

# Application of Time Series Analysis for Assessing Reservoir Trophic Status

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**Abstract.**—This study is to develop and apply a practical procedure for the time series analysis of reservoir eutrophication conditions. A multiplicative decomposition method is used to determine the trophic variations including seasonal, circular, long-term and irregular changes. The results indicate that (1) there is a long high peak for seven months from April to October yearly; (2) the long-term trend (T) increases with time (t) as following relationship:  $T = 51.4231 + 0.0605t$ ; and (3) circular change period decreases from two or three years to about one year. The methodology is feasible to present the eutrophication changes numerically.

## Introduction

Te-Chi is one of the main reservoirs in Taiwan. However, since 1981 its water quality has distinctly deteriorated due to excessive land use, improper felling of trees, soil washed out, as well as impact of pesticides and fertilizers, etc. Meanwhile, the concentrations of total phosphorus obviously exceeded the reservoir and lake's eutrophic limits of the U.S. Environmental Protection Agency (EPA) 0.020 mg/L. Therefore, a lot of investigators surveyed the water quality of Te-Chi reservoir from 1983 up to now. However, how does the data exhibit that the trophic status is improved, increased, or the same? Consequently, the objective of this study is to develop and apply a practical procedure for the time series analysis of reservoir eutrophication conditions using the data collected over a period more than ten years (R.O.C. Committee of Water Resources 1993; R.O.C. Tai Power et al. 1988-1992; R.O.C. Te-Chi Reservoir Watershed Management Committee et al. 1993-1996).

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## Methods

For the numerical model of the eutrophication, Carlson (1977) developed a Trophic Status Index (TSI) including three expressions of total phosphorus (TP), chlorophyll a, and transparency to evaluate the trophic status of water. Because the data of TP was more complete than the others in the past investigations (R.O.C. Committee of Water Resources 1993; R.O.C. Tai Power et al. 1988-1992; R.O.C. Te-Chi Reservoir Watershed Management Committee et al. 1993-1996), we used a multiplicative decomposition (MD) method as well as TSI (Trophic Status Index) for TP, TSI (TP), to analyze time series on the trophic status in Te-Chi reservoir. In general, TSI (TP) can be written as (ROCEPA 1991):

$$[S](TP) = 10 \left[ \frac{3.70 - 0.98 \ln[TP]}{\ln 2} \right] \quad (1)$$

where

(TP) = concentration of total phosphorus, mg/m<sup>3</sup>.

A time series is an ordered sequence of observations. The ordering is usually through time, particularly in terms of some equally spaced time intervals. A typical MD time series is constructed by the following four components (Chen et al. 1997):

1. Long-term trend: To describe a long-term growth or failing. This is the fundamental trend of long-term fluctuations including direction and strength.
2. Seasonal change: To describe a regular variation for specific time period that is usually equal to one year.
3. Circular change: To describe the periodic fluctuations over one year. The magnitude of the time period is irregular.
4. Irregular change: To describe a random or unexpected variation that is also a accidental fluctuation.

Theoretically, time series model is assumed to contain only the above four components. Additionally, we also assume that their net effect for observations is their product. If  $T$  = long-term trend,  $S$  = seasonal change,  $C$  = circular change, and  $I$  = irregular change, we can write the time series of  $Y$  as:

$$Y = T \cdot S \cdot C \cdot I \quad (2)$$

Equation (2) is usually called the multiplicative model, which is particularly useful to decomposing the time series. The units of  $T$  and  $Y$  are all the same to observations, and  $S$ ,  $C$ , and  $I$  are often unit less decimals (Chen et al. 1997).

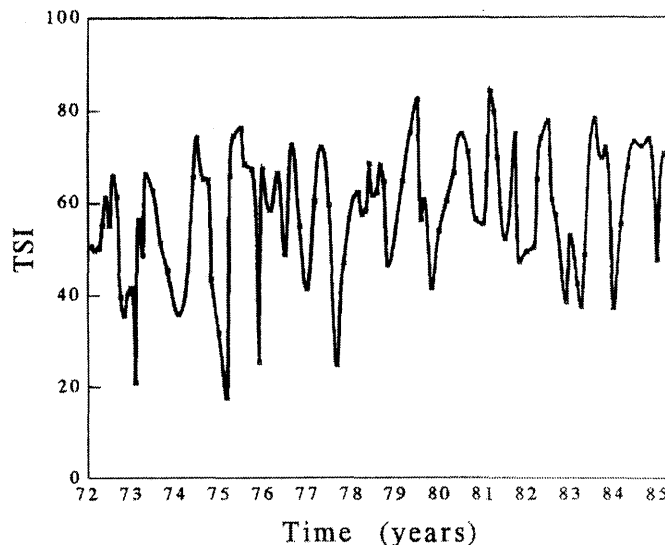


Figure 1. Variation of TSI (TP) with time.

## Results and Discussion

### Moving Average Calculation of 12 Months

The time series constructed by TSI (TP) of Te-Chi reservoir give a seasonal period of 12 months. In order to eliminate this seasonal variation, it is essential to seek the moving average (MA) of 12 months. Next, we use the central technology to obtain the MA of 2 months from each pair of neighboring MA, which is to identify the same time between MA and TSI (TP). After operation, the time series have not contained both factors of seasonal and irregular variation, i.e., the centralized MA represents a combination of long-term trend and circular change ( $T \times C$ ). Additionally, because of  $Y = T \times C \times S \times I$ , the combination of seasonal and irregular change ( $S \times I$ ) is also estimated by  $Y / (T \times C)$  (Chen et al. 1997). In this study,  $Y$  equals TSI (TP) of Te-Chi reservoir, and a plot of the TSI (TP) vs. time is shown in figure 1.

### Seasonal Index Estimation

The values of  $S \times I$  can be used to obtain seasonal variation ( $S$ ) of the time series.  $S \times I$  values are arranged in rows from month to month shown in table 1. The table indicates that the fluctuant trend is more regular. The means of  $S \times I$  in each row (or the same month) can eliminate most of irregular change. Thus, the 12 means are useful to represent the seasonal variation indexes ( $S_m$ ) of 12 months. Then,  $S_m$  is corrected to the seasonal adjustment index ( $S_c$ ), and the relationship between  $S_m$  and  $S_c$  (Chen et al. 1997) is:

$$S_c S_m \left[ 12 / \sum_{f=1}^{12} S_m \right], t = 1, 2, \dots, 12 \quad (3)$$

Variation of  $S_c$  with time presented in figure 2 indicated that the peak months of the eutrophic status in Te-Chi reservoir continued for approximately 7 months from April to October.

### Estimation of Seasonal Adjustment Index and Long-term Trend

From the seasonal adjustment index ( $S_c$ ), we can deduce the long-term trend ( $T$ ). Now, TSI (TP) ( $= Y$ ) divided by the relative month's  $S_c$  is set to  $d$ , that is:

$$d = Y / S_c \quad (4)$$

In addition, because the relationship between  $T$  and time is linear, we can assume:

$$T = b_0 + b_1 t \quad (5)$$

Again, we use the least squares method to obtain:

$$b_0 = \left[ \sum_{f=1}^{161} d / 161 \right] - b_1 \left[ \sum_{f=1}^{161} t / 161 \right] \quad (6)$$

and

$$b_1 = \left[ 161 \sum_{f=1}^{161} t d - \left( \sum_{f=1}^{161} t \right) \left( \sum_{f=1}^{161} d \right) \right] / \left[ 161 \sum_{f=1}^{161} t^2 - \left( \sum_{f=1}^{161} t \right)^2 \right] \quad (7)$$

Table 1. The data of seasonal variation (S).

Year	72	73	74	75	76	77	78	79	80	81	82	83	84	S <sub>m</sub>	S <sub>c</sub>
Month															
1		0.85	-	0.57	1.13	0.69	-	-	0.90	0.80	0.81	1.05	0.58	0.82	0.85
2		5	0.65	7	4	9	-	-	2	7	3	2	3	5	4
3		0.43	7	-	-	-	1.07	0.99	-	-	-	-	-	0.54	0.56
4		3	-	0.31	1.02	1.07	3	8	0.99	1.26	0.81	0.84	0.87	5	4
5		1.20	-	6	9	5	0.94	-	8	2	0	5	5	0.95	0.98
6		3	-	1.18	-	-	6	1.17	-	1.19	1.06	0.73	-	3	6
7	1.08	1.04	1.12	6	1.17	1.39	0.95	7	1.06	4	6	4	1.07	1.02	1.06
8	3	5	8	1.31	6	7	4	-	1	1.04	1.20	0.84	2	9	5
9	1.33	1.38	1.31	9	-	-	1.12	1.31	-	4	8	5	-	1.14	1.18
10	3	9	6	-	0.82	1.15	1	1	1.17	-	-	-	1.14	9	9
11	1.27	-	-	1.37	5	0	1.00	0.91	1	0.79	1.31	-	2	1.12	1.16
12	1	1.26	1.19	2	-	-	9	0	-	4	0	1.41	-	5	4
	0.81	0	0	1.18	1.26	0.46	1.01	0.98	1.07	-	1.01	9	1.09	1.14	1.18
	6	-	1.23	3	7	9	9	0	6	-	9	-	0	5	5
	0.72	0.98	5	-	-	-	1.11	-	-	1.25	0.97	1.19	-	1.14	1.18
	5	9	0.79	1.07	0.94	0.89	3	0.68	0.80	8	1	0	1.09	7	7
	0.82	-	5	3	9	1	1.04	3	6	0.80	-	1.16	9	1.04	1.07
	6	0.89	-	-	-	-	8	-	-	4	-	5	-	2	9
		8		0.41				0.74			0.74			1.10	1.14
				4				0			2			3	2
														0.86	0.90
														9	0
														0.66	0.68
														1	4

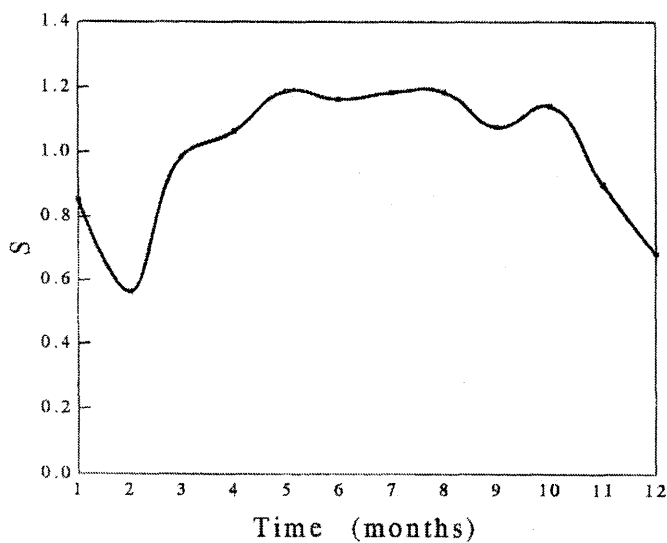


Figure 2. Seasonal variation (S) of eutrophication.

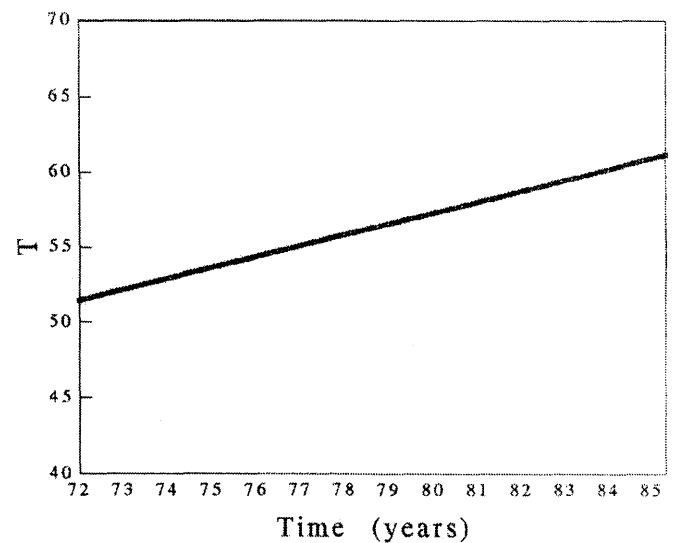


Figure 3. Long-term trend (T) of eutrophication.

In this study, the linear regression model of T is:

$$T = 51.4231 + 0.0605 t, \quad t = 1, 2, 3, \dots, 161 \quad (8)$$

More clearly, figure 3 exhibited that the long-term trend (T) of the eutrophic status in Te-Chi reservoir increased slowly with increasing time.

### Circular and Irregular Change Analysis

By substituting the values of TSI (TP) ( $= Y$ ),  $S (= S_t)$ , and T on both sides of  $Y = T \times C \times S \times I$ , we can get a combination of circular and irregular change, i.e.,  $C \times I$  (Chen et al. 1997).

Next, because the irregular component (I) doesn't generally extend more than 3 months, we adopted the moving average (MA) of 9 months to eliminate the effect of I, and to estimate the effective C values of circular fluctuation. Again, the I values are also given by calculating  $(C \times I)/C$  (Chen et al. 1997). Figures 4 and 5 presented the variation of C and I with increasing time, respectively.

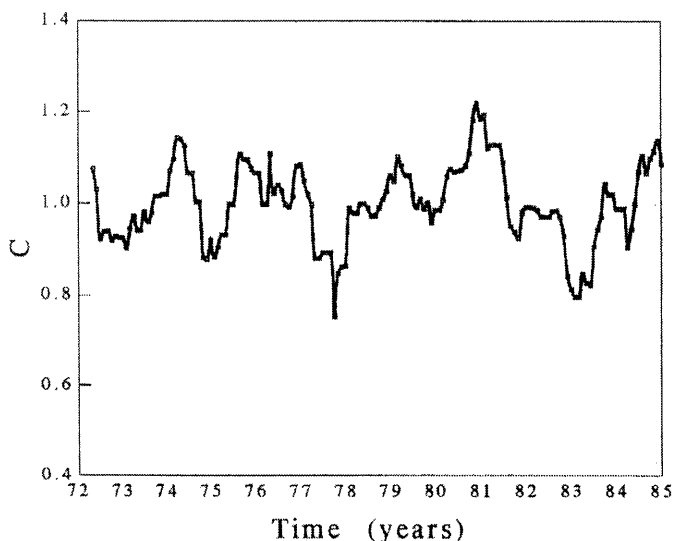


Figure 4. Circular variation (C) of eutrophication.

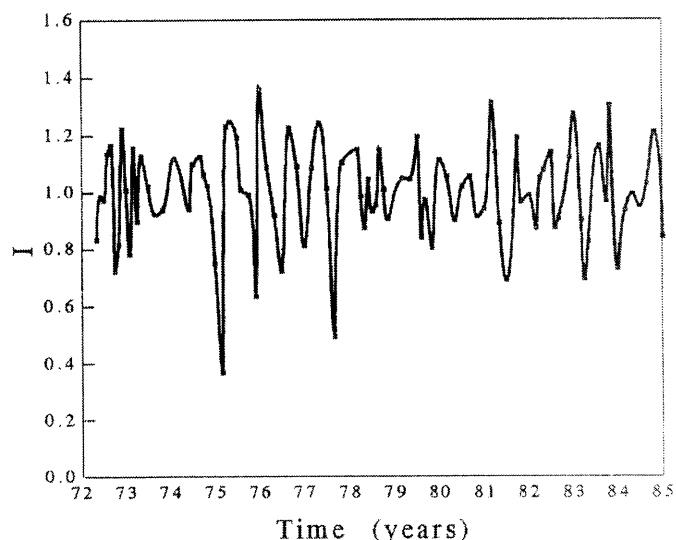


Figure 5. Irregular variation (I) of eutrophication.

## Conclusions

The results of the time series analysis indicated that the eutrophic status deteriorates continuously for the water quality of Te-Chi reservoir. Therefore, collection and treatment of agricultural wastewater, setting of effluent standards, no discharge of effluent into reservoirs, and severe execution are all necessary. Simultaneously, setting of the protective zone at the upstream from reservoirs, and reduction of land use in the watershed of reservoirs also shall be done well. Additionally, excellent soil and water conservation practices can extend the service time of reservoirs, improve water quality, and reduce health and safety risks for downstream people.

## Acknowledgments

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