Submerged-Weir Discharge Studies

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Contents in Brief—*Based on an application of the superposition principle and results of experimental work completed at the hydraulics laboratory of the Pennsylvania State College, where the author formerly served as an assistant professor of civil engineering, a general discharge formula for sharp-crested, submerged weirs has been developed. Results of the tests are said to prove: (1) that triangular and parabolic weirs are more accurate measuring devices than proportional and rectangular types, and (2) that sharp-crested submerged weirs can be used in practice with confidence, if certain design and operational specifications are satisfied.*

ENGINEERS HESITATE to use submerged weirs for liquid measurement and control in open channels because of lack of design and performance data. Submerged weirs conserve elevation and should have wide application in the design of gravity systems where savings in head loss mean savings in construction costs.

The amount of research concerned with submergence phenomena is small relative to its importance in hydraulic design. None of the early theories has been substantiated successfully by experimental results. For this reason, an investigation was undertaken in April 1943 at the hydraulic laboratory of The Pennsylvania State College to study the effect of submergence on weir discharge, and to develop design and performance characteristics for all common shapes of sharp-crested and broad-crested weirs.

After completing the work on the sharp-crested weirs described in Fig. 2, the investigation was interrupted in September 1943, because of war training programs. A few months after the start of these tests, J. K. Vennard and R. F. Weston (Submergence Effect on Sharp-crested Weirs, *ENR*, June 3, 1943, vol. p. 814) compared and summarized the experiments of six investigators (Bazin, Cone, Cox, Francis, and Fteley and Stearns), and presented an excellent simplification of the calculations in applying their results.

In order to derive an expression for submerged weir flow, it is necessary to make some simplifying assumptions. One such assumption was first made by Nancay Dubuat (*Principles d'Hydraulique*, vol. 1, p. 203, 1816). He considered the flow over a submerged weir to be made up of two parts: (1) a free flow with effective head equal to the difference in upstream and downstream levels; and (2) a submerged orifice flow likewise operating under a head equal to the difference in upstream and downstream levels. However, his equation has not been verified by tests of later experimenters.

As a first approximation to the proper form of submerged discharge, Q, as a function of the upstream and downstream heads h_1 and h_2 above the weir crest, a more simplifying assumption than that of Dubuat might be made. Assuming that the net flow over the weir is the difference of the free-flow discharge due to head h_1 minus the free-flow discharge due to head h_2 (Fig. 1), then,

$$Q = Q_1 - Q_2 \qquad (1)$$

This assumption implies that the head, h_2, does not directly affect the flow of water due to h_1 and likewise

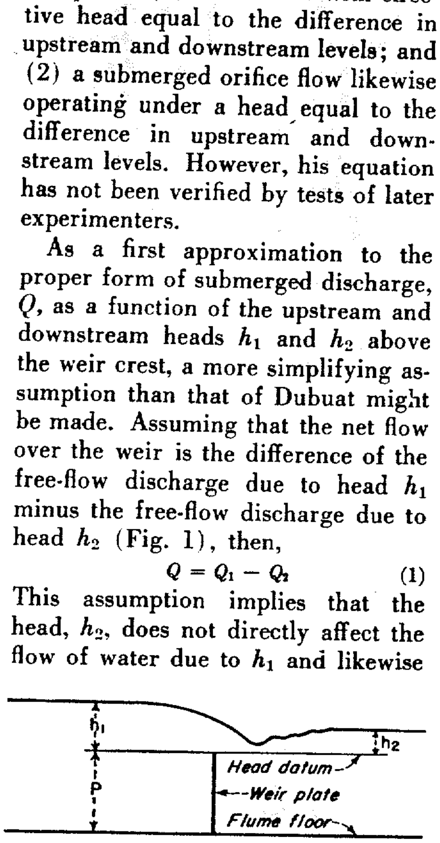

Fig. 1. Conditions affecting submerged flow are shown in this sketch.

that head, h_1, does not prohibit counterflow due to h_2. Its use is thus equivalent to an application of the principle of superposition which is frequently used in evaluating the combined effect of several independent conditions. The equation

$$\frac{Q}{Q_1} = 1 - \frac{Q_2}{Q_1} \qquad (2)$$

should not be expected to give a quantitative relation for determining Q, since interaction and other perturbing influences have been entirely neglected. These effects would probably contribute additional higher order terms in $\frac{Q_2}{Q_1}$.

The experimental tests show that $\frac{Q}{Q_1}$ is related functionally to $1 - \frac{Q_2}{Q_1}$ but that the appropriate function is not the simple linear one of equation (2). The results show that this relationship may be expressed in the form

$$\frac{Q}{Q_1} = f\left(1 - \frac{Q_2}{Q_1}\right) = k\left(1 - \frac{Q_2}{Q_1}\right)^m \qquad (3)$$

or since

$$Q_1 = C_1 h_1{}^{n1} \text{ and } Q_2 = C_2 h_2{}^{n2}$$

$$\text{then, } \frac{Q}{Q_1} = k\left(1 - \frac{C_2 h_2{}^{n2}}{C_1 h_1{}^{n1}}\right)^m \qquad (4)$$

where k and m are constants to be determined from data and the C's and n's are the coefficients and exponents that appear in the free-flow discharge equation obtained from previous calibration or by use of an appropriate standard weir formula. The values of C's and n's for the weirs tested are shown in Fig. 2.

For any given type of weir the coefficients, C_1 and C_2, and the exponents, n_1 and n_2, should be equal. Then, equation 4 reduces to

$$\frac{Q}{Q_1} = k\,(1 - S^n)^m \qquad (5)$$

where S = submergence ratio, $\frac{h_2}{h_1}$. The constant k and the exponent m, which account for the interaction effects,

were determined separately for each weir type by the algebraic method of averages. The results for the seven weir types when averaged arithmetically showed k equal to 1.00 and m equal to 0.385 for a practical submergence range of 0.00 to 0.90. The maximum variation of each from the average is less than 1 percent.

Thus, the general discharge equation for all sharp-crested weirs with crest curvatures expressed by continuous single-valued functions is

$$Q = Q_1 (1 - S^n)^{0.385} \tag{6}$$

For all sharp-crested weirs regardless of crest curvature, the more general form of equation 3 should be used. Assuming the same constants to apply, the equation for this case is

$$Q = Q_1 \left(1 - \frac{Q_2}{Q_1}\right)^{0.385} \tag{7}$$

WEIR NO.	DESCRIPTION	SHAPE AND SIZE	HEAD DATUM HEIGHT, P, FT.	FREE FLOW DISCHARGE FORMULAS, CFS	EXPONENT USED IN COMP.	NUMBER OF TESTS
1	SYMMETRICAL PROPORTIONAL	$\frac{X}{b} = 1 - \frac{2}{\pi} \tan^{-1}\sqrt{\frac{Y}{a}}$ WHERE: a = 0.03 FT. b = 1.00 FT. WITH X & Y IN INCHES	1.01*	$Q_0 = 0.863\, h_0$	1.00	42
2	RECTANGULAR FULL-WIDTH	3.02'	2.00	$Q_0 = 3.35 L\, h_0^{1.49}$	1.50	59
3	RECTANGULAR CONTRACTED	1.25' 0.50' 1.25'	1.00	$Q_0 = 2.96 L\, h_0^{1.44}$	1.44	38
4	RECTANGULAR CONTRACTED	1.00' 1.00' 1.00'	1.25	$Q_0 = 3.00 L\, h_0^{1.45}$	1.45	9
5	PARABOLIC	$Y = 3X^2$, IN FEET	0.83	$Q_0 = 2.04\, h_0^{1.98}$	2.00	33
6	90° TRIANGULAR	$Y = X$, IN FEET	2.00	$Q_0 = 2.54\, h_0^{2.51}$	2.50	40
7	CUSP PARABOLIC	$Y = \sqrt{25X}$, IN FEET	0.83	$Q_0 = 0.594\, h_0^{3.32}$	3.32	59
NOTE	For all weirs h_1 was measured 3.00 ft. upstream from weir and h_2 6.00 ft. downstream. Three piezometer holes on a transverse line in the flume floor were connected to each point gage stilling tube. *Head datum 0.010 ft. above weir crest.					

Fig. 2. Seven types of weirs were tested in this series of experiments.

Experimental tests

The seven experimental weirs described in Fig. 2 were mounted in turn midway in a steel flume 3 ft. wide, 3 ft. deep, and 25 ft. long. Discharge was measured by a calibrated, standard 90 deg. triangular, sharp-crested weir placed near the upstream end of the flume, and the degree of submergence was controlled by three 6-in. vertical outlets passing through the flume floor at the down-stream end and adjustable in elevation.

The same testing procedure was used for all weirs. After calibrating each weir for free flow by comparison with the triangular weir, the discharge was held constant and the degree of submergence changed 10 to 15 times from 0.00 to about 0.95. Upstream and downstream head observations were taken for each submergence setting using point gages reading in 4-in. stilling tubes. For each type of weir the same procedure was followed using three additional constant discharge rates. Detailed observations were recorded on the behavior of the nappe and downstream flow conditions.

The experimental results show that the dimensionless terms, $\left(\frac{Q}{Q_1}\right)$ and $\left(1 - \frac{Q_2}{Q_1}\right)$, have a consistent functional relation for all sharp-crested weir types. A logarithmic plotting of simultaneous values of $\left(\frac{Q}{Q_1}\right)$ and $(1-S^n)$ for each experimental weir (Fig. 3) shows the general conformity

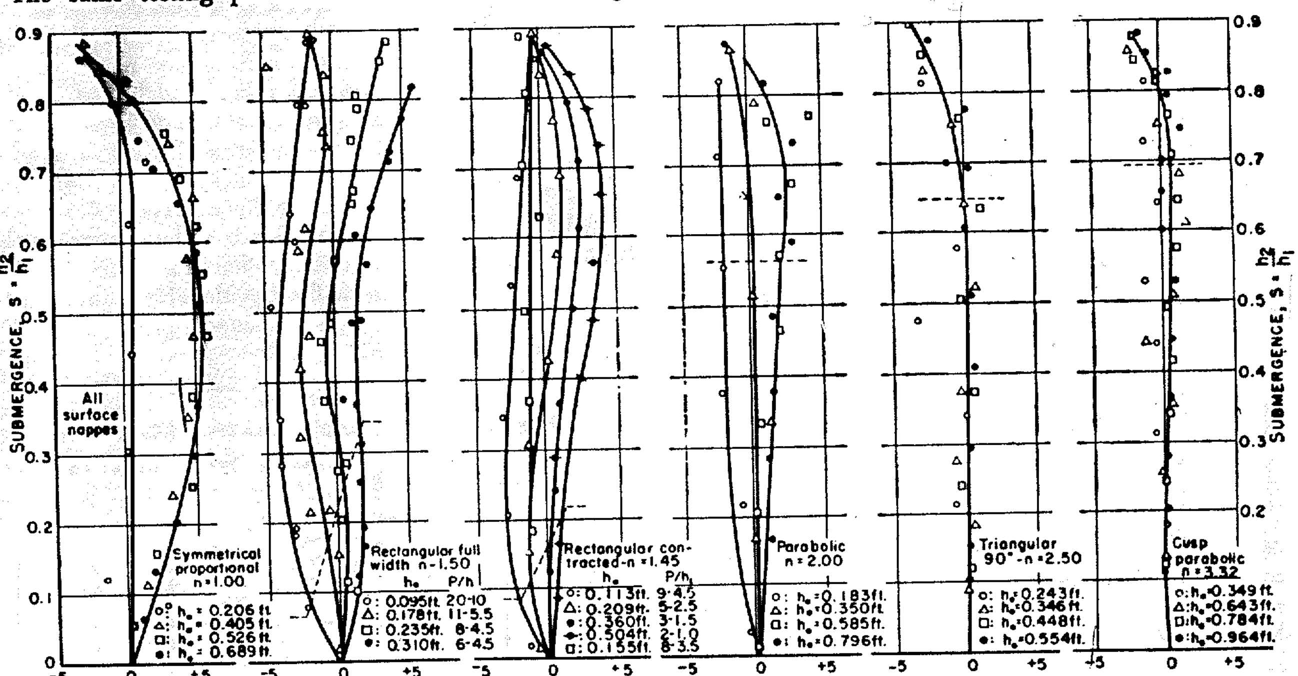

Fig. 3. The percent correction that must be applied to Q computed from equation (6) to obtain the measured discharge may be found from these graphs.

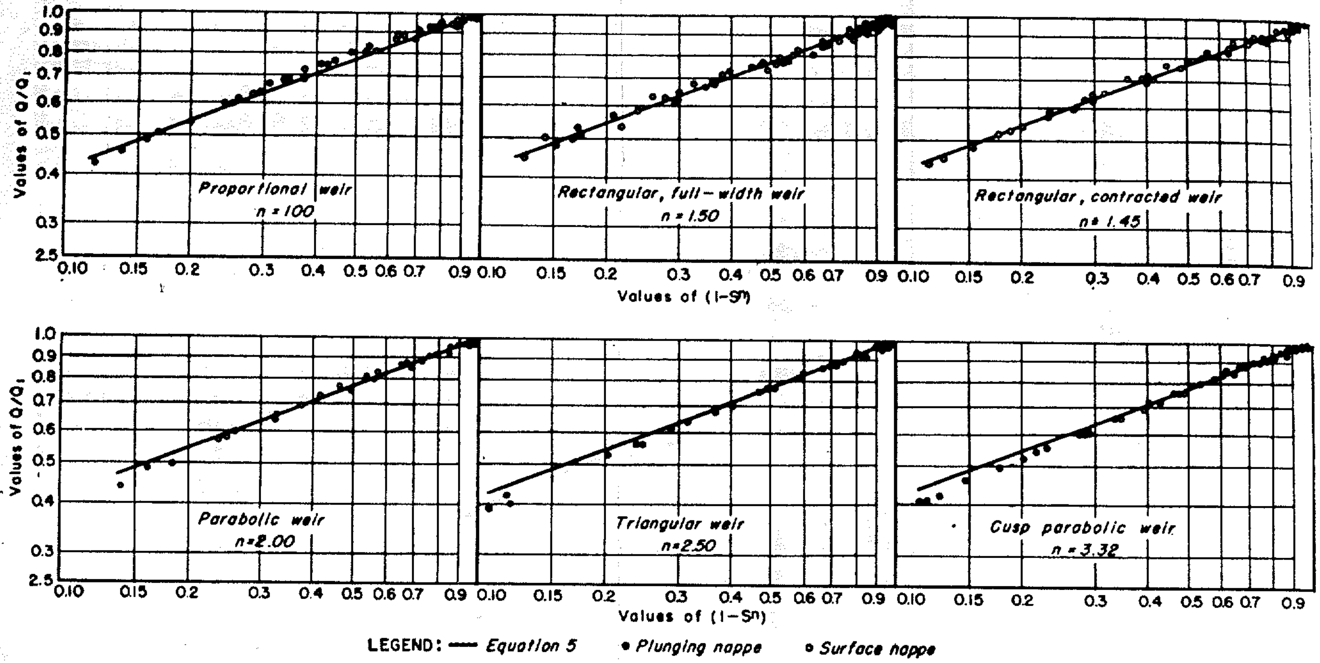

Fig. 4. Plotted on logarithmic scales, simultaneously obtained values of $\frac{Q}{Q_1}$ and $(1\text{-}S^n)$ conform closely to equation (5) with k = 100 and m = 0.385.

of all data with equation (5). The constant k and the exponent m were evaluated as 1.00 and 0.385, respectively, by methods described. It should be noted that as S increases the term $(1—S^n)$ decreases. All tests at submergences greater than 0.90 were excluded from the calculations so that the results would typify more closely practical submergence conditions.

The conformity of each test with equation (6) is illustrated in Fig. 4. The degree of submergence S is plotted against the percent correction that must be applied to Q computed by equation (6) to give the measured discharge rate. Average curves are shown for each constant discharge series, together with their ranges in $\frac{P}{h_1}$ and equivalent free flow heads, h_0. The point of transition from surface to plunging nappe conditions is also indicated.

V-notches perform best

As shown in Fig. 3, the results for weirs having top-heavy water sections are more consistent. No distinction is made among the various discharge series for the triangular and cusp parabolic weirs. However, for rectangular and proportional weirs, the variation between the low and high discharge series is about 5 percent and less for the parabolic weir.

These variations are the result of confining the nappe in narrow, shallow tailwater basins. Free circulation of the water underneath the nappe is restricted, regions of subnormal pressure are produced, and the same discharge flows through the notch at a lower h_1. This phenomenon is analogous to the operation of an unventilated free flow weir and has been explained clearly by Vennard and Weston. The rectangular, full-width weir was so erratic at low values of S and $\frac{P}{h_1}$ that tests in the two largest discharge series were excluded entirely for submergencies less than 0.10. The velocity of approach has not been considered in the computations for S and $\frac{P.}{h_1}$

Two types of nappe conditions have been identified with submerged weir discharge: (a) the plunging nappe and (b) the surface or flowing nappe. In the first the nappe plunges beneath the water surface and rises to the surface at a point downstream where some stream paths are directed upstream. In the second the nappe remains on the water surface, all stream paths are directed downstream and standing waves are developed. The transition from one nappe condition to the other is identified by a breaking of the standing wave crests similar in appearance to a hydraulic jump. This transition condition is defined as a rolling nappe.

The transition from one nappe condition to the other has little or no effect on the functional relationships when h_2 is measured just beyond the surface turbulence caused by the

TABLE I—VALUES OF $\frac{Q}{Q_1}$ FOR THREE COMMON SHARPE-CRESTED WEIR NOTCHES BY EQUATION (6), SHOWING COMPARISON WITH RESULTS OF OTHER WORKERS

Degree of Submergence S P/h₁	Proportional Notch Equation (6)	Rectangular Notches Equation (6) 2–10	Cone 6	Co 4–30	Francis, Fteley, Stearns 3–10	Stevens	Bazin 6	V-Notch Equation (6)
0.10	0.96	0.99	0.99	1.00	0.99	1.00	1.00	1.00
0.20	0.92	0.96	0.96	0.96	0.96	0.97	0.97	0.99
0.30	0.87	0.93	0.93	0.92	0.92	0.93	0.93	0.98
0.40	0.82	0.89	0.89	0.88	0.88	0.89	0.89	0.96
0.50	0.77	0.85	0.85	0.83	0.83	0.84	0.84	0.93
0.60	0.70	0.79	0.79	0.77	0.77	0.78	0.79	0.88
0.70	0.63	0.71	0.71	0.70	0.69	0.70	0.72	0.82
0.80	0.54	0.62	0.62	0.60	0.59	0.59	0.63	0.72
0.90	0.41	0.48	0.46	0.45	0.45	0.51	0.48	0.57

nappe (Fig. 2 and 3). For these studies this point was found to be 6.0 ft. downstream from the weir. The best point for a particular weir must be determined experimentally. In general, the point will move farther downstream as the rate of discharge increases.

The shape of the water-section has an interesting and logical effect on the occurrence of a nappe condition. The proportional weir, which has a bottom-heavy water-section, had surface nappes for all tests. More of the water mass is under a higher head which produces a flatter curvature of the nappe underside. The stream paths of the nappe are no nearly horizontal that a plunging condition never develops. However, the nappe, plunged during low submergences for all other weirs. For any one weir the plunging condition persists for a greater submergence range as Q is increased (see dashed lines of Fig. 3). As n increases from 1 to 7/2, the water-section changes from the bottom-heavy proportional type to the top-heavy cusp parabolic type Fig. 2. Under the latter condition more of the water mass is under a lower head, the nappe trajectory has a steeper downward slope, and the plunging condition persists for greater submergences (Fig. 2 & 3).

Comparisons and conclusions

Results agree favorably with those of earlier workers. Values of $\left(\frac{Q}{Q_1}\right)$ found by equation (6) are compared in Table I with a summary of existing data on rectangular notches, compiled by Vennard and Weston and Stevens (Stevens, J. C., "Sharp-Crested Weir Flows," *ENR*, July 29, 1943, vol. p. 187). Also values of $\left(\frac{Q}{Q_1}\right)$ for two other types of sharp-crested weirs are given.

As a result of this study, the following conclusions may be drawn:

1. The experimental work indicates that $\left(\frac{Q}{Q_1}\right)$ and $\left(1-\frac{Q_2}{Q_1}\right)$ have the same functional relationship for each weir tested. In addition, it is reasonable to expect that this relationship should be applicable to the general group of sharp-crested weirs.

2. The effect of submergence on weir discharge decreases as n increases, but the stability and consistency of such effect are improved as n increases. Therefore, triangular and parabolic weirs, are more accurate when submerged than are proportional and rectangular weirs.

3. Recommendations for practical application of equations (6) and (7) are:

(a) Measure h_2 just beyond the surface turbulence caused by the nappe, generally 6 to 10 ft. downstream from the weir.

(b) Measure h_1 at least $4h_1$ upstream.

(c) For proportional weirs about 5 percent accuracy can be expected if $\frac{P}{h_1}$ is greater than 1; for rectangular weirs 3 percent if $\frac{P}{h_1}$ is greater than 3, and for fully contracted triangular and parabolic weirs less than 2 percent.

(d) The velocity of approach can be neglected in computing S *and* $\frac{P}{h_1}$.

4. The accuracy of submerged weirs will be improved if the tail-water basin is sufficiently wide and deep to permit free circulation of the water underneath the nappe. In this way, interaction effects are stabilized.

5. An application of the principle of superposition might be valuable in studying the effect of submergence on other hydraulic structures—such as, orifices, slide gates and culverts.

Throughout the testing program, valuable suggestions and assistance were received from F. T. Mavis, formerly head of the Civil Engineering Department of the Pennsylvania State College; and from J. A. Sauer, head of the Engineering Mechanics Department.

The work is now being extended by the writer in graduate study at the University of Wisconsin. It will include a broad study of submergence phenomena downstream from the weir and testing of a series of different sharp-crested weirs. Moreover, submergence tests will be conducted on broad-crested weirs, orifices, sluice gates and culverts. J. G. Woodburn, Professor of Hydraulic Engineering, University of Wisconsin who is in charge of the program, has been most helpful in the preparation of this paper.

Parking Lots Located Within City Blocks

Cities everywhere are bucking acute parking problems but so far few have put into use some of the most convenient and economical space generally available—the cluttered areas behind buildings, inside blocks.

This statement by the American Society of Planning Officials highlights recent studies done on interior-block parking and is substantiated by cities which have used this method to help meet downtown parking needs.

Quincy, Mass.; Garden City, N. Y. and Kalamazoo, Mich. operate parking lots in the center of business block, utilizing land valued at lower rates than land fronting on streets.

Land for the 1,200 ft. lot in Quincy cost about $150,000 and was acquired by condemnation of property lying between railroad tracks and stores.

Kalamazoo financed its interior block parking lot with an assessment against a special benefit district. In Garden City about 800 cars can be parked in the city's eight paved and landscaped interior block lots.

The Yale University traffic bureau has conducted a study showing the feasibility of low-cost parking in downtown New Haven if block interiors are used. Land valuations are $21 less per sq. ft. for interior property, in one sample block, than for street-front property. Officials estimate that 100 cars could be parked inside the block for nearly $500,000 less than in a street-front lot.

Similar developments in the West include the rear-parking area provided by store owners in a privately-controlled development along Wilshire boulevard in Los Angeles. In Oakland, the planning commission has proposed to build 16 groundlevel parking lots on the edge of the business district, some within blocks.

The Oakland parking plan involves cost estimates of $3.1 million for acquiring the land and building facilities for 6,138 auto spaces. The Anaheim, Cal., planning commission also includes interior block parking in its city masterplan.