in m³, D_{BH} = diameter at 1.37 m (in cm),

H_T = total tree height in m, D_{upper} = upper stem diameter merchantability limit in cm, and $K = 0.00007854 \cdot [1 - (2 / b_3)]$.

^d Poletimber size trees: $Y_{POLE} = -0.378830 + 0.001855 \cdot (D_{BH})^2 \cdot H_T$, where Y_{POLE} = poletimber volume.

Sawtimber size trees: $Y_{SAW} = 0.82350 + 0.001630 \cdot (D_{BH})^2 \cdot H_T$, where Y_{SAW} = sawtimber volume.

^e $\text{Log}(Y_S) = 2.2020 \cdot \text{Log}(D_{BH}) + 0.4434 \cdot \text{Log}(H_T) - 1.8374$, where Y_S = stem volume.

^f The 10-cm d.b.h. class consists only of trees with d.b.h. ranging from 12.5 to 14.9 cm.

for trees with a d.b.h. > 60 cm is difficult due to their scarcity in the test dataset; however, it appears that NIMS equations might produce estimates that are comparable in accuracy to our equations for these trees.

Conclusions

Regression equations that satisfactorily estimate stem volume from d.b.h. and total tree height, and from d.b.h. alone, will facilitate future forest inventory field work and analysis in Puerto Rico. Not surprisingly, equations developed from the local inventory data generally outperformed the equations used in NIMS, which were developed from data collected in south Florida.

Applying the regression equations developed from the 1990 forest inventory dataset produces stem volume estimates acceptably close to those obtained by direct calculation, the method used in previous resource bulletins for Puerto Rico. However, to estimate volume growth and change over the past 10 years, 1990 tree volumes should be re-estimated using these new regression equations and the directly calculated 1990 values should not be employed in comparisons.

These volume equations for Puerto Rico may have application on other Caribbean islands that share the same species and forest types, and the nonlinear taper equations have the flexibility to accommodate different upper stem diameter merchantability limits. However, some very important restrictions will have to be adhered to if these equations are applied to other forest inventory datasets in the Caribbean:

1. Use of these volume equations requires the assumption that stand d.b.h. and height distributions, tree forms, and log lengths are similar to those in the 1990 Puerto Rico forest inventory dataset.
2. Unsound cull trees (for example, those too rotten or poorly formed to be usable) and portions of trees would have to be accounted for separately and subtracted from the gross total volume for a more accurate estimate of net timber and sound cull volume.
3. Palms and tree ferns were excluded from the model-building and testing datasets, so other equations or methods would have to be used to estimate their volume.
4. These equations estimate only stem volume and do not take into account smaller material such as branchwood, which can be used as fuel, or in craft items, small furniture components, or the like, and could have substantial value in the local markets.

Acknowledgments

The authors are grateful to Joseph Donnegan of the USDA Forest Service's Pacific Northwest Research Station for his review of the manuscript. We also thank the following USDA Forest Service, Southern Research Station personnel for their assistance: Mark Brown, Sonja Oswalt, and Stanley Zarnoch for their reviews of the manuscript; Raymond Sheffield, James Rosson, and Larry Royer for their guidance and suggestions; and Andy Hartsell for his help in replicating previous volume estimation methods.

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Species List

| Common name ^a | Scientific name |
|--------------------------|-------------------------------|
| Cabbagebark tree | <i>Andira inermis</i> |
| Oxhorn bucida | <i>Bucida burceras</i> |
| Pumpwood | <i>Cecropia schreberiana</i> |
| Cordia | <i>Cordia</i> spp. |
| Mountain immortelle | <i>Erythrina poeppigiana</i> |
| American muskwood | <i>Guarea guidonia</i> |
| Mahoe | <i>Hibiscus elatus</i> |
| Sacky sac bean | <i>Inga laurina</i> |
| River koko | <i>I. vera</i> |
| Mango | <i>Mangifera indica</i> |
| Sweetwood | <i>Ocotea</i> spp. |
| Matchwood | <i>Schefflera morototonii</i> |
| African tuliptree | <i>Spathodea campanulata</i> |
| Honduras mahogany | <i>Swietenia macrophylla</i> |
| West Indian mahogany | <i>S. mahagoni</i> |
| Malabar plum | <i>Syzygium jambos</i> |
| White cedar | <i>Tabebuia heterophylla</i> |

^aNomenclature retrieved (January 25, 2005) from the Integrated Taxonomic Information System online database (<http://www.itis.usda.gov>).

Brandeis, T.J.; Kuegler, O.; Knowe, S.A. 2005. Equations for merchantable volume for subtropical moist and wet forests of Puerto Rico. Res. Pap. SRS-39. Asheville, NC: U.S. Department of Agriculture Forest Service, Southern Research Station. 15 p.

In Puerto Rico, where locally grown woods are primarily used for furniture and crafts production, estimation of wood volume makes it possible to estimate the monetary value of one of the many commodities and services forests provide to society. In the Puerto Rican forest inventories of 1980 and 1990, workers calculated stem volume directly by applying geometric formulae to bole sections of merchantable trees. Field crews recorded several diameter and height measurements along the bole of each tree. If tree volume estimates were based on fewer tree measurements, this would significantly increase field crew productivity. For this reason, tree volume equations have been derived from Puerto Rican forest inventory data by directly calculating stem volume, then creating regression equations that estimate inside and outside bark merchantable stem volume from tree diameter at breast height and total height.

Keywords: Forest Inventory and Analysis, merchantable stem volume, Puerto Rico, tropical forest, volume equations.



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