Local scouring in low and high gradient streams at bed sills

Affouillement local dans des courants à faible et forte pente sur des seuils de fond

MARIO A. LENZI, *Assoc. Prof., Dept. of Land and Agroforest Environment, Univ. of Padua, via Romea, Agripolis, 35020 Legnaro (Padova), Italy*

ANDREA MARION, *Assist. Prof., Dept. of Hydraulic, Maritime and Geotechnical Engineering, Univ. of Padua, via Loredan 20, 35100 Padova, Italy*

FRANCESCO COMITI, *PhD Stud., Dept. of Land and Agroforest Environment, Univ. of Padua, via Romea, Agripolis, 35020 Legnaro (Padova), Italy*

ROBERTO GAUDIO, *Research Eng., I.R..P.I , National Research Council (C.N.R.), Via Cavour, 87030 Rende (Cosenza), Italy*

ABSTRACT

The main characteristics of local scouring downstream of bed sills, forming a staircase-like system in high-gradient streams with non-uniform alluvium, have been investigated through 13 clear-water laboratory runs. Three initial longitudinal slopes and different flow rates were considered, keeping the same distance between the baffles. The grain size distribution of the sediment is that of a real alpine torrent scaled to the model dimensions. The measured scour depth, length and shape are compared to previous results concerning low gradient and uniform sediment gradings. A dimensional analysis approach appears to remain valid; nevertheless some simplifications cannot be made, since the jet regime plays an important role both for the depth and the length of the scour, and consequently affects the scour shape. Two equations are proposed for the estimation of the maximum scour depth and length. The equations are from previous data sets on low-gradient tests and a new one of experimental results on high-gradient cases.

RÉSUMÉ

Les principales caractéristiques d'affouillement à l'aval de seuils de fond, formant un système en marches d'escalier dans des courants à forts gradients avec des alluvions non uniformes, ont été étudiées à travers 13 essais de laboratoire en eau claire. Trois pentes longitudinales initiales et différents débits furent étudiés en gardant la même distance entre les déflecteurs. La distribution des tailles de grains de sédiments est celle d'un réel torrent alpin à l'échelle des dimensions du modèle. Les mesures de profondeur, longueur et forme de l'affouillement sont comparées aux résultats antérieurs concernant de faibles pentes et des granulométries uniformes. Une approche par analyse dimensionnelle semble rester valable : cependant quelques simplifications ne peuvent plus être faites car le régime de jet joue un rôle important à la fois pour la profondeur et la longueur de l'affouillement, et par suite en affecte la forme. Deux équations sont proposées pour l'estimation du maximum de la profondeur et de la longueur de l'affouillement. Les formules proviennent des ensembles de données précédentes sur les essais à faible pente, et une nouvelle formule des résultats expérimentaux dans les cas de forte pente.

Keywords: bed sills, channel erosion, local scouring, high-gradient streams, laboratory flume.

1. Introduction

Mountain streams are often subject to channel incision. One method to stabilise them is to use a sequence of transverse grade-control structures, or bed sills. Local scour downstream of the sills can endanger their stability and create the risk of failure if the foundations are not designed taking into account the maximum scour depth.

Local scour by free jets is difficult to treat theoretically due to the complexity of its dynamics. Consequently experimental studies play a major role in relating the scour features (depth, length, shape, time-development) to the hydraulic and sediment variables. Field measurements present logistic problems along with difficulties in evaluating hydraulic and sediment parameters, therefore most of the research concerning this particular form of scour has been carried out using laboratory tests. The majority of the studies have addressed the problem of single, isolated drop structures ([13], [25], [30], [27], [26], [6], [9], [17], [30], [10], [28]); much less is known about the case of staircase-like systems of low check-dams or bed sills, the most popular technique to prevent the stream bed from being excessively degraded and incised ([5], [32], [33], [24], [16], [15]).

Referring to mountain steep streams (torrents), peculiar hydraulic and sediment features are observed: highly heterogenous grain size distribution of the bed material and very low relative submergence. Armouring processes and high roughness coefficients are the principal results of such conditions ([20],[21]).

The issue concerning the choice of a representative diameter for local scouring with graded sediment mixtures has not been completely solved yet. Most authors suggest the choice of D_{90} as the effective diameter ([33], [6], [9]), some others indicate D_m (median diameter) or D_{85} [27] instead. A very interesting evaluation in the case of scouring by horizontal submerged jets was presented by Aderibigbe and Rajaratnam [1]. They found that the best correlation between the non-dimensional scour depth and length and the densimetric Froude number (the only parameter seemingly affecting their results) was using D_{95}. Such diameter was also found to be roughly the median size of the bed material inside the final scour hole.

Another concern in predicting the scour depth and length for slop-

Revision received June 27, 2002. Open for discussion till April 30, 2003.

ing channels is the tailwater depth and its link with the jet trajectory ([31], [29]). In all studies that tested the role of the tailwater depth, this was found to affect the scour process directly ([27], [9]). The jet impact angle plays a major part in determining the scour depth along with its geometry ([6], [28]): the more vertical the jet, the deeper the scour and the steeper the downstream side of the scour hole, with an angle roughly corresponding to the jet angle.

In order to determine how the characteristics of high-gradient torrents affect the scouring process, a series of laboratory experiments (13 runs) has been carried out using the *Sloping Sediment Duct* facility at HR Wallingford Ltd. (United Kingdom).

2. Definition of the problem

The problem was defined by Gaudio et al. [16]. Considering the system sketch in Fig. 1, the function for the maximum clear-water, long-term scour depth can be written as:

$$y_s = f(g, \nu, \rho_w, \rho_s', q, h_u, D, a_1) \tag{1}$$

where g is the gravity acceleration, ν is the kinematic viscosity of water, ρ_w is the density of water, ρ_s' is the submerged density of sediments, q is the water discharge per unit width, h_u is the water depth of uniform flow condition, D is a characteristic grain size and a_1 is the "morphological jump", defined as:

$$a_1 = (S - S_{eq})L \tag{2}$$

where S is the initial longitudinal bed slope, S_{eq} is the equilibrium bed slope and L is the distance between sills. The equilibrium slope for clear-water can be addressed by the Shields' condition for threshold of motion for fully developed turbulence flows:

$$\theta_c = \frac{h_u S_{eq}}{\Delta D} = \text{constant} \tag{3}$$

where θ_c is the critical dimensionless shear stress and $\Delta = \rho_s'/\rho_w$ is the relative submerged density of sediments. Therefore:

$$S_{eq} = \frac{\theta_c \Delta D}{h_u} \tag{4}$$

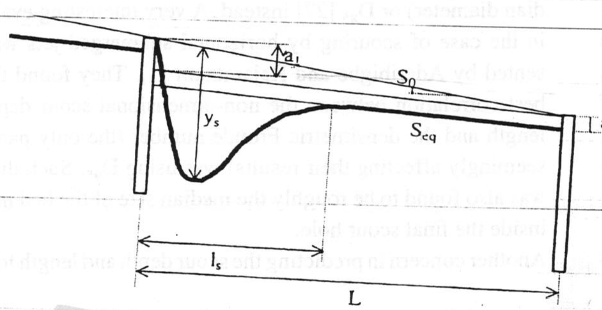

Fig. 1. Definition sketch of the system. The initial channel slope S_0 is analogous to S in the text.

Coupling Eq. (4) and a resistance formula like the Manning's equation for wide channels (with n roughness coefficient):

$$q = \frac{h_u^{5/3} S_{eq}^{1/2}}{n} \tag{5}$$

the uniform flow depth and the equilibrium slope can be expressed as:

$$h_u = \frac{(nq)^{6/7}}{(\theta_c \Delta D)^{3/7}} \tag{6}$$

$$S_{eq} = \frac{(\theta_c \Delta D)^{10/7}}{(nq)^{6/7}} \tag{7}$$

Eq. (6) expresses the dependance of the uniform flow depth from other physical parameters. This allows h_u to be dropped from Eq. (1).

Introducing the critical specific energy on the sills:

$$H_s = 1.5 \cdot \sqrt[3]{q^2/g} \tag{8}$$

the application of the Buckingham's Π theorem to Eq. (1) without h_u and choosing g, ρ_w and q as fundamental variables – leads to:

$$\frac{y_s}{H_s} = \phi\left[\frac{q}{\nu}, \Delta, \frac{a_1}{\Delta D}, \frac{a_1}{H_s}\right] \tag{9}$$

Neglecting the influence of viscosity for fully turbulent flows and assuming that the relative submerged density of the sediment is constant, Eq. (9) can be simplified as follows:

$$\frac{y_s}{H_s} = \Phi\left[\frac{a_1}{H_s}, \frac{a_1}{\Delta D}\right] \tag{10}$$

It can be observed that the first parameter represents the ratio between the energy loss associated to the drop and the flow energy on the sill, whilst the second is the ratio between the morphological jump and a term proportional to the mobility of the bed particles.

The dimensional analysis discussed above, assumes a substantially constant geometry for the overall flow pattern. This is actually a rough approximation as the geometry seems to be affected by the slope, through the development of different values of the jet angles. Eq. (10) could in principle include another non-dimensional parameter, e.g. those included in the jet angle equation found in [6]. In this work, only Eq.(10) has been investigated.

Marion et al. [24] and Gaudio and Marion [15] tested different slopes ranging from 0.0062 to 0.0160, distances between the sills, L, from 2 to 6.5 m, two uniform gravel gradings (D_{50} = 4.1 mm and 8.5 mm, respectively) and a uniform coarse sand distribution (D_{50}=1.8 mm). The formation of a small step ("sill step, a_2") in front of each sill was observed and measured. It was then related to the subcritical regime through the Froude number, Fr, as follows [16]:

$$a_2 = h_u - h_c = h_u(1 - Fr^{2/3}) \quad (11)$$

The step a_2 was shown to be a quantity dependent from the variables listed in Eq. (1), and therefore it was not added to that equation.

It appeared that in both low-gradient tests only the second parameter of Eq.(10) affected the results. The regression formulae presented in [15] was the following:

$$\frac{y_s}{H_s} = 0.180\frac{a_1}{\Delta D_{50}} + 0.369 \quad (12)$$

which covers the range $1.3 \leq a_1/(\Delta D_{50}) \leq 9.1$, with a correlation coefficient R=0.94.

Similarly, adopting the same dimensional analysis for the scour length l_s, the authors obtained the following empirical relation (valid only for the gravel sediments):

$$\frac{l_s}{H_s} = 1.87\frac{a_1}{\Delta D_{50}} + 4.02 \quad (13)$$

which covers the same range, with R=0.94.

As in the other studies, the non-dimensional scour holes were found to be self-affine, with the maximum scour depth occurring at a distance of $0.3 \cdot l_s$ to $0.4 \cdot l_s$ from the sill.

It is important to point out that in all these tests the scour length was never long enough to be constrained by the following sill. Thus the distance L did not produce effects of interference on the development of the scour.

The goal of the present research was to assess whether equations similar to (12) and (13) were applicable in mountain high-gradient streams with heterogenous grain size distribution, or whether two parameters are needed as expressed by Eq.(10).

3. Experimental set-up and data processing

Thirteen clear-water tests were carried out using a mobile-bed physical model set up in the *Sloping Sediment Duct* in the laboratory of HR Wallingford. This facility allows the easy modification of channel slope for a wide range of values (±65%). The flume is 5.57 m long, 0.6 m wide, 0.245 m deep, has a metal bottom and alternate glass and metal side walls [12].

The grain size distribution of the sediment used in tests is shown in Fig.2, where the characteristic diameters are also reported. It was a very heterogeneous mixture, ranging from coarse sand to small cobbles. The geometric standard deviation $\sigma_g = \sqrt{D_{84}/D_{16}} = 5.38$ was much greater than the threshold proposed by Breusers and Raudkivi [7] for the definition of nonuniform gradings σ_g=1.35. The relative submerged density was Δ=1.63.

The size grading reproduces at a 1:40 scale the sediment distribution of a reach of the Maso di Spinelle Torrent, a major tributary of the Brenta River, in Valsugana (Trentino region, Italy). Along this stream several boulder check-dams have recently been built,

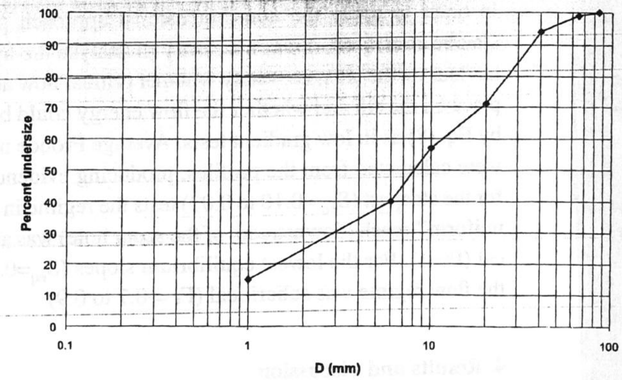

Fig. 2. Grain size distribution curve.

adopting a geomorphologic criterion ([11], [22]).

In all the tests the initial bed was set at a constant slope, achieved by tilting the flume. The slope was determined from the difference in elevation from a horizontal water surface. Three slopes were tested: 0.0785, 0.1145 (which is the actual reach gradient) and 0.1480. The range was chosen in order to cover the most common slopes occurring in natural step-pool systems. Three wooden baffles (1 cm-thick, and the width of the flume) were inserted into the granular fill of the flume. The distance between the sills was 1.050 m. In order to guarantee long-term equilibrium conditions the test duration was set at 18 hours for all tests. The duration was selected on the basis of direct observation and by using a video-camera to record the local scouring zone. An electrical point gauge with an acoustic device was used to measure the bed profile and the water surface elevation: for the former the longitudinal interval was 2.5 cm in the scour hole and 5 cm elsewhere, for the latter it was 10 cm. The profiles were taken along the centre-line only when the scour holes were two-dimensional, otherwise two other longitudinal bed profiles were measured. The maximum depth in the cross-section was also measured. The scour length was determined visually as the point where the equilibrium slope was obtained downstream of the hydraulic jump zone.

Although the initial bed was set-up with extreme care, the scouring process exhibited some asimmetry because the development of lateral bars produced a concentration of flow in a narrower section. This phenomenon was more significant with low to medium flow rates. This is in agreement with the well-known result that planform instabilities arise with low water depth/channel width ratios.

The dimensions of the scour holes were never large enough to occupy most of the distance between the sills. Therefore no interference between the sills occurred during the scouring process ([8], [23]).

The maximum scour depth, y_s (Fig. 1), was calculated downstream of the central sill, identifying the cross-section where the maximum depth occurred and computing the cross-section average depth (on 13 values). The equilibrium slope S_{eq} was evaluated from the bed profiles. The values of the sill step a_2 were also evaluated from the bed profiles. Once the equilibrium slope was obtained, the morphological jump a_1 was calculated using Eq. (2).

In order to apply the non-dimensional approach proposed by Gaudio et al. [16], it was necessary to analyse the hydraulic behaviour on the sills, i.e. verify whether critical flow actually took place on the sill and whether the flow energy could be expressed by Eq. (8) as in low gradient tests. Average Froude numbers, Fr, were calculated from the profiles, producing evidence that even for the steepest (S_{eq}= 0.10 to 0.07) tests the regime in the "quasi-uniform" reach (downstream of the scour hole) was at most critical (Fr≈1). For the lowest equilibrium slopes (S_{eq}=0.07 to 0.04) the flow regime was subcritical (Fr = 0.7 to 0.9).

4. Results and discussion

Maximum scour depth

Tab. 1 shows the maximum scour depth, y_s, measured in each test along with other measured quantities; Tab. 2 reports the calculated quantities and the main non-dimensional parameters.
It must be pointed out that the data from the present research are subject to an "intrinsic" scatter, due to the high non-uniformity of the sediment grading (i.e. the presence of protruding cobbles in few centimetres-deep pits) and to the fact that some tests led to three-dimensional scour. In Fig. 3 the scour depths are plotted along with data reported by Marion et al. [24] and Gaudio and Marion [15], in accordance to their original single parameter formulation. In their work, the grain size was uniform and the choice of a representative size was straightforward (D_{50}). It is evident that the new points do not match with the previous results and, therefore, the formulae obtained with mild slopes and uniform sediments cannot be generalised to high slopes and graded material. The choice of different representative diameter (e.g. D_{90}) did not produce any better agreement. It was hypothesised, therefore, that the adoption of a single parameter formula like Eq.(13), although applicable to mild slopes, is not appropriate when analysing scour at high slopes.
A multiple regression analysis was then performed using the statistical software package (*Statistica* 5.1). The best fit to the experimental data was obtained with the following non-linear equation:

$$\frac{y_s}{H_s} = \left[0.4359 + 1.4525\left(\frac{a_1}{H_s}\right)^{0.8626} + 0.0599\left(\frac{a_1}{\Delta D_{95}}\right)^{1.4908}\right] \quad (14)$$

Tab. 1. Initial condition and measured quantities, (h_{sill}, flow depth at the downstream edge of the sill; for the sediment characteristics, see Fig. 2)

Test	*Q* (m³/s)	*q* (m²/s)	*S*	S_{eq}	*L* (m)	y_s (m)	l_s (m)	h_{sill} (m)	a_2 (m)
H1	0.0111	0.0185	0.0785	0.068	1.050	0.050	0.21	0.024	0.007
H2	0.0125	0.0208	0.0785	0.065	1.050	0.066	0.30	0.027	0.010
H3	0.0143	0.0238	0.0785	0.053	1.050	0.082	0.38	0.028	0.010
H4	0.0165	0.0275	0.0785	0.046	1.050	0.095	0.42	0.032	0.015
H5	0.0175	0.0292	0.0785	0.044	1.050	0.106	0.42	0.033	0.020
H6	0.0044	0.0073	0.1145	0.096	1.050	0.035	0.15	0.013	0
H7	0.0075	0.0125	0.1145	0.088	1.050	0.064	0.19	0.018	0
H8	0.0098	0.0163	0.1145	0.076	1.050	0.075	0.25	0.021	0
H9	0.0125	0.0208	0.1145	0.062	1.050	0.106	0.30	0.026	0.004
H10	0.0143	0.0238	0.1145	0.053	1.050	0.122	0.35	0.029	0.004
H11	0.0040	0.0067	0.1480	0.104	1.050	0.071	0.18	0.013	0
H12	0.0060	0.0100	0.1480	0.090	1.050	0.095	0.20	0.017	0
H13	0.0090	0.0150	0.1480	0.073	1.050	0.133	0.30	0.022	0.004

Tab. 2. Calculated quantities and non-dimensional parameters.

Test	H_s (m)	a_1 (m)	$\frac{a_1}{H_s}$	$\frac{a_1}{\Delta D_{95}}$	$\frac{y_s}{H_s}$	$\frac{l_s}{H_s}$
H1	0.049	0.011	0.225	0.161	1.020	4.28
H2	0.053	0.014	0.267	0.207	1.244	5.65
H3	0.058	0.027	0.462	0.391	1.416	6.56
H4	0.064	0.034	0.535	0.498	1.488	6.58
H5	0.066	0.036	0.546	0.529	1.597	6.33
H6	0.026	0.019	0.734	0.284	1.323	5.67
H7	0.038	0.028	0.737	0.406	1.696	4.90
H8	0.045	0.040	0.899	0.590	1.668	5.56
H9	0.053	0.055	1.039	0.805	1.998	5.65
H10	0.058	0.065	1.115	0.943	2.107	6.04
H11	0.025	0.046	1.861	0.675	2.861	7.05
H12	0.033	0.061	1.872	0.890	2.921	6.15
H13	0.043	0.079	1.848	1.150	3.121	7.04

ıt some tests led to
depths are plotted
] and Gaudio and
gle parameter for-
ırm and the choice
D_{50}). It is evident
vious results and,
opes and uniform
and graded mate-
eter (e.g. D_{90}) did
ıesised, therefore,
ı like Eq.(13), al-
ppropriate when

ned using the sta-
st fit to the exper-
ı-linear equation:

$$\left(\frac{a_1}{D_{95}}\right)^{1.4908} \quad (14)$$

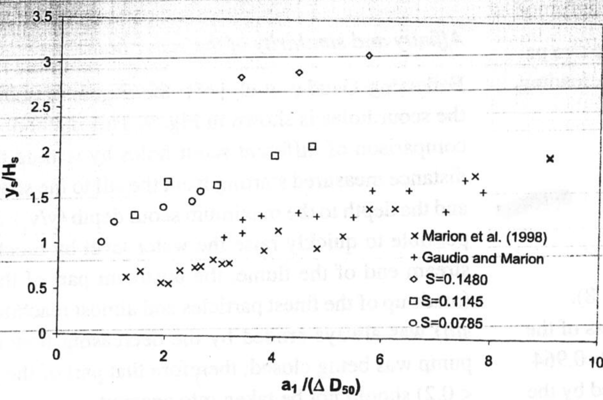

Fig. 3. Non-dimensional maximum scour depths as a function of $a_1/(\Delta D_{50})$.

which has a correlation coefficient R=0.951 and explains 90.5% of the variance (Figs. 4 and 5).

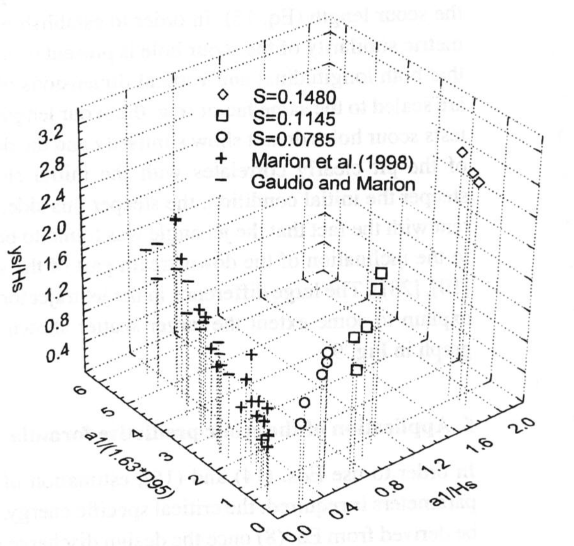

Fig. 4. Non-dimensional maximum scour depths as a function of $a_1/(\Delta D_{95})$ and a_1/H_s.

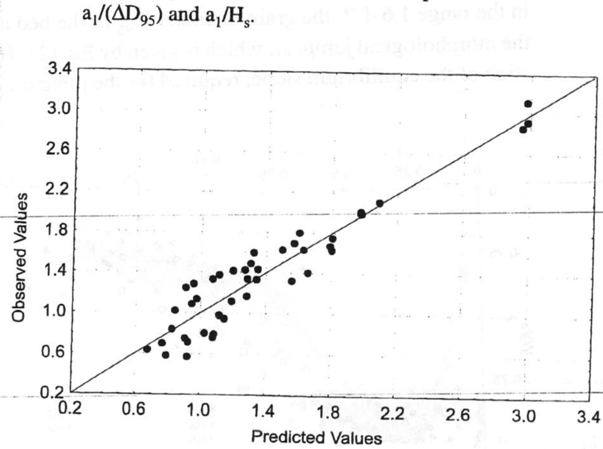

Fig. 5. Comparison between predicted (Eq. 14) and observed values of the dimensionless maximum scour depth (data from the present study, from Marion et al., 1998, and from Gaudio and Marion).

Tab. 3. Experimental range of the non-dimensional parameters in the low and in the high gradient tests.

	a_1/H_s	$a_1/\Delta D_{95}$
This study	0.225 – 1.872	0.161 – 1.150
Marion et al. [24]	0.101 – 0.512	0.720 – 5.31
Gaudio and Marion [15]	0.155 – 0.377	2.208 – 5.644

The use of the D_{95} produced the best correlation, nevertheless this turned out to be almost insensitive to the choice of the representative grain size.

In Tab. 3 are reported the experimental ranges of the two parameters a_1/H_s and $a_1/(\Delta D_{95})$ for the previous low-gradient tests and the new steep runs.

Scour length

In Figs. 6a and 6b the non-dimensional lengths of the scour hole are plotted, adopting the D_{50} and D_{90} respectively; they are compared with the results obtained for uniform gravel and low-gradient tests [24].

The suggestion that the choice of D_{90} provides a good fit to the data might be misleading, as the points displaced in Fig. 6b fall

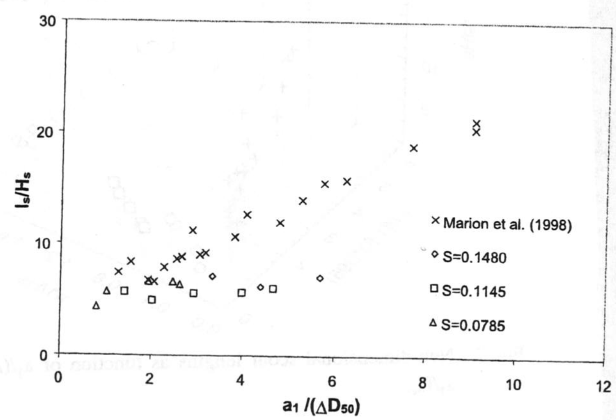

Fig.6a. Non-dimensional scour lengths as a function of $a_1/(\Delta D_{50})$.

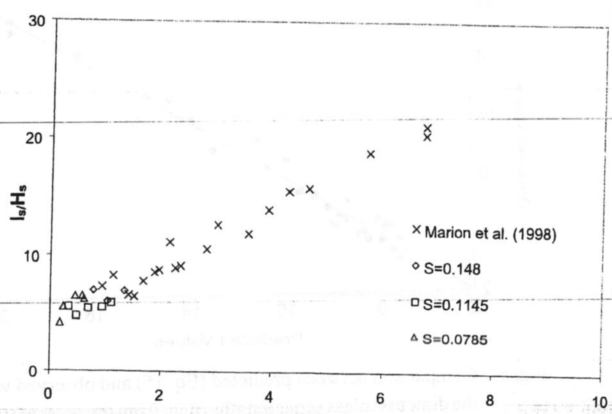

Fig. 6b. Non-dimensional scour lengths as a function of $a_1/(\Delta D_{90})$.

into limited ranges of $a_1/(\Delta D_{90})$ without overlapping.

Thus, as for the scour depths, an analysis considering the two parameters a_1/H_s and $a_1/(\Delta D)$ has been carried out (Fig. 7), leading to the following best-fit, multiple regression formula:

$$\frac{l_s}{H_s} = 4.479 + 0.023\left(\frac{a_1}{H_s}\right)^{-1.808} + 2.524\left(\frac{a_1}{\Delta D_{95}}\right)^{1.129} \quad (15)$$

with R=0.986 and an explained variance of 97.3% (Fig. 8).

For the same pair of variables but using D_{90}, the goodness of the fit did not change, whereas using D_{50} it decreased to R=0.964. Hence it seems that the length of the scour hole is affected by the coarsest grains more than the depth of the scour.

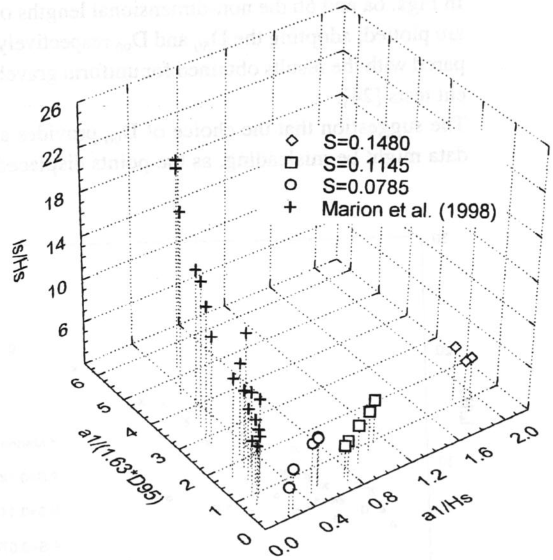

Fig. 7. Non-dimensional scour lengths as function of $a_1/(\Delta D_{95})$ and a_1/H_s.

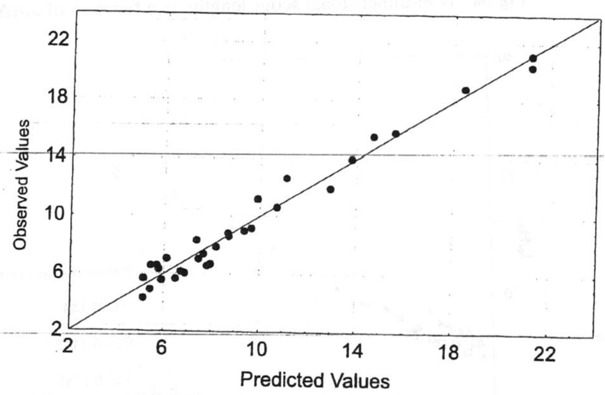

Fig. 8. Comparison between predicted (Eq. 15) and observed values of the dimensionless scour lengths (data from the present study and from Marion et al., 1998).

Affinity and similarity of the scour holes

Following Gaudio et al. [16], the degree of self-affinity among the scour holes is shown in Fig. 9. This methodology allows the comparison of different scour holes by scaling the longitudinal distance measured starting from the sill to the scour length (x_1/l_s) and the depth to the maximum scour depth (y/y_s). Since it was not possible to quickly raise the water level by blocking the downstream end of the flume, the upstream part of the scour profile (made up of the finest particles and almost reaching the top of the sill) was always eroded by the decreasing flow rates when the pump was being closed; therefore that part of the profile ($0< x/l_s < 0.2$) should not be taken into account.

New test profiles show to be somewhat self-affine (with the exception of test H5, which consists of the highest flow rate experimented). However they only partly overlie with those from the low-gradient tests. The approximately affine nature of the scour hole enables the assessment of the scour hole volume using the prediction equations for the maximum scour depth (Eq. 14) and the scour length (Eq. 15). In order to establish whether the geometric similarity of the scour hole is present or not, it is required that both longitudinal and vertical dimensions of the scour hole are scaled to the same factor (e.g. the scour length, Fig. 10). New tests scour holes do not show similarity and the downstream side of the pit clearly correlates with the initial slope values: the steeper the initial condition, the steeper this side. This is consistent with the fact that the jet angle was found to be roughly equal to the inclination of the downstream part of the scour hole ([6], [10], [28]). The large difference in the jet trajectory can therefore explain to some extent the larger scatter shown by the affinity graph in Fig. 9.

5. Application of the scour predictive formula

In order to use Eqs. (14) and (15), estimation of the following parameters is required: the critical specific energy, H_s, which can be derived from Eq. (8) once the design discharge is determined; the relative submerged particle density, Δ, which can be assumed in the range 1.6-1.7; the grain diameter D_{95} of the bed alluvium; the morphological jump, a_1, which is given by Eq. (2). The evaluation of the equilibrium slope, required for the jump a_1, is a criti-

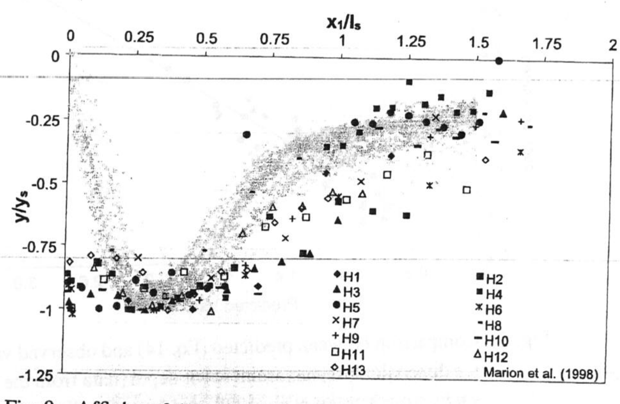

Fig. 9. Affinity of the scour holes.

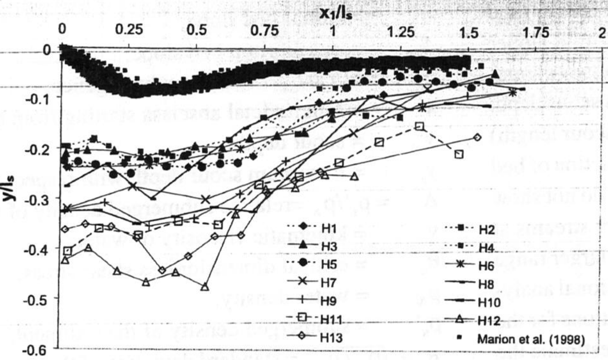

Fig. 10. Similarity of the scour holes.

cal point.

Marion et al. [24], whose experiments were characterised by high relative submergence ratio, $S_r=h_u/D_{84}>10$, found good agreement between the measured equilibrium slope and the one calculated through Eq.(7) with $\theta_c=0.040$ and $D=D_{50}$, assuming that the Manning's coefficient could be determined using the expression:

$$n=\frac{D_{90}^{1/6}}{26} \tag{16}$$

On the contrary, the present tests feature very low submergence ratios S_r, roughly ranging from 0.7 to 1.7, as found in natural mountain torrents. Therefore Manning's coefficient has been evaluated through the Keulegan's equation developed for macro-roughness conditions:

$$n=\frac{D_{90}^{1/6}}{15.1} \tag{17}$$

which gives n=0.038. The critical Shields' mobility parameter, θ_c, was set equal to 0.076, following the indications found in the scientific literature for coarse grain size distributions [19]. Finally, the most appropriate diameter to be used in Eq. (7) was assessed by searching for different diameter the best fit between the experimental points and the theoretical curve assuming the above values for n and θ_c. This calibration process led to the identification of the D_{65} (equal to 0.016 m) as the "characteristic" grain size for the equilibrium slope.

Fig. 11 shows the measured equilibrium slope and the curve expressed by Eq. (7) versus the unit discharge q: it can be observed that the agreement is fairly good except the three runs with the smallest discharges, which have submergence ratios S_r less than 1.

The evidence that for these tests the equilibrium slope is less than the predicted one might be due to the larger flow resistance induced by surface effects (i.e. small hydraulic jumps in correspondence with protruding cobbles) when the submergence drop below the unity, as found by Bathurst ([3], [4]).

Better results in predicting the equilibrium slope might presumably be achieved using other roughness formulations ([18], [4], [14], [2]) which are not discussed in this study. However, in practical application the adopted design discharge is large (return period of 50-100 years), and therefore the submergence ratio, S_r, is expected to be above unity.

Numerical example:

A mountain stream subject to incision is to be protected with bed sills and the distance between the structures has been chosen to be L=20 m: determine the clear water long-term maximum scour depth and the scour length given the following torrent characteristics:

- design discharge with 100-yr return period: $Q_{100}=30\ m^3/s$;
- channel average slope: S=0.11;
- channel width: B=10 m;
- sediment characteristic diameters: $D_{65}=0.40$ m; $D_{90}=0.75$ m; $D_{95}=0.90$ m.

The following parameters are assumed:

- Shields incipient motion parameter: $\theta_c=0.076$;
- relative submerged density of the sediments: $\Delta=1.63$.

Calculation:

- unit width discharge: $q=Q/B=3\ m^2/s$;
- Manning roughness coefficient (Eq. 17): $n=0.063\ s/m^{1/3}$;
- equilibrium slope (Eq. 7, with D_{65}): $S_{eq}=0.057$;
- morphological jump (Eq. 2): $a_1=1.06$ m;
- critical specific energy above the sill (Eq. 8): $H_s=1.44$ m;
- non-dimensional parameters: $a_1/H_s=0.74$; $a_1/(\Delta D_{95})=0.72$;
- non-dimensional scour depth (Eq. 14): $y_s/H_s=1.59$;
- maximum scour depth: $y_s=1.59\times1.44$ m=2.29 m;
- non-dimensional scour length (Eq. 15): $l_s/H_s=6.26$;
- scour length: $l_s=6.26\times1.44$ m= 9.01 m.

6. Conclusions

Bed sill sequences are useful to prevent erosion in incised channels, but the local scour hole downstream of the structures needs an adequate estimation, particularly in the case of high-gradient streams, where the scour depth is enhanced by the effect of larger

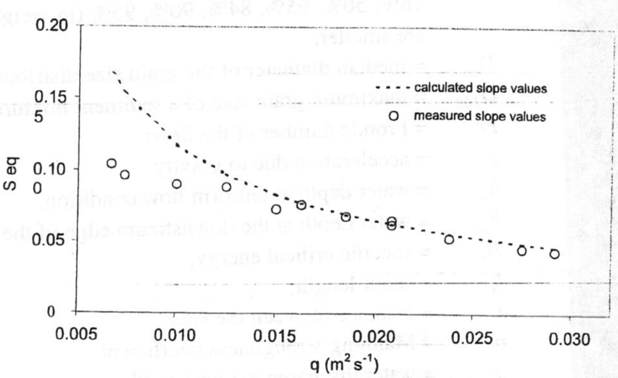

Fig. 11. Measured and calculated equilibrium slope values as a function of the unit discharge.

ratios between the morphological jump (i.e. the drop energy) and the flow energy. Previous results for stream at low-mild slopes have been extended to high-gradient torrents, using a two-parameter non-dimensional formulation for the maximum scour depth and length. The scour dimensions (in particular the scour length) are shown to be affected by the size of the coarsest fraction of bed material. The scour holes developing on steep slopes do not show similarity. They are more roughly self-affine than for streams at low-mild slopes; seemingly this is mainly due to the larger range of jet geometry (i.e. the jet impact angle). The dimensional analysis presented along with the two semi-empirical equations for the scour length and depth represent a designing-tool to estimate the scour dimension in different channel types (gravel-bed and boulder-bed). Nevertheless further investigation is required in order to extend the experimental range testing both other grain size distributions and channel slopes.

Acknowledgements

The funding for this work was provided by the Commission of the European Communities Directorate General for Science, Research and Development as part of the Programme "Training and Mobility of Researchers – Access to Large Scale Facilities" under contract No. ERB-FMGE-CT95-0082, by the European Commission Programme "Eroslope II", contract No. ENV4-CT96-0247, and by the University of Padua – Research Project 2001-2002 "Hydraulic control and restoration of mountain streams according to morphological and environmental criteria".
The authors are thankful to Gianluca Giacometti for directing the work at HR Wallingford Ltd.
Dr. Roger Bettess is kindly acknowledged for valuable suggestions during the experimental phase and for the review of this manuscript. Fabio De Napoli is also acknowledged for his help in flume setting.

Notations

a_1 = $(S_0-S_{eq})L$, morphological jump;
a_2 = sill step;
B = channel width;
D = grain size;
D_{16}, D_{50}, D_{65}, D_{84}, D_{90}, D_{95} = grain size such that respectively 16%, 50%, 65%, 84%, 90%, 95% (in weight) of grains are smaller;
D_m = median diameter of the grain size distribution;
D_{100} = maximum grain size of a sediment mixture;
Fr = Froude number of the flow;
g = acceleration due to gravity;
h_u = water depth at uniform flow condition;
h_{sill} = water depth at the downstream edge of the sill;
H_s = specific critical energy;
l_s = scour length;
L = distance between the sills;
n = Manning's roughness coefficient
q = water discharge per unit width;
Q = water discharge;
S = initial bed slope;
S_{eq} = equilibrium bed slope;
S_r = h_u/D_{84} = relative submergence;
x = longitudinal abscissa starting from the sill;
y = scour depth;
y_s = maximum scour depth with respect to the initial bed;
Δ = ρ_s'/ρ_w = relative submerged density of the sediment;
ν = kinematic viscosity of water;
θ_c = critical dimensionless shear stress;
ρ_w = water density;
ρ_s' = submerged density of the sediment;
$\sigma_g = \sqrt{D_{84}/D_{16}}$ = standard deviation of the grain size distribution;

References

[1] ADERIBIGBE, O., RAJARATNAM, N. (1998), Effect of sediment gradation on erosion by plane turbulent wall jets, *Journal of Hydraulic Engineering*, ASCE, 124(10), 1034-1042.

[2] BAIAMONTE, G., FERRO, V. (1997), The influence of roughness geometry and Shields' parameter on flow resistance in gravel-bed channels, *Earth Surface Processes and Landforms*, 22(8), 759-772.

[3] BATHURST, J. C., LI, R., SIMONS, D. (1981), Resistance equation for large-scale roughness, *Journal of Hydraulic Engineering*, ASCE, 107(HY12), 1593-1613.

[4] BATHURST, J. C. (1985), Flow resistance estimation in mountain rivers, *Journal of Hydraulic Engineering*, ASCE, 111, 625-643.

[5] BECCHI, I. (1971), Comportamento idraulico della sistemazione a salti di fondo di alvei naturali (in Italian), *L'Energia Elettrica*, XLVIII (2), 73-78.

[6] BORMANN, N., JULIEN, P. Y. (1991), Scour downstream of grade-control structures, *Journal of Hydraulic Engineering*, 117(5), 579-594.

[7] BREUSERS, H. N. C, RAUDKIVI, A. J. (1991), *Scouring. Hydraulic Structures Design Manual*, IAHR, A. A. Balkema, 143 pp.

[8] COMITI, F. (1999), Studio sperimentale dei processi di scavo a valle di soglie di fondo (in Italian), Degree Thesis in Forest and Environmental Sciences, University of Padua, 98 pp.

[9] D'AGOSTINO, V. (1994), Indagine sullo scavo a valle di opere trasversali tramite modello a fondo mobile (in Italian), *L'Energia Elettrica*, 71(2), 37-51.

[10] D'AGOSTINO, V. (1996), La progettazione delle controbriglie (in Italian), Proceedings *XXV Convegno di Idraulica e Costruzioni Idrauliche*, Turin, 16-18th September 1996, 107-118.

[11] D'AGOSTINO, V., CERATO, M., DA RE, F., LENZI, M. A. (1997), La sistemazione idraulica dei torrenti con briglie in massi (in Italian), *Dendronatura*, 2, 39-51.

[12] DAMGAARD, J. S., WHITEHOUSE R. J. S., SOULSBY, R. L. (1996), A sloping sediment duct for the study of sediment transport, Proceedings *25th International Conference on Coastal Engineering*, Orlando, Florida, 3913-3920.

[13] DODDIAH, D., ALBERTSON, M. L., THOMAS, R. (1953), Scour from jets, *5th Congress of the International Association*

for Hydraulic Research, Minneapolis, USA, 161-169.

[14] Ferro, V., Giordano, G. (1991), Experimental study of flow resistance in gravel-bed rivers, *Journal of Hydraulic Engineering*, ASCE, 117(10), 1239-1246.

[15] Gaudio, R., Marion, A. (in revision), Time evolution of scouring downstream of bed sills.

[16] Gaudio, R., Marion, A., Bovolin, V. (2000), Morphological effects of bed sills in degrading rivers, *Journal of Hydraulic Research*, IAHR, 38(2), 89-96.

[17] Habib, E., Mossa, M., Petrillo, A. (1994), Scour downstream of hydraulic jump, *Modelling, Testing & Monitoring for Hydro PowerPlants Conference*, Budapest, Hungary, 561-602.

[18] Jarrett, R. D. (1984), Hydraulics of high-gradient streams, *Journal of Hydraulic Engineering*, ASCE, 110(11), 1519-1539.

[19] Krey, H. (1925), Grenzen der Übertragbarkeit der Versuchsergebnisse, *Z. angew. Math. Mech.*, 5(6).

[20] Lenzi, M. A., D'Agostino, V. (1998), Distanze di trasporto di ciottoli marcati in un torrente alpino ad elevata pendenza (in Italian), *L'Acqua*, 4, 55-68.

[21] Lenzi, M. A., D'Agostino, V. (2000), Step pool evolution in an alpine torrent, in U. Maione, B. Majone Lehto and R. Monti R. (Eds.), *New trends in water and environmental engineering for safety and life: Eco-compatible solutions for aquatic environment*. A. A. Balkema Publ., CD-ROM, 12.

[22] Lenzi, M. A., D'Agostino, V., Sonda D. (2000a), *Ricostruzione morfologica e recupero ambientale dei torrenti* (in Italian), BIOS, Cosenza, 208 pp.

[23] Lenzi, M. A., Marion, A., Comiti, F., Gaudio, R. (2000b), Riduzione dello scavo a valle di soglie di fondo per effetto dell'interferenza tra le opere (in Italian), Proceedings *XXVII Convegno di Idraulica e Costruzioni Idrauliche*, IDRA2000, Genoa, Italy, Vol. III, 271-278.

[24] Marion, A., Gaudio, R., Bovolin, V. (1998), Scour downstream of bed sills, HR Wallingford Research Report TR70, Wallingford, United Kingdom, 50 pp.

[25] Martins, R. B. F. (1973), Contribution to the knowledge on the scour action of free jets on rocky river beds, *Transactions of the 11st International Congress on Large Dams*, Vol. II, Question 41, Reply 44, Madrid, 799-814.

[26] Mason, P.J. (1989) Effects of air entrainment on plunge pool scour, *Journal of Hydraulic Engineering*, ASCE, 115(3), 385-399.

[27] Mason, P.J., Arumugam, K. (1985), Free jet scour below dams and flip buckets, *Journal of Hydraulic Engineering*, 111(2), ASCE, 220-235.

[28] Mossa, M. (1998), Experimental study on the scour downstream of grade-control structures, Proceedings of *XXVI Convegno di Idraulica e Costruzioni Idrauliche*, Catania, Italy, 9-12 September, 581-594.

[29] Rajaratnam, N. (1981), Erosion by plane turbulent jets, *Journal of Hydraulic Research*, IAHR, 19 (4), 339-358.

[30] Stein, O. R., Julien, P. Y. (1994), Sediment concentration below free overfall *Journal of Hydraulic Engineering*, ASCE, 120(9), 1044-1053.

[31] Tarapore, Z.,S. (1956), Scour below a submerged sluice gate, Master of Science Thesis, University of Minnesota, Minneapolis.

[32] Volkart, P., Tshopp, J., Bisaz, E. (1973), The effect of sills on river bed, Proceedings of *International Symposium on River Mechanics*, IAHR, 1, Bangkok, 167-178.

[33] Whitthaker, J. G. (1987), Sediment transport in step-pool streams, in C. R. Thorne, J. C. Bathurst, R. D. Hey (Eds.), *Sediment Transport in Gravel Bed Rivers*, Wiley & Sons, 545-570.

for Hydraulic Research, Minneapolis, USA, 164-169.
[14] [illegible] V., GIORDANO, G. (1997). Experimental study of [illegible] in gravel-bed rivers. Journal of Hydraulic Engineering, ASCE, 123(10), 1278-1286.
[15] [illegible] R., MARION, A. (in revision). Time evolution of scouring downstream of bed sills.
[16] GAUDIO, R., MARION, A., BOVOLIN, V. (2000). Morphological effects of bed sills in degrading rivers. Journal of Hydraulic Research, IAHR, 38(2), 89-96.
[17] [illegible], E., MOSSA, M., PETRILLO, A. (1994). Scour downstream of hydraulic jump. Modelling, Testing & Monitoring for Hydro Powerplants Conference, Budapest, Hungary, 561-600.
[18] JARRETT, R. D. (1984). Hydraulics of high-gradient streams. Journal of Hydraulic Engineering, ASCE, 110(11), 1519-1539.
[19] [illegible] H. (1935). [illegible] der [illegible] der Vor-[illegible]. Z. angew. Math. Mech., 56(?).
[20] LENZI, M. A., D'AGOSTINO, V. (1998). Distanze di [illegible] dissipazione nei torrenti alpini ad elevata pendenza (in Italian), L'Acqua, 4, 55-62.
[21] LENZI, M. A., D'AGOSTINO, V. (2000). Step pool evolution in the alpine torrent. In U. Maione, B. Majone Lehto and R. Monti (Eds.), New trends in water and environmental engineering for safety and life: Eco-compatible solutions for aquatic environment. A. A. Balkema Publ., CD-ROM, 12 p.
[22] LENZI, M. A., D'AGOSTINO, V., SONDA, D. (2000). Ricostruzione morfologica e recupero ambientale dei torrenti (in Italian). BIOS, Cosenza, 208 p.
[23] LENZI, M. A., MARION, A., COMITI, F., GAUDIO, R. (2000). Riduzione dello scavo a valle di soglie di fondo per effetto di interferenze tra le opere (in Italian). Proceedings XXVII Convegno di Idraulica e Costruzioni Idrauliche, IDRA 2000, Genova, Italy, Vol. III, 271-278.
[24] MARION, A., GAUDIO, R., BOVOLIN, V. (1998). Scour downstream of bed sills. HR Wallingford Research Report TR 70, Wallingford, United Kingdom, [illegible] p.
[25] MARTINS, R. F. B. (1973). Contribution to the knowledge on the scour action of free jets on rocky river beds. Proceedings of the 11th International Congress on Large Dams, Madrid, Question 41, Reply 44, Madrid, 799-814.
[26] MASON, P. J. (1989). Effects of air entrainment on plunge pool scour. Journal of Hydraulic Engineering, ASCE, 115(3), 385-399.
[27] MASON, P. J., ARUMUGAM, K. (1985). Free jet scour below dams and flip buckets. Journal of Hydraulic Engineering, 111(2), ASCE, 220-235.
[28] MOSSA, M. (1998). Experimental study on the scour downstream of grade-control structures. Proceedings of the 26th Convegno di Idraulica e Costruzioni Idrauliche, Catania, Italy, 9-12 September, [illegible]-594.
[29] RAJARATNAM, N. (1981). Erosion by plane turbulent jets. Journal of Hydraulic Research, IAHR, 19(4), 339-358.
[30] STEIN, O. R., JULIEN, P. Y. (1994). Sediment concentration below free overfall. Journal of Hydraulic Engineering, ASCE, 120(9), 1044-1059.
[31] TARAPORE, Z. S. (1956). Scour below a submerged sluice gate. Master of Science Thesis, University of Minnesota, Minneapolis.
[32] VOLKART, P., [illegible], J., BISAZ, E. (1973). The effect of sills on river bed. Proceedings of International Symposium on River Mechanics, IAHR, Bangkok, 16?-178.
[33] WHITTAKER, J. G. (1987). Sediment transport in step-pool streams. In C. R. Thorne, J. C. Bathurst, R. D. Hey (Eds.), Sediment Transport in Gravel-Bed Rivers, Wiley & Sons, [illegible].