project record

APRIL 1979

SAN DIMAS EQUIPMENT DEVELOPMENT CENTER

Vehicular Classification of Forest Soils and Slopes

7924 1202

FOREST SERVICE ■ U.S. DEPARTMENT of AGRICULTURE

VEHICULAR CLASSIFICATION OF FOREST SOILS AND SLOPES

by

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INTRODUCTION

Appropriate prime movers work on forest slopes to remove or treat forest residues, improve timber stands, and heal damage from flood and fire. Additionally, recreational vehicles using forest slopes have tripled in size, power, and mobility over the past few decades. The ability of a slope to sustain and recover from vehicular loads depends upon the vehicle, how it is operated, and on site characteristics. These characteristics include slope steepness, sub- and surface-soil type, root structure, ground temperature, debris and snow cover, water content, and permeability needs. The latter influences erosion and root development rates.

The prediction of production costs or of final results to the land is not easy when there is a poor match of equipment to work site. Also, the development of powerful off-road vehicles means that sites formerly disturbed only when equipment was present for timber thinning, harvesting, etc. are now traversed much more frequently. Consequently, the establishment of criteria for controlling vehicular use on forest soils and slopes has become very important.

When this project was initiated, mobility classification of steep-slope vehicles was limited to vehicular ground pressure, hard-ground stability parameters, and tractive ability for various gradients. Other pertinent site characteristics were not usefully related to vehicular characteristics. When potential damage levels at a work site were unknown in terms of vegetal or erosion effects, vehicular operational parameters from other sites were used as guidelines. These guidelines fail to control slope damage due to shear and sinkage when vehicles and sites are poorly related. Practical predictive needs require more general relations between vehicles and sites.

Vehicular engineers/designers, soils laboratory experts, and foresters/scientists all need to interface by relating their research efforts to a common parameter to *control* (rather than having to go to the extreme and *forbid*) vehicular damage to forest soils and slopes. The common parameter is shear displacement (δ) of the soil.

PRIOR INVESTIGATIONS

Military

Intensive investigations of vehicular soil mechanics to ensure that military wheeled and tracked vehicles completed their missions started during World War II. Postwar studies of how continued use of selected military vehicles gradually made roads impassible resulted in:

- New, simple instruments—such as cone penetrometers for predicting off-road soil strengths.
- Methods of predicting rut depths after various numbers of vehicular passes.

Unfortunately, these study results cannot be transformed into useful vehicular site damage relations without further work because:

- Damage is commonly measured in military terms of rut depth and/or how many trips made the road impassible. These are so far beyond the limits of desirable nonmilitary site damage as to make them unusable from the Forest Service's standpoint.

- Answers obtained for effective military use resulted from testing a wide range of soils with specific military vehicles. Data on varying vehicle loads, vehicle types, tire pressures, wheel slips, etc. needed for more universal use are often missing.

Studies of vehicular snow mechanics by the U.S. Army Cold Regions Research and Engineering Laboratory are more general than most military research in this area in that snow modeling has been included. However, the Forest Service is not yet ready for the snow parameter; other variables have to be delineated first.

The most valuable aspect of San Dimas Equipment Development Center's (SDEDC's) review of prior military research was the informal expert advice obtained from those within the military who were contacted. Without exception, they felt that slip was a key parameter to unraveling the other effects of a vehicle/soil experience. However, they differed considerably, according to varying backgrounds, on the effects of slip on further soil changes and on the value of laboratory vs. field experiments for studying these effects. Interchanging cause and effect, they were quite consistent in expecting slip changes to occur as a result of soil changes. In other words, the effects of slip that cause changes appeared to require much further study; the use of slip changes to detect and measure soil changes usually appeared more promising to them.

Several prominent Canadian and American investigators took a strongly different viewpoint. These, with slight differences in the preferred measuring methods, took the view that all changes were energy accountable, and that investigations that did not attempt to relate soil changes to the energy spent in causing the changes would lead to repetitious and wasteful blind alleys. Further, these investigators agreed that mere static concepts of safe and fail stress levels would be incapable of measuring the extent of damage after initial soil failure commenced. A vast accumulation of past military research on soil failure mobility limits appears to be incapable of extension for this reason.

In one way or another, all of those contacted took exception to the word "damage." Soil compaction is considered to be an improvement when it takes place in a roadbed and damage when it takes place off the road. They felt that the effects of vehicles should be reported as changes, without resort to moralistic definitions of good and bad, which often depend upon the use of the soil and the downstream effects of soil changes. Unfortunately, this two-way viewpoint, though valid for compaction effects, may encourage underestimates of the effects of other, clearly destructive, vehicular impacts.

Our contacts viewed the importance of vehicular dynamic characteristics mostly along lines dictated by the needs of their profession. Civil and materials engineers, with a few bold exceptions, confined their interest to the static parameters of ground pressure and total weight. They considered these adequate for an initial study of vehicular damage. Mobility experts and designers of vehicles, tires, and tracks considered the following to be of paramount importance: weight transfer, tire or track configuration, percentage of weight available for tractive effort, ground conformability, and the natural frequencies and damping of various tire and suspension systems. Developers of vehicles put their faith in performance tests and usually had a strong distrust of vehicular predictions derived from static soil mechanics. The use of soil mechanics to modify vehicles was usually valued only by those knowledgeable in both vehicle performance and soil mechanics.

Agricultural

SDEDC's contacts with the agricultural community were limited to the National Tillage Machine Laboratory (NTML), Auburn, Ala.; no discussions were held with agricultural equipment designers. NTML has been pursuing the tillage arts of preventing and removing vehicular (machine) compaction effects. Their studies focus on soil dynamics, with intensive investigation into the amounts of applied and wasted energy needed for plowing. Currently, they make slip measurements in terms of slip energy to avoid further assumptions of rolling radii and other dimensions that become obscured when working wheels sink deeply into soft soils. While NTML's study of vibratory effects appears to be confined to vibrating plows, they lead the field in accounting for energy expenditures in soft soils.

FOREST SERVICE INVESTIGATIONS

Forest Service soil scientists are attempting, from root development and runoff requirements, to establish tolerable permeability levels for vehicular compaction. Also, the Missoula Equipment Development Center (MEDC) is investigating vehicular compactive effects on level ground (ED&T project No. 7075) while SDEDC conducted an initial study to classify vehicles (ED&T project No. 8058). Coordination between the Centers highlighted the following areas of agreement:

1. Soil/moisture relations are important and the most promising entry to investigating vehicular compactive effects on cohesive soils would be Proctor moisture-density curves. A similar investigation of vehicular effects on the relative density at varying moistures of non-cohesive soils may be less promising because of their well-known sensitivity to vibrations.

2. Relating vehicular operations to other than observable immediate engineering site effects would not be feasible. Subsequent natural and cultural extensions and the healing of initial damage appeared to require special training and long-term observations by bio-scientists. The best goal of engineering investigations appeared to be that of establishing a valid starting point for these observations.

3. Slope effects of vehicles should be a natural extension of their level-ground effects, and shear stresses and weight transfers imposed by topographic gradients would be the major difference between slope and level-ground vehicular operations.

4. Soil properties for slopes appear to be most easily obtainable from controlled level-ground experiments.

5. Improved instrumentation is needed to detect and measure soil changes from vehicular operations as they occur.

In followup to the latter point, SDEDC contacted a developer of seismic instrumentation to learn if any commercial interest could be generated in this direction—see appendix I.

The MEDC project is relating soil compaction to water content, soil type, litter depth, and vehicular ground pressure, while the SDEDC effort related slope-induced stresses and slips to site damage. These stresses and slips are being incorporated into the level-ground experiments; therefore, the MEDC data probably can be used to predict compaction on hillside slopes. A general solution must relate site damage to the same vehicular inputs of soil compactions and displacements, whether on level ground or slopes. Two Centers looking separately at level and hillside facets, without shear and slip, could prevent the discovery of the general solution that is needed.

MECHANICAL CAUSES OF SITE DAMAGE

A vehicular wheel or track changes a soil and its cover by compression, shear, or tear. The first effect noted from flat-plate experiments (fig. 1) is sinkage. The problem immediately arises of determining how much of the sinkage is due to (1) compaction of the soil directly beneath the plate and (2) pushing of a column (rigid body) of soil downwards. The downward push evidently reduces local compaction by transferring part of the compaction to lower layers. When the loaded plate is moved horizontally, the sinkage is significantly increased by horizontal shearing effects.

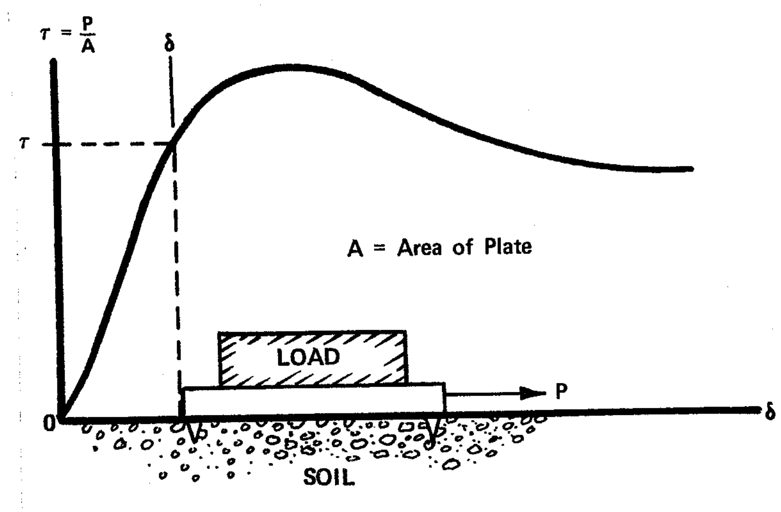


Figure 1. Flat-plate shear displacement test.

Increased horizontal force causes increased horizontal movement of the plate. Similar experiments with a hollow shear box of loaded soil show that horizontal movement of lower layers of soil take place until peak force is developed, after which the shearing of soil becomes more localized in the vicinity of the failure plane. These phenomena are consistent in both flat plate and shear box tests. The only practical difference is that the shear box localizes displacements closer to the failure plane; the flat plate displaces further from movement of lower soil layers.

Plant roots—if not completely broken—tend to be pulled upward from lower levels, with some tendril breakage, as the upper part of the plant is horizontally displaced along with the soil and turf above the failure plane.

If each small element of a tire or each pad of a crawler track is considered to act like a flat plate, then slip is no longer a constant. It now varies from zero, at the beginning of vehicular ground contact, to full displacement at the end of contact. Operating at 5 percent slip, an ideal tire on a earth mover with, say, a 2-ft ground contact line, would move the soil 0.05 x 2 ft = 0.1 ft; while an ideal 10-ft long crawler tractor would cause a horizontal ground displacement of 0.05 x 10 ft = 0.5 ft.

Admittedly, these two examples are contrived—for instance, the tire would introduce additional "squirm" slips as it flexed to adapt its toroidal shape to the flat ground. Also, the crawler track would probably slip much less than indicated. We are merely observing how the soil gets moved. The flat-plate shear test yields a curve of $\tau = P/A$ plotted against the displacement δ of the soil. We can now examine (fig. 2) an ideal tire or track of ground contact length L, weighted to the same uniform ground pressure as the flat plate (fig. 1). (The "v" subscript indicates "vehicular.")

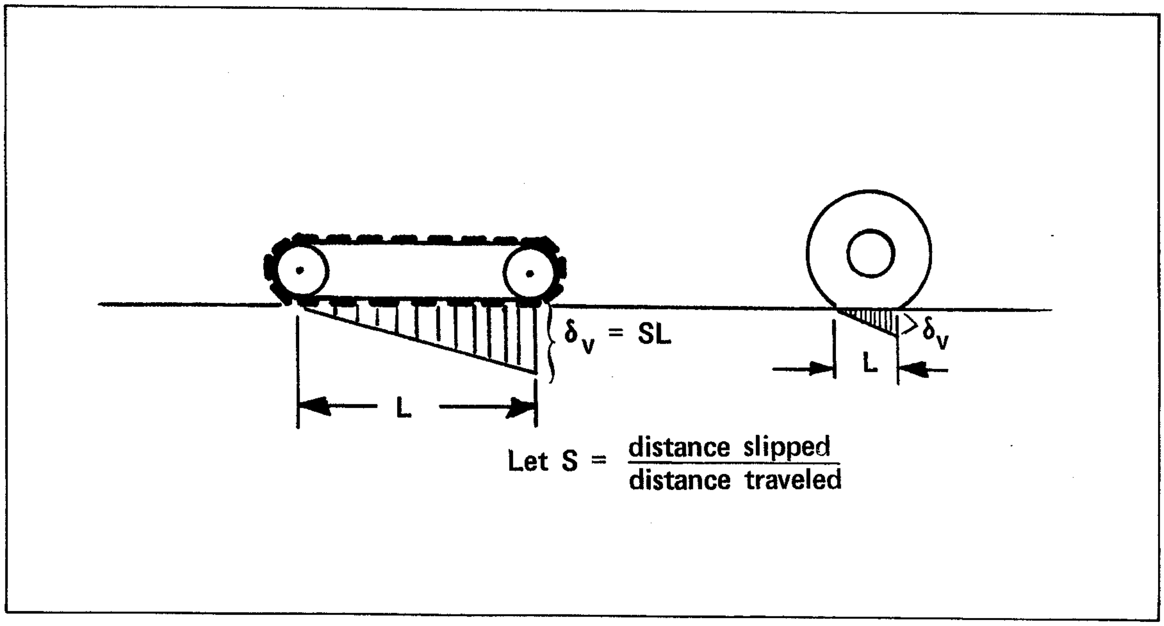


Figure 2. Linear distribution of soil shear displacement.

Total slip displacement δ is percent slip (S) times ground contact length (L), $\delta = SL$, which is 0.5 ft for S = 0.05 and L = 10 ft in the crawler track example already given. For steady-state motion, this slip displacement develops linearly along the track from zero at the front end to full $\delta_v = SL$ at the rear. Each pad on the crawler track will trace out the τ curve of figure 1 but, at any instant, the mean shear value τ_v of the total vehicular wheel or track will be given by the mean value of the shaded area in figure 3.

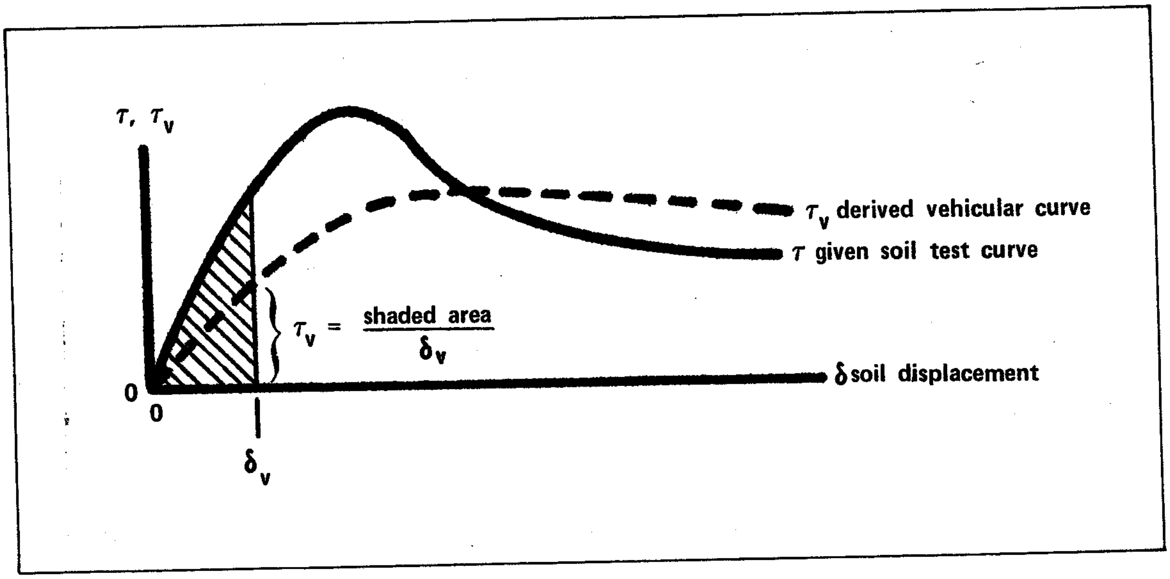


Figure 3. Derivation of vehicular shear stress curve from soil test curve.

Since we now have (in fig. 3 plus in the material presented in appendix II) a means of predicting how far a wheel or track under a tractive load will shear a given soil, we can assemble a list of principles for developing its implications:

1. Root damage from soil shear probably depends upon the length of the shear and upon the root-holding rigidity of lower soil layers.

2. The length of soil shear is proportional to the percent slip and the length of longitudinal ground contact of the slipping tire or track.

3. At light loads, side slips induce the same side force as longitudinal slips—except for the effects of the rut wall (and minor differences in tread design). The same force/slip (and force/shear) relations hold whether a vehicle on a hillside is operating forward, sideways, or a vector combination of these.

4. For the sake of generalization and vehicular convention, the $\tau_v - \delta$ curves can be replotted into conventional μ-slip ("mu-slip") curves from the constant scale factors:

$$\mu = \text{tractive coefficient} = \frac{\text{horizontal force}}{\text{vertical force}} = \frac{\tau_V}{\text{ground pressure}}$$

$$\text{and } s = \text{relative slip} = \frac{\text{soil displacement}}{\text{ground length}} = \frac{\delta_V}{L}.$$

5. Curves of μ-slip show that tractive forces cannot induce motion without inducing slip and, therefore, soil displacement. Greater forces always induce increased slips, until the peak μ is reached—after which further increases in slip fail to produce further increases in tractive force. This "spinning of wheels" is a runaway condition in which slip cannot be calculated from force requirements.

6. All tractive forces needed to overcome vehicular motion resistance, tow implements or trailers, and climb or circle a hillside can be added vectorially. The stress magnitude of the resultant force, applied to the τ_V curve of figure 3, determines the magnitude δ of soil displacement, while its direction defines the direction of soil displacement δ.

7. Both soil shear and μ-slip tests show that, in looser soils, more displacement and slip is needed to develop the same shear or tractive force. Larger sinkages and corresponding larger rolling resistances also occur in looser soils, so that greater forces—requiring yet larger slips—are required to propel the vehicle (fig. 4).

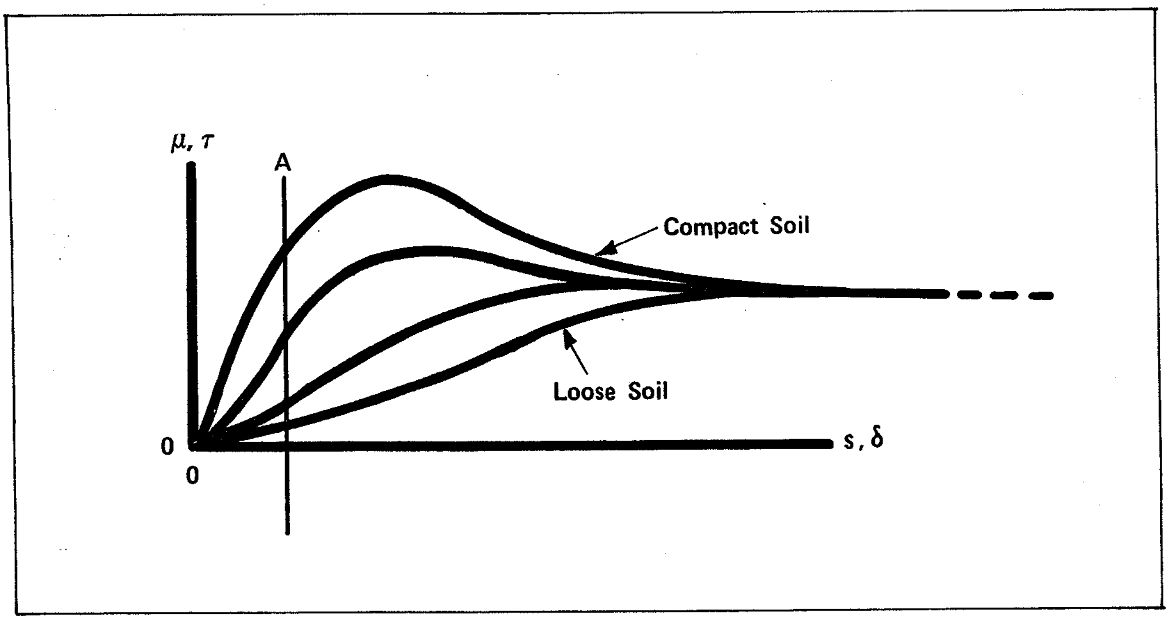


Figure 4. Compaction-caused changes in initial slope of soil and vehicular test curves.

If we wish to limit soil displacement to a controlled value—say, at A in figure 4—we must reduce the tractive effort, when soils are looser, by reducing the tractive load and/or increasing the tire or track contact area. The method of relating vehicular slips to flat-plate soil tests (appendix II) gives a new rationale for using laboratory measurements for field predictions of traction and sinkage. A second method (appendix III) was developed for relating changes in shear and tractive slip to changes in compaction by "parameterizing" the initial slopes of the test curves (fig. 4).

INTERPRETATION OF RELATIVE DAMAGE

Sinkage, or rutting, compacts a *granular* soil so that repeated vehicular trips are made with much less rolling resistance and sinkage. Repeated trips in *cohesive* soil, however, continue to deepen the initial rut, until the road becomes impassible. The amount of compaction in lower soil layers is not generally related to rut depth (at the present time), but soil experiments always show that a slipping tractive force always increases the sinkage over that of a free-rolling wheel.

When civil engineers prepare specifications for road or airport construction, they favor having construction vehicles operated so as to create a uniform compaction over the entire roadway or runway. In agricultural and forestry work, many favor confining construction and user vehicles to prescribed paths and prepared roadways to localize compaction. This practice may be adequate in level-ground or sidehill work, but could cause erosive runoffs in a downhill rut unless "Kelly humps" or similar rehabilitative measures are taken before leaving the area. Clearly, site conditions must enter into the choice of random vs. confined vehicular operations.

If repeated trips are made with significantly decreased sinkage and slip energy waste, we can infer that further damage of any kind is taking place at a much reduced rate, and visa-versa. Measurement of vehicular operational effects using the relative changes made appears to be more rational. These changes will always be relatable to the slip energy that is expended in making them. Therefore, vehicular slip measurements can be useful indicators of relative damage at the time damage is taking place. Most important, the slip parameter can be used for forecasting the accumulative effects of repeated trips.

As a final thought, a destructive driver can cause profound changes in fragile soils through vehicular damage. If this driver is not redirected, any foreseeable improvements in vehicles will have reduced beneficial effect.

SUMMARY AND CONCLUSION

Previous military studies of vehicular effects on soils and slopes have been mobility oriented. Consequently, military studies are not easily usable for rational control of vehicular damage on forestland slopes. Slope effects can be controlled by limiting slips to nondamaging levels. This criterion will automatically inhibit both steep-slope and level-ground operations when soil conditions are prohibitive. The effects of repeated trips appear to be estimable by adding soil displacements from each pass.

Establishment of appropriate damage levels must come about by relating biodamage effects to the amount of soil shear causing the damage. Sinkage and compaction on slopes can be reproduced on level ground; the effects of mild slopes will be confined to shear stresses and movements induced in the soil. These shear stresses can be introduced as tow forces in level-ground studies.

Damage classification must be relative to an existing site before vehicular operation. Absolute levels of damage cannot be developed through research, since no zero conditions of damage exist in the natural development of existing lands. The rate of shear changes in a soil depend upon the amount of slip between the tire or track and the soil. Soil shear is a function of percent slip times the ground contact length. Conversion of soil laboratory studies to vehicular effects can be made by rational use of this fundamental relationship. The percentage of allowable slip for a specified amount of soil displacement will be inversely proportional to the length of tire or track footprint that is being slipped.

The SDEDC goal was to find a simple means for predicting and controlling hillside damage from vehicular operations. We now propose to control both hillside and level-ground damage by controlling soil slip. This development and related findings of silviculturalists, agrologists, soil and material scientists, hydrologists, and others into an effective land and vehicular management effort cannot take place—unless the separate findings and needs are relatable to common parameters and operating levels.

SDEDC concludes that vehicular damage control by specified slips, compactions, and sinkages can be effected as soon as the now separate investigations begin to be related to the soil shear displacement parameter that has been presented in this *Project Record.* The transformations shown in appendixes II and III are adequate for first-generation technical bridges to connect the now separate investigations of soils and vehicles.

APPENDIX I—COMMENTS ON MEDC STUDY PLAN

On the pages that follow are SDEDC's comments on a draft manuscript of an MEDC study plan for their ED&T project No. 7075. The concluding page of the SDEDC memo contains information from Rockwell Autonetics on using seismic instruments to obtain needed data for Forest Service studies in the area of vehicular damage control on forest soils and slopes.

UNITED STATES DEPARTMENT OF AGRICULTURE
FOREST SERVICE

Equipment Development Center, San Dimas, California 91773

REPLY TO: 1630 Written Information

MAY 4 1978

SUBJECT: Manuscript Review -- 7075, "Predicting Soil Compaction on Forest Lands "

TO: Technical Publications Editor, MEDC

I am sorry to be so late in answering your March 16th letter. I've been on annual leave and have had subsequent delays in contacting advisors because of their travel activities.

Both Dave Jones and I like the main idea of the Study Plan. We agree that the Proctor moisture-density curves belong at the center of a serious study of compaction. On this basis, Dave and I have marked up the margin of the draft copy with appropriate (we think) comments on sample sizes and other details and are returning it with this letter.

Going further, I think the Study Plan is too narrow. It proposes to tell us when to shut down the machines because the ground is too wet, but fails to tell us which machines should be shut down and which can be allowed to run. It also fails to tell us how hard we can work the machines that we allow to continue running and doesn't suggest any measures we can take to mitigate their use.

I cannot foresee controlling compaction if we cannot describe and control the machines that cause it and detect the damage as it occurs. The plan is an excellent piece of work to start with, but needs additions. It fails to establish a basis for controlling compaction because of the things that have been left out:

Size Effect - I can hire kids with bicycles to compact a fill. Their 55 psi tires will, according to theory, compact a fill more densely than large 20-ton pneumatic rollers with 20 psi tire pressure. Of course, the bicycles will merely compact a thin top layer, the rollers will have much deeper effect. Ground pressure isn't the whole story.

Administrators in the Yellow Knife District in the Northwest Territory are considering switching over from ground pressure to gross vehicle weight for more realistic evaluations of permit applications for oil exploration machines operating on the Arctic tundra. Dr. James Taylor, from the Auburn Tillage Laboratory, is presenting a paper at the ASAE meeting in Utah this month dealing with the size effect of large tractors. This may be pertinent to your investigation, and I suggest that you contact Jim before eliminating size effect from the study.

Tractive Effects - Mobility curves of tractive effort (force) for different tire and track slips are very similar to the force-slip curves generated by dragging a loaded plate along the ground in a direct shear soil test. Reece showed large increases in sinkage as the plate was dragged further. Track designers calculate the change of trim as the rear of the track settles deeper from linearly increasing front to rear track slip. Sinkage isn't compaction, but it's close enough to make me suspicious of any study that assumes that the 18 to 22 percent slip, at which peak traction occurs on many soils, has negligible effect on either the intensity or penetration of compaction, since the changes of trim frequently exceed the initial settlement that occurs without slip. Load transfer doesn't explain the large changes in trim; static load transfers don't have that much effect.

Force and slip measurements will be needed to relate your level ground data to operations on slopes. Since percent slip is the most important variable for controlling slope damage, curves of traction vs. slip are essential.

Vibratory Effects - Vibratory compactors work well at and slightly above optional Proctor moisture contents, but also compact cohesionless soils effectively over a wide range of moisture contents. Pronounced resonant increases of compactive penetration are common when the vibrator operates at the natural frequency of the soil. The Proctor curves, though applicable, do not explain why a vibratory roller is so much more effective than an ordinary pneumatic roller. Similar effects of naturally vibrating bogies and chassis pound out roads so rapidly in quarries that large springs and shock absorbers have replaced the unsprung trucks in mining and quarry work. These effects will probably be more apparent on harder ground, but it appears likely that some chassis frequencies will be more damaging than others. Screening these out should be a goal of the study.

Detection of Vehicular Dynamic Effects - As you requested during our phone conversation, I contacted several knowledgeable people about the possibilities in using seismic detection apparatus to help screen the bad vehicles from the good ones. The state of this art has been advancing so rapidly that recent developments may be quite promising.

Henry Hodges, a well qualified mobility expert from Carson City, Nevada, opined that if you leave out vehicle dynamics the study will come to a dead end, since it won't span known vehicular variances. Henry did quite a bit of getting vehicular signatures some years back (working with the Lockheed people). These have since come into common military use, such as in minefields. The idea is to ignore your jeep, but to blow up the bad guys who are chasing you. Henry found the predominant frequencies to range between 15 and 45 Hz. Apparently, identification of vehicles is no problem, identification of compaction-sensitive frequencies seemed sensible to Henry. The idea of using the change in signals to detect when harmless bounce changes to permanent soil deformations intrigued him. Henry suggested some preliminary investigative work should be done before planning a rigid program.

Henry advised some small preliminary work to screen out non-essential noise and concentrate on the meaningful frequencies.

I also contacted Craig Kirkpatrick and Bill Kohlenberger of Rockwell Autonetics to see what they thought of the possibilities of using seismic instruments. They hadn't heard of the agricultural and forest compaction problem, but showed interest, and the discussion went something like this:

1. Vehicle identification - No problem, this is an established practice.

2. Can they detect and measure slippage? Sounds intriguing, but they would want to see some of the signals before committing themselves.

3. Can they detect by signal changes when elastic vibrations change to permanent soil deformations? Bill and Craig thought this should show up in impedence changes, if the basic signal contained the necessary information. As Bill put it, "If it's there we can get it, information processing is our business, and we have shelf software for extracting it. If it's not there, we can't." Bill said they would have to get and process some signal data before committing himself.

4. Can they measure deeper compaction changes? Bill said they could get around the refraction seismograph problem of requiring increasing densities as you go deeper by sinking an oscillator to below the inversion level. He said experimental work would be needed before he could say.

Overall, they showed considerable interest in the problem and would like to work with you and Hodges on it, but need some support. They can meet with you to define the problem, but when they go home they must have a big enough picture for the company to see some worthwhile downstream activity to justify the effort. They are a technology-intensive firm, and have a pretty strong policy against shallow investigations. They go all the way or not at all, and must see something ahead to justify starting.

Bill said his best chance of participating would be if he could get their ground vehicles division people interested in the problem. Perhaps Jim Taylor could furnish a view of the agricultural side of the problem to help generate this kind of interest.

The excellent summary of the Task Force's conclusions, and the bibliography at the end of your proposal were very useful in generating Bill's interest; our activities are far removed from their normal line of work, and you will have to fill this void for them.

Leonard B Della-Moretta

LEONARD B. DELLA-MORETTA
Industrial Engineer

A flat-plate shear test yields a curve of soil shear stress, $\tau = \frac{P}{A}$, plotted against the shear displacement, δ, of the soil. The flat-plate shear test also yields a curve of sinkage "Z," usually measured in inches, against the shear displacement, δ, of the soil (fig. II-1).

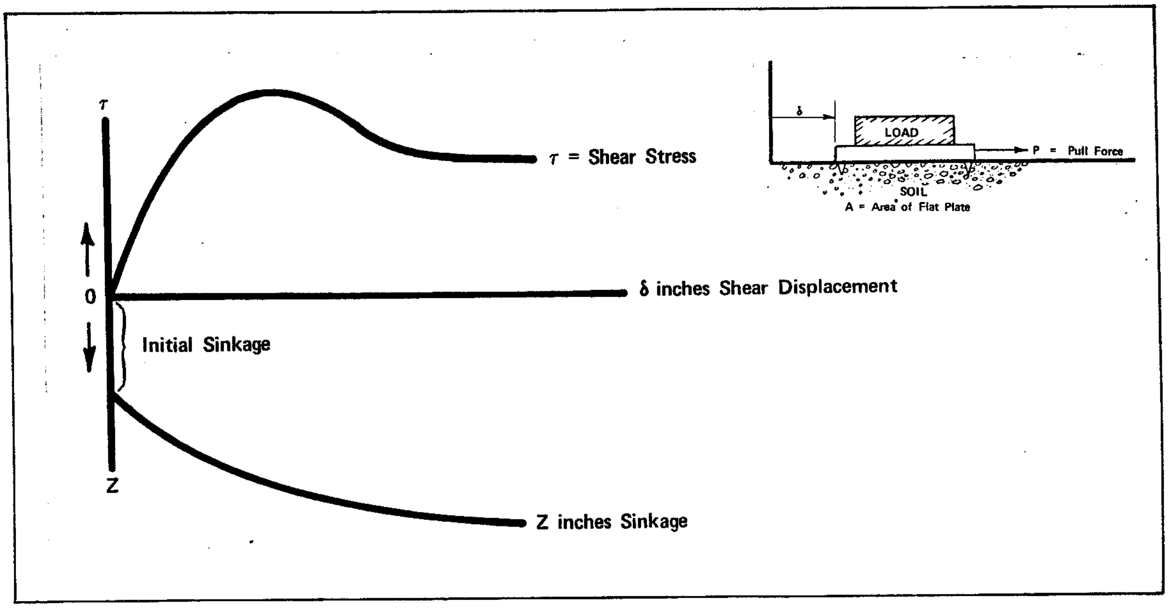


Figure II-1. Variation of shear stress and sinkage with increasing soil displacement.

We assume an *ideal* wheel or track of ground length, L, loaded at the same uniform ground pressure as the flat plate used to obtain the τ and Z curves of figure II-1 (figure II-2).

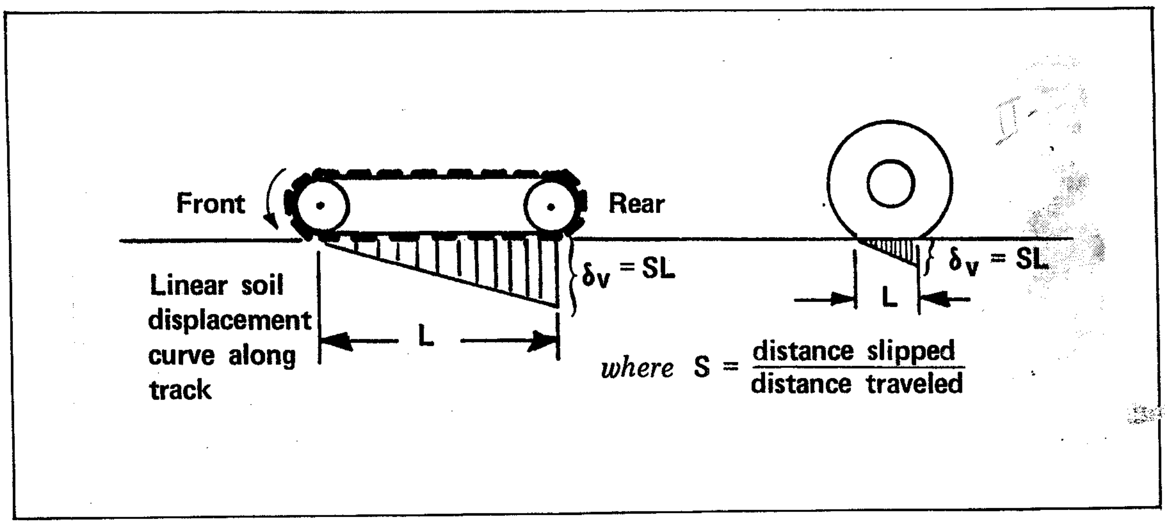


Figure II-2. Linear distribution of soil displacement under a wheel or track.

Under ideal steady-state motion, the soil's shear displacement will vary linearly from zero at the front of the track to full δ_v at its rear. If we regard each small segment of the track to be a small flat shear plate undergoing linearly increasing slip as the tractor moves ahead, each small element will trace out the given $\tau - \delta$ and $Z - \delta$ curves. The mean shear value τ_v of the vehicle track will be the mean value of the shaded area in figure III-3 and given by the integral

$$\tau_v = \frac{1}{\delta_v} \int_0^{\delta_v} \tau \, d\delta \qquad (1)$$

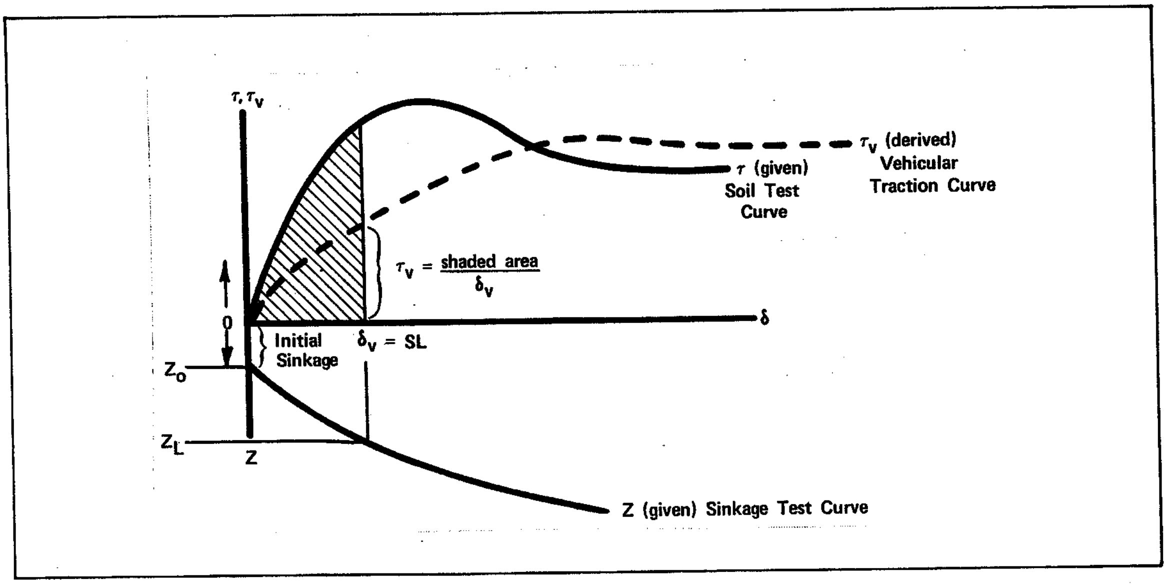


Figure II-3. Deriving a point on vehicular traction curve from area under soil test curve.

The sinkage along the track (assuming tire or track flexibility) will vary from the initial sinkage, Z_o, at the front of the track to full sinkage, Z_L, at its rear. The change of trim, from front to rear of the track, will be approximately given by the chord from these two points on the $Z - \delta$ test curve.

The derived $\tau_v - \delta$ curve can be converted to a conventional automotive μ-slip curve by the linear scale factors:

$$\mu = \frac{\tau_v}{\text{ground pressure}} \quad \text{and} \quad S = \frac{\delta_v}{L} . \qquad (2)$$

Inverse Transformation

We can reverse the procedure to derive a hypothetical flat plate's $\tau - \delta$ shear curve from vehicular $\tau_v - \delta$ test curve by differentiating (1) to obtain:

$$\frac{d\,\tau_v}{d\,\delta_v} = \frac{d}{d\delta_v}\left[\frac{1}{\delta_v}\int_o^{\delta_v} \tau\, d\,\delta\right] = \frac{1}{\delta_v^2}\int_o^{\delta_v} \tau\, d\,\delta + \frac{1}{\delta_v}\tau$$

$$= -\frac{\tau_v}{\delta_v} + \frac{\tau}{\delta_v}$$, from which we obtain

the hypothetical flat plate soil-test curve:

$$\boxed{\tau = \tau_v + \delta_v \frac{d\,\tau_v}{d\,\delta_v}\ .} \qquad (3)$$

Transforming a Flat-Plate Soil Test Curve ($\tau - \delta$) to a Vehicular Traction Curve ($\tau_v - \delta$).

The transformation is accomplished in four steps (fig. II-4):

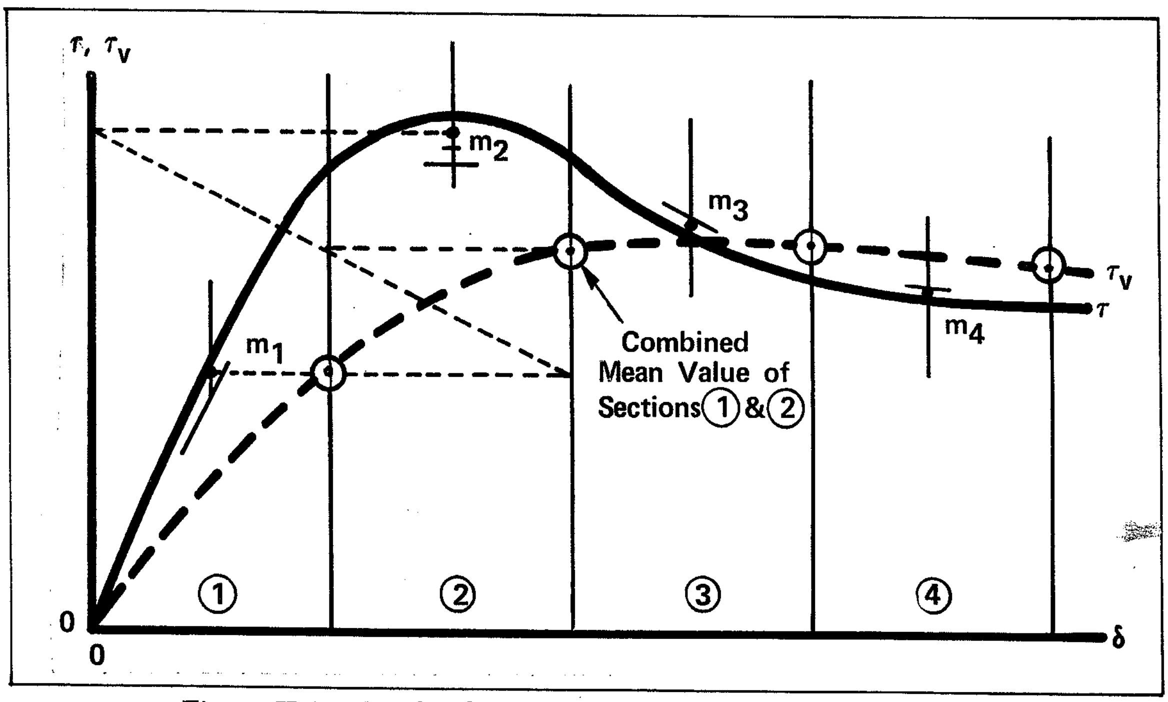
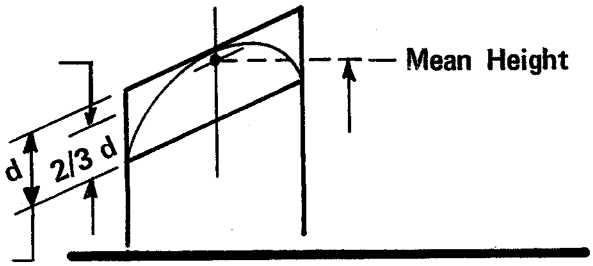


Figure II-4. Graphical construction of vehicular traction curve.

1. Divide the given $\tau - \delta$ curve into convenient sections by vertical lines.

2. Find the mean value of each section by use of the property "The area of a parabolic segment equals 2/3 the area of the parallelogram circumscribing it" (labeled m_1, m_2, etc. in figure II-4):



3. Use the mean values of segments 1 and 2 to obtain their combined mean value. Use this combined value with the mean value of segment 3 to get the mean value of segments 1, 2, 3, etc., using the construction in figure II-5. Dotted lines illustrate it for (1) and (2) in figure II-4.

4. Combine mean values of areas (A) and (B) (figure II-5):

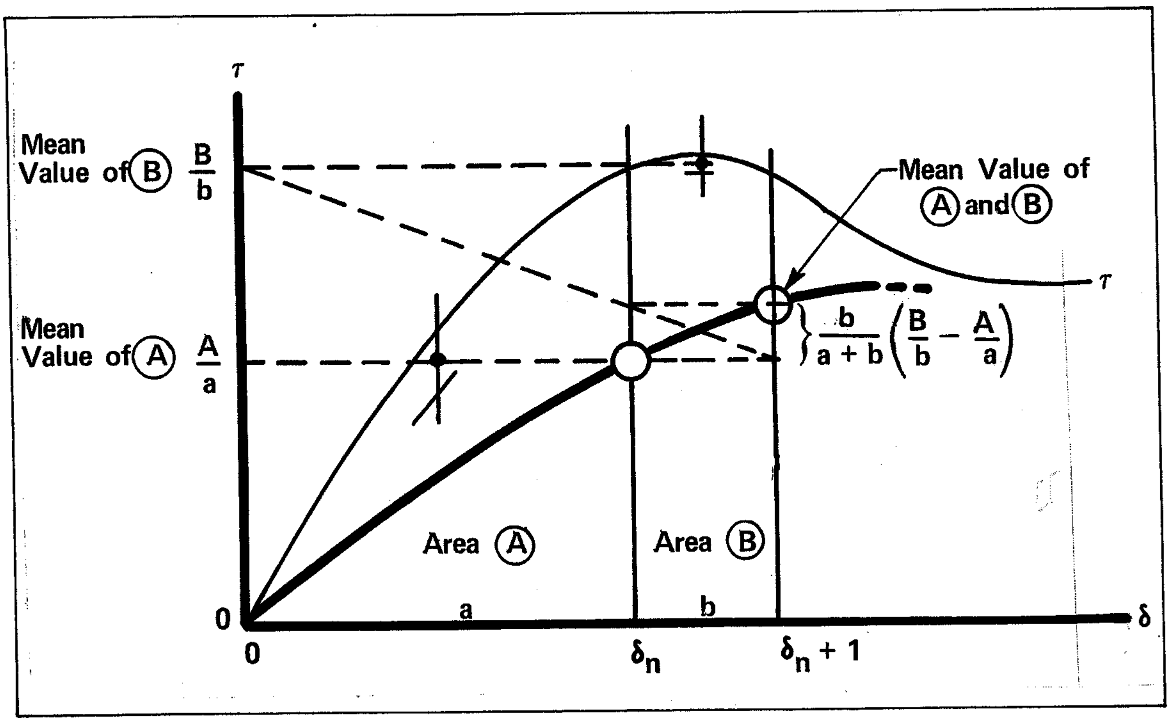


Figure II-5. Deriving graphical method of constructing vehicular traction curve.

$$A = \text{Area of segment whose base is } a = \delta_n$$

$$B = \text{Area of segment whose base is } b = \delta_{n+1} - \delta_n$$

By definition:

$$\tau_{v\,n+1} = \frac{A+B}{a+b} = \frac{A}{a} + \frac{A+B}{a+b} - \frac{A}{a}$$

$$= \frac{A}{a} + \frac{Ba - Ab}{(a+b)\,a}$$

$$= \frac{A}{a} + \frac{b}{a+b}\left(\frac{B}{b} - \frac{A}{a}\right).$$

APPENDIX III—ELEMENTARY TRANSFORMATION OF TIRE μ-SLIP AND SOIL SHEAR STRESS/STRAIN TEST CURVES

On the pages that follow is a paper that was presented at "Tire Rolling Losses and Fuel Economy—An R&D Planning Workshop," sponsored by the Society of Automotive Engineers (SAE) Highway Tire Committee, October 1977.

Elementary Transformation of Tire μ-Slip and Soil Shear Stress/Strain Test Curves

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BY RECENT ESTIMATE the Forest Service, U. S. Department of Agriculture, each year sees approximately 600-million tons of forest products hauled from forest to mill over an average distance of 60 miles. That portion of the automotive energy transmitted to the road surface that does not accomplish tractive effort causes heat (usually non-damaging) and, more seriously, movement of the surface, or some component of the surface. While a certain amount of asphalt surface flexure may be acceptable, or even desirable, waste energy that removes all the gravel from a logging road or creates a dust cloud as the fines become airborne is not a desirable effect. If the Forest Service's annual investment in road maintenance could be reduced by just 1 percent, there is a potential savings of nearly one-million dollars (1)*.

Road damage causes increased rolling resistance. If this damage is measured in terms of rolling resistance, then it is equivalently being measured in units of energy which are the only units that can measure the destructive processes that create road damage. Consequently, better predictions of the waste automotive energy are being sought. These mixed Tire and Soil Mechanics investigations lead to more intensive study of the μ-slip curves that delineate waste automotive energy—the energy that drives the related wear and deterioration mechanisms.

The tractive coefficient μ (the horizontal tractive force ÷ vertical force) can be related to the shear stress τ of a soil shear stress/strain curve. Similarly, the slip coefficient s (the distance slipped ÷ distance traveled) resembles the shear strain γ of the soil shear test—particularly so at *mild slips* when the measured "slip":

- Is distributed as shear strain in adjacent layers of tire and road;

*Numbers in parentheses designate References at end of paper.

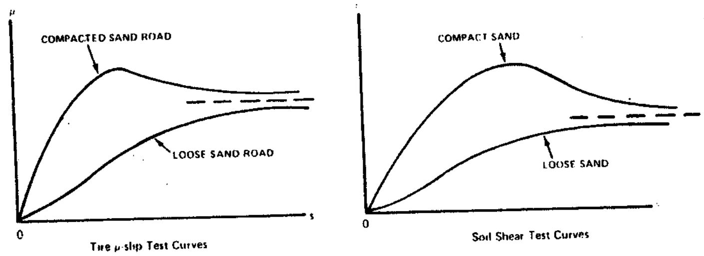

Fig. 1—Tire and soil test curves

- Accumulates from hysteresis of both materials as slip on the recording instrument during each load/unload cycle.

Both the μ-slip and soil shear curves (fig. 1) start from the origin, with initial slopes that appear related to the initial states of the test materials. Both curves approach horizontal asymptotes to the right at levels that appear to be intrinsic material properties, relatively independent of initial material states. Also, for both curves the product of the vertical and horizontal coordinates is unit energy. For example, the product of μ and s represents the energy per unit weight of vertical load wasted into the tire/road system per unit distance traveled.

The purpose of this paper is to show how three mathematical operations (curve fitting, curve interpolation, and transformation of fitted function) can provide a useful family of data based on tests that define tire traction vs. slip or soil shear stress vs. strain. Without *a priori* assumptions about what the curves should look like (beyond what was stated pertaining to slopes and asymptotes):

- The curves can be modeled algebraically to fit experimental curves to any required degree of accuracy;

ABSTRACT

A new method for functional representation of tire μ-slip and soil shear stress/strain test curves is presented; it does not distort the curves' characteristic initial slopes and final asymptotic values. The test curves, and their derivatives, can be fitted to any desired accuracy by increasing the degree of the polynomial exponential expression that represents them. Constants for curve fitting are found by linear equations suitable for least squares solution. Further, a method is presented for multivariable transformation of the derived curves through interpolation of their constants. Curves corresponding to any desired combination of parameters, spanned by a set of test curves, can be generated by these transformations.

- Intermediate curves, between a set of experimental curves, can be obtained by interpolation for any desired set of conditions;
- A curve can be transformed for different conditions by extending the methods used for interpolation.

CURVE FITTING

With x representing the horizontal and y the vertical coordinates of the figure 1 curves, a function having the desired properties of characteristic initial slope and horizontal asymptote, and capable of being fit with any deisred accuracy to experimental points lying between these, is

$$y = e^{\frac{\sum_{o}^{n} a_i x^i + \ln \ln (1+x)}{1+x^n}} \qquad (1)$$

for $o < x < \infty$.

Using $n = 3$ or 4 appears adequate to represent common experimental curves with some smoothing of experimental irregularities. Higher values of n can be taken as desired to preserve small details when these are meaningful. A theorem by Weierstrass (2) on polynomial interpolation assures us that the function y (Eq. 1) can be fit to the test curve to any desired degree of accuracy by simply increasing n; because, by taking the logarithms of both sides, the logarithm of y can be fit to any desired degree of accuracy. The derivative of the function can represent the slope of the curve to any desired accuracy, from similar arguments.

The boundary limits are:

$$\text{as } x \to o,\ y \to o,\ y' \to e^{a_o}$$
$$\text{as } x \to \infty,\ y \to e^{a_n},\ y' \to o. \qquad (2)$$

From Eq. 2:

$$a_o = \ln y'(o) \text{ and } a_n \doteq \ln y(\infty). \qquad (3)$$

From Eq. 1, taking logarithms yields

$$(1+x^n) \ln y = \sum_{o}^{n} x^i a_i + \ln \ln (1+x). \qquad (4)$$

Therefore, a sufficient condition for constants a_i to fit experimental point x_A, y_A is

$$\sum_{o}^{n} x_A^i a_i = (1+x_A^n) \ln y_A - \ln \ln (1+x_A). \qquad (5)$$

Any $n+1$ points yield $n+1$ equations which can be formed from Eq. 5 and solved linearly for the a_i to fit the given points. If more than $n+1$ points are taken, the equations can be converted in the ordinary way to normal equations for least squares derivation of the constants a_i.

The derivative of the fitted curve can be found by differentiating Eq. 4 to obtain

$$y' = \frac{y}{1+x^n}\left(\sum_{1}^{n} i\, x^{i-1} a_i + \frac{1}{(1+x)\ln(1+x)} - n\, x^{n-1} \ln y\right). \qquad (6)$$

CURVE INTERPOLATION

If, by the preceding methods, we fit two experimental curves taken at, say, tire pressure A and tire pressure B, to nth degree expressions, we will obtain two sets of constants a_{iA} and a_{iB} ($i = o, \ldots, n$) that define the two curves. If we then want to form a similar curve for tire pressure P, lying between A and B, we can interpolate each a_i constant between its A and B values. Thus, if tire pressure P were 0.1 of the interval from A to B, then the a_i constant, for example, would be linearly interpolated as $a_{iP} = a_{iA} + 0.1\,(a_{iB} - a_{iA})$, and similarly for each a_i.

The interpolatory transformation is perfectly general. The a_i constants can be interpolated by conventional polynomials, when three or more curves are given, for curvilinear interpolation of the constants a_i as functions of the parameter defining the separate given curves.

More generally, if a set of curves is obtained from a set of experiments involving several parameters, the constants obtained by fitting each curve to the same nth degree equation can be interpolated as functions of the several variable parameters. Then, by calculating each a_i for any desired combination of parameter values, the μ-slip or soil shear curves can be developed for any chosen set of conditions spanned by the experimental set of test curves. The general effect on μ of changing any independent parameter can then be found by taking the partial derivative of the compound function with respect to that parameter.

For multivariable interpolation of the constants as function of mth degree polynomials of r parameter variables, it is necessary that $(m+r)!/m!r!$ curves be fitted to obtain enough data points for each a_i to permit evaluation of the $(m+r)!/m!r!$ constants appearing in that a_i's interpolation polynominal. Linear, or nearly linear, parameters permit lower degrees of m to be used.

Parameters associated with different test instruments can be compared by such tests under multivariable conditions. If a parameter used to define the separate curves is not interpolable, it is simply not a good parameter for those conditions, and should be discarded or transformed to a more nearly linear parameter before use in a large multivariable model.

TRANSFORMATION OF FITTED FUNCTION

Our methods so far involve only interpolation between known experimental curves and, therefore, rest only upon the boundary assumptions of characteristic initial slope and horizontal asymptote, and the method of interpolation. The possibility exists, however, of going farther by deriving a second curve from only one experimental curve and a separately established form for transformation of its constants.

Due to newness of Eq. 1, the illustrative examples that follow are necessarily hypothetical. Appropriate experiments will be needed for acceptance or rejection of these or similar transformations.

EXAMPLE 1.—If we assume that a cohesionless soil or tire/soil combination's intrinsic shear properties are invarient with respect to the initial state of looseness or compaction, then the effects of looseness or compaction on a μ-slip or soil shear curve can be portrayed by varying the a_o coeffi-

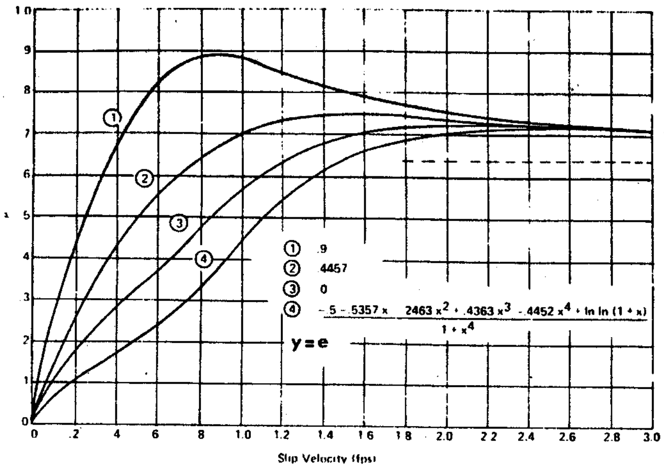

Fig. 2—Effect of varying the a_o coefficient

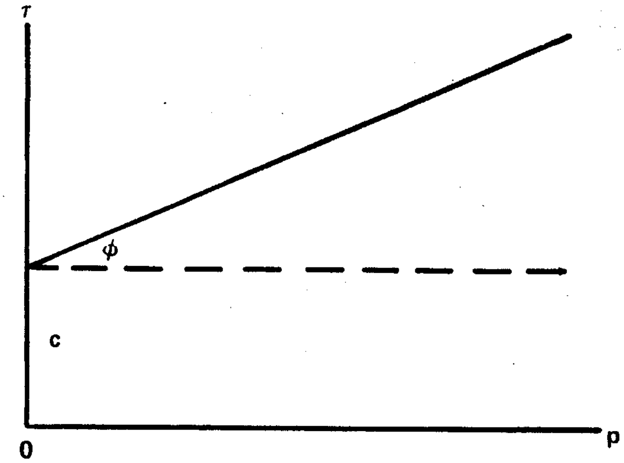

Fig. 4—Coulomb's Empirical Law for soil failure stress

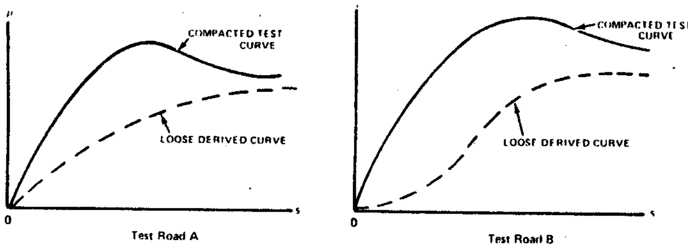

Fig. 3—Calculated effects of decompacting road surfaces

cient in Eq. 1 (since x^o is dependent of x) while holding the other coefficients constant. The family of curves so obtained (fig. 2) strongly resembles textbook representations of sands' μ-slip and direct shear test curves at different compactions.

EXAMPLE 2.—By the method of example 1, μ-slip tests taken on two different compacted soil-aggregate roads can be transformed to portray test curves of the same roads in a loosened condition (shown dotted in fig. 3).

Test road B's derived loose curve suggests that the loosened road surface can be healed by recompaction from traffic imposing μ stresses lower than the μ at the loose curve's point of inflection. The test curve for road A lacks the initial hollow that suggests the healing property. The presence or absence of the initial hollow depends upon whether the a_i coefficient of the fitted test curve is positive or negative. This coefficient is obtained from the test curve and preserved during its transformation. Field tests of road surface aggregate quality may be possible from this transformation.

EXAMPLE 3.—The failure shear stress of engineering soils is commonly expressed in Coulomb's Empirical Law (3):

$$\tau = c + p \tan \phi, \tag{7}$$

where p is the normal pressure on the soil's failure plane and c and tan ϕ are experimental constants (fig. 4).

Either the maximum tractive coefficient μ, or the maximum shear stress τ, obtained from test curves of either taken at different ground pressures A and B, permit Coulomb constants to be found by simultaneous solution of the A and B cases of Eq. 7 to obtain:

$$\tan \phi = \frac{\tau_B - \tau_A}{p_B - p_A} \text{ and } c = \frac{p_B \tau_A - p_A \tau_B}{p_B - p_A}. \tag{8}$$

The maximum μ values needed for τ_A and τ_B in Eq. 8 can be obtained by plotting the fitted μ-s function when that portion of the test curve is missing or poorly defined.

From the empirical nature of Coulombs' Law, the motives for obtaining constants for automotive use from an automotive test are: to preserve dimensional similitude of the tire's footprint, to preserve a common method of measuring the nominal ground pressure in the tire's footprint, and to include the boundary layer shear stress at the tire/road interface in the test results.

EXAMPLE 4.—The μ-slip function, Eq. 1, is commonly used to depict a wheel's tractive output effort in the form

$$\mu_{out} = \frac{P}{W} \tag{9}$$

where P = drawbar pull

and W = weight on the wheel.

The input μ-slip function can be obtained by tests from

$$\mu_{in} = \frac{T\Theta}{W(1+s)}, \tag{10}$$

where T = driving torque applied to the wheel

and Θ = angle, in radians, turned by the wheel per unit travel distance.

By deleting the ln ln (1 + x) term from the numerator of Eq. 1, we obtain

$$y_{in} = e^{\frac{\sum_{o}^{n} a_i x^i}{1+x^n}} \tag{11}$$

suitable for fitting Eq. 10 since Eq. 11 is not restrained to passing through the origin (fig. 5).

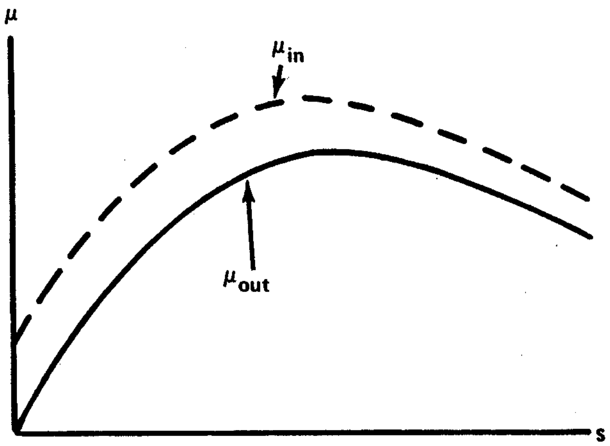

Fig. 5—Input and output μ-slip curves

The boundary limits of Eq. 11 yield

$$y(o) = e^{a_o}$$
$$y(\infty) \doteq e^{a_n} \quad (12)$$
$$y'(o) = e^{a_o} a_1.$$

If desired, the rolling resistance (RR) fraction and the tractive efficiency E_T curves can be obtained from figure 5 and plotted on the same diagram from the relations

$$(RR) = \mu_{in} - \mu_{out} \quad (13)$$

and

$$E_T = \frac{\mu_{out}}{(1+s)\,\mu_{in}}. \quad (14)$$

General expressions for (RR) and E_T as multivariable functions of several parameters can be formed by fitting and interpolating the μ_{in} and μ_{out} functions to multivariable tests in the manner that has been described.

CHANGE OF VARIABLE—The curve-fitting process can be troublesome when the known x values are small numbers because, in Eq. 5, and in least squares adaptations of Eq. 5, the known x values are raised to high powers of x on the left side of the equation. Consequently, numbers like 0.0000000081 appear and compound with similar small numbers to cause a near-zero determinant of the system's coefficients.

A simple remedy is to change the x scale before fitting, so that the x values will be of the same order of magnitude as the y values. Then change the x scale back again when plotting the curve.

AFTERMATH

Researchers Brenner and Kondo (4) found that a multiplicative model of tire wear from several causes was significantly better than an additive model. Joint investigations by the Forest Service, USDA, and the Nevada Automotive Test Center (5) subsequently related tire wear to slip energy.

In view of these developments, it is encouraging that the μ-slip function, which was derived only from boundary assumptions of slope and asymptote, should assume a multiplicative from similar to that developed by workers in these other areas. The role of waste energy in the destruction of roads, tires, and the natural environment creates a further need for even more energy (usable and waste) to repair this damage. Such damage may greatly compound the cost of fuel lost in the initial energy waste.

Effective reduction of initial energy losses will benefit the road, the tire, and also the natural environment, which must someday become our major energy source.

ACKNOWLEDGMENT

The author is grateful that Mr. Henry Hodges, who first exposed him to the uses and meaning of the μ-slip curve, has agreed to compare and report at the Symposium on Tire Rolling Losses and Fuel Economy on the function's ability to represent actual test curves, and to treat the tire implications of these curves in a companion paper.

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