Chapter 4

Benefit Functions

This chapter presents the benefit functions associated with different water uses in a river basin. Benefit functions fit into two categories: downward sloping prices (the first part of this chapter) and constant prices (in the second part of the chapter). The following uses are considered: 1) hydropower with variable-energy head; 2) hydropower with fixed-energy head; 3) water for irrigation demand areas; 4) water for municipal and industrial demand areas; 5) water for instream water recreation activities; and 6) water for reservoir recreation activities. The basic operational characteristics and modeling assumptions for the system components are in the previous chapter.

Downward Sloping Price Benefit Functions

Variable-Head Powerplant

Hydropower generation is a function of the powerplant discharge, the hydraulic head under which the turbines operate, and the efficiency of the turbine-generator group. In turn, the effective head on the powerplant changes with elevation changes of the water surface in the reservoir forebay and tailrace. The energy rate function (*erf*), as defined in (3.8), can be simplified by assuming effective head to be a linear function of reservoir storage. In this formulation, storage serves as a surrogate for reservoir water surface elevation, and any effect of backwater on the powerplant tailrace level is ignored. The *erf* is expressed in megawatt-hours (MWh) per million cubic meters (Mcm) of flow through the turbines:

$$erf\,[\text{MWh/Mcm}] = \eta\,(a_1 + b_1 S) \qquad (4.1)$$

where η encompasses all the mechanical and electrical efficiencies of the generation unit, and a_1 and b_1 are parameters of the linear model.

In this formulation, energy production during a given time interval depends on the average reservoir storage during that same time interval. Average storage $\bar{S}_i$ during time interval i can be estimated as the storage at the beginning of the period S_i^o, plus half of the inputs and outputs from

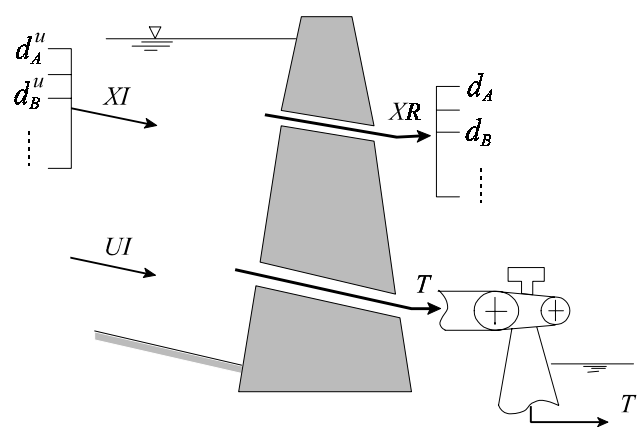

Figure 4.1 Inflows and outflows for a variable-head powerplant.

the reservoir during the period. Using the notation established in Chapter 3 and figure 4.1, the average storage during the time interval is written as:

$$\bar{S}_i = S_i^o + \frac{(UI_i + XI_i)}{2} - \frac{(T_i + XR_i + UR_i)}{2} \tag{4.2}$$

where the powerplant release T has been segregated from the general group of controlled reservoir releases XR. To maintain unit consistency throughout the model formulation, flow variables denote water volume.

The price for energy sold by powerplants is usually stipulated by the power market to which the utility sells its energy. It is assumed that an additional powerplant does not significantly change the power pool, and that it has no effect on the energy price structure. This is generally true except for small, isolated electrical systems where an additional power utility can significantly affect energy supply and thus price.

For most electrical systems the price of energy depends on the time of day that the energy is delivered to the electrical grid. The highest prices are paid for energy delivered during the high demand on-peak hours. Additional amounts of energy, if available, are sold at lower unit prices during off-peak demand periods and during the night. This is the core of the classical hydro-scheduling problem. The three classes of energy are represented in figure 4.2 by a step function, showing the marginal price paid for energy (where a mill is one-tenth of a cent) as a function of the plant utilization factor.

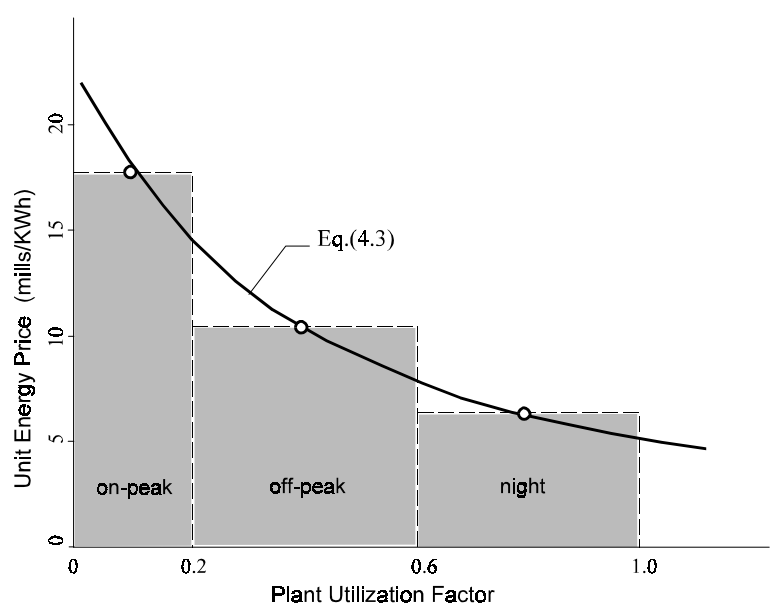

Figure 4.2 Demand function for hydro-energy.

The utilization factor indicates how the available water is used by the hydropower plant, within the constraint of powerplant capacity and characteristics. In this model, the utilization factor is the percent of plant operation time during the time interval of analysis (monthly, weekly, daily). In turn, the operation time can be replaced by the flow percent passing through the turbines, assuming that the generation units operate at a nominal flow rate. Note that the structure of prices may remain constant all year or change from season to season; for instance, summer prices for energy can be higher than winter prices.

The diminishing returns for hydropower in figure 4.2 can be analytically represented by the exponential function in (4.3), which provides adequate flexibility to represent downward sloping demand curves:

$$b\,[\$/\mathrm{MWh}] = \frac{p_S}{P}\, a_2 \exp(-T/b_2) \tag{4.3}$$

where T denotes the powerplant release (volume released during the time period), a_2 and b_2 are parameters of the exponential function, and p_S accounts for possible seasonal changes in energy prices with respect to the average annual price P.

The marginal price function for hydropower assumes that power releases are drawn first during on-peak hours, then during off-peak hours, and whatever volume of water is left will be used to generate night (dump) energy. Because the total number of on-peak, off-peak, and night hours of energy demanded by the electrical grid changes depending on the time interval of simulation (daily, weekly, or monthly) , the exponential demand curve in figure 4.2 must be built for the case under consideration and will yield different values for parameters a_2 and b_2.

Following Laufer and Morel-Seytoux (1979), integration of the product of the energy rate function (4.1) times the marginal price function (4.3) with respect to the powerplant release T_i, yields the hydropower return function that computes the benefit during a given time interval i:

$$B_i^{\mathrm{HPW\,VH}}(\eta, T_i, \bar{S}_i) = \int_0^{T_i} erf(\bar{S})\; b(z)\; dz \tag{4.4}$$

where z is a dummy variable of integration and all other terms have been defined earlier. Substituting the final expressions of $erf(\bar{S})$ and $b(T)$ into the integral above, the following expression is obtained:

$$B_i^{\mathrm{HPW\,VH}} = \int_0^{T_i} \eta \frac{p_i}{P} a_2 \exp(-z/b_2)\; [a_1 + \frac{b_1}{2}(2S_i^o + UI_i + XI_i - z - XR_i - UR_i)]\; dz \tag{4.5}$$

By further expanding (4.5), (4.6) is obtained:

$$B_i^{\text{HPW VH}} = \eta \frac{p_i}{P} \frac{a_2 b_1}{2} (2a_1/b_1 + 2S_i^o + UI_i + XI_i - XR_i - UR_i) \int_0^{T_i} \exp(-z/b_2)\, dz$$
$$- \eta \frac{p_i}{P} \frac{a_2 b_1}{2} \int_0^{T_i} z \exp(-z/b_2)\, dz \tag{4.6}$$

Carrying out the integration and regrouping some terms, the economic benefit from the operation of a single powerplant during time period *i* is obtained:

$$B_i^{\text{HPW VH}} = \eta \frac{p_i}{P} \frac{a_2 b_1 b_2}{2} \{[2a_1/b_1 + 2S_i^o + UI_i + XI_i - XR_i - UR_i]\,[1 - \exp(-T_i/b_2)]$$
$$+ [(T_i + b_2)\exp(-T_i/b_2) - b_2]\} \tag{4.7}$$

Note that the storage S_i^o at the beginning of period *i* depends on the history of inflows and outflows from the reservoir up to that period, expressed as:

$$S_i^o = S^o + \sum_{k=1}^{i-1} (UI_k + XI_k) - \sum_{k=1}^{i-1} (T_k + XR_k + UR_k) \tag{4.8}$$

where again the turbine release T has been separated from the term *XR* that represents other controlled reservoir releases. Substituting (4.8) into (4.7), the total benefit accruing from hydropower generation (with variable-head) in its expanded form is obtained:

$$B_i^{\text{HPW VH}}[\$] = \eta \frac{p_i}{P} \frac{a_2 b_1 b_2}{2} \{[2a_1/b_1 + 2S^o + 2\sum_{k=1}^{i-1} (UI_k + XI_k) - 2\sum_{k=1}^{i-1} (T_k + XR_k + UR_k)$$
$$+ (UI_i + XI_i - XR_i - UR_i)]\,[1 - \exp(-T_i/b_2)]$$
$$+ [(T_i + b_2)\exp(-T_i/b_2) - b_2]\} \tag{4.9a}$$

where all terms have been defined earlier. The lengthy equation (4.9a) is a simple expression that computes the hydropower benefit as a function of the powerplant release, the history of inflows to and outflows from the reservoir that regulates flows for the powerplant, and a series of parameters that represent physical and economic data associated with the hydropower utility. This expanded form of the benefit function promotes the partial derivatives in Appendix A. Equation (4.9a) can be written in a more compact form by expressing it in terms of $\bar{S}_i$, the average reservoir storage during the interval *i*:

$$B_i^{\text{HPW VH}}[\$] = \eta \frac{p_i}{P} a_2 b_2 \{[a_1 + b_1 \bar{S}_i - \frac{b_1 b_2}{2}][1 - \exp(-T_i/b_2)] + \frac{b_1}{2} T_i\} \tag{4.9b}$$

The derivation of (4.9a) is based on the assumption that reservoir evaporation is a known quantity, embedded within the term *UR*. The implications of this simplification are discussed in detail in Chapter 5. While it is possible to solve the water allocation problem using the hydropower benefit function derived above, it is also possible to formulate the hydropower function more rigorously, where evaporation losses are included explicitly in the simulation of the reservoir dynamics (see Chapter 5). The result of this more elaborate formulation modifies (4.9b) by introducing a new parameter β as shown in (4.9c):

$$B_i^{\text{HPW VH}}[\$] = \eta \frac{p_i}{P} a_2 b_2 \{[a_1 + b_1 \bar{S}_i - \frac{b_1 b_2}{2} \beta_i][1 - \exp(-T_i/b_2)] + \frac{b_1}{2} \beta_i T_i\} \tag{4.9c}$$

Equations (4.9b) and (4.9c) express two alternative formulations to calculate hydropower benefits under variable hydraulic head. The equation for the term β is introduced in Chapter 5, equation (5.31)

Fixed-Head Powerplant

The hydropower benefit function in (4.9) can be simplified for a run-of-river powerplant for which the energy rate function (4.1) reduces to a single value because of the fixed energy head acting on the hydraulic turbines, that is:

$$erf\,[\text{MWh/Mcm}] = \eta\, a_1 \tag{4.10}$$

The marginal price of energy is again indicated as in (4.3), where *T* denotes the total volume of water passing through the turbines (figure 4.3).

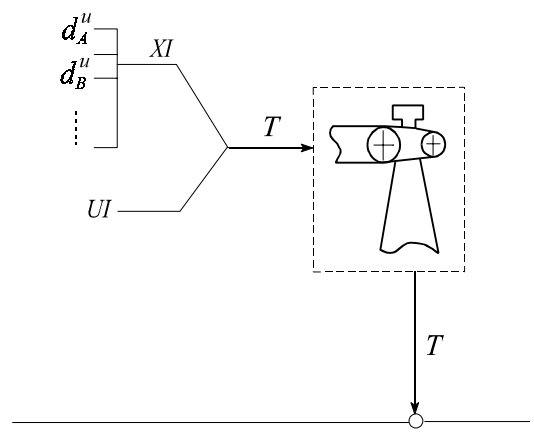

Figure 4.3 In/outflows for a fixed-head powerplant.

Again, the integral of the product of the energy rate times the demand function with respect to the powerplant release yields the hydropower benefit function. Using the final forms of *erf* and *b* in (4.10) and (4.3), the following expression is obtained:

$$B_i^{\text{HPW FH}} = \int_0^{T_i} \eta a_1 \frac{p_i}{P} a_2 \exp(-z/b_2)\, dz \tag{4.11}$$

where *z* is a dummy variable of integration. Carrying out the integration, the economic benefit from the operation of a fixed-head powerplant during time interval *i* is:

$$B_i^{\text{HPW FH}}[\$] = \eta \frac{p_i}{P} a_1 a_2 b_2 [1 - \exp(-T_i/b_2)] \tag{4.12}$$

In contrast to a variable-head powerplant where a single controlled reservoir release is allowed to enter the powerplant, the flow T into a fixed-head powerplant can be a function of several controlled and uncontrolled flow variables. The most general case is depicted in figure 4.3 and expressed mathematically as:

$$T_i = f[XI_i, UI_i] = f[(d_{A_i}^u, d_{B_i}^u, \ldots), UI_i] \tag{4.13}$$

where XI is substituted by $d_{A_i}^u, d_{B_i}^u, \ldots$ to indicate possible multiple controlled flows entering the powerplant, and UI is all uncontrolled flows reaching the powerplant. Substituting (4.13) into (4.12), the expanded form of the fixed-head hydropower benefit function is:

$$B_i^{\text{HPW FH}}[\$] = \eta \frac{p_i}{P} a_1 a_2 b_2 \{1 - \exp[(-c_A^u d_{A_i}^u - c_B^u d_{B_i}^u - \ldots - UI_i)/b_2 \tag{4.14}$$

where $c_A^u, c_B^u, \ldots$ are the coefficients of the upstream decision variables $d_A^u, d_B^u, \ldots,$ respectively.

Irrigation Demand Area

As discussed in Chapter 2, the benefit from an irrigation demand area (IRR) is determined by the net increase in income from the cultivated irrigated land. Typical of agricultural areas, the IRR demand curve includes large amounts of low value water. In this model, the marginal value of a given unit of flow used for irrigation, expressed in dollars per million of cubic meters ($/Mcm), is assumed to follow an exponential decreasing function (figure 4.4):

$$b^{\text{IRR}}[\$/\text{Mcm}] = a_3 \exp(-A/b_3) \tag{4.15}$$

where A represents the total volume of water entering the IRR area as indicated in figure 4.5, and a_3 and b_3 are the parameters of the exponential function. In general, the decaying model in (4.15) provides good fittings of the marginal-benefit curves derived in practice, except for the lower portion of the curves. While the exponential function becomes asymptotic to the axis, actual demand curves will intercept the horizontal axis (figure 2.2).

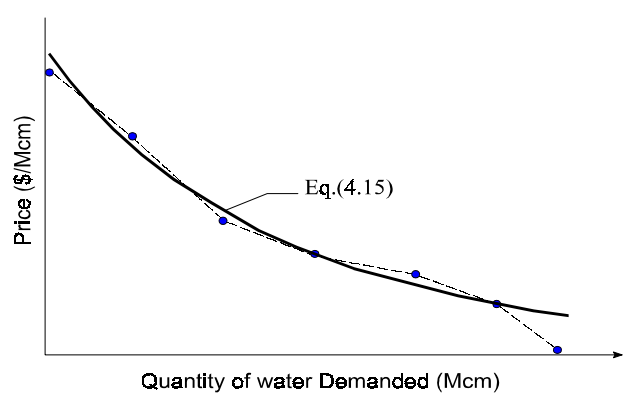

Figure 4.4 Annual irrigation demand.

Part of the flow diverted into the agricultural zone is consumptively used, but a fraction of A returns to the stream via subsurface flow or drainage systems. Once the return flow reaches a stream, it is available for use downstream. This fraction of the incoming flow, indicated by r in figure 4.5 is the return flow coefficient of the irrigation zone. The return flow coefficient is assumed constant for all seasons and can be highly variable from one irrigation area to another depending on irrigation practices and soil characteristics.

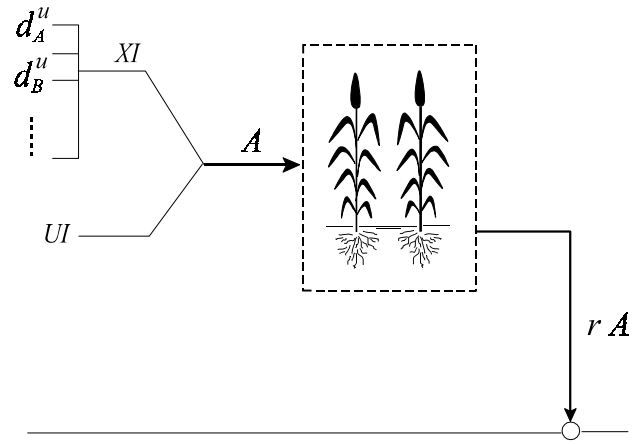

Figure 4.5 In/outflows to an irrigation demand area.

The integration of (4.15), which is the area under the demand curve, yields the total benefit from the IRR demand area (i.e., the total willingness to pay for irrigation water in the demand area):

$$B_i^{\mathrm{IRR}} = \int_0^{A_i} a_{3_i} \exp(-z/b_{3_i})\, dz \tag{4.16}$$

where z is a dummy variable of integration. Solving the integral in (4.16), the IRR benefit function for a single time interval i is obtained, where the parameters a_3 and b_3 can vary by time interval (i.e., seasonally):

$$B_i^{\mathrm{IRR}}[\$] = a_{3_i} b_{3_i} [1 - \exp(-A_i/b_{3_i})] \tag{4.17}$$

As depicted in figure 4.5, the total flow A entering the IRR area can be several controlled XI and uncontrolled UI inflows. Mathematically:

$$A_i = f[XI_i, UI_i] = f[(d_{A_i}^u, d_{B_i}^u, \ldots), UI_i)] \tag{4.18}$$

where the term XI is expressed by $d_{A_i}^u, d_{B_i}^u, \ldots$ to indicate possible multiple controlled flows entering the agriculture irrigation zone. Substituting (4.18) into (4.17), the expanded form of the benefit function is:

$$B_i^{\mathrm{IRR}}[\$] = a_{3_i} b_{3_i} \{1 - \exp[(-c_A^u d_{A_i}^u - c_B^u d_{B_i}^u - \ldots - UI_i)/b_{3_i}]\} \tag{4.19}$$

where $c_A^u, c_B^u, ..$ are the coefficients of the upstream decision variables $d_A^u, d_B^u, ..$ respectively.

Municipal and Industrial Demand Area

Empirical studies indicate that the quantity of water demanded by the municipal and industrial (M&I) sector is sensitive to price (Martin and Thomas 1986, Howe, 1982, Howe and Linaweaver 1967, Foster and Beattie 1979, Lyman, 1992) but not as sensitive as irrigation demand. Compared with agricultural, the M&I sector demands limited quantities of water but is willing to pay relatively higher prices; M&I demand tends to be relatively inelastic. The analytical form of the M&I demand curve available in the model also follows an exponential decaying model:

$$b^{M\&I}\ [\$/\text{MCM}] = a_4 \exp(-D/b_4) \tag{4.20}$$

where D represents the total volume entering the M&I area and a_4 and b_4 are parameters of the exponential function. In general, only a small portion of the flow D diverted into the demand zone is consumptively used. The unused portion, indicated by the return flow coefficient r in figure 4.6, becomes wastewater flow that, after treatment, is available to other users.

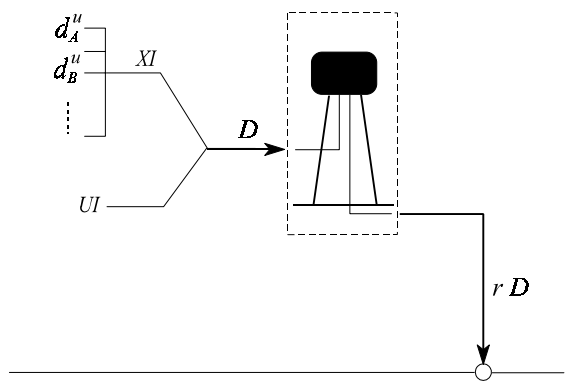

Figure 4.6 In/outflows to a municipal and industrial zone.

The integral of the demand curve (4.20) yields the total benefits stemming from the municipal and industrial water use:

$$B_i^{\text{M\&I}}[\$] = \int_0^{D_i} a_{4_i} \exp(-z/b_{4_i})\ dz \tag{4.21}$$

where z is a dummy variable of integration. Solving the integral, the M&I benefit function for a single time interval i is:

$$B_i^{\text{M\&I}}[\$] = a_{4_i} b_{4_i} [1 - \exp(-D_i/b_{4_i})] \tag{4.22}$$

Note that in (4.22) the parameters a_4 and b_4 can vary seasonally. As in the irrigation case, the amount of water D entering the M&I area can be several controlled and uncontrolled flows (figure 4.6). Mathematically, the M&I diversion is expressed as:

$$D_i = f[XI_i, UI_i] = f[(d_{A_i}^u, d_{B_i}^u, \ldots), UI_i] \tag{4.23}$$

where the controlled inflow *XI* is replaced by $d_{A_i}^u, d_{B_i}^u, ..$ to indicate possible linear combinations of several control variables contributing to flows to the M&I zone. Substituting (4.23) into (4.22), the expanded form of the M&I benefit function is:

$$B_i^{M\&I}[\$] = a_{4_i} b_{4_i} \{1 - \exp[(-c_A^u d_{A_i}^u - c_B^u d_{B_i}^u - ... - UI_i)/b_{4_i}]\} \tag{4.24}$$

where $c_A^u, c_B^u, ..$ are the coefficients of the upstream decision variables $d_A^u, d_B^u, ..$ respectively.

Instream Recreation Area

Because instream recreational (IRA) opportunities are not generally sold in a market, estimating the benefits from water use for recreation requires unique economic valuation approaches such as the travel cost method and the contingent valuation method (Freeman 1993). Brown et al. (1991) list many of the studies performed over the last 20 years that used these methods to focus on the recreational value of instream flow.

For example, Duffield et al. (1992) used a dichotomous choice, contingent valuation survey to interview recreationists along the "blue ribbon" trout fishery rivers in Montana. Interviews were conducted through the summer, and flow conditions were recorded for each interview day. The information allowed the investigators to develop a relationship between willingness to pay (WTP) for instream recreational participation and alternative flow conditions. According to Duffield et al. (1992), the marginal value of a given flow unit is the effect of instream flows on recreational experience quality, quantity of use, and lagged effects on recreation conditions. It is possible that only one or two of the three terms may be empirically significant for a given resource. Similar to the results provided by other studies, Duffield et al. (1992) identified a nonlinear relation of total recreation benefits to flow rate, which shows a concave shape that increases with flow to a point but then decreases for further increases in flow (see figure 4.7). For example, at very low flows rapids are not a challenge for whitewater boaters, at moderate flows floating quality

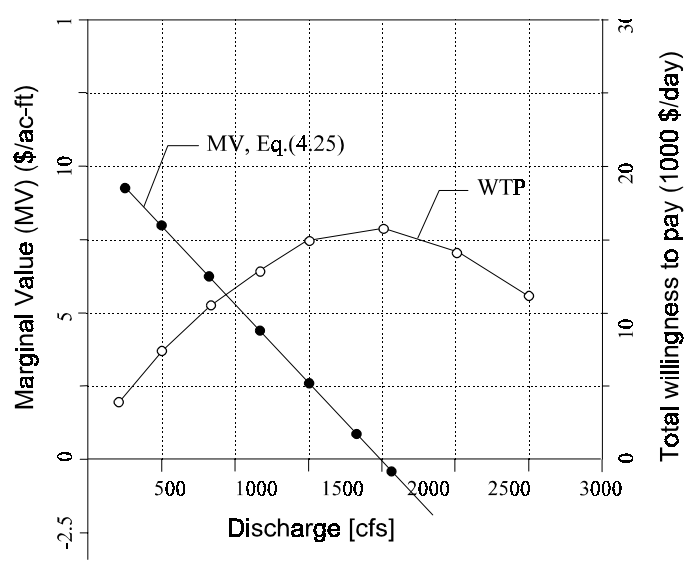

Figure 4.7 Demand function for instream water recreation (1988 dollars) (Duffield et al. 1992).

improves, but at very high flows the rapids become washed out or too dangerous. This concave relation applies to all instream recreation activities, but the flow levels at which recreation quality is maximized differ by activity (Brown et al. 1991).

The analytical form of the demand curve for IRA adopted in this model (4.25) was from the aforementioned study and is shown in figure 4.7. The marginal recreational value of instream flow, expressed in dollars per million cubic meter ($/Mcm), is assumed to follow a linear decreasing model:

$$b^{\mathrm{IRA}}\,[\$/Mcm] = a_5 - b_5 \cdot \qquad (4.25)$$

where R represents the total flow (volume during the time period) passing through the recreation area and a_5 and b_5 are parameters of the linear function. Because no consumptive water use takes place in the IRA reach (except for evaporation and infiltration losses from the stream, which are typically small and ignored), the same amount of flow R is available downstream from the recreational user (see figure 4.8).

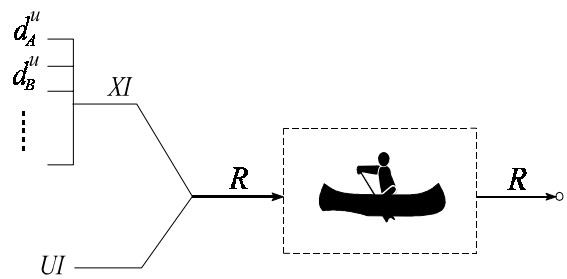

Figure 4.8 In/outflows for an instream recreation area.

The integral of the demand curve with respect to the instream flow yields the total benefits derived from the IRA river reach:

$$B_i^{\mathrm{IRA}}\,[\$] = \int_0^{R_i} (a_{5_i} - b_{5_i} z)\, dz \qquad (4.26)$$

where z is a dummy variable of integration. Solving the integral in (4.26), the IRA objective function for a single time interval i is:

$$B_i^{\mathrm{IRA}}\,[\$] = a_{5_i} R_i - \frac{b_{5_i}}{2} R_i^2 \qquad (4.27)$$

that also indicates that the parameters a_5 and b_5 can vary seasonally.

The total flow R passing through the IWR river reach can be several controlled and uncontrolled flows as depicted in figure 4.8 and expressed mathematically in (4.28), where the controlled inflow term XI is expressed by $d_{A_i}^u, d_{B_i}^u, ..$ to indicate the possibility of multiple controlled variables contributing flows to the recreation area:

$$R_i = f[XI_i, UI_i] = f[(d_{A_i}^u, d_{B_i}^u, ...), UI_i] \qquad (4.28)$$

Substituting (4.28) into (4.29), the expanded form of the benefit function is:

$$B_i^{\mathrm{IRA}}[\$] = a_{5_i}(c_A^u d_{A_i}^u + c_B^u d_{B_i}^u + \ldots + UI_i) - \frac{b_{5_i}}{2}(c_A^u d_{A_i}^u + c_B^u d_{B_i}^u + \ldots + UI_i)^2 \tag{4.29}$$

where $c_A^u, c_B^u, ..$ are the coefficients of the upstream decision variables $d_A^u, d_B^u, ..$ respectively.

Reservoir Recreation Activities

A recreation benefits curve relates the marginal benefit, expressed in dollars per visitor-day, with the number of participating people expressed as visitors-day (James and Lee, 1971, p.416). For a given reservoir capacity (which defines the amount of area suitable for recreation), total annual potential benefits can be calculated by integrating the marginal curve.

Reservoir recreation was included in this model assuming that the reservoir already exists (i.e., its size was determined by other water uses), and that the aggregate value of the recreation activity is related to the water level in the lake. In this manner, the reservoirs recreation benefit (B^{RRA}), expressed as the total economic benefit across all users, is a direct function of the water stored in the reservoir (in turn a function of the water level), and is assumed to follow a hyperbolic tangent function, (equation 4.30a):

$$B_i^{RRA}[\$] = a_{6_i}[\tanh(b_{6_i}\bar{S}_i - c_{6_i}) + 1.0] \tag{4.30}$$

where $\bar{S}_i$ represents the average storage during time period *i,* and a_6, b_6 and c_6 are coefficients of the model. The average storage is calculated knowing the inflows and outflows from the reservoir as indicated by equations (3.2) and (3.3). Equation (4.30a) allows us to set recreation benefits in commensurate terms with other water uses.

Recreation activities may be limited between a maximum (S_{rec_max}) and a minimum (S_{rec_min}) reservoir level (figure 4.9), which may not coincide with reservoir level restrictions for other water uses (e.g., hydropower).

Similar to the hydropower case (compare equations (4.9b) and (4.9c)), the two approaches used in AQUARIUS for modeling reservoir evaporation (implicit and explicit), introduces a change in the derivation of the benefit function for reservoir recreation.

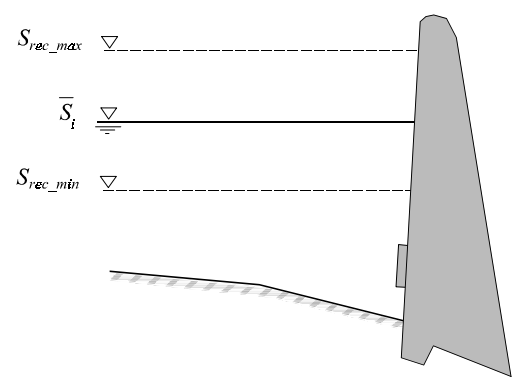

Figure 4.9 Maximum and minimum storage levels in a reservoir with recreation activities.

Although in this particular case the expression (4.30) remains valid for either approach, the term $\bar{S}_i$ is calculated using equation (4.2) for the former approach and (5.31) for the latter. Details about the two approaches for modeling reservoir evaporation are presented in Chapter 5.

The hyperbolic benefit function yields an S-shaped curve that most closely fits available data on the effect of lake level on recreation use (Allen et al. 1998). Figure 4.10 shows a fitting example in which the maximum and minimum reservoir recreation levels are 9,000 and 6,500 Mcm, and the estimated benefits are as indicated by the markers. The fitted benefit function is asymptotic to the lowest and highest reservoir recreation levels. No recreation benefits are realized below the lowest level. Benefits increase rapidly as the reservoir storage increases from its lowest level, and reach a maximum when the reservoir is near its maximum capacity.

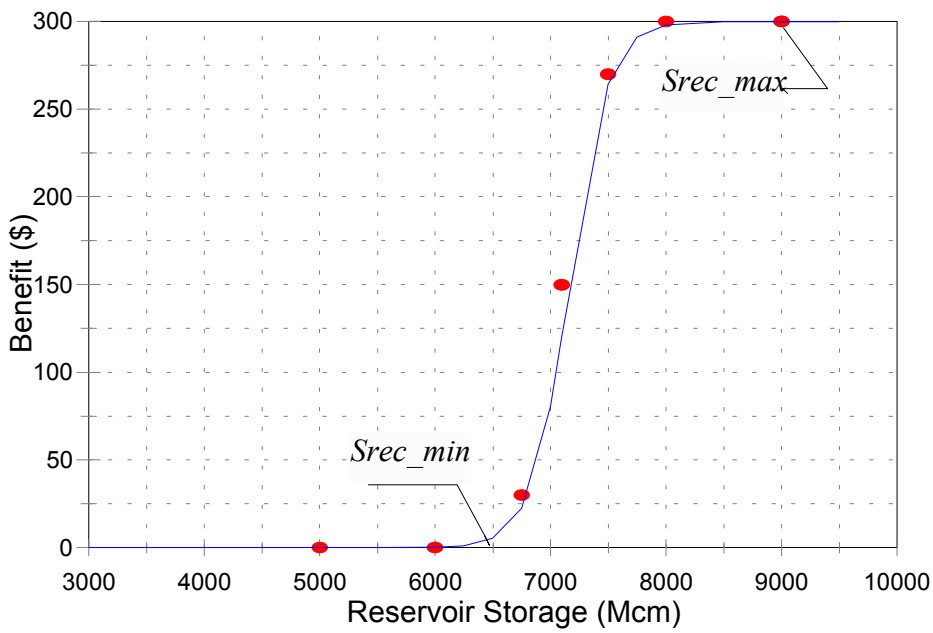

Figure 4.10 Fitted reservoir recreation benefit curve

In order to fit the benefit function to a set of known or assumed points giving the relation of total benefit to reservoir storage, it is helpful to begin with some pre-estimated parameters. The seasonal parameters a_6, b_6 and c_6 can be set once the lowest and highest recreation storage levels and the maximum total benefit are defined for each season. Parameter a_6 is estimated as half of the maximum benefit. Parameter b_6 is a small scaling coefficient (e.g., 0.001) that controls the slope of the S-shaped curve. Parameter c_6 can be initially estimated as the average of the highest and lowest recreation storage amounts multiplied by the valuer of the parameter b_6. After some visual calibration, the parameters for the case shown in figure 4.10 resulted: a_6 = 150 \$, b_6 = 0.003, and c_6 = 21.5.

Constant Price Benefit Functions

Variable-Head Powerplant

The energy rate function *erf* remains as defined in equation 4.1, while returns for hydroenergy are represented by the constant function indicated in (4.31) and displayed in figure 4.11,

$$b\,[\$/\text{MWh}] = \frac{p_S}{P}\,a_2 \tag{4.31}$$

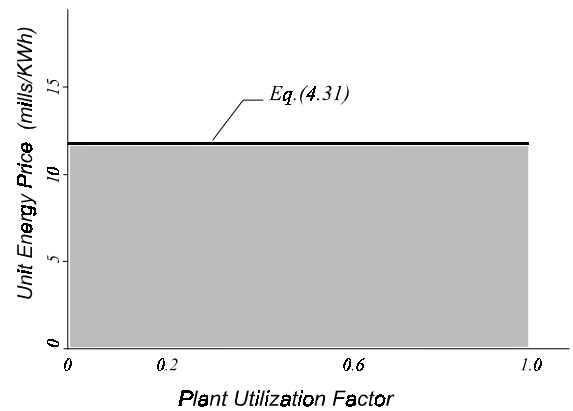

Figure 4.11 Constant Demand function for energy.

where as defined earlier, p_s accounts for seasonal changes in energy prices with respect to the average annual price P. When no distinction is made between possible changes in energy prices during on-peak and off-peak hours, the integration of the product of the energy rate function (4.1) times the constant marginal price function (4.31) with respect to the powerplant release T, yields the economic benefit from the operation of a single powerplant during time period i :

$$B_i^{\text{HPW VH}} = \eta\,\frac{p_i}{P}\,\frac{a_2\,b_1}{2}\,\{[2a_1/b_1 + 2S_i^o + UI_i + XI_i - XR_i - UR_i]\,T_i - T_i^2/2\} \tag{4.32}$$

As indicated earlier, the storage S_i^o at the beginning of period i depends on the history of inflows and outflows from the reservoir up to that period (equation 4.8). Substituting (4.8) into (4.32), the total benefit accruing from hydropower generation with variable-head and constant price is obtained:

$$B_i^{\text{HPW VH}}[\$] = \eta\,\frac{p_i}{P}\,\frac{a_2\,b_1}{2}\,\{[2a_1/b_1 + 2S^o + 2\sum_{k=1}^{i-1}(UI_k + XI_k) - 2\sum_{k=1}^{i-1}(T_k + XR_k + UR_k) + (UI_i + XI_i - XR_i - UR_i)]\,T_i - T_i^2/2\} \tag{4.33a}$$

Equation (4.33a) can be written in a more compact form by expressing it in terms of $\bar{S}_i$, the average reservoir storage during the time interval i,

$$B_i^{\text{HPW VH}}[\$] = \eta\,\frac{p_i}{P}\,\frac{a_2\,b_1}{2}\,\{(\frac{2\,a_1}{b_1} + 2\,\bar{S}_i)\,T_i - \frac{T_i^2}{2}\} \tag{4.33b}$$

Once again, and as explained for equation (4.9), the explicit modeling of reservoir evaporation modifies equation (4.33b) to yield the alternative expression (4.33c). The calculation of the parameter β_i is introduced in Chapter 5, equation (5.31)

$$B_i^{\text{HPW VH}}[\$] = \eta \frac{p_i}{P} a_2 b_1 \{ (\frac{a_1}{b_1} + \bar{S}_i) T_i + \frac{\beta_i}{4} T_i^2 \} \tag{4.33c}$$

Fixed-Head Powerplant

The hydropower benefit function (4.33) can be simplified for a run-of-river powerplant for which the energy rate function reduces to a single value because of the fixed energy head acting on the hydraulic turbines. Using the simplified forms of *erf* and *b*, equations (4.10) and (4.31) respectively, the economic benefit from the operation of a fixed-head powerplant with constant energy prices during time interval *i* results:

$$B_i^{\text{HPW FH}}[\$] = \eta \frac{p_i}{P} a_1 a_2 \ T_i] \tag{4.34}$$

After considering all potential controlled and uncontrolled flow variables entering the powerplant, (4.34) is rewritten in its expanded form as:

$$B_i^{\text{HPW FH}}[\$] = \eta \frac{p_i}{P} a_1 a_2 \{ c_A^u d_{A\ i}^u + c_B^u d_{B\ i}^u + \ldots + UI_i \} \tag{4.35}$$

Irrigation Demand Area

Given a constant price per million cubic meters of water used in irrigation, the irrigation benefit function is as expressed in (4.36) and illustrated by the demand curve in figure 4.12:

$$b^{\text{IRR}}[\$/\text{Mcm}] = a_3 \exp(-A/b_3) \tag{4.36}$$

The integration of (4.36) yields the total benefit from the IRR demand area (i.e., the total willingness to pay for irrigation water in the demand area):

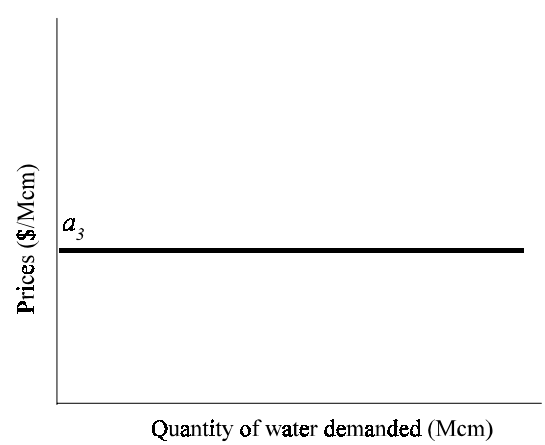

Figure 4.12 Constant irrigation demand.

$$B_i^{\mathrm{IRR}} = \int_0^{A_i} a_{3_i}\, dz \tag{4.37}$$

Solving the integral in (4.37), the IRR benefit function for a single time interval *i* is obtained (equation 4.38a), and in its expanded form (equation 4.38b):

$$B_i^{\mathrm{IRR}}\,[\$] = a_{3_i} A_i \tag{4.38a}$$

$$B_i^{\mathrm{IRR}}\,[\$] = a_{3_i} (c_A^u d_{A_i}^u + c_B^u d_{B_i}^u + \ldots + UI_i) \tag{4.38b}$$

Municipal and Industrial Demand Area

Given a constant price per million cubic meters of M&I water, the M&I demand function reduces to equation (4.39):

$$b^{M\&I}\,[\$/\mathrm{Mcm}] = a_4 \tag{4.39}$$

The integration of the demand curve yields the total benefits stemming from the municipal and industrial water use:

$$B_i^{\mathrm{M\&I}}[\$] = \int_0^{D_i} a_{4_i} \exp(-z/b_{4_i})\, dz \tag{4.40}$$

Solving the integral in (4.40), the M&I benefit function for a single time interval *i* is obtained (4.41a), and in its expanded form (4.41b):

$$B_i^{\mathrm{M\&I}}[\$] = a_{4_i} D_i \tag{4.41a}$$

$$B_i^{\mathrm{M\&I}}[\$] = a_{4_i} (c_A^u d_{A_i}^u + c_B^u d_{B_i}^u + \ldots + UI_i) \tag{4.41b}$$

Instream Recreation Area

Given a constant price per million cubic meters of instream recreation water, the IRA demand function reduces to equation (4.42):

$$b^{\mathrm{IRA}}\,[\$/Mcm] = a_5 - b_5 R \tag{4.42}$$

The integration of the demand with respect to the instream flow yields the total benefits derived from the IRA river reach:

$$B_i^{\mathrm{IRA}}\,[\$] = \int_0^{R_i} a_{5_i}\, dz \tag{4.43}$$

Solving the integral in (4.43), the IRA benefit function for a single time interval *i* is obtained (equation 4.44a), and in its expanded form (equation 4.44b):

$$B_i^{\mathrm{IRA}}\,[\$] = a_{5_i} R_i \tag{4.44a}$$

$$B_i^{\mathrm{IRA}}\,[\$] = a_{5_i} (c_A^u d_{A_i}^u + c_B^u d_{B_i}^u + \ldots + UI_i) \tag{4.44b}$$