5. Sample size

One of the most common objectives of bed-material sampling is the characterization of the distribution of particle sizes present in a given stream channel. The ideal way to describe the particle-size distribution of a streambed would be to count and measure every particle within the reach. This is essentially impossible, and therefore one must rely on taking samples of the bed material and use the sample results to estimate the general characteristics of the sampling area.

The question is, how many particles or how much sediment should be sampled? The answer is a compromise between sampling precision and sampling effort. As the number of particles collected increases, the precision with which the bed material can be described increases as well. The precision obtained in the sample must be sufficient to measure the effects being investigated by the project goals (e.g., Is there a significant increase or decrease in streambed fines? Has the armor layer changed, etc.?). If the sampling program is not sufficient to meet these goals, the validity of the field results must be called into question. As the sample sizes collected increase, the costs and effort associated with the field work also increase and will eventually become prohibitively large. Another factor to consider is that the increased precision obtained by collecting ever larger sample sizes does not follow a linear relationship. The benefit obtained from collecting an additional 10 particles is much greater when the existing sample size is 20 as opposed to when the sample size is 200.

The characteristics of the bed material being sampled is also an important factor in determining sample size. When there is little variability in the material, i.e., when the bed is well sorted , smaller samples will suffice to precisely describe the bed. With greater variability, i.e., poorer sorting, the sample sizes must be increased to obtain the same precision. Similarly, a smaller sample size suffices if the bed is homogeneous, which means that the particle-size distribution is more or less the same throughout the sampling area.

Because sample size determines both the cost and the benefits of field measurements, careful attention should be paid to this aspect before going out into the field. The minimum sample size necessary to ensure a specific sampling precision should be calculated beforehand and then be evaluated for cost requirements. However, in order to estimate the minimum sample size for some preset precision, one must have at least an approximate estimation of the bed-material standard deviation or sorting – which in itself requires sampling. This circularity may be resolved by performing a pilot data-collection study or through estimation based on experience with streams that have bed-material characteristics similar to the stream being studied.

Various methods for computing a relationship between sample size and precision for bed-material samples are described in the literature. The methods fall into three groups:

- Empirical recommendations,
- Computations based on an assumed normal distribution, and
- Computations that do not assume any underlying distribution type.

Older literature on sediment sampling often has empirical recommendations of sample size that are based on the D_{max} particle size and developed for particular bed-material properties. These recommendations usually do not assign a certain level of precision to a certain sample size and are not generally applicable. More recent literature bases sample-size computation on an assumed normal distribution of the bed-material particle size. These procedures are generally applicable if the assumptions of a normal distribution holds, but the descriptions can be highly technical and difficult to understand without background knowledge of statistics. Other sample-size recommendations do not assume an underlying distribution type and are generally applicable. This document compiles a variety of sample-size computations, explains their application, and compares the results. This chapter provides the user with background information that assists in selecting a sample-size procedure suitable for a specific study objective.

Methods used to compute minimum sample size are different for number-based sampling (Section 4.11 and 4.1.2), areal sampling (Section 4.1.3), and volume-based sampling (Section 4.2). For this reason, sample-size computations are discussed separately for each sampling method.

5.1 Factors affecting sample size

The computation of sample size is affected by a variety of factors which include:

- Assumptions made about the underlying distribution type of the bed-material particle-size (approximately normal, log-normal, or no assumptions are made regarding the distribution type (Section 2.1.4.3)) and these assumptions determine which statistics need to be used;

- Bed-material characteristics:
  - standard deviation s of the particle-size distribution (typically estimated by the Inman sorting coefficient s_I or by the moment method s_{frq}),
  - heterogeneity of bed material within the reach (variability among samples),
  - limited parent population size (N) in relation to sample size (n) in a small sampling area;

- Acceptable error of measurements which may relate to:
  - absolute, percentage, or standard error,
  - particle sizes in ϕ or mm units,

- the distribution mean or any percentiles (e.g., D_5, D_{50}, or D_{95}), and
- be affected by the number of operators involved;

- Acceptable chance that the computed result is wrong
  - expressed by a predetermined confidence level α;

- Bias (systematic deviation of sampling results from population characteristics) due to:
  - unrepresentative particle selection (operator bias),
  - unrepresentative sampling from heterogeneous deposits (sampling bias),
  - unrepresentative presence of particles from the largest size class (statistical bias).

Assumptions made about the underlying distribution type

Assumptions made regarding the population distribution type of the bed material sampled determine the kind of statistics used for sample-size computations. Traditionally, it is assumed that unimodal, log-transformed bed-material particle-size distributions derived from a sufficiently large sample size approach a normal distribution in ϕ-units. Assuming an approximately normal distribution has the advantage that commonly available sample-size statistics can be used which are based on normal distributions (Section 5.2.2). Bed-material samples, however, rarely have a true normal or Gaussian distribution (Kothyari 1995). The user needs to evaluate whether the goodness-of-fit to a Gaussian distribution is close enough to warrant the assumption of approximate normality (Section 2.1.4.3). Church et al. (1987) and Rice and Church (1996b) cautioned that true Gaussian distributions for log-transformed particle-size distributions are unlikely for gravel-bed streams. The user could either use an empirical approach to determine a "sufficient" sample size, or use a bootstrap (resampling) approach (Section 5.2.3.4) that provides a relation between sample size and error. Sample size – error relations computed from a bootstrap approach are independent of an underlying distribution type and may differ substantially from similar relations computed using Gaussian-based statistics.

Bed-material characteristics

For a specified accuracy and precision, sample size n should increase as the variability of the parent population increases, i.e., as the sorting of the bed material becomes poorer or the standard deviation becomes larger. If the bed-material composition is spatially heterogeneous and varies markedly between different locations of the sampling reach, samples collected from the reaches are likely to be highly variable as well. The large standard deviation between individual samples necessitates collecting a large number of samples for a desired accuracy and precision. Small mountain streams with large particles might have only a limited number of particles available for sampling. In this situation, the population size N is limited in relation to sample size n. This limitation takes a statistical effect as N becomes less than about 100 times the necessary number of particles n and causes a decrease in the sample size necessary for a specific precision. Bed-material characteristics further affect the relationship between sample size and accuracy when

operators tend to favor mid-size particles while neglecting fines and large cobbles in poorly sorted bed material.

Accuracy and precision

Usually when samples are taken, the user wants to know the accuracy and precision of a sample. Accuracy refers the size of the deviations from the true value. The accuracy of bed-material sampling may never be known because the true distribution of bed-material particles in the reach could only be determined by collecting every particle in the reach. Using the example of target practice (Fig. 5.1), with the target being the representative description of the particle-size distribution of a deposit, accuracy is the closeness of the shots to the target center (Fig. 5.1 d). Precision refers to the size of deviations from the mean value obtained by repeated applications of the sampling procedure, i.e., the ability to repeatedly hit the same area (hopefully the center) of the target (Fig. 5.1c and d).

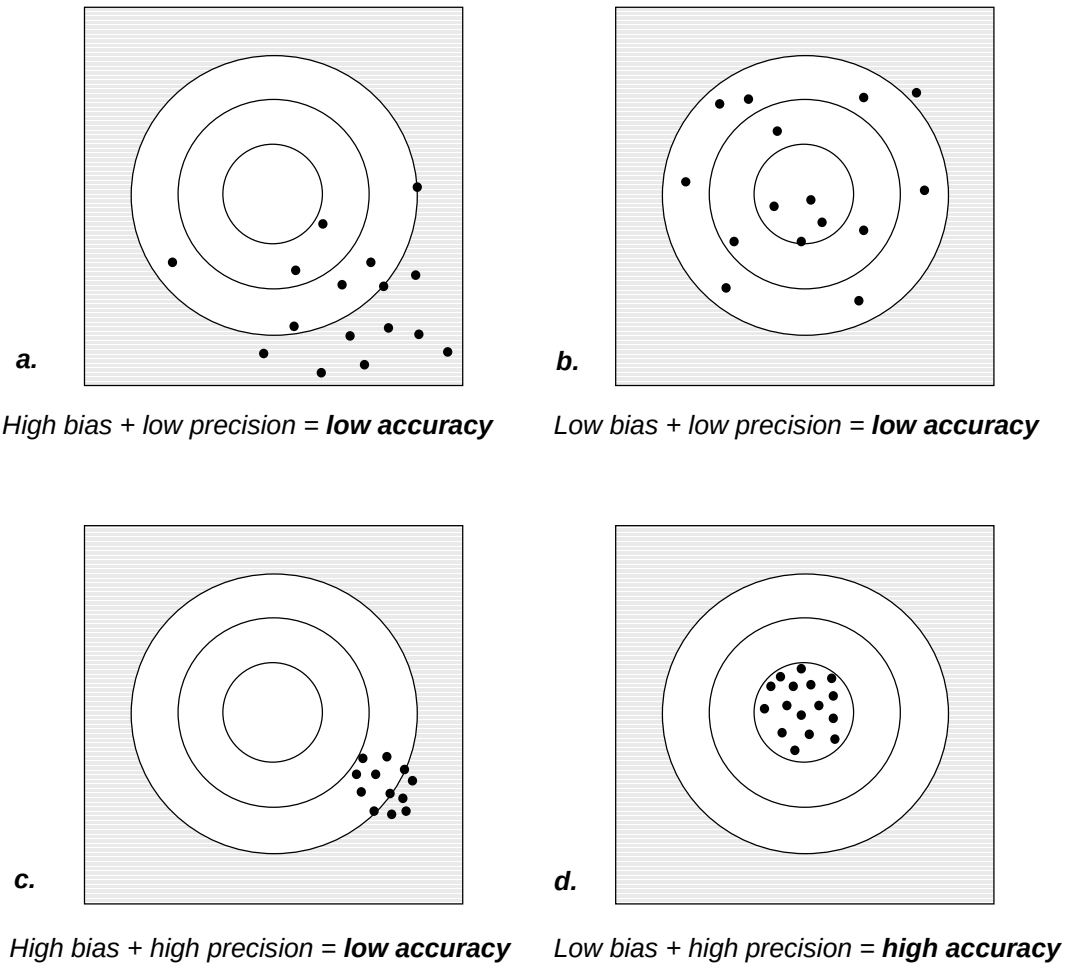

Fig. 5.1: Patterns of shots at a target. (Redrawn from Gilbert (1987), by permission of John Wiley and Sons, Ltd.).

Precision describes how dispersed or tightly bunched the shots are (Fig. 5.1 a and b) and indicates the inter-sample variability (i.e., standard deviation between samples). Precision is used to quantify how many repeated samples are needed to arrive at a stable sampling result. Accuracy and precision are interrelated. Samples of low precision also have a low accuracy (Fig. 5.1 a and b). Samples of high precision are not necessarily accurate if all samples are off set from the true result by some constant amount, i.e., by bias. Precise samples can only be accurate in the absence of bias (Fig. 5.1 d).

Bias

Bias is the systematic deviation of a sampling result from the true population characteristics (Fig. 5.1 a and c). Bias can stem from a variety of sources. *Operator* bias results when the operator selects mid-sized, "handy" particles and excludes "inconveniently" small, or large particles, or particles form poorly accessible streambed locations. Operator bias can be ameliorated by training and using an appropriate sampling methodology (Sections 4.1.1.3 – 4.1.1.6, 5.2.2.8), but since it cannot be eliminated, operator bias increases with the number of operators and with sample size. *Statistical* bias is caused by sampling too few particles from the largest size class and is ameliorated by a large sample size (Sections 5.4.1.1, 5.4.1.3, 5.4.1.4). *Sampling* bias means to sample particle distributions not representative of the parent distribution in the reach and may result from sampling spatially heterogeneous beds in an unrepresentative way. This can be avoided by using spatially segregated sampling schemes (Sections 6.3 and 6.5).

5.2 Pebble counts: number-based sample-size recommendations

5.2.1 General form of number-based sample-size equations

For approximately Gaussian shaped particle-size distributions that are not very skewed, mean and median are similar. In this case, a one-step procedure can be applied to estimate the sample size necessary to obtain a desired precision of the sample mean particle size ϕ_m or D_m. The general form of a sample-size equation is:

$$n = \left(\frac{t \cdot \sigma}{e}\right)^2 \tag{5.1}$$

n is the sample size, i.e., the number of particles to be sampled, t is a statistical numerical value, σ is the population standard deviation, and e is the acceptable error around the mean. These terms will be described in more detail.

Standard deviation

The population standard deviation σ describes how wide the range of values is within the population, specifically the range of values comprised within the central 68% of all data

(Section 2.1.5.4). However, the population standard deviation is rarely known, and must therefore be substituted by an estimate of standard deviation that is derived from the sample. σ may be estimated from the sample standard deviation $s \equiv \sqrt{s^2}$, where s^2 is the sample variance. The sample standard deviation s is computed from the absolute or percent frequencies of the particle number frequency-distribution

$$s = \sqrt{s^2} = \sqrt{\frac{1}{n\text{-}1} \sum_{i=1}^{k} n_i \, (\phi_{ci} - \phi_m)^2} \qquad \text{(see Eq. 2.56)}$$

where n is the total number of particles (or 100 %), k is the number of size classes, n_i is the number of particles per size class (or the percentage), ϕ_{ci} is the center of class in ϕ-units of the ith size class, and ϕ_m is the arithmetic mean particle size in ϕ-units. This computation of standard deviation is also called the "second moment" method. The expression becomes complicated for grouped data such as particle-size distributions (see Eq. 2.58) and is therefore commonly substituted by the Inman sorting coefficient s_I that describes the range of particle sizes contained within the central 68% of all data (Eq. 2.46, Section 2.1.5.4).

$$s_I = \frac{|\phi_{84} - \phi_{16}|}{2} \qquad \text{(see Eq. 2.46)}$$

Numerical values of standard deviation and the Inman sorting coefficient are identical for true Gaussian distributions, but deviate somewhat if the particle-size distribution is not exactly normal (see Table 2.14).

The sample standard deviation or the Inman sorting coefficient is usually not known before the sampling project starts, and need to be obtained from a pilot study. A preliminary value of sample size is then computed for a preset precision using the estimated value of s or s_I and a sample of the computed size is collected. The standard deviation s or s_I is computed for the collected sample, and sample size is recalculated. If sample sizes based on the pilot study and on the actual sample are different, the process of computing and comparing sample-size requirements needs to be repeated until the difference in required and collected sample size is insignificant.

Error

The error around the mean can be expressed in absolute or in percentage terms. The user may specify an acceptable error for the sample mean, e.g., ±0.15 ϕ around the mean in ϕ-units (ϕ_m) or ±10% around the mean in mm (D_m), and compute the sample size necessary to attain this goal. Similarly, the error associated with a given sample size may be

calculated by solving sample-size equations for the error *e*. The inverse square relation between *n* and *e* is such that a fourfold increase in sample size reduces the error by half, whereas allowing for twice the error reduces the sample size by a factor of 4. Other possibilities to express sampling errors include standard errors and errors around percentiles in terms of mm, ϕ, % or in terms of a percentile range (Section 5.2.3).

Sample statistics

The term *t* in Eq. 5.1 is a statistical numerical value known as Student's *t*. For bed-material sampling studies, Student's *t* is preferred over other statistics because the population σ is usually not known and approximated by *s*, the sample standard deviation. The *t*-variate cuts off (100 α/2)% of the upper tail of a *t*-distribution with *n*-1 degrees of freedom. The numerical value of Student's *t* depends on two parameters: confidence level: and sample size. The confidence level α describes the certainty (or the percent of all cases) in which a specified precision will be obtained by sampling the required sample size. A value typically chosen is $\alpha = 0.05$ which pertains to a 95% confidence level which means that the particle size of interest will be within a predetermined limit in 95% of all cases. Table 5.1 shows the relation between percent confidence, α-levels, and the resulting value for $t_{1-\alpha/2,\, n-1}$ for large *n*. Note that for large *n*, values of $t_{1-\alpha/2,\, n-1}$ are identical

Table 5.1: Relation between precision (expressed in terms of confidence levels, or percent chance that error is exceeded), the corresponding α-levels, and values of $t_{1-\alpha/2}$ or $Z_{1-\alpha/2}$ for $n \rightarrow \infty$.

% confidence that error is not exceeded $1-\alpha$ (%)	% chance that error is exceeded α %	α-level α	Percentile of normal distribution $1-\alpha/2$	Distance betw. median and percentile in terms of standard deviation $t_{1-\alpha/2,\, n\rightarrow\infty}$, or $Z_{1-\alpha/2}$
0	100	1.0	0.5	0.0
30	70	0.7	0.65	0.385
50	50	0.5	0.75	0.675
68.2	31.8	0.318	0.841	1.000
80	20	0.20	0.90	1.280
85	15	0.15	0.925	1.440
90	10	0.1	0.95	1.645
91	9	0.09	0.955	1.695
92	8	0.08	0.96	1.750
93	7	0.07	0.965	1.810
94	6	0.06	0.97	1.880
95	**5**	**0.05**	**0.975**	**1.960**
98	2	0.02	0.99	2.327
99	1	0.01	0.995	2.576
99.5	0.5	0.005	0.9975	2.810
99.9	0.1	0.001	0.9995	3.270
99.96	0.04	0.0004	0.9998	3.490
99.99	0.01	0.0001	0.99995	3.603

to the values of $Z_{1-\alpha/2}$ which is the standard normal deviate that cuts off (100 α/2)% of the upper tail of a standard normal distribution. Values of $Z_{1-\alpha/2}$ are used instead of $t_{1-\alpha/2,\, n-1}$ if the population standard deviation σ is known and n is larger than 100.

For a specified confidence level, the relationship between t and sample size n is not linear. For sample size larger than 200, t takes a constant value of 1.96 for a 95% confidence level (for $\alpha = 0.05$, $t_{1-\alpha/2,\, n-1} = t_{0.975,\, n-1} = 1.96$). But for small sample sizes, t changes significantly with sample size. For a sample size of 5 which allows n-1 = 4 degrees of freedom, $t_{0.975,\, n-1} = 2.776$, and increases to 12.7 for a sample size of 2. Table 5.2 provides t-values for various degrees of freedom which are equal to n-1, and a 95% confidence level for which $t_{1-\alpha/2} = t_{0.975}$. Values for t for other confidence levels and samples sizes can be obtained from statistical tables available in standard statistics books (e.g., Gilbert 1987).

Table 5.2: Values for Student's t for various degrees of freedom (n-1) and a 95% confidence level (α = 0.05) with $t_{1-\alpha/2} = t_{0.975}$

n-1	$t_{1-\alpha/2,\, n-1}$	n-1	$t_{1-\alpha/2,\, n-1}$	n-1	$t_{1-\alpha/2,\, n-1}$	n-1	$t_{1-\alpha/2,\, n-1}$	n-1	$t_{1-\alpha/2,\, n-1}$
1	12.706	11	2.201	21	2.080	35	2.032	85	1.991
2	4.303	12	2.179	22	2.074	40	2.021	90	1.990
3	3.182	13	2.160	23	2.069	45	2.015	95	1.988
4	2.776	14	2.145	24	2.064	50	2.010	100	1.987
5	2.571	15	2.131	25	2.060	55	2.005	105	1.985
6	2.447	16	2.120	26	2.056	60	2.000	110	1.983
7	2.365	17	2.110	27	2.052	65	1.998	115	1.981
8	2.306	18	2.101	28	2.048	70	1.996	120	1.980
9	2.262	19	2.093	29	2.045	75	1.995		
10	2.226	20	2.086	30	2.042	80	1.993	∞	1.96

Sample size

The necessary sample size n may have to be computed iteratively if n is smaller than approximately 200 because the value of t depends on sample size (Table 5.2). This is not a concern for pebble counts which comprise more than 200 particles. However, when using the general sample-size equation Eq. 5.1 to compute the number of subsamples required for a specified precision (two-stage sampling, Section 5.2.3.1), n may be smaller than 10, and t varies pronouncedly with n when n is small (Table 5.1).

The calculated sample size refers to the confidence level specified by the t value. If a t-value for a 95% confidence level is used, i.e., $t_{1-\alpha/2,\, n-1} = t_{0.975,\, n-1}$, a sample size is computed for which there is a 95% chance that the absolute difference (positive or negative) between the estimated sample mean and the true population mean is less than the specified acceptable error.

The computed sample size is usually rounded to the next higher integer value, because sample sizes are whole numbers, not decimals. Either 7 or 8 samples are collected, but not 7.3. It is left at the user's discretion of whether a sample size of 13.1 is rounded to 13 or 14. Rounding is denoted by the symbol ≅ in this document.

5.2.2 Prespecified error around the mean

The variables in the right-hand term of the general sample-size equation (Eq. 5.1) can be slightly altered, so that Eq. 5.1 can be used to compute the sample size around the mean for a variety of different applications. Sample errors around the mean may be specified as absolute error $e_{\pm\phi m}$ in ϕ-units, as percent error $e_{\%Dm}$ around the mean in mm, and as percent error $e_{\%\phi m}$ around the mean in ϕ-units (Sections 5.2.2.1 – 5.2.2.3). The confidence level may be changed, and consequently the numerical value of t. In all example computations provided in this section, sample sizes are computed for a 95% confidence level ($\alpha = 0.05$), a value that is commonly selected. However, some study objectives may specify a different confidence level. A normal distribution of bed-material is assumed when using Eq. 5.1, but a slight variation of the error term makes it possible to use the equation for logarithmically distributed samples (Section 5.2.2.4). Another assumption for Eq. 5.1 is an unlimited supply of particles to be sampled. Again, a slight modification of Eq. 5.1 allows the user to compute sample size when the number of particles that may be sampled is limited, for example, in a small sampling area (Section 5.2.2.5). All variations of Eq. 5.1 used in Section 5.2.2.1 to 5.2.2.5 are listed in Table 5.5 in Section 5.2.2.6. Example computations are performed with all equations introduced in Sections 5.2.2.1 - 5.2.2.5 for the same particle-size distributions so that computed samples sizes may be compared (Section 5.2.2.6).

5.2.2.1 Absolute error around the mean in ϕ-units

The sample size for a specified absolute error around the mean particle size of a sample in terms of ϕ-units (e.g., $e_{\pm\phi m} = \pm 0.2$ ϕ-units) is computed from:

$$n = \left(\frac{t_{1-\alpha/2;n-1}}{e_{\pm\phi m}} \cdot \quad s_I \right)^2 \tag{5.2}$$

n is the sample size for which there is a small (e.g., 5%) chance only ($\alpha = 0.05$) that the absolute difference (positive or negative) between the estimated sample mean and the true population mean is larger than or equal to the acceptable absolute error $e_{\pm\phi m}$.

Example 5.1:
Given is the particle-size distribution from Table 2.3 and Fig. 2.12 in Section 2.1.4.1:

D_{84} = 104 mm ϕ_{84} = -6.70 $s_I = |\phi_{84} - \phi_{16}|/2$ = 1.94
D_{50} = 32 mm ϕ_{50} = -5.00
D_{16} = 7 mm ϕ_{16} = -2.82
D_m = 27.2 mm ϕ_m = -4.76

If the user has no idea about the approximate value of n, the t-value for an indefinitely large sample size $n \to \infty$ is used in a first trial. For $\alpha = 0.05$, $t_{0.975,\, n\text{-}1} = 1.96$, and sample size n for an acceptable absolute error $e = \pm 0.2\ \phi$-units becomes:

$$n = \left(\frac{1.96}{0.2} \cdot 1.94\right)^2 = 361.5 \cong 362.$$

If the acceptable error is increased to $e_{\pm\phi m} = \pm 0.5\ \phi$, n becomes 57.8, rounded up to 58. In this case, a t-value of 1.96 would not be appropriate and computations need to be repeated with a t-value for n-1 = 57, which is close to 2.00 (Table 5.2). Using $t_{0.975,\, n\text{-}1} = 2.00$, n = 60.2, and is rounded to 61. This computed n is almost similar to the n for which the t-value was selected. Usually, about three iterations are required to reach this convergence.

Eq. 5.2 indicates that a pebble count in a poorly sorted streambed ($s_I \approx 2$) requires almost 400 particles for a 95% certainty that the mean of the sample is by no more than ±0.2 ϕ-units different from the population mean. An error of ±0.2 ϕ means that in 95% of all samples, the sampled mean can be expected to be within the range of -4.56 to -4.96 ϕ (i.e., between 23.6 and 31.1 mm) of the true mean of -4.76 ϕ (i.e., 27.2 mm).

Fig. 5.2 shows relations between sample size and the absolute error $e_{\pm\phi m}$ in ϕ units around the mean using t-statistics and a 95% confidence level for samples with various sorting coefficients. These curves may be used to estimate the number of particles required for a desired precision in pebble counts in streams with different sorting coefficients. Sample size in Fig. 5.2 was calculated iteratively to account for the variation of $t_{1\text{-}\alpha/2,\, n\text{-}1} = t_{0.975,\, n\text{-}1}$ with n.

5.2.2.2 Percent error around the mean in mm

Eq. 5.2 can be adjusted to apply to particle sizes in mm (ISO 1992). In this case, sorting is expressed as the logarithmic geometric standard deviation $s_{g;sq}$ (see Eq. 2.54, Sect. 2.1.5.4), and the error is expressed in terms of the log of the percentage error around the D_m in mm added to 1.

$$n = \left(\frac{t_{1\text{-}\alpha/2;n\text{-}1}}{\log\,(1+e_{\%Dm})} \cdot s_{g;sq}\right)^2 \qquad (5.3)$$

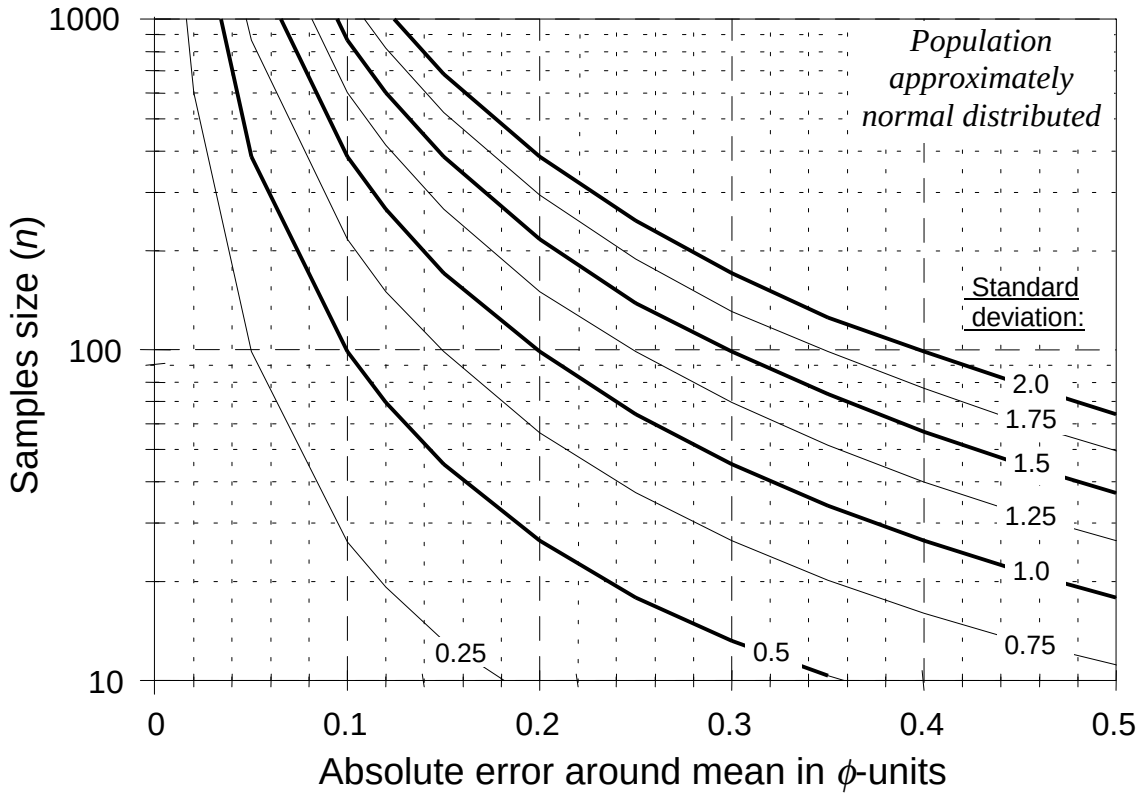

Fig. 5.2: Relation between sample size n and absolute error $e_{\pm\phi m}$ around the mean in ϕ-units based on Student's t-values for approximately normal distributed bed material for various sorting coefficients, and α =0.05. Sample size n was computed iteratively to adjust for the variability of $t_{0.975,\, n\text{-}1}$ with sample size.

Example 5.2:
A pilot study estimates D_{84} = 104 mm, and D_{16} = 7 mm. The logarithmic geometric standard deviation (Sect. 2.1.5.4) is $s_{g;sq} = 0.5 \log (D_{84}/D_{16}) = 0.586$.

The absolute error of ±0.2 ϕ in Eq. 5.2 corresponds approximately to a ±13.9 % error around the D_m of 27.2 mm (see below) which was chosen as the tolerable error. In log units, an error of ±13.9 % is expressed as log (1+0.139). n is computed by solving the equation:

$$n = \left(\frac{1.96}{\log 1.139} \cdot \quad 0.586 \right)^2 = 415.7 \cong 416$$

Comparison of absolute error in ϕ and percent error in mm

Sample-size computations in ϕ units and in the corresponding units of mm are not truly equivalent, because an error that is symmetrical around a mean in ϕ units is not

symmetrical around the mean in mm, and vice versa. For example, an absolute error of ±0.2 ϕ units around a mean particle size of ϕ_m = -5.0 ϕ encompasses the range of -4.8 to -5.2 ϕ. The equivalent range in mm is 27.9 to 36.8. The percent difference between 27.9 mm and the mean particles size D_m of 32 mm is (27.9 - 32) · 100/32 = -12.8%, whereas the percent difference between 36.8 mm and the D_m of 32 mm is (37.7 - 32) · 100/32 =15.0%. This asymmetry may be negligible for relatively small errors, but becomes quite pronounced as the absolute error in ϕ increases. An error of ±1.0 ϕ around a ϕ_m of -5.0 ϕ (=percentage error of 20%) encompasses the size range of -4 to -6 ϕ that is equivalent to the range of 16 to 64 mm, and describes an error of -50 to +100% around the D_m of 32 mm (see Fig. 5.9 in Section 5.2.3.4).

5.2.2.3 Percent error around the mean in ϕ-units

The percent error $e_{\%\phi m}$ around the mean in ϕ-units is the absolute error divided by the mean and computed from $e_{\%\phi m} = e_{\pm\phi m}/\phi_m$. Sample size for a percent error $e_{\%\phi m}$ with a 5% chance (α =0.05) that the difference between sample ϕ_m and population mean ϕ_m is smaller than the prespecified percent error may be computed from

$$n = \left(\frac{t_{1-\alpha/2;n-1}}{e_{\%\phi m}} \cdot \frac{s_I}{\phi_m}\right)^2 = \left(\frac{t_{1-\alpha/2;n-1}}{e_{\%\phi m}} \cdot CV\right)^2 \qquad (5.4)$$

Note that sample standard deviation (or sediment sorting) is divided by ϕ_m as well. s/ϕ_m defines the coefficient of variation CV, also termed the relative standard deviation. Using the Inman sorting coefficient s_I to describe the standard deviation s, CV may be computed from:

$$CV = \frac{s}{\phi_m} = \frac{\frac{|\phi_{84} - \phi_{16}|}{2}}{\phi_m} = \frac{|\phi_{84} - \phi_{16}|}{2\phi_m} \qquad (5.4a)$$

Estimates of ϕ_{84}, ϕ_{50}, and ϕ_{16} may be obtained from a pilot study of a 100-particle pebble count. Table 5.3 presents coefficients of variation (CV) for bed material of different sorting coefficients and different mean particle sizes.

Example 5.3:
From Example 5.1, s_I is taken as 1.94, and sample ϕ_m is -4.76 ϕ.
An absolute error of ±0.2 ϕ-units (Example 5.1) becomes a percent error $e_{\%\phi m}$ = ±0.2/-4.76 = 0.042 or 4.2%. n is computed from:

$$n = \left(\frac{1.96}{0.042} \cdot \frac{1.94}{-4.76}\right)^2 = (46.67 \cdot 0.408)^2 = 361.7 \cong 362$$

The sample size calculated by Eq. 5.4 is equal to the sample size calculated by Eq. 5.2.

Table 5.3: Values of *CV* for poorly sorted bed material with various sorting coefficients and various mean particle sizes

D_m (mm)	ϕ_m (ϕ-units)	*Sorting coefficient* s_I *1.0*	*1.5*	*2.0*
16	-4.0	0.25	0.38	0.50
22.6	-4.5	0.22	0.33	0.44
32	-5.0	0.20	0.30	0.40
45	-5.5	0.18	0.27	0.36
64	-6.0	0.17	0.25	0.33

5.2.2.4 Percent error in ϕ and mm for approximate lognormal distributions

Although particle-size distributions in ϕ units tend to roughly approach normal distributions, it is conceivable that a particular particle-size distribution might obtain a better fit to a log-normal distribution than a normal distribution. In this case, Eq. 5.5 may be used to estimate the sample size for a prespecified percent error around the mean (Gilbert 1987).

$$n = \frac{t^2_{1-\alpha/2;n-1} \cdot s_I^2}{\ln(e_{\%\phi m}+1)} \tag{5.5}$$

Example 5.4:
In accordance to Example 5.3, an absolute error of ±0.2 ϕ-units around the mean of -4.76 ϕ becomes a percent error of 0.042 or 4.2 %. Sample size is computed from:

$$n = \frac{1.96^2 \cdot 1.94^2}{\ln(0.042+1)} = \frac{3.84 \cdot 3.76}{0.041} = 351.4 \cong 352$$

A similar form of Equation 5.5 may be applied if particle sizes in mm units approached a lognormal distribution. The graphic geometric standard deviation $s_{g;sq} = (D_{84}/D_{16})^{0.5}$ is used in this case and has the value of 3.85 for D_{84} = 104, and D_{16} = 7 mm (Eq. 2.54 in Section 2.1.5.4). A percentage error of 0.042 (4.2%) in terms of ϕ-units (see above) is approximately similar to a percent error of 13.9% in mm-units for the particle-size distribution in the example.

$$n = \frac{t^2_{1-\alpha/2;n-1} \cdot s_{g;sq}^{\ 2}}{\ln(e_{\%Dm}+1)} \tag{5.6}$$

Example 5.5:

$$n = \frac{1.96^2 \cdot 3.85^2}{\ln(0.139+1)} = \frac{3.84 \cdot 14.83}{0.129} = 438.9 \cong 439$$

5.2.2.5 Limited number of particles available for sampling ($N \neq \infty$)

When sampling small geomorphological units (e.g., bars and riffles), or sedimentary units (areas of homogeneous bed-material composition) there may be a shortage of particles that can be sampled. The number of particles present on the particular geomorphological or sedimentary unit (the population size N) might not be much larger, or even smaller than the sample size n computed with Eqs. 5.2 - 5.6. In this case, sample size n needs to be adjusted for limited population size N. This is accomplished by dividing the equation for unlimited sample size by 1 + the quotient of the original sample-size equation and population size N. Equations that include a term for limited population size N provide virtually the same results as equations for unlimited N, if N exceeds n by a factor of 1000 or more. If $N = 100\ n$, n is reduced by less than 1% compared to the n computed without adjustment for N, and if $N = 10\ n$, n reduces by 10%. Thus, as N decreases, sample size n also decreases.

Absolute error around mean in ϕ-units

For a prespecified absolute error $e_{\pm\phi m}$ in ϕ-units, and a limited number of particles N in the target population available for sampling, n is computed from (Gilbert 1987)

$$n = \frac{\left(\frac{t_{1-\alpha/2;n-1}}{e_{\pm\phi m}} \cdot s_I\right)^2}{1 + \left(\left(\frac{t_{1-\alpha/2;n-1}}{e_{\pm\phi m}} \cdot s_I\right)^2 / N\right)} \tag{5.7}$$

Example 5.6:

A grid count is done on a small bar with an area $A_b = 2\text{ m} \cdot 2.6\text{ m} = 5.2\text{ m}^2$. Almost all surface particles are within the size range of 35 - 45 mm. If the mean particle b-axis size is 45 mm, and particles are mostly ellipsoid (a-axis = 1.5 b-axis) and flat lying, the area covered by one particle A_p can be approximated by $A_p = \pi\,(0.045/2 \cdot 0.068/2) = 0.0024\text{ m}^2$. Thus, the number of surface particles on this bar is estimated to be $N = A_b/A_p = 2164$.

$$n = \frac{\left(\frac{1.96}{0.2} \cdot 1.94\right)^2}{1 + \left(\left(\frac{1.96}{0.2} \cdot 1.94\right)^2 / 2164\right)} = \frac{361}{1+(361/2164)} = 309.4 \cong 310$$

Equation 5.2 (for unlimited population size) produced a sample size of 362. Because of the limited number of particles in the population, Eq. 5.7 calculates a lower sample size of n = 310. The effect of population size N on sample size becomes negligible as N exceeds 100 n (ca. 50,000), and n increases to 359. For the example above, this occurs as the sampling area reaches 119 m^2 (Table 5.4).

Table 5.4: Example of change in sample size n with a change in population size N.

N	Sampling Area (m^2)	Side length for square (m)	n
100	0.24	0.5	78
500	1.2	1.1	210
1,000	2.4	1.5	265
5,000	12	3.5	337
10,000	24	5	349
50,000	119	11	359
100,000	238	15	360
500,000	1190	35	361

Percentage error around the mean in mm

If the adjustment for limited population size is applied to Eq. 5.3, the sample size required for a specified percentage error around the D_m (in mm) is computed from:

$$n = \frac{\left(\frac{t_{1-\alpha/2;n-1} \cdot s_{gl}}{\log(1+e_{\%Dm})}\right)^2}{1 + \left(\frac{t_{1-\alpha/2;n-1} \cdot s_{gl}}{\log(1+e_{\%Dm})}\right)^2 / N} \tag{5.8}$$

Percent error around the mean in ϕ-units

Similar to Eq. 5.4, sample size for a specified percent error around the mean in a limited population size and an approximately normal distribution of particle sizes in ϕ-units is estimated from:

$$n = \frac{\left(\frac{t_{1-\alpha/2;n-1}}{e_{\%\phi m}} \cdot \frac{s}{\phi_m}\right)^2}{1 + \left(\left(\frac{t_{1-\alpha/2;n-1}}{e_{\%\phi m}} \cdot \frac{s}{\phi_m}\right)^2 / N\right)} \tag{5.9}$$

Percent error around the mean in ϕ-units, approximate lognormal distribution assumed
Parallel to Eq. 5.5, sample size for a specified percent error around the mean of an approximately lognormal distribution of particle sizes in ϕ-units and a limited population size is obtained from:

$$n = \frac{t^2_{1-\alpha/2;n-1} \cdot s^2}{[\ln(e_{\%\phi m}+1)] + \left(\frac{t^2_{1-\alpha/2;n-1} \cdot s^2}{N}\right)} \tag{5.10}$$

Percent error around the mean in mm, approximate lognormal distribution assumed
As in Eq. 5.6, sample size for a specified percent error around the mean of an approximately lognormal distribution of particle sizes in mm and a limited population size is obtained from:

$$n = \frac{t^2_{1-\alpha/2;n-1} \cdot s^2}{[\ln(e_{\%Dm}+1)] + \left(\frac{t^2_{1-\alpha/2;n-1} \cdot s_{g;sq}{}^2}{N}\right)} \tag{5.11}$$

5.2.2.6 Comparison between sample-size equations for errors around the mean

Equations introduced in this section are summarized in Table 5.5. When applied to the same particle-size distribution, all equations compute sample sizes between 352 to 439 for an absolute error around the mean of ±0.2 ϕ, which is equivalent to a 4.2 % error (around the mean in ϕ), and approximately equal to an error of ±13.9 % around the mean in mm. To compare the results of the five equations (5.2 to 5.6) over a wide range of errors, sample sizes were computed for errors between ±0.1 and 1.0 ϕ, and plotted in Fig. 5.3. Eqs. 5.2 ($e_{\pm\phi m}$) and 5.3 ($e_{\%\phi m}$) yield identical relations between sample size and error, while sample size computed for a corresponding percent error in mm units, ($e_{\%Dm}$) (Eq. 5.3) is slightly higher. The sample size – error relations have a somewhat different shape if the particle-size distribution is assumed to approach a lognormal distribution instead of a normal one.

Table 5.5: Variations of the standard sample-size equation for computing absolute or percent errors around the mean particle-size in mm (D_m) or ϕ-units (ϕ_m) for approximately normal and lognormal distributions, and for unlimited and limited population particle numbers. Numbers in parentheses refer to equation numbers in the document.

Particle size units	Particle-size distribution type	Error around the mean	Unlimited population Size ($N > 100\ n$)	Limited population Size ($N < 100\ n$)
ϕ	approx. lognormal	absolute	$\left(\frac{t \cdot s_I}{e_{\pm\phi m}}\right)^2$ (***eq. 5.2***)	$\frac{\left(\frac{t}{e_{\pm\phi m}} \cdot s_I\right)^2}{1 + \left(\left(\frac{t}{e_{\pm\phi m}} \cdot s_I\right)^2 / N\right)}$ (***eq. 5.7***)
mm	approx. lognormal	percent	$\left(\frac{t \cdot s_{g;sq}}{log\ (1+e_{\%Dm})}\right)^2$ (***eq. 5.3***)	$\frac{\left(\frac{t \cdot s_{g;sq}}{log\ (1+e_{\%Dm})}\right)^2}{1 + \left(\frac{t \cdot s_{g;sq}}{log\ (1+e_{\%Dm})}^2 / N\right)}$ (***eq. 5.8***)
ϕ	approx. normal	percent	$\left(\frac{t \cdot s_I}{e_{\%\phi m} \cdot \phi_m}\right)^2$ (***eq. 5.4***)	$\frac{\left(\frac{t}{e_{\%\phi m}} \cdot \frac{s_I}{\phi_m}\right)^2}{1 + \left(\left(\frac{t}{e_{\%\phi m}} \cdot \frac{s_I}{\phi_m}\right)^2 / N\right)}$ (***eq. 5.9***)
ϕ	approx. lognormal	percent	$\frac{t^2 \cdot s_I^2}{ln\ (1+e_{\%\phi m})}$ (***eq. 5.5***)	$\frac{t^2 \cdot s_I^2}{\ln\ (1+e_{\%\phi m}) + \left(\frac{t^2 \cdot s_I^2}{N}\right)}$ (***eq. 5.10***)
mm	approx. lognormal	percent	$\frac{t^2 \cdot s_{g;sq}^2}{ln\ (1+e_{\%Dm})}$ (***eq. 5.6***)	$\frac{t^2 \cdot s_{g;sq}^2}{\ln\ (1+e_{\%Dm}) + \left(\frac{t^2 \cdot s_{g;sq}^2}{N}\right)}$ (***eq. 5.11***)

t = values for Student's t statistic, = $t_{1-\alpha/2;\ n-1}$; s_I = Inman's sorting coefficient; $s_{g,sq}$ = geometric standard deviation, square-root approach; $e_{\pm\phi m}$ = absolute error around the mean particle-size computed in ϕ-units. $e_{\%Dm}$ = percent error around the mean particle-size; $e_{\%\phi m}$ = percent error around the mean particle-size computed in ϕ-units.

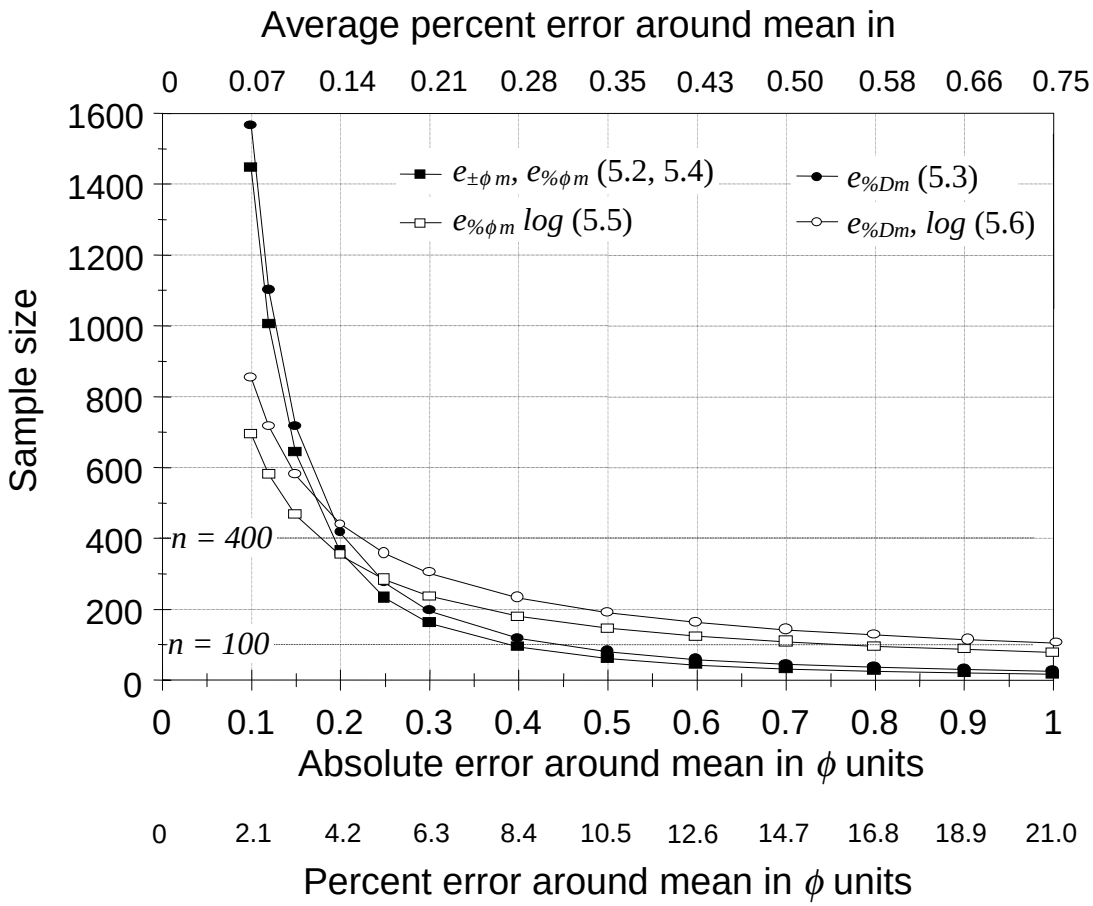

Fig. 5.3: Relation between sample size and error around the mean computed for the example particle-size distribution (D_{50} = 32 mm, s = 1.94; Table 2.3, Fig, 2.12) with five sample-size equations (Eq. 5.2 – 5.6). Equation numbers are indicated in brackets. The x-axis scale "Absolute error around the mean in ϕ-units" refers to Eqs. 5.2 and 5.4. The x-axis scale "Percent error around the mean in ϕ-units" refers to Eq. 5.5, whereas the x-axis scale "Average percent error around the mean in mm-units" refers to Eqs. 5.3 and 5.6.

5.2.2.7 Effect of bed-material sorting and error on sample size

The effects of bed-material sorting and acceptable error on sample-size requirements are quite pronounced (Fig. 5.2). Bed-material sorting typically ranges between 0.5 (well sorted lowland gravel-bed rivers) and 2.5 (poorly sorted headwater streams). Acceptable errors typically range between 5 and 50%. The numerical value for Student's t varies by no more than 1% for sample sizes larger 25. Assume that a well sorted (s_I = 0.5) lowland stream requires a sample size of 25 particles for an acceptable error around the mean. Sample size for the same mean particle size and the same acceptable error increases by a factor of 4 to 100 particles in a moderate to poorly sorted streambed (s_I = 1), and again by a factor of 4 to 400 particles in a poorly to very poorly sorted (s_I = 2) mountain gravel-bed stream. Similarly, for the same particle-size distribution, an increase of acceptable error from 10, to 20, to 40% decreases the sample-size requirement from 400, to 100, to 25

particles, respectively. Either a doubling in sorting, or halving of the acceptable error leads to a fourfold increase in sample size. These numerical examples demonstrate that statements of sample-size requirements cannot be taken out of context, but must be evaluated in light of streambed sorting and the acceptable error.

5.2.2.8 Influence of multiple operators on sampling accuracy

None of the sample-size recommendations presented so far account for errors introduced by operators. Operator errors may be attributed to two main factors: (1) incorrect measurement of particle size (Section 2.1.3.6), and (2) biased particle selection among operators (Section 4.1.1.3 – 4.1.1.6). Both factors increase the variability of the sample (i.e., standard deviation) and consequently increase the sample-size requirement or reduce the accuracy. In contrast to the statistical error, the operator error becomes relatively more important as sample size increases. This is because the statistical error decreases with sample size, but operator error is the same for all sample sizes (Hey and Thorne 1983). Since sampling accuracy is comprised of the errors made by all operators involved, sampling accuracy decreases as more operators are involved in the sampling.

Prompted by observed operator errors, Marcus et al. (1995) compared results of five replicate samples each obtained by eight different operators at two different sites with five replicate samples each from a single operator. They found that when multiple operators took the samples, the standard deviation around a given particle size increased at a rate about twice as high with particle size than standard deviations of replicate samples from a single operator (Fig. 5.4). For a particle size of 2 mm, samples collected by the single operator had a standard deviation of ±2.6 mm. This value increased to ±4.2 mm when the replicate samples were collected by several operators. Likewise, particles with a 250 mm diameter had a standard deviation of ±30 mm for a single operator. The value nearly doubled to ±54 mm for multiple operators. This difference shows that it is more problematic to detect a change in bed-material size over time or between sites when several operators are involved.

As the standard deviation for multiple-operator samples exceeds the standard deviation for samples from a single operator (Marcus et al. 1995) by a factor of 1.8, the sample size needs to be larger by a factor of $1.8^2 = 3.2$ when multiple operators are employed. Consequently, using two or more operators in the belief that the larger sample will provide a more accurate estimate of the population characteristics has the opposite effect: it increases the sample size necessary for the sample level of accuracy. The maximum benefit of intensive sampling is achieved *only* if *all* samples are collected by a single operator, unless operator bias can be substantially reduced (e.g., by training and using a sampling frame), or be eliminated altogether.

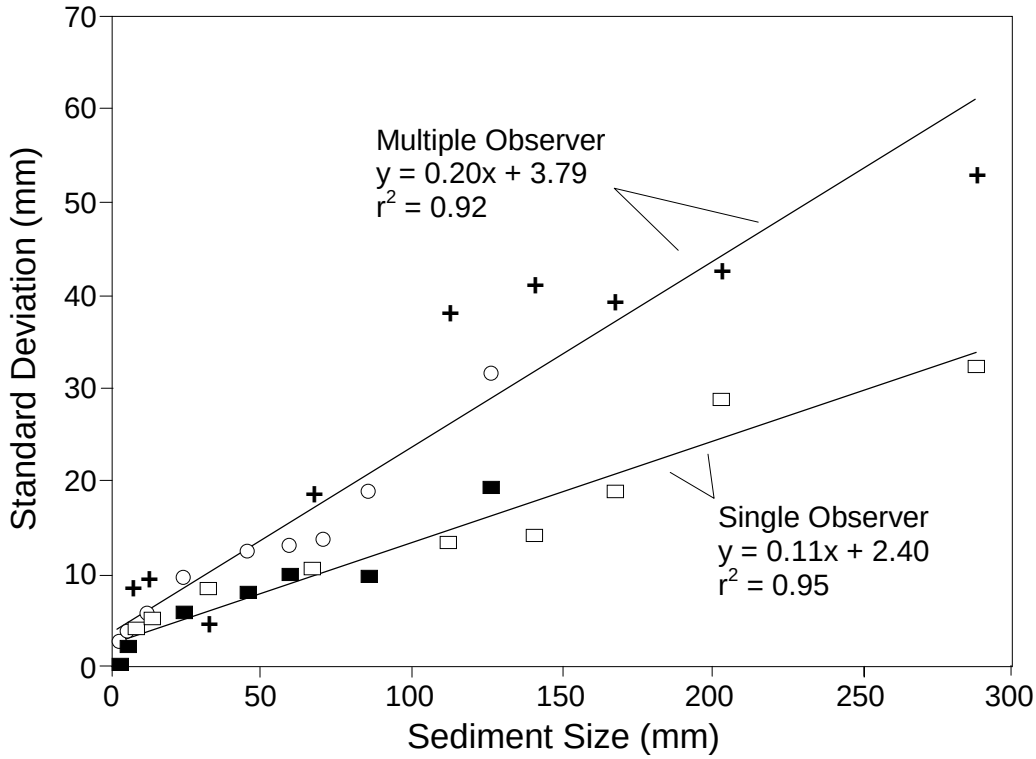

Fig. 5.4: Relation of mean size of 10 percentiles (D_{min}, D_5, D_{10}, D_{25}, D_{50}, D_{75}, D_{84}, D_{90}, D_{95}, D_{max}) to standard deviation for replicate samples collected by one (■, □) and by multiple observers (○, +). ■ and ○ refer to one sampling site, □ and + to another. (Redrawn from Marcus et al. (1995), by permission of the American Geophysical Union).

5.2.2.9 Computation of sample size and error in the field

A study site may not be close to the office, and it might be inconvenient, if not unfeasible, to return to the study site at a later time to augment a sample size that is too small. Thus, it is recommended to compute the relation between sample size and error around the mean in the field. A laptop computer is needed and a prepared spreadsheet that computes a cumulative frequency distribution, the ϕ_{16} and ϕ_{84}, the Inman sorting coefficient, and the absolute error around the mean for a given sample size (Fig. 5.5). The spreadsheet should likewise be set up to compute the sample standard deviation using the moment method (Section 2.1.5.4).

As particles are added to the frequency distribution of the sample (n_i), sample size, the particle size of the percentiles of interest, the sorting coefficient, and the currently obtained absolute error are automatically updated. Particles need to be added to the sample until the computed error is less than a specified value, for example ±0.2 ϕ.

D	ϕ	n_i	$n_{\%}$	$\Sigma n_{\%}$	
2	-1.0	4	1.6	0.0	
2.8	-1.5	5	2.0	1.6	ϕ_{16} = -2.82
…	…	…	…	…	ϕ_{84} = -6.70
128	-7.0	3	1.2	99.6	$s_{,l}$ = 1.94
180	-7.5	1	0.4	100.0	
		n=246	100		

Table of t-values for 95% confidence interval

n-1	$t_{1-\alpha/2,n-1}$
1	12.701
2	4.303
…	…
120	1.98
∞	1.96

$$e_{\pm\phi m} = \frac{t_{1-\alpha/2;n-1} \cdot s_l}{\sqrt{n}} = \frac{1.96 \cdot 1.94}{\sqrt{246}} = 0.24\ \phi$$

Fig. 5.5: Schematic presentation of a spreadsheet that could be used to compute the absolute error around the mean particle size in ϕ-units directly in the field.

5.2.3 Specified error for all percentiles

The equations presented in the previous sections determine the sample size required for estimating the error between the sample and the population mean. However, the user might need to know the error associated with specific percentiles or with all percentiles of the distribution. A two-step sampling procedure (Section 5.2.3.1) can be used to compute the relation between sample size and precision for the median or percentiles close to the median. A binomial approach (Section 5.2.3.2) can be used for specific percentiles (e.g., D_{10}, D_{90}), while a multinomial approach (Section 5.2.3.3) is used for computing the precision of the entire distribution. A bootstrapping approach (Section 5.2.3.4) can be used compute the precision around specified percentiles through a resampling procedure.

5.2.3.1 Two-stage sampling approach (ISO 1992)

Two-stage sampling is a procedure for bed-material sampling proposed by the International Organization of Standards (ISO 1992). The first step of the procedure involves collecting several subsamples (e.g., 5) each of equal size (e.g., 50 or 100 particles each). The median or, a percentile close to it, is computed for all subsamples as well as the standard deviation. In the second step a common sample-size equation (e.g., Eq. 5.2 in Section 5.2.2.1) is used to determine the number of subsamples needed to ensure that the difference between the computed median particle-size and the population median particle-size does not exceed a specified error value. The total number of sampled particles is the number of subsamples times the number of particles per subsample.

Computation for particle sizes in mm

Following the general steps previously described, a number of subsamples (q) are taken from a homogeneous deposit, each sample containing n particles. For each of the q subsamples, the particle size of the median is determined, either graphically from

cumulative distribution curves, or by logarithmic interpolation between percentiles (Section 2.1.4.2). ISO (1992) proposes to compute the D_{50} in units of mm (although ϕ units are preferable, see below). It is assumed that the q values of D_{50} are approximately normal distributed. The standard deviation s_{50} of the q values of D_{50} is determined from:

$$s_{50} = \sqrt{\frac{\sum_{i=1}^{q}(D_{50} - D_{50m})^2}{q\text{-}1}} \tag{5.12}$$

where D_{50m} is the arithmetic mean particle size in mm of the D_{50} obtained from the q subsamples.

The number of subsamples q, each with the same sample size n, required for a 5% chance (if $\alpha = 0.05$) that the absolute difference (positive or negative) between the sampled median D_{50} and the population median (i.e., the mean median established from the subsamples) is larger or equal to the acceptable absolute error $e_{\pm D50}$ is computed using Eq. 5.13. The sample size q is likely to be rather small, perhaps less than 10. The value for t varies markedly with sample size as long as samples sizes are small. It is therefore important to use an appropriate value for $t_{1\text{-}\alpha/2,q\text{-}1}$ (Table 5.2). The appropriate value for t is found by iteratively solving Eq. 5.13 until q equals the subsample size q.

$$q = \left(\frac{t_{1\text{-}\alpha/2;q\text{-}1}}{e_{\pm D50}} \cdot \quad s\right)^2 \tag{5.13}$$

The total number of particles to be sampled is the number of particles in each subsample n, multiplied by the number of q subsamples.

> <u>Example 5.7:</u>
> Eq. 5.13 is solved iteratively because the number of subsamples is typically smaller than 30 in bed-material samples, and the value for $t_{1\text{-}\alpha/2,q\text{-}1}$ varies especially for small sample sizes. Assume that five subsamples had D_{50} particle sizes of 45, 64, 76, 90, and 108 mm, with a mean D_{50} of 77 mm, and a standard deviation s_{50} of 24.1 mm. The tolerable absolute error around the D_{50} is ±10 mm. In the absence of an *a priori* estimate of the appropriate sample size for a ±10 mm error, a value of q = 20 subsamples is selected in the first trial of solving Eq. 5.13 (any other value would have been fine, too) and yields a subsample size of q = 26. The estimated subsample size (q_{est}=20) and the computed subsample size (q_{comp}=26) do not match after the first trial. The second trial uses

the estimated subsample size q = 26 to estimate the appropriate value of t. The newly computed subsample size is q = 25. Using the t-value for q = 25, the value computed for q is 25. Equivalence is reached between the estimated and computed q for a subsample size of 25 after the third trail.

Trial	q_{est}	q-1	$t_{1-\alpha/2;q-1}$	q_{comp}
1	20	19	2.093	25.4 ≅ 26.
2	26	25	2.060	24.7 ≅ 25
3	25	24	2.064	24.7 ≅ 25.

A total of 1,250 particles, i.e., 25 subsamples of 50 particles each, have to be sampled to ensure that the D_{50} particle size is within ±10 mm of the true D_{50} particle size.

Computations in units of ϕ

It is recommended to apply the two-stage approach to particle-sizes in units of ϕ, rather than to units of mm. The term D_{50} in Eqs. 5.12 and 5.13 is then substituted by values of ϕ_{50}. The advantage of computations in ϕ units is that the medians in ϕ obtained from several subsamples approximate a normal distribution better than median values D_{50} in mm. For a normal distribution, sample means will be normally distributed for any subsample size n, whereas for lognormal or skewed distributions, sample means attain a normal distribution only for subsample sizes of 30 and larger (Triola 1995, p. 252-257).

<u>Example 5.8:</u>

The ϕ_{50} particle sizes of four subsamples of 50 particles each are -5.2, -5.1, -5.0, and -4.8 ϕ. The standard deviation s of the four values of ϕ_{50} is 0.171 ϕ. For an acceptable absolute error of ± 0.2 ϕ, Eq. 5.13 yields

$$q = \left(\frac{3.182}{0.2} \cdot \quad 0.171\right)^2 = 7.4 \cong 8$$

q = 8 subsamples (of 50 particles each, = 400 particles total) are required for an acceptable error around the ϕ_{50} of ±0.2 ϕ-units.

The two-step sampling method is most suitable when estimating the required sample size for a specified error around the median in the field. For sample sizes of 100 or 50, the median particle size can be quickly calculated on paper. Sample standard deviation is a preprogrammed function of many scientific calculators. Prepared forms or spreadsheets may be helpful when computing the sample size necessary for a specified error in the field.

5.2.3.2 Binomial distribution approach (Fripp and Diplas 1993)

The one-step methods compute the absolute error in mm or ϕ-units, or the percentage error around the mean, whereas the two-step methods compute the absolute or percent error around a specified percentile. For both procedures it was assumed that either the entire distribution approximated a normal distribution, or that the percentile values from the various subsamples approached normal distributions. A binomial and a multinomial approach (Section 5.2.3.3) can be used for computing the error around a given percentile in terms of a percentile range. For example, a percentile error of ±10% means that the particle size of the D_{75} may be within the particle size range of the D_{65} and D_{85} of the population. Binomial and multinomial approaches assume no specific underlying distribution type.

The binomial approach presented by Fripp and Diplas (1993) is based on the binomial probabilities of the percent finer or percent coarser cumulative particle-size distribution (i.e., the grain-size curve). The approach is used for computing the percentile error around a given percentile $e_{\pm p}$. The computed sample size n ensures that the particle size of a given percentile p is within a specified error range between two percentiles that are ± some percentage larger and smaller than the percentile p. n is calculated from:

$$n = \frac{(Z_{1-\alpha/2})^2 \cdot p \cdot (1 - p)}{e_{\pm p}^{\ 2}} \tag{5.14}$$

Z is the standard normal deviate that cuts off (100 α/2) % of the upper tail of a standard normal distribution. Z-values for various values of 1-α/2 can be obtained from statistical tables provided in general statistics books. The value of $Z_{1-\alpha/2}$ for the commonly chosen 95% confidence interval with $\alpha = 0.05$ is 1.96 (see Table 5.1 for the relation between confidence interval, α-levels, and corresponding values for $Z_{1-\alpha/2}$). p is used as a decimal value of the percentile of interest (i.e., 0.5 for D_{50}), and the subscript $_p$ refers to a specified percentile.

Example 5.9:
The sample size required to remain below a ± 10% percentile error around the D_{16}, i.e., the D_{16} particle size is to be within the range of the sample D_6 to D_{26}, is

$$n = \frac{1.96^2 \cdot 0.16 \cdot (1 - 0.16)}{0.1^2} = 51.6 \cong 52$$

Eq. 5.14 can be solved for the error term and be used to compute the error associated with a given sample size. The error around the 50th percentile of a distribution of particle-sizes in mm (D_{50}) for a sample size of n = 100 is

$$e_{\pm p} = 1.96 \cdot \sqrt{\frac{0.5 \cdot (1 - 0.5)}{100}} = 0.098 = 9.8\%$$

An error of $e_{\pm p} \approx 10\%$ indicates that the particle size of the sample D_{50} is within the D_{60} and the D_{40} of the population distribution

The binomial approach calculates independent confidence intervals, one at a time, for each particle size-class and can therefore not be used to represent the entire distribution. A multinomial approach is needed to compute error bands around an entire particle-size distribution.

5.2.3.3 Multinomial distribution approach (Petrie and Diplas 2000)

To overcome the limitations of a binomial approach, Petrie and Diplas (2000) presented a multinomial approach which can be used for placing confidence intervals around all particle-size classes in a cumulative frequency distribution curve of a pebble count sample. The population cumulative frequency distribution (i.e., the percent finer or percent coarser curve) can then be expected to be within the confidence interval in a specified percentage of all cases (e.g., in 90% of all cases for a 90% confidence interval). Similar to a binomial approach, a multinomial approach does not assume a specific underlying distribution type.

Relation between sample size and error

The number of particles n necessary to ensure that a percentile of interest p is within an allowable confidence interval, $e_{\pm p}$ (i.e., percentile error) is computed from:

$$n = \frac{(e_{\pm p} + p) \cdot (1 - e_{\pm p} - p) \cdot \chi^2_{\alpha/k;1}}{e_{\pm p}^{\,2}} \qquad (5.15)$$

$e_{\pm p}$ is the error in percentage points around the percentile p of a particle-size distribution. An error of ±10 percent around the D_{50} ($e_{\%p50} = 0.1$), for example, means that the particle size of the sample D_{50} may be within the D_{40} and the D_{60} of the population distribution. $\chi^2_{\alpha/k;1}$ is the upper $(1-\alpha/k) \cdot 100$ percentage point of the chi-square distribution for one degree of freedom and can be obtained from standard statistical tables. α is the confidence coefficient and k is the number of size classes of the particle-size distribution. If table values are not available, the value for $\chi^2_{\alpha/k;1}$ can be approximated using a regression function that relates published values of $\chi^2_{\alpha/k;1}$ to α/k and yields ($r^2 = 0.99$):

$$\chi^2_{\alpha/k;1} = -1.435\,(-1.755 \cdot \ln(\alpha/k)) \qquad (5.16)$$

Example 5.10:

D_{50} is the percentile of interest p. The tolerable error $e_{\pm p}$ between the sample estimate of D_{50} and the confidence interval around the D_{50} is ±10 percent, i.e., $e_{\pm p}$ = 0.1. This means that the particle size of the sample D_{50} could be within the particle size of the D_{40} and the D_{60} of the population distribution. The desired confidence level is α = 0.05, and the distribution has 10 size classes so that α/k = 0.005. The table value of $\chi^2_{\alpha/k;1}$ for one degree of freedom and α/k = 0.005 is 7.88. Eq. 5.16 computes $\chi^2_{\alpha/k;1}$ as 7.86. Using the value 7.88, the necessary sample size is (Eq. 5.15)

$$n = \frac{(0.1 + 0.5) \cdot \ (1 - 0.1 - 0.5) \cdot \ 7.88}{0.1^2}$$

$$= \frac{0.6 \cdot \ 0.4 \cdot \ 7.88}{0.01} = 189.1 \cong 190.$$

The relation between sample size and percentile errors around the D_{50} is plotted in Fig. 5.6 for different numbers of particle size-classes k. A pebble count particle-size distribution from a coarse gravel or cobble-bed stream typically has 15 to 20 size classes when particle sizes are measured in 0.5 ϕ intervals. According to Eq. 5.15, a 100-particle sample with15 size classes has a percentile error $e_{\pm p50}$ of ±16% and can only ensure that the D_{50} particle

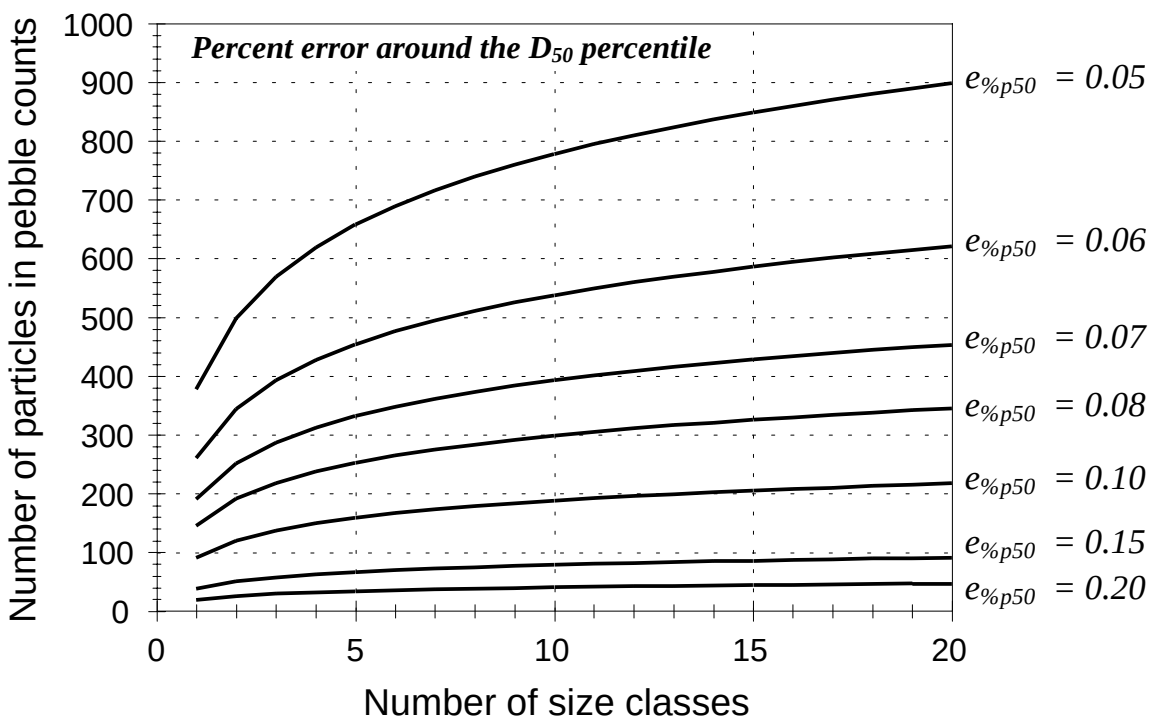

Fig. 5.6: Sample size necessary for various percentage errors around the D_{50} for different numbers of particle size-classes k of a particle-size distribution.

size is within the range of the population D_{34} to D_{66}. A 400-particle sample reduces this error to about ± 7%, narrowing the range of the D_{50} particle size of the sample to within a range between the D_{43} and the D_{57} population particle-size.

Confidence bands

Confidence bands can be plotted around a distribution using the equation

$$e_{\pm p} = \sqrt{\frac{\chi^2_{\alpha/k;1} \cdot \; p_i \cdot \; (1 - p_i)}{n}} \tag{5.17}$$

The absolute percentile error around a given percentile varies only with the number of size classes k and the selected α-value, and is the same for any particle-size distribution as long as the values of α and k are identical. Fig. 5.7 plots error bands for a 95% confidence

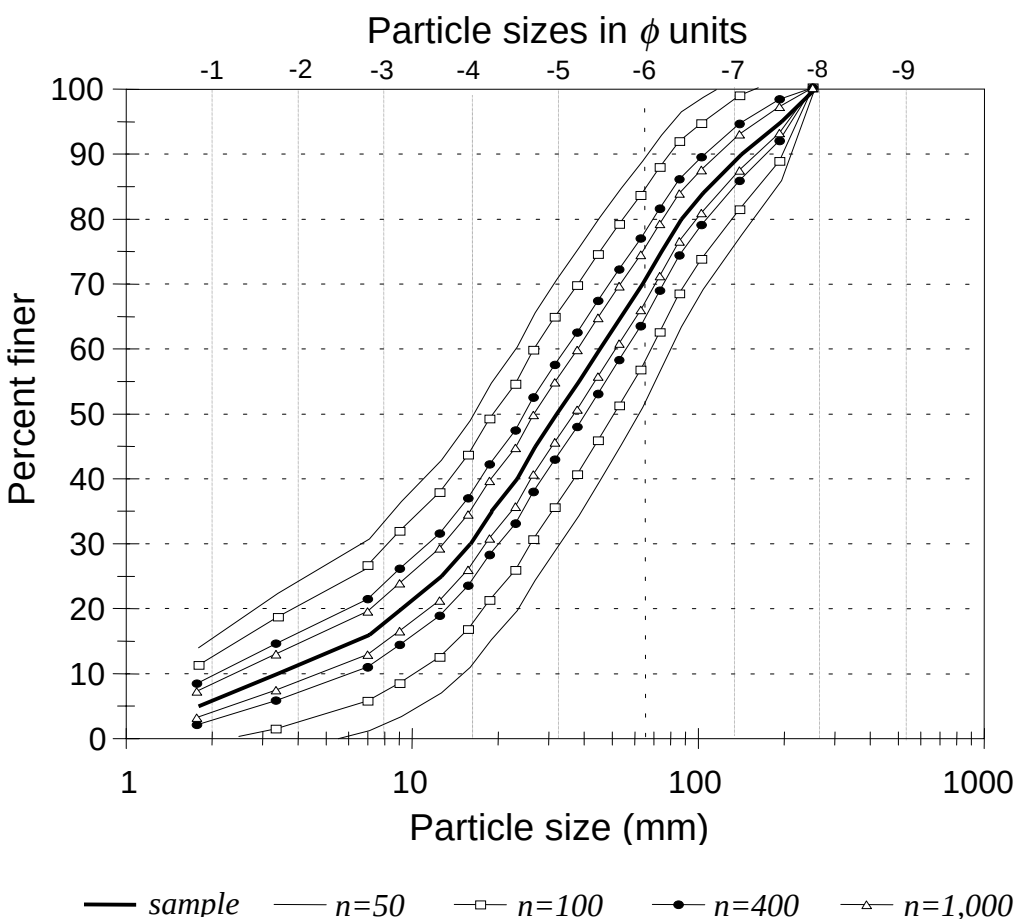

Fig. 5.7: Error bands ($e_{\pm pi}$) for a 95% confidence level around the example particle-size distribution given in Section 2.1.4.1 with 15 size classes (α/k = 0.05/15 = 0.0033; $\chi^2_{\alpha/k;1}$ = 8.57). Vertical lines indicate particle size classes in 1.0 ϕ-units.

level around the example particle-size distribution with 15 size classes presented in Section 2.1.4.1. Error bands are not entirely symmetrical around the distribution. For the example distribution used in Fig. 5.7, a sample size of n = 100 produces a percentile error around the D_{80} of ±11.7%, i.e., the D_{80} particle size could fall between the D_{92} and the D_{68} particle size. The percentile error around the D_{16} is ±10.5%, meaning that the D_{16} particle size may fall between the $D_{5.5}$ and the $D_{26.5}$ population particle size.

5.2.3.4 Bootstrap approach: no assumed distribution type (Rice and Church 1996b)

Rice and Church (1996b) proposed a computer sampling, or "bootstrap" method for determining the sample size required for a prespecified error (standard error or error in ϕ units) around a given percentile. Bootstrapping determines the relation between error and sample size from repetitive computer sampling of a parent distribution that constitutes several thousands of actually measured bed-material particle *b*-axes. Bootstrapping, like the binomial and multinomial approaches (Section 5.2.3.2 and 5.2.3.3), does not require assumptions about the underlying parent distribution type. Computations are therefore free of any error introduced by assuming an inappropriate underlying distribution type and have the advantage that the computed sample-size requirements are tailored to a specific bed-material composition found at a specific sampling site.

Computation of the bootstrap percentile standard error and the absolute percentile precision

A measurement of 3,500 particle *b*-axes provides a data base that is sufficiently large to be a good approximation of the population distributions. A large number of replicate samples r, e.g., r = 200 is drawn, each with the sample size n (sampling without replacement for each individual sample). A particle-size frequency distribution and a probability density function are constructed for each sample, and all percentiles of interest are determined. Thus, for each sample size n there are 200 repeated samples for a given bootstrap percentile D_{pb} (subscript $_b$ refers to bootstrap analyses), e.g., 200 values of D_{16b} established for a sample size of n = 50, 100, 500, etc. particles. The 200 replicates define a distribution of $D_{pb,r=200}$ - values with an arithmetic mean $D_{pmb,r=200}$ and a standard deviation $s_{pb,r=200}$. The standard deviation is the bootstrap percentile standard error s_{pb} for the bootstrap percentile D_{pb} and is computed from (Rice and Church 1996b):

$$s_{pb} = \frac{\sum^{r} (D_{pb} - D_{pmb})^2}{r - 1} \tag{5.18}$$

where D_{pmb} is the mean particle size of a specified percentile p in the bootstrap analysis. The procedure is repeated for each percentile of interest for various sample sizes. Once

the bootstrap standard error around a given percentile is known for various sample sizes, a power regression function can be established between s_{pb} and sample size n

$$s_{pb} = a_p \cdot \sigma n^c \tag{5.19}$$

for each percentile. σ is the population bed-material standard deviation, a_p and c are coefficients obtained from the regression function.

For a *known* distribution, a percentile standard error s_p can be computed from:

$$s_p = \frac{\sqrt{p \cdot (1 - p)}}{y_p} \cdot \frac{\sigma}{\sqrt{n}} \tag{5.20}$$

The index $_p$ refers to the specified percentile, p is the decimal value of the percentile (i.e., 0.5 for D_{50}), y_p is the ordinate (y-value) of the population probability density function at the given percentile. y_p is not known if the distribution is not known, which makes it impossible to use Eq. 5.20 without prior knowledge of the particle-size frequency distribution. However, the first term of Eq. 5.20

$$\frac{\sqrt{p \cdot (1 - p)}}{y_p} = a_p = \text{constant} \tag{5.21}$$

assumes a constant value a_p for each percentile for all sample sizes. The value a_p can be obtained from the least-square regression function of the relation between standard error and sample size (Eq. 5.19) computed from the bootstrap results. Substituting the first term of Eq. 5.20 with a_p allows the computation of the standard error around a percentile in an unknown distribution:

$$s_p = a_p \cdot \frac{\sigma}{\sqrt{n}} \tag{5.22}$$

In order to adjust computer sampling without replacement to a finite population from which the samples are drawn, a correction factor needs to be applied to the percentile standard error for a preset sample size n and population size N. This adjustment modifies Eq. 5.20 to

$$s_p' = \frac{\sqrt{p \cdot (1 - p)}}{y_p} \cdot \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N - n}{N - 1}} \tag{5.23}$$

The standard percentile error corrected for finite population size is then computed from

$$s_p' = a_p \cdot \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N - n}{N - 1}} \tag{5.24}$$

For a population sample size N of 100 n or more, the last term approaches 1 and can be omitted.

Rice and Church (1996b) used the bootstrap approach to compute the relation between sample size and error for various particle-size percentiles of a gravel-bed river in Canada (Mamquam River, Fig. 5.8). The particle-size distribution has a standard deviation of s = 1.17 ϕ and is slightly skewed towards a tail of fine particles ($sk_{aF\&W}$ = 0.165 (Eq. 2.61); sk_{frq} = 0.55 (Eq. 2.70)), a characteristic common to many gravel beds. The graphs showing the relationship between sample size and standard error (Fig. 5.10 and 5.11) are

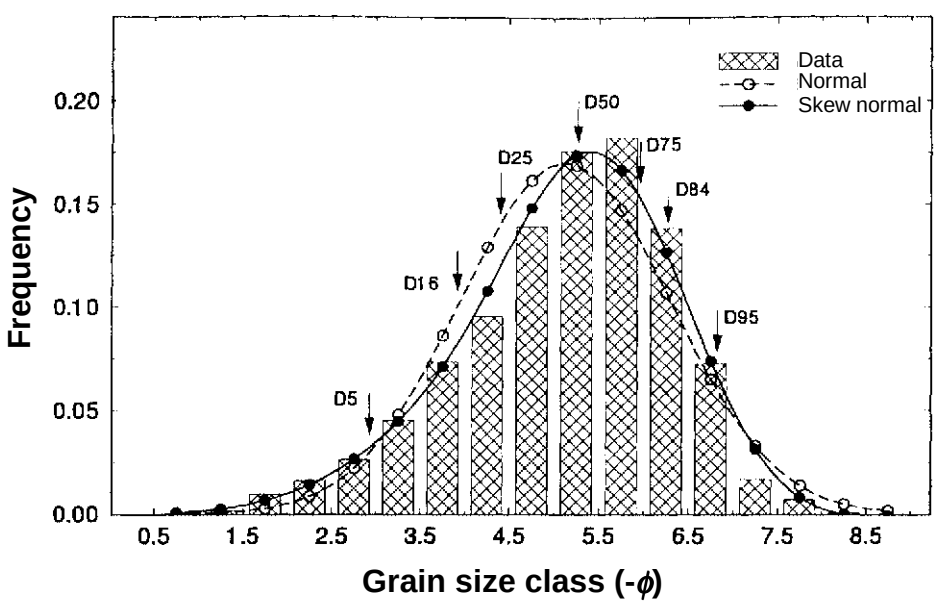

Fig. 5.8: Particle-size distribution for a sample from a bar head in the Mamquam River, British Columbia; mean annual flood is 152 m^3/s. D_{95} = 111 mm (-6.8ϕ), D_{84} = 79 mm (-6.29ϕ , D_{50} = 38 mm (-5.25ϕ), D_{16} = 15 mm (-3.91ϕ), and D_5 = 7.5 mm (-2.91ϕ); σ = 1.17ϕ , skewness = 0.165 ($sk_{a,F\&W}$) or 0.55 (sk_{frq}). According to a Kolmogorov-Smirnov test there is a less than 1% chance (α = 0.01) that the distribution is Gaussian. (Reprinted from Rice and Church (1996b), by permission of the Society for Sedimentary Geology).

discussed in the following section where the statistical (bootstrap) bootstrap error will be compared to statistical errors computed by assuming an underlying distribution type. The standard error computed from Eq. 5.24 can be converted into an absolute error in ϕ-units by

$$s_p = \frac{e_{\pm\phi p}}{t_{(1-\alpha/2,n-1)}} \quad \text{and vice versa} \quad e_{\pm\phi p} = s_p \cdot t_{(1-\alpha/2,n-1)} \tag{5.25}$$

Values of $t_{(1-\alpha/2,n-1)}$ for various samples sizes are listed in Table 5.2. Table 5.6 presents absolute errors in ϕ obtained by converting the bootstrap standard errors for sample sizes 50, 100, 400, and 1000 particles and a 95% confidence level. Error values in Table 5.6 may be used as a general estimate of absolute errors expectable around various percentiles for bed material that is slightly skewed towards a tail of fine particles and that has a sorting coefficient close to 1.17.

Table 5.6: Absolute error $e_{\pm\phi p}$ in ± ϕ-units for a 95% confidence level for percentile estimates of the Mamquam River, with a distribution slightly skewed towards a tail of fine particles and a standard deviation of s = 1.17 ϕ (from Rice and Church 1996b).

Sample size	D_5	D_{16}	D_{25}	D_{50}	D_{75}	D_{84}	D_{95}
50	0.89	0.61	0.52	0.37	0.33	0.35	0.44
100	0.62	0.40	0.36	0.26	0.23	0.25	0.30
400	0.30	0.21	0.19	0.12	0.11	0.11	0.12
1000	0.19	0.13	0.12	0.07	0.07	0.06	0.07

Note that the computed bootstrap error is purely statistical. It does not include errors stemming from unrepresentative sampling by operators. The (statistical) bootstrap error around the seven percentiles between D_5 and D_{95} for a sample size of n = 400 (gray shaded box in Table 5.6) is plotted in Fig. 4.2 (Section 4.1.1.3) and compared to the total error observed in parallel pebble counts in mountain streams.

Standard and absolute errors in ϕ-units can be converted into percent errors in mm-units. Fig. 5.9 may be used for these conversions.

Percentile standard errors: bootstrap computation versus computations with assumed distribution types

In symmetrical Gaussian distributions, standard percentile errors s_p and absolute errors in ϕ-units $e_{\pm\phi p}$ around percentiles have two properties: (1) they are paired such that errors around the ϕ_{95} and the ϕ_5, and errors around the ϕ_{84} and ϕ_{16}, etc. are equal; and (2) the tails of the distribution (ϕ_5 and ϕ_{95}) have higher errors than the mean (ϕ_{50}).

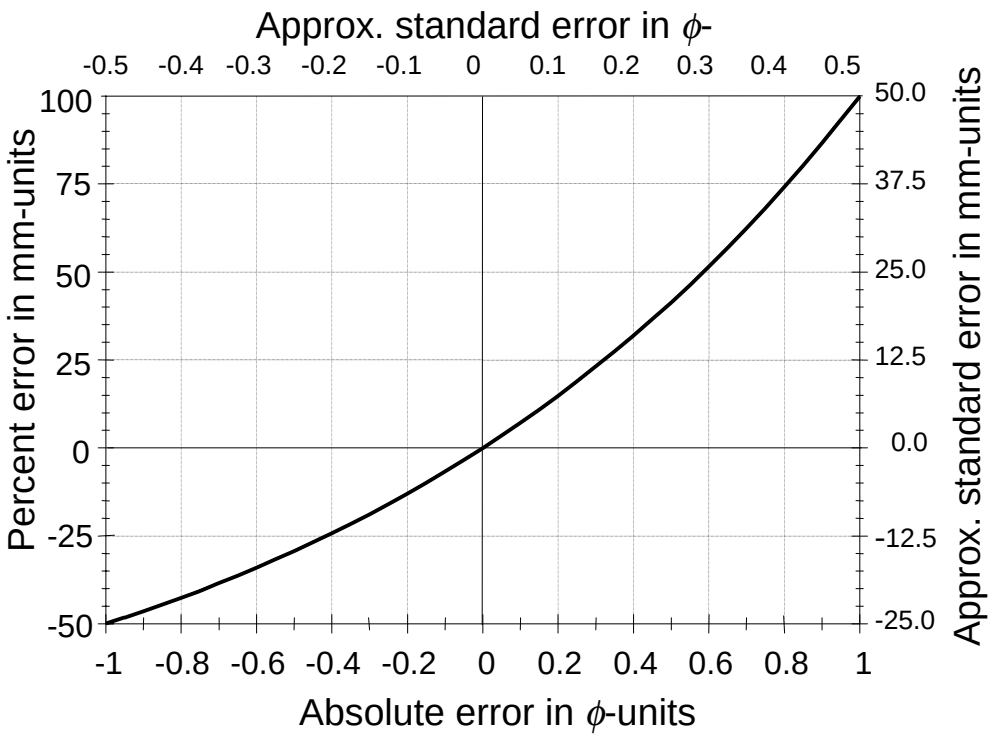

Fig. 5.9: Conversion of absolute and standard errors, in ϕ-units to percent error in mm-units. $t_{1-\alpha/2,n-1}$, or $Z_{1-\alpha/2}$ were approximated by the numerical value of 2.

The distribution of bed-material particle sizes in gravel-bed rivers is rarely symmetrical, not even when large sample sizes with smooth frequency distributions are collected in large streams (Fig. 5.8). Coarse gravel beds are usually slightly skewed towards a tail of fines particles. Therefore, standard or absolute errors are usually not paired, nor is the standard error of any percentile identical to the one computed when an underlying Gaussian distribution is assumed. In bed-material size distributions that are skewed towards a fine tail, the computed bootstrap error around the ϕ_{84} is smaller than the error around the D_{84} computed from an assumed symmetrical normal distribution (Fig. 5.10). By contrast, the bootstrap error around the ϕ_{16} is larger than the error around the ϕ_{16} computed for an assumed Gaussian distribution. The more asymmetrical the particle-size population, the larger the difference between the error of paired percentiles, e.g., the D_{16} and D_{84}.

Similarly, the more asymmetrical the particle-size population the larger the difference between the standard or absolute errors for given percentiles obtained by bootstrapping compared to those obtained by assuming an underlying Gaussian distribution.

Note however, that the absolute error $e_{\pm\phi 50}$ obtained from the bootstrap approach for the D_{50} particle size at the Mamquam River is similar to the absolute error around the mean $e_{\pm\phi m}$ computed by the general sample-size equation (Eq. 5.2).

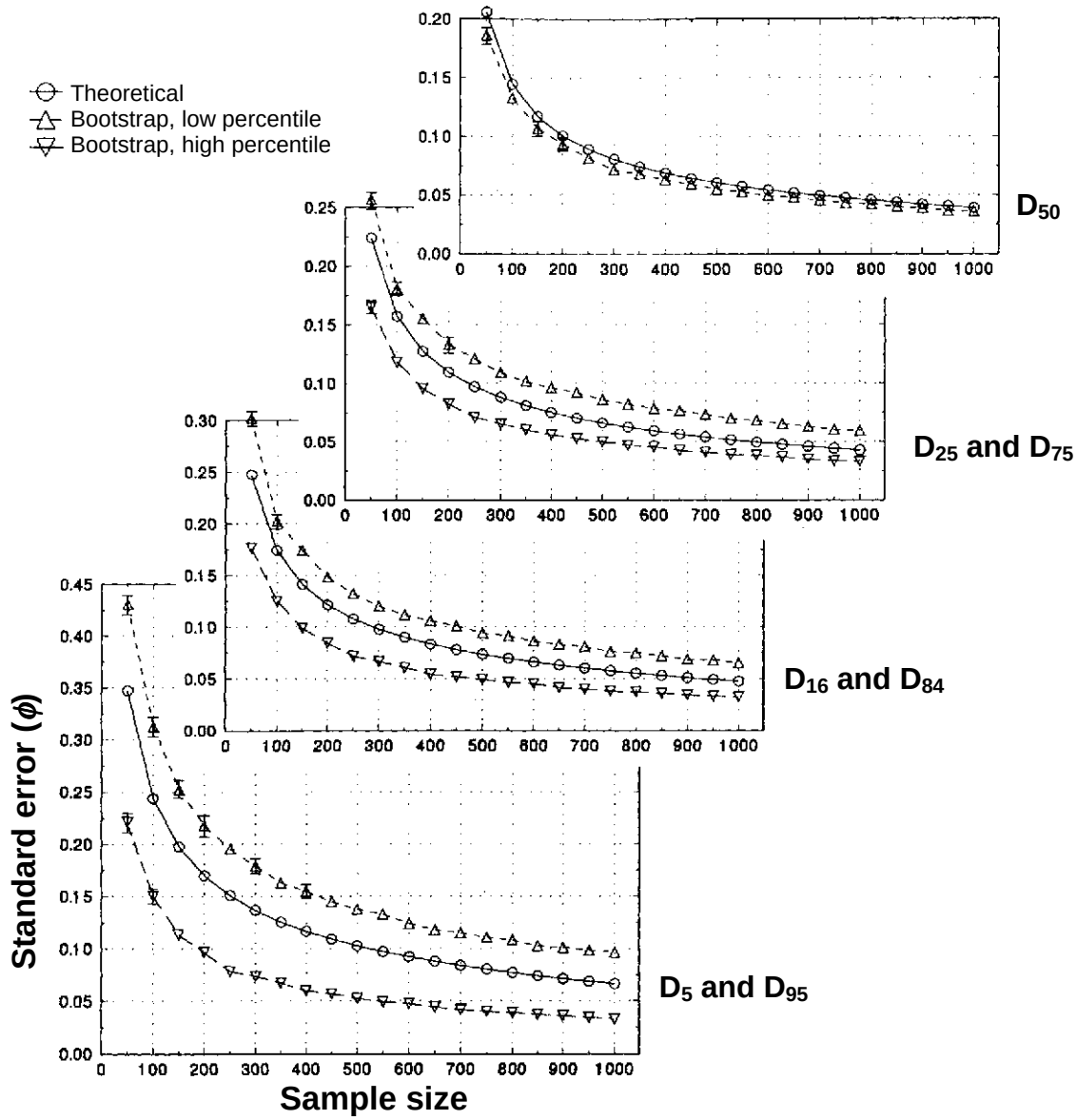

Fig. 5.10: Percentile standard errors for various sample sizes obtained from bootstrapping and from assuming an unskewed Gaussian distribution. (Reprinted from Rice and Church (1996b), by permission of the Society for Sedimentary Geology).

Rice and Church (1996b) compared not only the bootstrap percentile errors with the percentile errors computed for an assumed normal distribution, but also for a skewed normal distribution. The question was whether fitting a skewed normal distribution to the parent population would remove the difference between the bootstrap error and the error computed analytically for a best-fit skewed normal distribution. The assumption of a skewed normal distribution did not achieve a better agreement between bootstrap error and analytical error (Fig. 5.11). Particularly disconcerting was the unreliability of the

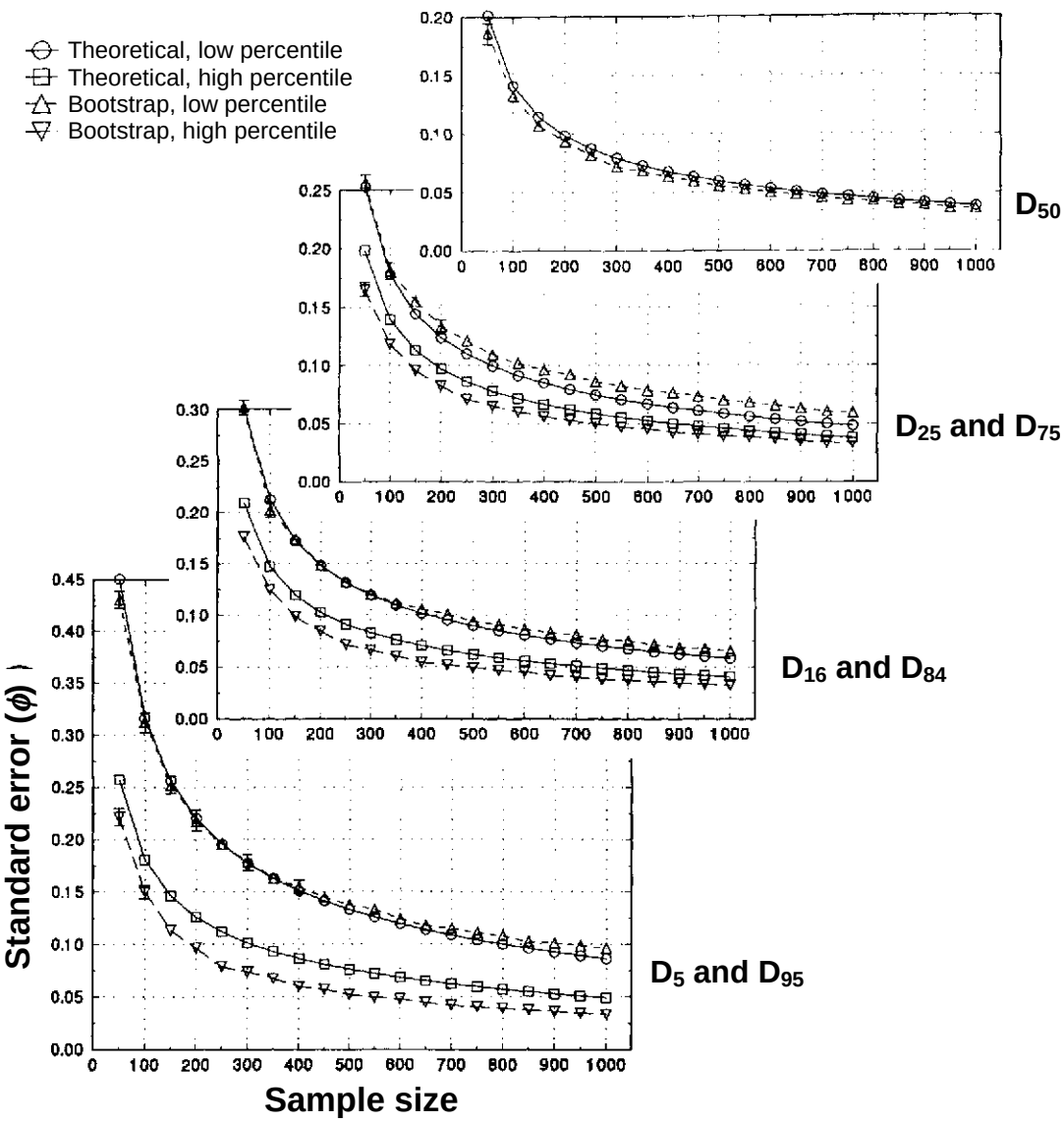

Fig. 5.11: Percentile standard errors for variable sample size obtained from bootstrapping and from assuming a skewed normal distribution. (Reprinted from Rice and Church (1996b), by permission of the Society for Sedimentary Geology).

improvements: while errors around some percentiles were well predicted when assuming a underlying skewed normal distribution, errors around other percentiles were greatly over- or underpredicted (Fig. 5.11). The only percentile for which there is relatively little difference between sample-size requirements from a bootstrap approach and those computed from assuming an underlying symmetrical or skewed Gaussian distribution is the ϕ_{50}. This similarity indicates that bootstrapping is not necessary if the percentile in question is the D_{50}, or if a pilot study indicates that surface bed-material sizes could be approximated by a Gaussian distribution. If the distribution does not approach a normal distribution, and the percentile of interest is a high or low percentile, then bootstrapping or

a multinomial approach should be used. Alternatively, a two-step approach for percentiles in ϕ-units may be used (Section 5.2.3.1). Bootstrapping becomes particularly useful for skewed and bimodal distributions, since the latter has no formal sample-size criteria (Rice and Church 1996b).

Software that includes bootstrapping procedures is commercially available, such as the program "Resampling Stats" from Resampling Stats Inc[1]. which has an add-in version for the spreadsheet program Microsoft Excel.

Collecting the large sample necessary for bootstrapping is problematic in mountain streams

A thorough bootstrapping analysis requires taking a large field sample to characterize the parent population. Rice and Church (1996b) used a sample of more than 3,500 particles. Sample sizes that large may be possible to obtain only in the beds of large streams that have large areas of homogeneous particle-size mixtures. Sampling several thousand particles is a problem in mountain gravel bed-rivers. If, for example, a stream is 10 m wide, and four particles are sampled per 1-m section along a transect using the sampling frame (Section 4.1.1.6), 40 particles can be sampled per transect. Almost 90 transects would have to be sampled to obtain 3,500 particles such as in the study by Rice and Church (1996b). If transects were spaced at about 2 m intervals, a homogeneous reach almost 200 m long would have to be sampled. Homogeneity over a 200 m stream segment could perhaps be expected in a plane-bed stream, but not in a riffle-pool stream that, if 10 m wide, has about 4 riffle-pool sequences over a 200 m distance. However, even though a bootstrap approach may not be feasible in a mountain gravel-bed river, the knowledge gained from the bootstrap study by Rice and Church (1996b) about percentile errors in skewed distributions as opposed to symmetrical ones is quite valuable and should be considered when estimating errors around high or low percentiles in skewed distributions.

5.2.3.5 Summary: the relation between sample size and error

Beneficial effect of sampling tapers off for large sample sizes

Sampling precision increases as the reciprocal of the square root of sample size n (standard error $s_p \approx 1/\sqrt{n}$). Thus, sampling precision improves dramatically as n increases at small values of n, but the improvement becomes insignificant for high values of n. For the bed material of the Mamquam River with $\sigma = 1.17\ \phi$, Rice and Church (1996b) determined the cutoff point beyond which further sampling does not significantly improve sampling precision is at a sample size of 400 particles.

Relation between sample size, sorting, and error

The relation between sample size, sorting, and error $n = (t \cdot s/e)^2$ is such that halving the acceptable error margin e, or doubling of bed-material sorting s leads to approximately a

[1] *Resampling Stats, Inc., 612 N/ Jackson St., Arlington, VA 22101; Web-page: http://www.resample.com;*

fourfold increase in sample size n, and a doubling in sorting doubles the error for a given sample size (Section 5.2.2.7). The relation between sample size, sorting, and error around the mean is visible in Fig. 5.2. A 400-particle surface sample in moderately-sorted bed material ($s = 1\ \phi$) yields an approximate absolute error of $\pm\ 0.1\ \phi$ around the mean, whereas sampling only 100 particles increases the absolute error margin to approximately $\pm 0.2\ \phi$. In more poorly sorted bed material with $s = 2\ \phi$, sampling 400 particles leads to an absolute error around the mean of $\pm 0.2\ \phi$, and 100 particles to an error of $\pm 0.4\ \phi$.

Comparison: one-step and bootstrap approach

The relation between sample size, sorting, and error around the mean is similar to the relation between sample size, sorting, and error around the D_{50} established by Rice and Church (1996b) in a bootstrap approach. For the Mamquam River with a standard deviation of 1.17 ϕ, a sample size of 400 particles resulted in an absolute error around the D_{50} of $\pm 0.122\ \phi$, whereas an absolute error of $\pm 0.115\ \phi$ around the mean particle size ϕ_m was computed for a 400-particle sample by the general sample size equation (Eq. 5.2).

For percentiles other than the D_{50}, results from the bootstrap approach and an assumed normal distribution differ and the difference increases towards the tails of the distribution. The bootstrap approach indicates that for distributions skewed towards a tail of fine particles, sample error is significantly lower for high percentiles than for low percentiles. Consequently, it takes a considerably larger sample size to accurately characterize low percentiles (D_5, D_{16}) than high percentiles (D_{84}, D_{95}) in distributions skewed towards a fine tail. Percentiles between D_{50} and D_{95} require nearly the same sample size for a given precision (Table 5.6 and Fig. 4.2 in Section 4.1.1.3).

The poor precision of low percentiles for a given sample size in distributions skewed towards a fine tail results from the relative scarcity of fine gravel particles in coarse gravel and cobble-bed streams. In a 100-particle pebble count from a coarse gravel bed, the number of counts per size class typically varies between 0 and 5 for each of the finest 5 or 8 size classes (excluding sand). However, each of the coarsest 4 or 5 size classes (except the very largest size class) might have 10 or 20 counts. The addition of one more count in any of the fine size classes cause more change in the percentile particle size of that size class than the addition of one more count to a coarse size class that has already 10 or 20 counts. This results in more uncertainty in the quantification of the low percentiles.

Comparison: bootstrap and empirical results

Results from the bootstrap analysis compare well with results from empirical studies conducted in mountain gravel-bed streams (Section 4.1.1.3, Fig. 4.2) with distributions skewed towards fines and sorting coefficients of $\approx 1.2\ \phi$. When a sampling frame (Section 4.1.1.6) was used to reduce operator bias in particle selection in pebble counts, repeated pebble counts on riffles in various streams had total absolute errors $e_{\pm\phi p}$ of ± 0.1 to $\pm 0.15\ \phi$ around all percentiles between the D_{50} and D_{95}. This range of total absolute errors is quite similar to the bootstrap errors established for the slightly skewed distribution from the

Mamquam River and suggests when sampling large gravel and cobbles, operator errors do not significantly contribute to the total error.

The absolute error of ±0.42 ϕ obtained for the D_5 particle size of pebble counts in mountain streams exceeded the bootstrap error around the D_5 of ±0.30 ϕ. The poor accuracy for samples of small particles in pebble counts is attributable to the (inconsistent) operator bias against small particles and should be disconcerting for studies concerned with the amount of surface fines. Sampling accuracy for small particles requires not only larger sample sizes than are required for large percentiles, but requires sample sizes even larger than predicted from appropriate sample-size statistics to account for operator bias against fines.

Comparison: One-step and bootstrap with multinomial approach

Fig. 5.7 illustrates error bands computed around the example distribution presented in Section 2.1.4.1 (s = 1.94 ϕ, $sk_{aF\&W}$ = 0.17, sk_{frq} = 0.72) for various sample sizes using the multinomial approach. An absolute error in mm around a given percentile can be obtained from Fig. 5.7 as the horizontal distance between the error band and the sample distribution. The absolute error around the D_{50} is approximately ±0.4 ϕ units for a sample size of 400 and increases to nearly ±0.8 ϕ for a sample size of 100. Therefore, the absolute error predicted for the D_{50} from the multinomial approach is approximately twice as large as the absolute error around the mean computed from the one-step approach (Eq. 5.2).

The error bands computed with the multinomial approach for the skewed distribution described in Section 2.1.4.1 indicate a larger absolute error for small percentiles than for large percentiles. These figures are similar to the bootstrap results for the skewed distribution from the Mamquam River.

5.2.4 Detectability of change in percent fines (Bevenger and King 1995)

Natural or anthropogenic disturbances in the watershed or the riparian area may lead to elevated amounts of fine sediment in a streambed. The amount of fine sediment that impairs aquatic habitat depends on the species of concern, the benthic community, and bed-material properties. Monitoring fine sediment can be used to observe and evaluate the effects of change in the natural conditions of the watershed or in watershed management.

Fine sediment supplied to a mountain gravel-bed stream accumulates primarily in the interstitial spaces of the subsurface sediment and in backwater areas. Accumulations of fines in the surface sediment of the general streambed are relatively scarce. Taking volumetric samples of the subsurface is time and labor consuming, however. To simplify and accelerate the sampling process, Bevenger and King (1995) proposed sampling and analyzing the amount of surface fines in the bed using a (zigzag[2]) pebble-count procedure.

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[2] *The operator walks a zigzag course from bank to bank picking pebbles from the streambed at intervals spaced about 7 feet apart, and covers about a hundred meters of stream section (Bevenger and King 1995), (Section 6.2.2.1). The 7-foot interval was chosen to reduce serial correlation in the samples particles and more closely adhere to the statistical independence assumptions of the analysis.*

The statistical error associated with small percentiles is usually relatively large (Section 4.1.1.3). Bevenger and King (1995) therefore specified the sample size necessary for detecting differences in the percent fines obtained from two pebble counts using a 2 x 2 contingency table analysis. One of the pebble counts is carried out in a *reference* reach, which means before the reach was impaired or in an unimpaired reference reach that serves as "background". The second pebble count is performed in the *study* reach, which means in the reach in which the percent fines may have changed over time. The sample size necessary to detect a change in the percent fines of the study reach depends on four factors:

- Sample size at the reference site
  Sample size at the study site has to be larger if the sample size at the reference site is small, and can be smaller, if a large sample was taken at the reference site.

- Percent of fines at the reference site
  A larger sample must be taken at the study reach if the reference reach has a high percentage of fines (i.e., sandy gravel-bed streams). A smaller sample can be taken when the percent fines at the reference site is small (i.e., gravel beds with little sand).

- The minimum difference in the percent fines to be detected between the reference and study site
  Detecting a small difference in the percent fines between study and reference sites requires a larger sample size than is needed to detect a larger change in the percent fines.

- Acceptable risk levels in terms of Type I and Type II error
  Type I error is the risk of falsely concluding a significant difference between the two samples and is typically set at $\alpha = 0.05$. Type II error is the risk of falsely concluding that there is no difference and is typically set to $\beta = 4\alpha = 0.20$. Type I and Type II errors are inversely proportional for a given sample size. That is, a decrease in one necessarily results in an increase of the other. If the occurrence of a difference in the grain-size distribution is as important as the occurrence of no difference, then both α and β are set to 0.05.

5.2.4.1 Sample-size determination from diagrams

The required sample size depends on combinations of the four factors mentioned above. Thus, Bevenger and King (1995) provided multiple plots with several curves each (Figs. 5.12 and 5.13) to specify the sample size at the study site for different values of the four factors. The following five steps are taken to determine sample size from the diagrams:

1. Determine the risk levels for Type I and Type II error and select the appropriate figure (Fig. 5.12 or 5.13).

2. Determine the percent fines at the reference site and select the appropriate plot from Fig. 5.12 or 5.13. For 10% fines at the reference site (p_r = 0.10), select the plot "10% fines at reference site" in Fig. 5.12 or 5.13.

3. Determine the detrimental percent fines at the study site. For example, 19% fines at the study site (p_s = 0.19) may be a threshold value for impairing aquatic habitat. The necessary minimum detectable difference between study and reference site must then be $p_s - p_r = 0.19 - 0.1 = 0.09$ or 9%.

4. Determine the sample size taken or to be taken at the reference site and select the corresponding graph for 100, 150, 300, 450, or 600 particles on the diagram.

5. On the appropriate diagram in Figs. 5.12 or 5.13 locate a minimum detectable difference of 0.09 on the vertical axis, and determine the sample size at the study-site at the intersection of a minimum detectable difference of 0.09 with the respective graph for reference-site sample size.

For example, the plot for 10% fines at the reference site in Fig. 5.12 indicates that a 300-particle sample (stippled line) at the reference site requires another 160 particles to be

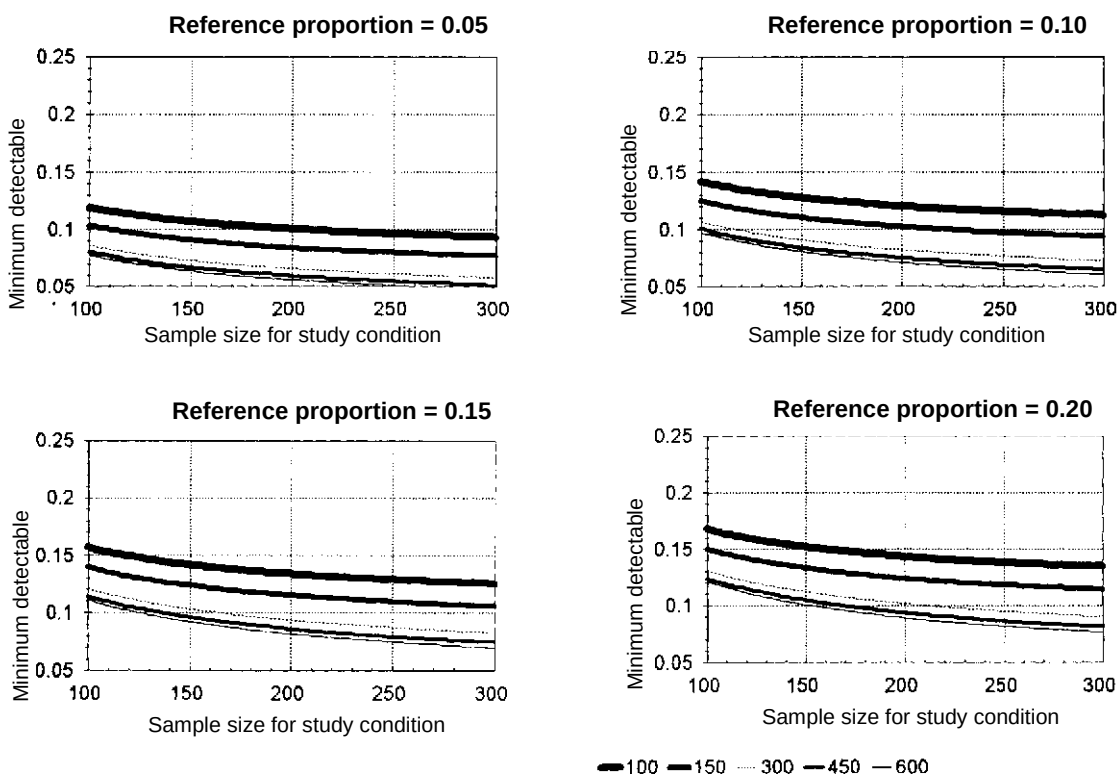

Fig. 5.12: Sample size necessary at the study site to detect a minimum difference in percent fines between the study and the reference site $p_s - p_r$ for various reference-site sample sizes and risk levels. Risk levels for type I and type II errors are set to $\alpha = 0.05$ and $\boldsymbol{\beta} = \mathbf{0.20}$. (Reprinted from Bevenger and King (1995)).

collected at the study site. A 600- particle sample at the reference site reduces the sample size at the study site to 120. Likewise, if 150-particle had been sampled at the reference site, a little over 300 particles have to be sampled at the study site.

If the tolerable percent fines at the study site was 12% (p_s = 0.12), and the reference site had 5% fines, the required minimum percent difference that needs to be detectable between study and reference site is p_s - p_r = 0.07. In this case, the plot for 5% fines at the reference site in Fig. 5.12 indicates that at 300-particle sample at the reference site requires at least 170 particles to be sampled at the study site.

Fig. 5.13 is used if both the Type I and Type II error are set to a confidence level of 95% (α and β = 0.05). If there are 10% fines at the reference site, and the tolerable percent fines at the study site is 20%, the minimum difference to be detected is p_s - p_r = 0.10. The plot for 10% fines at the reference site in Fig. 5.13 indicates that about 290 particles need to be sampled at the study site, if 300 particles had been sampled at the reference site.

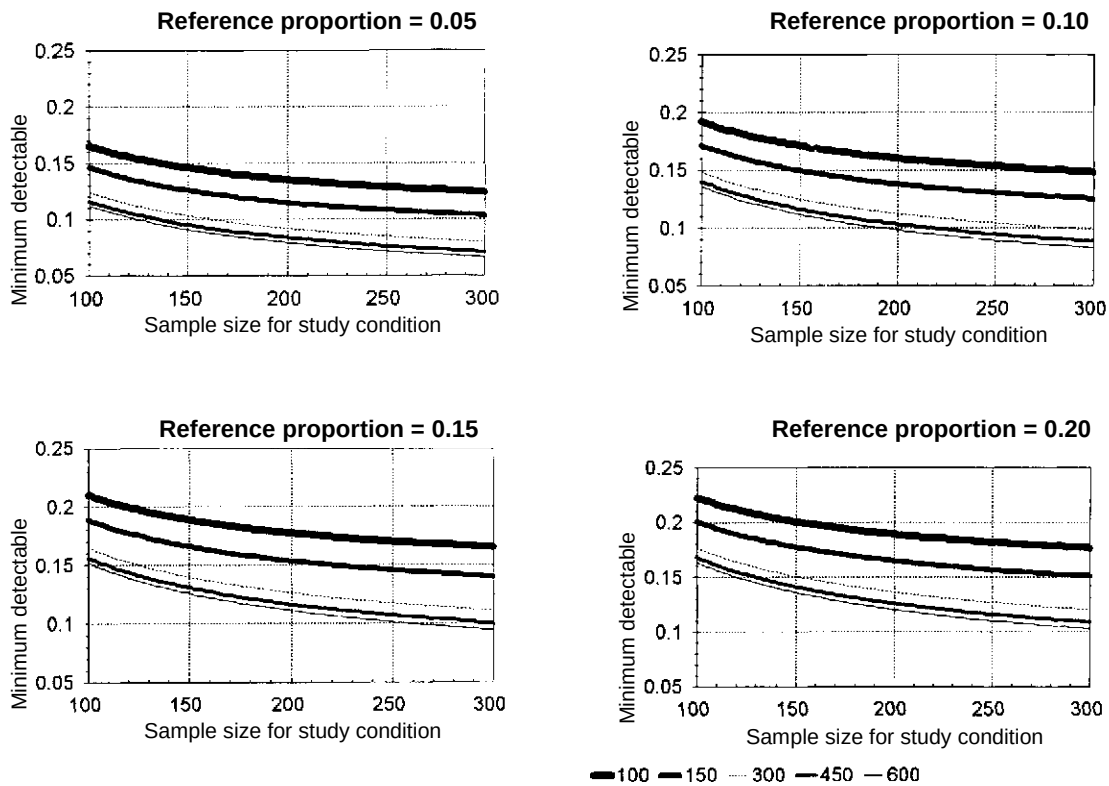

Fig. 5.13: Sample size necessary at the study site to detect a minimum difference in the percent fines between the study and the reference site p_s - p_r for various reference-site sample sizes and risk levels. Risk levels for Type I and Type II errors are set to α = 0.05 and $\boldsymbol{\beta}$ **= 0.05**. (Reprinted from Bevenger and King (1995)).

This yields a total sampling effort of 590 particles. If 600 particles had been collected at the reference site, 195 particles would have to be collected at the study site, and this increases the total sample size to 795 particles. If 100 particles were sampled at the reference site, an extrapolation of the graph for 100 particles would intersect the line for a minimum detectable difference of 0.1 at more than 1,000 particles for the study site and result in a total sample size of more than 1,100. Little information about a change in the percent fines is gained when small samples are collected both at the reference and the study site. If 100 particles were sampled at each site, the minimum detectable difference is only 0.18. Given 9% fines at the reference site, a 100-particle pebble count can at best detect a doubling of the percent fines at the study site (Potyondy and Hardy 1994; King and Potyondy 1993).

The total sampling effort can be minimized if the same number of particles are sampled at both sites. Table 5.7 indicates, for α and β = 0.05, that sampling 293 particles at both sites results in the smallest total sample size (586 particles). In order to optimize the study effort and to find the smallest total sample size that will detect a given difference pilot studies should be conducted to estimate the percent fines at both the reference and the study sites. The result can then be used for estimating the optimum sample size.

Table 5.7: Equal and unequal sample sizes for p_r = 0.10 and p_s = 0.20, and preselected values of α and β (from Bevenger and King 1995).

	Equal sample Size			Unequal sample Size					
	α			α					
	0.01	0.05	0.10	0.01		0.05		0.10	
β	n_r, n_s	n_r, n_s	n_r, n_s	n_r	n_s	n_r	n_s	n_r	n_s
0.01	566	417	347	848	424	635	318	534	267
0.05	419	293	236	617	309	439	220	357	179
0.10	349	236	185	510	255	350	175	278	139
0.20	275	177	134	394	197	257	129	197	99

5.2.4.2 Sample-size computation

The statistical background for the procedure presented by Bevenger and King (1995) is provided by Fleiss (1981). Sample size for a tolerable risk level is based on the acceptance or rejection of the null hypotheses that the difference between the proportion of the percent fines at the reference site p_r and the study site p_s is either 0 (no difference), or > 0 (i.e., there is a difference). Minimum sample size n_s for the pebble count at the study site can be calculated from Equations 5.26 and 5.27 (Fleiss 1981). If sample size at the reference site n_r is negotiable, and if there is no reason for different sample sizes between the two sites, an equal sample size should be selected for the reference and study sites as it results in the smallest combined sample size $n_s + n_r$. The smallest sample size for both the study and reference site can be computed from:

$$n_s = \frac{n'}{4}\left(1 + \sqrt{1 + \frac{4}{n' \cdot \ |p_s - p_r|}}\right)^2 \tag{5.26}$$

with

$$n' = \frac{\left(z_\alpha \sqrt{p_r + p_s \cdot \left(1 - \frac{p_r + p_s}{2}\right)} - z_{1-\beta}\sqrt{p_r \cdot \ (1 - p_r) + p_s \cdot \ (1 - p_s)}\right)^2}{(p_s - p_r)^2} \tag{5.27}$$

where z_α and $z_{1-\beta}$ refer to the ordinates of the standard normal distribution, and the subscripts α and β refer to the risk levels of the error Type I and II. Commonly used values for z_α and $z_{1-\beta}$ are:

α or β :	0.01	0.05	0.10	0.20
z_α (neg.values); $z_{1-\beta}$ (pos.values):	(-) 2.327	(-) 1.645	(-) 1.282	(-) 0.842

<u>Example 5.11:</u>
If the reference site has 10% fines < 8 mm (p_r = 0.10), and it is desirable to detect an increase in the percent fines to 20% or more at the study site (p_s = 0.20), with acceptable risk levels of α = 0.05 and β = 0.20, sample size at the study site is computed from Eq. 5.26 and 5.27:

$$n' = \frac{\left(-1.645\sqrt{0.1 + 0.2 \cdot \left(1 - \frac{0.1+0.2}{2}\right)} - 0.842 \cdot \sqrt{0.1 \cdot \ (1 - 0.1) + 0.2 \cdot \ (1 - 0.2)}\right)^2}{(0.2 - 0.1)^2}$$

$$n' = \frac{\left(-1.645 \cdot \sqrt{0.3 \cdot \ 0.85} - 0.842 \cdot \sqrt{0.1 \cdot \ 0.9 + 0.2 \cdot \ 0.8}\right)^2}{0.1^2}$$

$$n' = \left(\frac{-0.831 - 0.421}{0.1}\right)^2 = 157$$

and

$$n_s = \frac{157}{4} \cdot \left(1 + \sqrt{1 + \frac{4}{157 \cdot |0.2 - 0.1|}}\right)^2$$

$$n_s = 39.2 \cdot (1 + \sqrt{1.255})^2 = 177$$

Modified forms of Eqs. 5.26 and 5.27 are used if the sample size for the reference and study site are different; however, the ratio of sample size at the reference site and the study site needs to be known prior to the computation. The reader is referred to the source literature by Bevenger and King (1995) and Fleiss (1981) for this case.

5.2.4.3 Operator error in the percent fines adds to the statistical error

Statistical computations of sample size, including the computation of sample size by (Bevenger and King 1995) refer to the *statistical* error associated with a certain sample. Computation of sample size, including the computation of sample size by Bevenger and King (1995) only refer to the sample size needed to avoid a statistical error. However, the user must keep in mind that operators introduce further sampling errors that are not included in the computed statistical sampling error but nevertheless add to it. Operators commonly bias against fine particles (Section 4.1.1.3), because fine particles may be partially hidden between large particles, and because large particles are more likely to be touched and selected in a pebble count than fine particles. Fine particles also tend to accumulate in locations of the streambed that are poorly accessible, such as in pools or under overhanging branches near the banks. Inaccessibility makes fine particles less likely to be included in a pebble count. Operator errors and bias against fines are not included in a computed relation between sample size and statistically detectable error in the percent fines. The actual minimum detectable error in the percent fines is therefore smaller than computed from the statistical analysis. To account for this neglect, a sample size larger than predicted is required to detect a given change in the percent fines. Operator bias against fines as well as the variability of sampling results between operators can be reduced by using a sampling frame (Bunte and Abt 2001) (Section 4.1.1.6). Sampling the streambed in a systematic pattern along even-spaced transects spanning the full bankfull width of the stream further assists in reducing operator errors with respect to sampling fines.

5.3 Areal sampling: area-based sample-size recommendations

In contrast to pebble counts or grid samples that collect a predetermined number of particles from a transect or a grid, areal samples collect all surface particles contained in a specified (small) sampling area (Section 4.1.3). Thus, sample size may be described in terms of the size of the area that needs to be sampled. The size of the sampling area may be based on geometrical consideration, such as a multiple of the area covered by the D_{max} particle size. Alternatively, a two-stage sampling approach may be applied to specify the

number of subsamples needed from a homogeneous deposit to attain a specified sampling precision for the median particle size (Section 5.3.2). A multinomial approach may be used to compute the percentile error for the entire distribution (Section 5.3.3)

5.3.1 D_{max} and geometrical considerations

Diplas (1992a) and Diplas and Fripp (1992) suggested that an areal sample should cover a sampling area equal to at least 100 times the area of the D_{max} particle size in order to provide a relatively high precision for all percentiles. Fripp and Diplas (1993) increased this sample-size recommendation to 400 time the area of the D_{max} particle. This increase ensured that the volume of the sample would satisfy De Vries' (1970) "low precision" criterion with a relative error of 10% (Sect. 5.4.1.1).

The area of one individual areal sample is usually small (about 0.1 m^2) and several individual areal samples need to be combined for the total sample. The total sampling area A_{tot} can be estimated from a multiple of the exposed area of the largest particle(s)

$$A_{tot} = 400\ D_{max}^{\ 2} \tag{5.28}$$

<u>Example 5.12:</u>
The D_{max} particle size of a deposit is estimated at 40 mm, the upper range of particle sizes suitable for adhesive sampling (Section 4.1.3.2). If a spherical particle shape is assumed, the area covered by an individual particle A_p with a 40 mm *b*-axis size is

$$A_p = \pi \cdot \left(\frac{b}{2}\right)^2 = \pi \cdot \left(\frac{0.04\ \text{m}}{2}\right)^2 = 0.00126\ \text{m}^2$$

If an ellipsoidal particle shape with the *a*-axis 1.5 times the *b*-axis is assumed, the area covered by one particle increases to

$$A_p = \pi \cdot \left(\frac{a \cdot b}{4}\right) = \pi \cdot \left(\frac{0.06\ \text{m} \cdot 0.04\ \text{m}}{4}\right) = 0.00189\ \text{m}^2$$

An intermediate particle area of 0.00160 m^2 is obtained if a square particle shape is assumed with $A_p = b^2$. Using $A_p = b^2$, the total sampling area $A_{tot} = 400 \cdot 0.0016\ m^2 = 0.64\ m^2$, which is an area of 0.8 by 0.8 m.

If one areal sample covers approximately 0.1 m^2, Eq. 5.28 suggests that 6 – 7 of those areal samples should be collected in order to sample an area of sufficient size and to gain sufficient material for a particle-size analysis.

5.3.2 Two-stage sampling: specified error around the median

ISO (1992) proposes a two-stage approach for defining the minimum sampling area. ISO (1992) advises that the minimum sampling area for each individual sample A_s must be larger than 8 times the area of the D_{max} particle size in order to avoid bias towards the largest particles. For a D_{max} particle size of 40 mm, each individual areal sample should be at least 0.11 by 0.12 m = 0.013 m^2 in size.

Computation in mm units

A two-stage approach can be used to determine the relation between sample size and absolute error around central percentiles of the distribution (Section 5.2.3.1). A number of q areal subsamples are collected, and the median particle size is computed for each of the q subsamples, either graphically from cumulative distribution curves, or by linear interpolation between percentiles (Section 2.1.4.2). ISO (1992) suggests using the median particle size D_{50} in units of mm, assuming that the q values of D_{50} are approximately normally distributed. This guideline document recommend using the median particle size ϕ_{50} in units of ϕ (see below).

The sample standard deviation s_{50} of the q values for D_{50} is determined from

$$s_p = \sqrt{\frac{\sum_{i=1}^{q}(D_{50} - D_{50m})^2}{q\text{-}1}} \tag{5.29}$$

D_{50m} is the arithmetic mean particle size in mm of the D_{50} values obtained from the q samples. An appropriate value for $t_{1\text{-}\alpha/2,q\text{-}1}$ is selected from Table 5.2. Eq. 5.30 can then be used to calculate the number of subsamples q so that there is only a 5% chance (at α = 0.05) that the absolute difference (positive or negative) between the estimated values of the percentile in question D_{50} and the true population D_{50} is larger or equal to the acceptable absolute error $e_{\pm D50}$. The absolute error is the difference (in mm) between the sample and the population D_{50}. Note that Eq. 5.30 may have to be solved iteratively (see Example 5.13)

$$q = \left(\frac{t_{1\text{-}\alpha/2;q\text{-}1}}{e_{\pm D50}} \cdot \quad s_{50}\right)^2 \tag{5.30}$$

The total sampling area for one complete areal sample is $A_{tot} = A_s \cdot \quad q$.

Example 5.13:

D_{max} is estimated as 40 mm and D_{50} is the percentile of interest. Sample area A_s required for avoiding bias against large particles in a subsample is $8 \cdot 0.04^2$ m^2 = 0.0128 m^2. Five subsamples were collected with five closely-spaced values for D_{50} of 22, 25, 27, 30, and 32 mm. From Eq. 5.29, the sample standard deviation s for the D_{50} percentile is computed to be 3.96 mm. The acceptable error around the D_{50} particle size is 5 mm.

Eq. 5.30 needs to be solved iteratively when t-statistics are used. An arbitrary sample size of 10 subsamples is selected in the first trial of Eq. 5.30 and yields a sample size of 3.2. Estimated and computed subsample size q do not yet correspond. After four subsequent trails, correspondence is reached for a subsample size of 5.

Trial	q_{est}	q-1	$t_{1-\alpha/2;q-1}$	q_{comp}
1	10	9	2.262	3.2 ≅ 4
2	4	3	3.182	6.4 ≅ 7
3	7	6	2.447	3.7 ≅ 4
4	5	4	2.776	4.8 ≅ 5

Taking 5 subsamples from a total area of $A_{tot} = 5 \cdot 0.013$ m^2 = 0.065 m^2 (about 0.25 by 0.26 m) provides a 95% probability that the sample D_{50} size is within ± 5 mm of the population D_{50}. This is a sampling area about 10 times less than predicted from Eq. 5.28.

If the 5 subsamples were more different and had D_{50} sizes of 14, 19, 27, 33 and 39 mm, and a standard deviation of 10.139, the iterative solution of Eq. 5.30 yields $(2.101 \cdot 10.139/5)^2 = 18.15$ which is rounded up to 19. The total area covered by the subsamples is $A_s = 19 \cdot 0.013$ m^2 = 0.247 m^2 (about 0.49 m by 0.50 m). The total sampling area in this example is much larger than in the previous example because the spread (variance) of the 5 values of D_{50} is much larger. Nevertheless, the sampling area computed from Eqs. 5.29 and 5.30 is still less than half the total sampling area computed from Eq. 5.28.

Note that the two-stage approach computes only the precision for the particular set of subsamples used in the computation. The precision associated with a given sample size would have to be computed numerous times, each time with a newly collected set of subsamples, in order to compute the mean precision associated with a specified subsample size in a specified sampling area. This topic is discussed in more detail in Section 5.4.2.1.

Computations in units of ϕ

It is recommended to apply the two-stage approach to median particle-sizes in units of ϕ, rather than the median in mm, because values of ϕ_{50} from several subsamples approximate a normal distribution better than the values of D_{50} (Triola 1995; Section 5.2.3.1). D_{50} in Eq. 5.29 is then substituted by values of ϕ_{50}.

5.3.3 Multinominal approach

Sample sizes computed from simple geometric approximations such as sampling area A = 100 or 400 D_{max}^2 (Eq. 5.28) yield relatively large sample sizes in order to provide a relatively high precision for all percentiles. However, a concrete relation between sample size and error around a given percentile and selected confidence level is not obtained from Eq. 5.28. In order to specify a relation between sample size and error, Petrie and Diplas (2000) suggest a multinomial approach to compute the size of the sampling area. The multinomial approach is applied in two steps. The first step computes the number of particles needed for a grid sample (Section 5.2.3.3). The second step converts this number of particles to the size of a sampling area. A factor is needed for this conversion, and its numerical value depends on the packing of surface particles and the proportion of surface voids. For a voidless surface, the Kellerhals and Bray (1971) conversion coefficients (Section 4.3.1), may be used. Other conversion factors may be needed for other surface conditions (Sections 4.3.2 and 4.3.3). The reader is referred to the original literature by Petrie and Diplas (2000) for details.

5.4 Volumetric sampling: mass-based sample-size recommendations

Sample mass required for representative volumetric samples can be computed by three methods:

- As an empirical function of the D_{max} particle size,
- By computing the number of subsamples required (two-stage approach), and
- By analytical means based on an assumed underlying distribution type.

A large number of empirical equations exist in which sample mass is expressed as a function of the D_{max} particle size (Sect. 5.4.1). These equations are simple to apply, but different equations predict greatly different sample sizes. Sample-mass recommendations based on the D_{max} particle size do not require assumptions about an underlying frequency distribution type. Sample sizes predicted from empirical functions of D_{max} are generally large, but they do not provide information about the relationship between sample mass and error. Therefore, the precision of a sample remains unknown.

If the precision of a sample needs to be known, sample-mass equations should be employed that provide information on the relation between sample mass and precision. One possibility is a two-stage sampling approach in which a number of subsamples is collected (Section 5.4.2.1). Based on the central limit theorem, the precision of any percentile[3] in a distribution can be computed for various samples sizes. However, the precision obtained for a specific sample size, e.g., three subsamples, is not the same for any set of three subsamples from a deposit because each subsample is (slightly) different. Therefore, many sets of three subsamples would have to be collected to obtain the mean precision for a subsample size of three. The requirement for collecting a specified number of subsamples repeatedly can be bypassed by plotting the precision for various (unrepeated) subsample sizes. The data will scatter, but fitting a power- or exponential function through data points provides a surrogate relation between sample size and precision for a given percentile in a given deposit (Section 5.4.2.2).

A large physical sampling effort can be reduced by using a bootstrap procedure. Bootstrapping is a technique that collects repeated samples (by computer) from a parent population. The parent distribution might be generated by a computer based on specifications of the actual deposits (standard deviation and mean) that are obtained from a pilot study and an assumed distribution type (Section 5.4.3). The computational effort of bootstrapping is rather large and may require using a resampling program. The main drawback is that the computer-generated sample cannot be a perfect surrogate for a large sample from a distinct parent distribution. Bootstrapping may also be applied to a parent distribution of an actual bed-material sample (5.2.3.4) that is entered into the computer. The sample needs to be (usually prohibitively) large in order to accurately describe the parent population and all particles must be collected independently of each other.

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[3] *Two-stage approach is better suited for central percentiles. Peripheral percentiles require a larger sample size to reach normality.*

5.4.1 Sample mass as a function of largest particle size

Why use the D_{max} particle size?

Sample-size statistics that assume an underlying normal distribution indicate that a larger sample size is required to accurately describe the distribution tails than the central parts of the distribution. Consequently, a sample size that is sufficiently large to describe the distribution tails will also suffice to accurately describe the entire particle-size distribution.

The coarse tails of bed-material samples from gravel- and cobble bed streams are comprised of only a few large particles per size class which nevertheless contribute a rather large proportion of the total sample weight. Presence or absence of one or a few large particles in the distribution tail influences not only the percentiles of the coarse tail, but central and fine percentiles as well. Therefore, a volumetric sample needs to be sufficiently large so that coarse particles are representatively included in the sample. Because representatively sampling the coarse tail ensures accuracy for the entire distribution, sample mass is determined as a function of the D_{max} particle size. Because particle mass is a function of the third power of particle size, sample-size equations for volumetric samples are (usually) a function of the third power of D_{max}, i.e., D_{max}^3.

Defining the D_{max} particle size

The D_{max} particle size used for determining the mass of volumetric samples does not necessarily have to be the largest particle found in the sampling reach, but should be the size of the largest particles to be represented in the sample. The largest particle sizes to be represented in a sample depend on the study objective. When determining the D_{50} or another percentile for computations of bedload transport rates in a given streambed, untransportably large particles, e.g., boulders: unearthed from glacial deposits, or supplied from rock fall, should not be included in the sample. If the study objective is to compute the stream roughness, untransportably large boulders should be included in the analysis.

The largest particle size of concern that should be representatively included in bed-material samples for bedload studies is often the dominant, large particle size D_{dom}. In mountain streams with occasional supply of non-fluvial supply of large particles, the particle size of D_{dom} is roughly equivalent to the D_{90} particle size. D_{dom} is approximately the largest particle size transportable during frequently occurring large floods (e.g., bankfull flow or a flood with a two-year recurrence interval). The size of D_{dom} can be estimated from the mean *b*-axis size of about 30 large (except the very largest) particles deposited on the upstream end of gravel bars or on other fresh depositional surfaces that are not affected by backwater or wake hydraulics. Absence of alga cover and negligible embeddedness may be interpreted as signs of recent transport. Those indicators can be misleading and indicate particle sizes too large for D_{dom}, if the last flood greatly exceeded the commonly largest bankfull or biennial flood and deposited either extraordinarily large clasts, or buried the streambed with finer sediment. In this case, a tractive force diagram (Lane 1955; Leopold 1992, p. 194) may be used to estimate the size of D_{dom} for flow properties of commonly occurring floods.

In uncoupled streams, the D_{max} particle size may be transportable during the floods of concern. Sample-mass: equations may then be based on the center of class of the largest fluvially transported size class $D_{max,c}$, or the D_{95} particle size. The term D_{max} is used as the largest transportable size class in the discussion of sample mass in the following section, and not as the absolute largest particle size found in a reach.

5.4.1.1 Sample mass as cubic functions of D_{max}

Several sample-mass recommendations are available that predict sample mass as a function of D_{max}^3:,(e.g., ISO (1977) following De Vries (1970), Neumann-Mahlkau 1967), Church et al. (1987), Diplas (1992a), Diplas and Fripp (1992), Fripp and Diplas (1993). However, these cubic sample-mass equations are based on different criteria which include:

- Effect that adding or omitting the largest particle(s) has on the total sample mass,
- Error acceptable for the particle size of a large size fraction,
- Constant coefficient of variation for the sizes of individual particles within a size class over neighboring large size classes,
- Number of particles that should be contained in the largest size class, and
- Feasibility of obtaining a statistically required sample volume.

The different criteria produce different cubic sample-mass equations. To facilitate a better comparison of the numerical results, all cubic sample-mass equations are expressed in the same form of

$$m_s = a \cdot D_{max}^3 \cdot \rho_s = b \cdot D_{max}^3 \tag{5.31}$$

where m_s is sample mass and usually expressed in units of kg unless otherwise specified. a and b are coefficients, and ρ_s is the particle density. The unit of the D_{max} particle size is in meters for the equations in Section 5.4.1.1, however in Fig. 5.14, D_{max} is indicated in units of mm for familiarity. For simplicity, all particles are assumed to be spheres or ellipsoids[4], and the term $\pi/6$ is incorporated in the a coefficient. A particle density ρ_s of 2,650 kg/m^3 is assumed for particles and the numerical value is incorporated into the b coefficient. All cubic sample-mass equations are plotted in Fig. 5.14 (the numbers on graphs refer to equation numbers in Section 5) and listed in Table 5.8. Sample masses predicted by these equations for a specified D_{max} particle size range over three orders of magnitude, i.e., the percentage weight of the D_{max} particle size of total sample mass ranges between roughly 0.01 and 10%.

[4] *The volume of an ellipsoid with an axis ratio a:b:c of 3/2:1:2/3 is equal to the volume of a sphere with a diameter of 1. See Eq. 5.64 in Section 5.4.5.*

Percent error in total sample mass incurred by the largest particle

One of the criteria used to establish an appropriate sample mass is the amount of error produced by the unrepresentative presence or absence of the largest particle in the total sample mass. In small samples from poorly sorted deposits, the largest particle can account for a substantial fraction of the total sample weight. The arbitrary presence or absence of the largest particle thus substantially affects the weight of the total sample mass. If the resulting error in sample mass is not to exceed 1%, sample mass must be larger than 100 times the mass of the D_{max} particle. A regression function fitted to the graph provided by Neumann-Mahlkau (1967) determined the relationship between sample mass and D_{max} particle size to be

$$m_s = 138{,}000\ D_{n,max}^{\ 3} \tag{5.32}$$

where $D_{n,max}$ is the nominal diameter (in m) of the D_{max} particle size (Section 2.1.2), and m is sample mass (in kg). For spheres, or ellipsoidal particles with axes ratios of $a = 3/2\ b$, $b = D_{max}$, and $c = 2/3\ b$, the particle weight of D_{max} is equal to the weight of a particle with a nominal diameter of D_{max} ($D_{(n)max}$). If the potential error introduced by the largest particle is allowed to increase to 10% (i.e., the D_{max} particle size is allowed to assume 10% of the total sample mass), the regression function becomes (same units as above)

$$m_s = 13{,}800\ D_{n,max}^{\ 3} \tag{5.33}$$

Both functions are plotted in Fig. 5.14 and labeled 32 and 33.

Relative error

The sample-mass recommendation by De Vries (1970) is based on an analysis of the relative error $e_{\%pi}$ of the ith size fraction. The relation can be computed from:

$$e_{\%pi}^{\ 2} = \frac{D_i^{\ 3} \cdot \ \beta \cdot \ \rho_s}{p_i \cdot \ m_s} \tag{5.34}$$

where p_i is the probability by mass of the ith size fraction and β is a constant. Laboratory experiments using sand and small gravel < 14 mm estimated a mean value of $\beta = 0.8$. De Vries (1970) considered the D_{84} as characteristic of the large particle-size fraction. Thus, sample mass m_s as a function of the D_{84} particle size can be computed from:

$$m_s = \frac{D_{84}^{\ 3} \cdot \ 0.8 \cdot \ \rho_s}{e_{\%pi}^{\ 2} \cdot \ p_i} \tag{5.35}$$

De Vries (1970) suggests setting p_i = 10%. For a relative error $e_{\%pi}$ of 1% ("high precision"), the denominator in Eq. 5.35 is 10^{-5}. The sample size for various degrees of precision is

$$m_s = \frac{D_{84}^{\ 3} \cdot \ 0.8 \cdot \ \rho_s}{0.01^2 \cdot \ 0.1} = \frac{D_{84}^{\ 3} \cdot \ 0.8 \cdot \ \rho_s}{10^{-5}} = 0.8 \cdot \ 10^X \cdot \ D_{84}^{\ 3} \cdot \ \rho_s \tag{5.36}$$

with units in meters and kg. The exponent x equals 5 for a "high" precision of 1%, 4 for a "normal" precision of 3%, and 3 for a "low" precision of 10%.

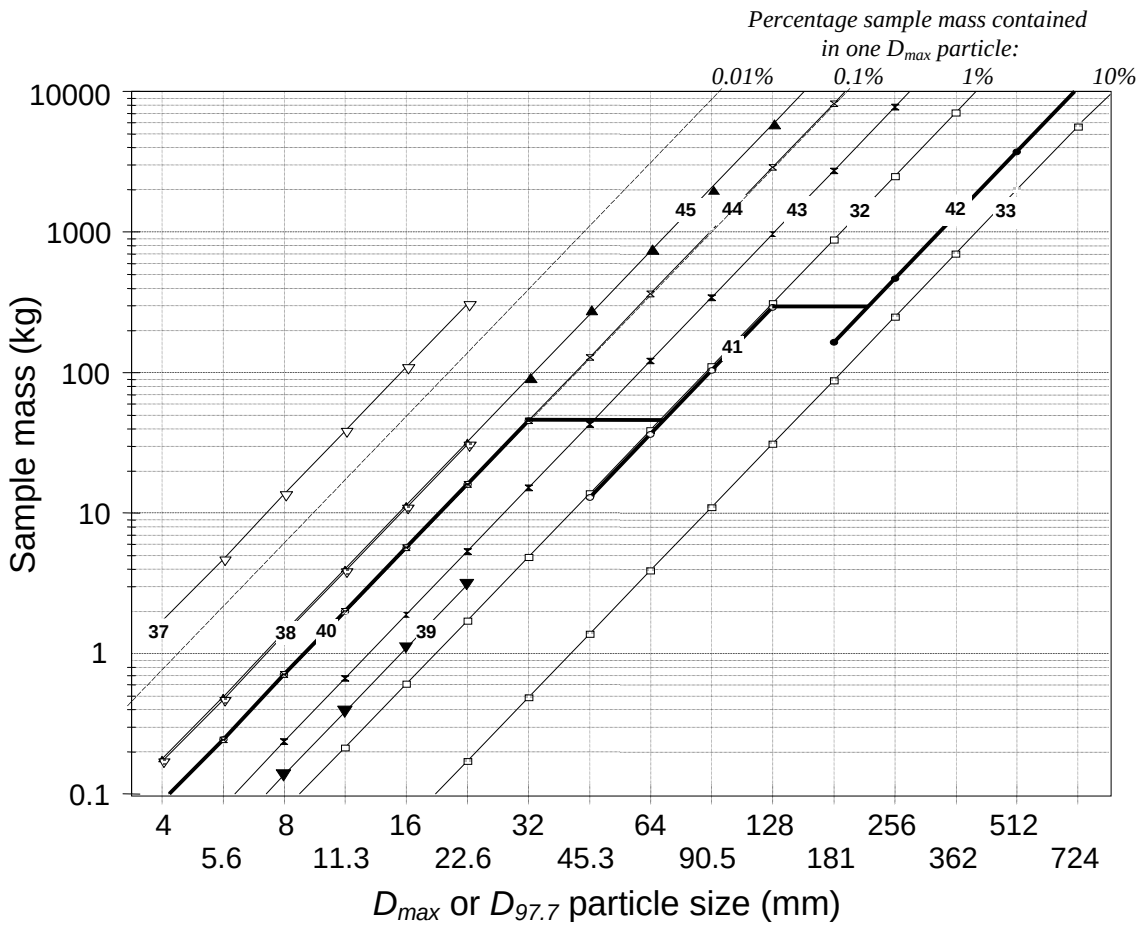


Fig. 5.14: Comparison of various sample-mass recommendations (kg) for gravel and cobble bed material based on cubic functions of D_{max}. The numbers on the graphs refer to the equation numbers in Section 5. The four lines labeled 0.01, 0.1, 1 and 10 refer to the percent sample mass contained in the mass of the D_{max} particle.

The sample-mass recommendations by De Vries (1970) were developed for sand and fine gravel. If De Vries' recommendations are applied to medium and large gravel particles and cobbles, the sample mass becomes very large. The International Organization of Standardization (ISO 1977) adopted the De Vries (1970) sample-size recommendations.

In order to compare the sample-mass recommendations by De Vries (1970) and ISO (1977) that use the D_{84} particle size with those that are based on D_{max}, the recommendations based on the D_{84} needs to be modified. If the D_{max} particle size is assumed to be equal to the $D_{97.7}$, and the sample standard deviation is 1.0 ϕ, then the $D_{97.7}$ particle size is approximately 1.0ϕ-units larger than the D_{84} (i.e., twice as large) (Fig. 2.19 in Section 2.1.5.4). The sample mass required for a D_{84} particle 8 mm in size in the original plot by De Vries (1970) is therefore assigned to a $D_{97.7}$ or D_{max} particle size of 16 mm in Fig. 5.14. The sample mass (kg) for high, normal, and low, precision recommended by De Vries (1970) (Eq. 5.36) can then be expressed as cubic functions of the $D_{97.7}$ particle size (m)

$$m_s = 26{,}500{,}000\ D_{97.7}^{\ 3} \qquad \text{for "high precision"} \tag{5.37}$$

$$m_s = 2{,}650{,}000\ D_{97.7}^{\ 3} \qquad \text{for "normal precision"} \tag{5.38}$$

$$m_s = 265{,}000\ D_{97.7}^{\ 3} \qquad \text{for "low precision"} \tag{5.39}$$

Constant variability of particle sizes per size class over all size classes

Church et al. (1987) presented a sample-mass criterion that is independent of an assumed underlying distribution type. Church et al. (1987) found that the coefficient of variation *CV* of particle sizes within a 0.5 ϕ-size class is approximately 10% if the size class contains more than 100 particles. To ensure a constant *CV* of 10% for all sizes classes including the largest, Church et al. (1987) empirically determined that the mass of the largest particle in the sample should not exceed 0.1% of the total sample mass. Consequently, the sample mass m_s (kg) should be 1,000 times the mass of the D_{max} particle size. This recommendation can be mathematically expressed as

$$m_s = 1{,}000\,\frac{\pi}{6}\,D_{max}^{\ 3} \cdot \rho_s = 524 \cdot D_{max}^{\ 3} \cdot \rho_s = 1{,}388{,}000\ D_{max}^{\ 3} \tag{5.40}$$

with D_{max} in m, and a particle density ρ_s of 2,650 kg/m^3 (see graph labeled 40 in Fig. 5.14). Similar to the "normal" precision criterion by De Vries (1970), Eq. 5.40 yields unmanageably large sample masses when applied to particle sizes larger than 32 mm. For coarse gravel with a D_{max} of 32 to 128 mm, Church et al. (1987) therefore suggest a less stringent criterion in which the mass of a D_{max} particle accounts for 1% of the total sample mass. This can be expressed by:

$$m_s = 100 \frac{\pi}{6} D_{max}^3 \cdot \rho_s = 52.4 \cdot D_{max}^3 \cdot \rho_s = 138{,}000\, D_{max}^3 \tag{5.41}$$

Equation 5.41 (graph 41 in Fig. 5.14) is identical to the sample-mass equation by Neumann-Mahlkau (1967) for the 1% precision criterion (Eq. 5.33, graph 33 in Fig. 5.14). As particle sizes exceed 128 mm, sample masses again become so large that Church et al. (1987) lowered the criterion to D_{max} = 5% of the total sample weight.

$$m_s = 20 \frac{\pi}{6} D_{max}^3 \cdot \rho_s = 10.47 \cdot D_{max}^3 \cdot \rho_s = 27{,}751\, D_{max}^3 \tag{5.42}$$

The three sample-mass criteria by Church et al. (1987) plot as parallel graphs in Fig. 5.14. In order to obtain one function applicable to all particle sizes, the three functions can be united by a staircase function which, in a second step, can be smoothed by a power regression function that is fitted through the corner points of the staircase functions. This procedure and the resulting sample-mass equation is discussed further under "Canadian standards" in Section 5.4.1.2.

Volumetric considerations

Diplas (1992a) and Diplas and Fripp (1992) based their sample-mass recommendation for volumetric samples on the following considerations: If 100 particles are sufficient for a line or a grid sample, and if a particle with the diameter D occupies an area larger than D^2, then the minimum area for an areal sample is $A_{min} = 100 \cdot D^2$. For an entire particle-size distribution, total sampling area could be defined as $A_{tot} = 100\, D_{max}^2$. If the minimum depth of a volumetric sample is set to 2 D_{max} (Sect. 4.2.2.2), sample mass m (kg) becomes

$$m_s = 200\, D_{max}^3 \cdot \rho_b = 460{,}000\, D_{max}^3 \tag{5.43}$$

where D_{max} is in meters and ρ_b is the sediment bulk density assumed to be 2,300 kg/m^3 (Table 2.16 in Section 2.5). Subsequent computations of precision and sample size prompted Fripp and Diplas (1993) to increase the minimum number of particles for a pebble count to 200 - 400 particles. Total sample area A_{tot} then increases to 200 or 400 D_{max}^2 (Eq. 5.28), with a sample mass of

$$m_s = 400 \text{ to } 800\, D_{max}^3 \cdot \rho_b = 1{,}380{,}000\, D_{max}^3 \tag{5.44}$$

if a multiplier of 600 is selected. Note that sample-mass recommendations in Eq. 5.44 are nearly identical to those proposed by Church et al. (1987) in their 0.1% criterion (Eq. 5.40).

Sampling until 5 D_{max} particles are contained in the sample

A simple field criterion for estimating the necessary sample mass that does not require monitoring sample weight is proposed by Ibbeken (1974). He suggests continuing to sample until at least 5 particles of the D_{max} size class are contained in the sample. This approach implies that the spatial distribution of D_{max} particles within a sediment deposit is truly random, and that there is no user bias towards or against sampling large clasts. In order to compare Ibbeken's criterion with those discussed above, a percentage weight needs to be assumed for D_{max} particles in the total deposit. If the percentage is set to 1%, and the D_{max} particle-size class is 180 mm with an average D_{max} particle weight of 8 kg, Ibbeken's sample-mass criterion yields $5 \cdot 8 \text{ kg} \cdot 100 = 4{,}000$ kg. In this sample of 4 metric tons, the mass of one D_{max} particle m_{Dmax} comprises 0.2% of the total sample weight. In terms of the notations used above, Ibbeken's sample-mass criterion can be rewritten as:

$$m_s = 2000 \cdot m_{Dmax} = 2000 \cdot \frac{\pi}{6} D_{max}^{\ 3} \cdot \rho_s = 2775{,}073\, D_{max}^{\ 3} \qquad (5.45)$$

If it is assumed that D_{max} particles make up 5% of the deposit, Ibbeken's sample-mass criterion yields $5 \cdot 8 \text{ kg} \cdot 20 = 800$ kg and the mass of one D_{max} particle m_{Dmax} would comprise 1% of the total sample weight. This result is identical to the sample mass criterion in Eq. 5.41.

5.4.1.2 National standards: non-cubic functions of D_{max} particle size

It is conceptually evident that sample mass should increase as a *cubic* function of particle size. Nevertheless, the resulting steep increase of sample mass with particle size leads to large and often unmanageable sample sizes for cobble-sized bed material. Most national standards therefore propose sample-mass recommendations that require a relatively high sample mass for small D_{max} sizes, but the increase of sample mass with particle size then continues at a lesser rate than it does with a cubic function. Regression functions fitted to the relations between sample mass and particle size yield either power functions with exponents between 1 and 1.5, or linear functions. Note that these relations are empirical and units on both sides of the equations do not necessarily match.

British, German, and American table value standards

Some of the national sample-mass recommendations are provided as table values only. Examples are the British BS 812, I standards (cited by Mosley and Tindale 1985), the German recommendations (DVWK 1988), and the American ASTM D75-71 standards

(cited by Mosley and Tindale (1985)). The British and German sample-mass recommendations are limited to particle sizes smaller than 60 mm, whereas the American ASTM D75-71 standards apply to particles smaller than 90 mm. For a visual comparison of sample mass, tabulated values and computed sample mass are plotted in Fig. 5.15.

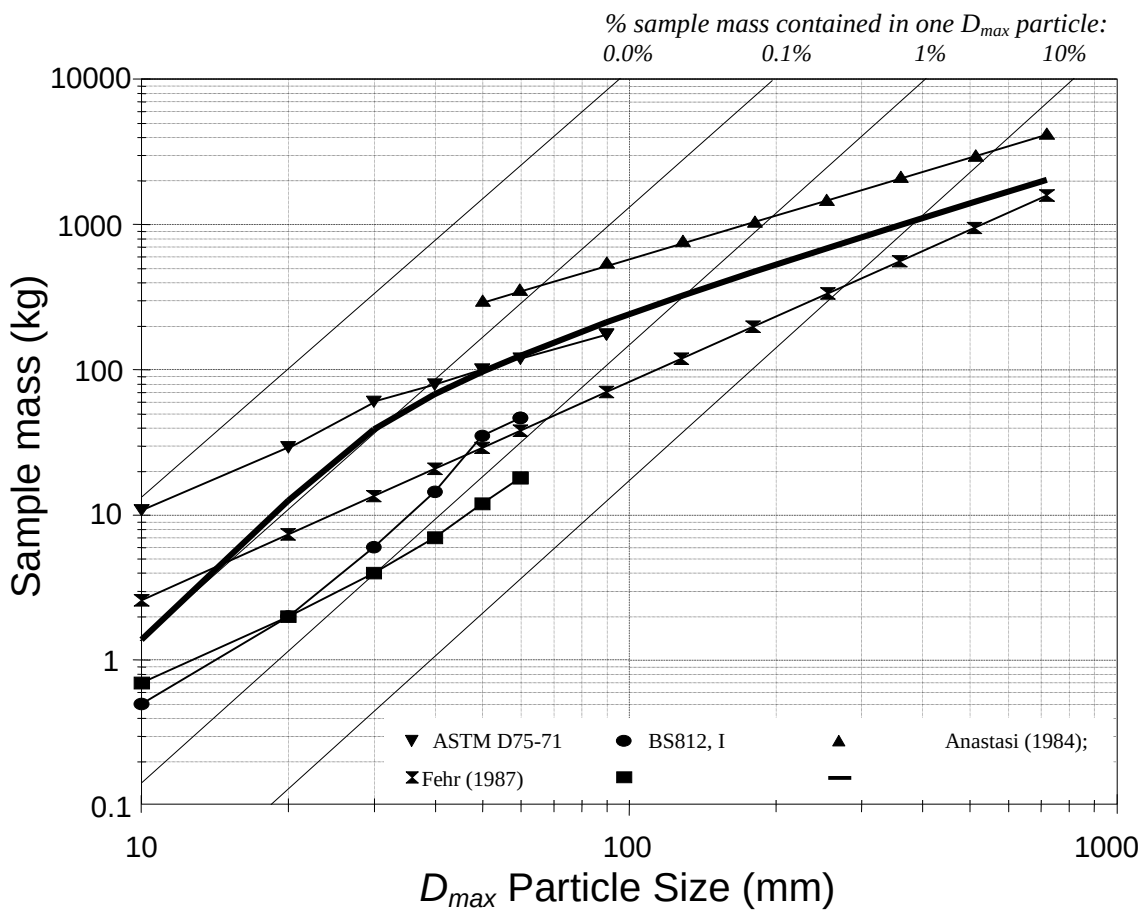


Fig. 5.15: Empirical sample-mass recommendations for gravel and cobbles expressed as power and linear functions of D_{max} (combined from various sources). The four lines indicate the percentages of 0.01, 0.0, 1, and 10 % of the total sample mass comprised in the mass of the D_{max} particle size (see Fig. 5.14).

American standards

Sample mass (kg) recommended by the ASTM D75-71 standards for particles smaller than 90 mm can be expressed by a linear regression equation

$$m_s = 2{,}069\ D_{max} - 6.7 \tag{5.46}$$

with D_{max} expressed in units of m (Fig, 5.15). The American ASTM C136-71 standard (cited by Church et al. 1987) has no restriction on particle size and determines sample mass m (kg) as

$$m_s = 2{,}596\ D_{n,max}^{1.5} \tag{5.47}$$

where $D_{n,max}$ (in m) is the nominal diameter (Section 2.1.2) of the particles retained on the largest sieve size (Fig. 5.15).

German standards

Sample-mass recommendations published by the Deutscher Verband für Wasserbau und Kulturtechnik (DVWK 1988) extend to particle sizes up to 60 mm and can be expressed by a power regression function (with m in kg, and D_{max} in m; Fig. 5.15)

$$m_s = 712.4\ D_{max}^{1.43} \tag{5.48}$$

Swiss standards

The empirical Swiss recommendations for sample mass in gravel-bed rivers are based on sample volume (V) (Anastasi 1984; Fehr 1987)

$$V_s = 2.5\ D_{max} \tag{5.49}$$

with V in m^3 and D_{max} in m. The mass of sediment contained in this sample volume varies with the bulk density ρ_b which is affected by the sorting and packing of the particles in the sample. Bulk density for gravel deposits ranges between 1,700 and 2,600 kg/m^3 (Table 2.16 in Section 2.5). For comparison with other sample-mass equations, ρ_b was set to 2,300 kg/m^3, a value proposed for gravel-sand mixtures by Carling and Reader (1982). Sample mass (m) in kg is then

$$m_s = 2.5\ D_{max} \cdot \rho_b = 5{,}750\ D_{max} \tag{5.50}$$

Canadian standards

Church et al. (1987) proposed using three sample-mass criteria depending on the D_{max} particle size (Section 5.4.1.1). However, use of three criteria can lead to confusion in samples-mass estimates. Sample-mass requirements for particles of 32 mm is 45 kg if the 0.1% criterion is applied (Eq. 5.40), whereas sample mass for 45 mm particles is only 13 kg, if a less stringent criterion of 1% is used (Eq. 5.41) (Fig. 5.14). The Canadian standards described by Yuzyk (1986), Yuzyk and Winkler (1991), and Zrymiak (in press) fitted a staircase function through the three graphs by Church et al. (1987) to unite the three criteria in a monotonic function. Another possibility to unite the three sample-mass criteria in one strictly monotonic function is to fit a power regression function through the

corner points of the staircase function (Fig. 4.20 in Section 4.2.3.2), yielding the sample-mass equation (r^2=0.999):

$$m_s = 2{,}881.6\ D_{max} - 47.56 \tag{5.51}$$

with m in kg, and D_{max} in m. The adjusted sample-mass equation for Church et al. (1987) falls midway between the range of the other non-cubic sample-mass equations (Fig. 5.15).

Summary

Sample-mass recommendations that are based on the size of the D_{max} particle vary over several orders of magnitude for a specified D_{max} particle size (Fig. 5.14. and 5.15). This variability is shown in Table 5.8 that presents cubic and non-cubic regression equations for sample-mass recommendations and compares sample-mass requirements for D_{max} particle sizes of 16 and 180 mm.

None of these recommendations have been formally adopted as the standard for sampling bed material in gravel-bed streams in the United States. The empirical sample-mass recommendations most frequently used and referenced are those by Church et al. (1987). The adjusted and strict-monotonic sample-mass equation for Church et al. (1987) describes the center of the range proposed by cubic and non-cubic sample-mass equations.

5.4.1.3 Error of the entire particle-size distribution due to the presence or absence of particles from the largest size class

Presence or absence of large particles not only affects total sample mass, but also alters the particle-size distribution in general. The presence of a statistically non-representative large particle is less likely than an absence, but has a disproportionate effect on the sampled particle-size distribution. The presence of an unrepresentative large D_{max} particle, that comprises a large percentage of the total sample mass, considerably coarsens the entire particle-size distribution compared to a parent population in which large particles are not overrepresented. This can be illustrated with Fig. 5.16, which is described in a different context below. Assuming the heavy black line in Fig. 5.16 represents the parent particle-size distribution of the deposit, the line termed “biased” indicates a sample distribution in which the largest particle comprises 30% of the total mass. Compared to the parent population, the D_{50} particle size is more than doubled, and the D_{75} size is even quadrupled in the sample in which large particles are overrepresented.

Chance absence of particles from the largest size class causes a sample particle-size distribution that is finer than the parent population. This effect is less pronounced than a chance overrepresentation, but it occurs statistically more often. The effect of chance absence of the D_{max} particle on the sample particle-size distribution is discussed in more detail in Section 5.4.1.4.

Table 5.8: Comparison of cubic and non-cubic regression functions for sample mass as a function of the D_{max} particle size (sample mass in kg and D_{max} particles size in m). Sample mass in parentheses indicates that the D_{max} particle size is beyond the intended range of the equation. See text for assumptions and units.

Regression Function	Author, *Criterion*	Equation Number	Sample Mass (kg) for D_{max} of: 16 mm	180 mm
Cubic sample-mass equations:				
$m_s = 26{,}500{,}000\ D_{max}^3$	De Vries (1970), ISO (1977), *high prec.*.	5.37	108	(154,550)
$m_s = 2{,}775{,}073\ D_{max}^3$	Ibbeken (1974), *5 D_{max} particles*	5.45	11.4	16,180
$m_s = 2{,}650{,}000\ D_{max}^3$	De Vries (1970), ISO (1977), *norm. prec.*	5.38	10.9	(15,450)
$m_s = 1{,}388{,}000\ D_{max}^3$	Church et al. (1987), $D_{max} = 0.1\%\ m_s$	5.40	5.7	8,090
$m_s = 1{,}380{,}000\ D_{max}^3$	Fripp and Diplas (1993), *400 particles*	5.44	5.7	8,050
$m_s = 460{,}000\ D_{max}^3$	Diplas and Fripp (1992) , *100 particles*	5.43	1.9	2,680
$m_s = 265{,}000\ D_{max}^3$	De Vries (1970), ISO (1977), *low prec.*	5.39	1.1	(1,550)
$m_s = 138{,}800\ D_{max}^3$	Church et al. (1987), $D_{max} = 1\%\ m_s$	5.41	(0.60)	810
$m_s = 138{,}000\ D_{max}^3$	Neumann-Mahlkau (1967), $m_s = 100\ D_n$	5.32	0.57	805
$m_s = 27{,}751\ D_{max}^3$	Church et al. (1987), $D_{max} = 5\%\ m_s$	5.42	(0.11)	160
$m_s = 13{,}800\ D_{max}^3$	Neumann-Mahlkau (1967), $m_s = 10\ D_n$	5.33	0.06	80
Non-cubic sample-mass equations:				
$m_s = 5{,}750\ D_{max}$	Anastasi (1984); Fehr (1987)	5.50	(92)	1030
$m_s = 2{,}069\ D_{max} - 6.7$	ASTM D75-71	5.46	26	(370)
$m_s = 2{,}882\ D_{max} - 47.6$	Church et al. (1987), adjusted	5.51	1.1	472
$m_s = 2{,}596\ D_{max}^{1.5}$	ASTM C136-71	5.47	5.2	200
$m_s = 712.4\ D_{max}^{1.43}$	DVWK (1988)	5.48	1.9	(61)

5.4.1.4 Sample-mass reduction: truncation and readjustment at the coarse end

All cubic, and even some of the non-cubic sample-mass equations recommend sample masses ranging from several metric tons to several hundreds of metric tons for bed material containing large cobbles and boulders. Such sample masses are not only unmanageably large, but would severely disturb the streambed as a consequence of their collection.

Church et al. (1987) recommend truncating volumetric samples in coarse beds and excluding from the sample particles larger than 256 mm, which typically weigh more than about 23 kg a piece. Particles larger than 256 mm are difficult, if not unsafe, to pick up for most persons and are therefore not likely to be representatively included in a volumetric sample, anyway. However, the presence of particles larger than the largest sampled size-class in the streambed should be recorded in the field notes. Any inadvertently collected large particle may then be discarded and only sufficient sediment is retained for an unbiased sample of the largest particle size present in the truncated sample.

The truncation and readjustment method of estimating the coarsest part of a cumulative frequency distribution is based on the assumption that the percent frequency of the largest one or two particles size classes is typically small in very large and representative samples from coarse gravel-bed streams. To obtain a smooth shape of the upper end of the cumulative distribution curve, the truncated sample needs to be extended to its relevant or full (pre-truncation) particle-size spectrum. This is accomplished by assigning small percentage frequencies to the truncated size classes (Fig. 5.16). The added percentages decrease for consecutively larger particle sizes. Estimates for those small percentages can

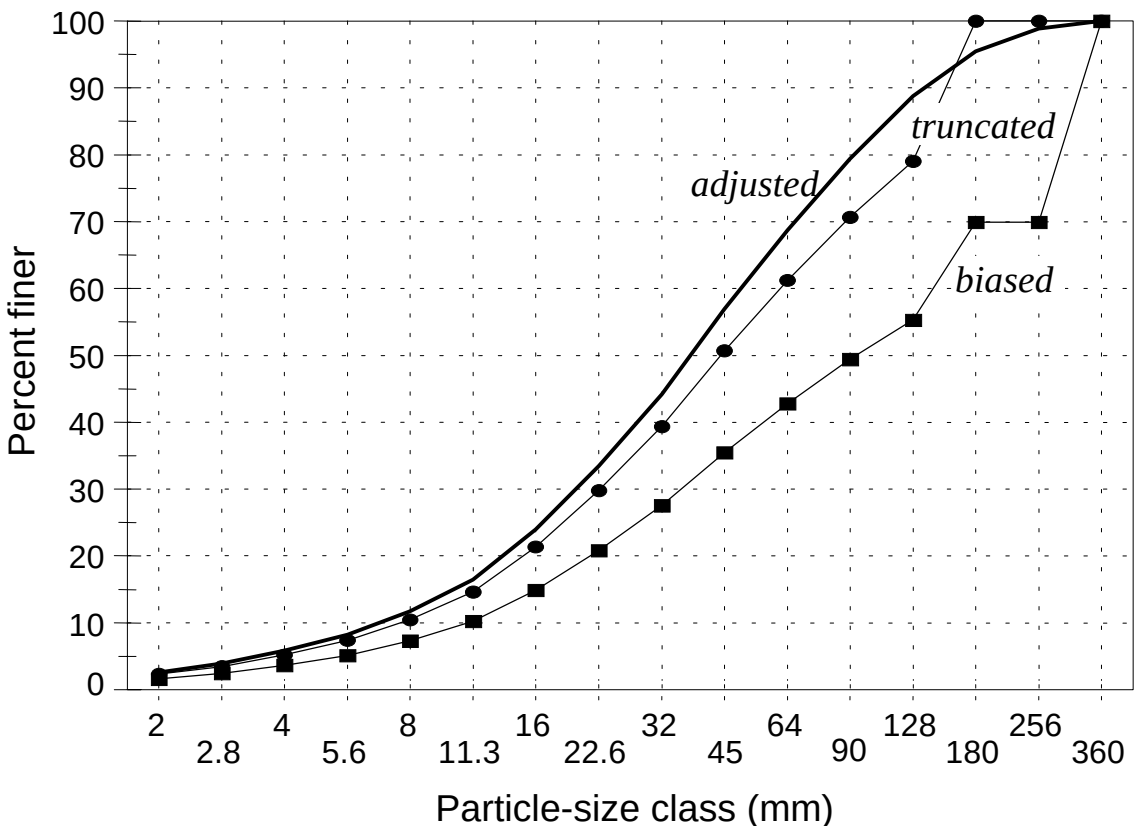


Fig. 5.16: Example of a biased sample from a deposit with a D_{max} size class of 256 mm. The sample was truncated by two size classes at 128 mm and then readjusted to its original D_{max} particle-size class of 256 mm.

be obtained by handfitting a smooth upper part of the cumulative distribution curve. The total percent frequency needs to be reset to 100% to compute the new cumulative frequency distribution.

If the assumption that large particles in unbiased samples account for only a small percentage of the total distribution mass is valid for a given sampling situation, truncation and readjustment can provide a more accurate approximation of the true bed-material size distribution than in small samples in which 20% or more of the sample weight is due to a single large particle, or in which the largest particles are not accounted for at all.

Truncation and readjustment is not applicable if there is reason to believe that the paucity or abundance of large particles in the sample is a result of fluvial processes, for example, a recent change in the local sediment budget. An abundance of large particles in subsurface sediment, or a scarcity of large particles in the surface sediment, may result from the burial of a former armor surface by a local deposition of mid-sized particles.

5.4.2 Sample mass as a function of acceptable percentile errors

Sample-mass considerations in previous sections are primarily aimed at avoiding bias due to the unrepresentative presence or absence of a single large particle. The resulting empirical sample mass equations do not provide information regarding sampling precision. If a relationship between sample mass and sampling precision is needed, it may be computed from a two-step approach. A two-step approach computes the number of subsamples necessary for a specified sampling precision around the median particle size based on the central limit theorem. The particle size of the means or medians (or of percentiles close to the median) in subsamples are approximately normal distributed). Sections 5.2.3.1 and 5.3.2 described two-stage sampling for pebble counts with number-based sample-size statistics and for areal samples. Section 5.4.2.1 describes how a two-stage approach is used to estimate the precision of volumetric weight-based samples.

5.4.2.1 Two-stage sampling approach (ISO 1992)

Individual volumetric samples taken with one of the sampling devices described in Section 4.2.3 are not likely to contain sufficient sediment for an acceptable level of precision in a size-distribution analysis. Therefore, ISO (1992) suggests collecting several subsamples. The mass of the largest particle D_{max} (in m) per subsample should be less than 3% of the subsample mass m_{ss} in order to avoid sample bias towards the larger fraction. This criterion for subsample mass m_{ss} (in kg) can be expressed by the function:

$$m_{ss} = 33.3 \frac{\pi}{6} D_{max}^{3} \cdot \rho_s = 46{,}205\, D_{max}^{3} \qquad (5.52)$$

where ρ_s is particle density of 2,650 kg/m^3.

Computation in units of mm

Similar to the two-stage sampling approaches proposed by ISO (1992) for grid and areal sampling (Sections 5.2.3.1 and 5.3.2), a number (q) of volumetric samples are collected from a homogeneous deposit. For each individual sample, the particle size of the median or a percentile close to it is computed, either graphically from cumulative distribution curves or by logarithmic interpolation between percentiles (Section 2.1.4.2). ISO (1992) proposes to compute the median D_{50} in units of mm. It is assumed that the q values of D_{50} are approximately normally distributed. This guideline document recommends performing the computations in ϕ-units (see below).

The sample standard deviation s_{50} of the q values of D_{50} is determined from:

$$s_{50} = \sqrt{\frac{\sum_{i=1}^{q}(D_{50} - D_{50m})^2}{q-1}} \tag{5.53}$$

D_{50m} is the arithmetic mean particle size in mm of the median D_{50} obtained from the q samples. Equation 5.53 for sample standard deviation is preprogrammed in most scientific pocket calculators and spreadsheet programs. For two subsamples, s_{50} is computed by

$$s_{50} = \frac{D_1 - D_2}{\sqrt{2}} \tag{5.54}$$

Eq. 5.55 can be used to determine the number of weight-based subsamples q required to remain below a 5% chance ($\alpha = 0.05$) that the absolute difference (positive or negative) between the estimated values of the D_{50} and the true population D_{50} is larger than or equal to the acceptable absolute error $e_{\pm D50}$. The absolute error is the difference (in mm) between the D_{50} in the sample and in the population. Using an appropriate value for Student's t from Table 5.2, the number of subsamples q is

$$q = \left(\frac{t_{1-\alpha/2;q-1}}{e_{\pm D50}} \cdot s_{50}\right)^2 \tag{5.55}$$

Total sample mass m_{tot} is the mass contained in each subsample m_{ss} multiplied by the number of q subsamples.

Example 5.14:
The D_{max} particle size of a deposit is estimated as 64 mm. Sample size m_{ss} for each subsample is 12 kg according to Eq. 5.52. Five subsamples were collected and have D_{50} particle sizes of 23, 29, 32, 38, and 44 mm. From Eq. 5.53, the standard deviation s_{50} for the D_{50} is 8.1 mm. An absolute error of $e_{\pm D50}$ = 5 mm around the D_{50} particle size is considered acceptable. Eq. 5.55 needs to be solved iteratively when *t*-statistics are used (see Table 5.2 in Section 5.2.1 for *t*-values). An arbitrary sample size of 20 subsamples is selected in the first trial of Eq. 5.55. The subsample size q_{est} = 20 is not equal to the computed subsample size q_{comp} = 12 after the first trial.

Trial	q_{est}	q-1	$t_{1-\alpha/2;q-1}$	q_{comp}
1	20	19	2.093	11.5 ≅ 12
2	12	11	2.201	12.7 ≅ 13
3	13	12	2.179	12.5 ≅ 13.

After the third trial, the subsample size q_{est} for which the *t*-value was selected has converged with the computed subsample size q_{comp} = 13. The total sample mass of 13 subsamples of 12 kg each = 156 kg has a 95% probability that the sample D_{50} size is approximately within ± 5 mm of the size of the population D_{50}.

Note that one physical sample, i.e., the amount of sediment that is collected by using a sampling device once, might not have a sufficient mass for an unbiased representation of large particles. For D_{max} particle sizes larger than 90 mm, the 3% criterion in Eq. 5.52 requires a subsample mass of 34 kg. Therefore, for large D_{max} sizes, it may be necessary to combine several physical samples into one subsample in order to reduce bias incurred by the statistically unrepresentative presence or absence of large particles. Combined subsamples are then used for the two-stage sampling approach.

Computations in units of ϕ

It is recommended applying the two-stage approach to median particle sizes in units of ϕ rather than to units of mm. Values of ϕ_{50} from several subsamples are expected to approximate a normal distribution better than the values of D_{50} (in mm) and should therefore be preferred over computations in units of mm (Triola 1995) (Section 5.2.3.1). D_{50} in Eqs. 5.53 and 5.54 is then substituted by values of ϕ_{50}.

Precision from two-stage approach is not general but refers to analyzed samples only

Each set of subsamples has a unique precision. For example, one set of three subsamples may have three D_{50} particle sizes of 45, 50, and 55 mm, while another set of three subsamples has the three D_{50} sizes of 43, 49, and 54 mm. Sample standard deviations will be slightly different for each set of subsamples, e.g., 5.0 in the first set of subsamples, and

5.5 in the second. Consequently, the computed sampling error (or precision) is different as well, yielding an absolute error of 12.4 mm around the D_{50} for the first, and of 13.7 mm for the second set of three subsamples.

The variability between individual subsamples increases due to bed-material heterogeneity in the sampling area or due to operator errors. The mean precision for a specified number of subsamples within a sampling area (e.g., three subsamples) is obtained if sets of three subsamples are collected repeatedly and precision is computed for each set. The precision is then averaged over all subsets of three samples and the result is the mean precision expected for a sample size of three. The same procedure is repeated for all sample sizes. The resulting data provide a description of the relationship between sample size and precision for a given percentile in a given deposit. The precision of this relationship increases with the number of subsamples over which precision is averaged for each subsample size. However, the repeated computation of precision for a large number of samples of the same sample size is a (prohibitively) large sampling effort.

Sampling efforts can be reduced by two procedures: One is using a regression function to determine the relationship between sample size and precision in a scatter plot. The second is a computer re-sampling procedure from a parent distribution for which the measured particle sizes are entered into a computer. Hogan et al. (1993) combined both procedures and developed a computerized two-stage sampling methodology (Section 5.4.2.2).

5.4.2.2 Computerized two-stage sampling (Hogan et al. 1993)

The first step for computerized two-stage sampling is to obtain a large bed-material sample (parent sample) that may be derived from combining several subsamples taken from within a homogeneous deposit. The parent sample serves as a population surrogate and should be as large as possible, because the larger the mass of the parent sample, the more accurate the surrogate. The sample is sieved, and the sizes of all particles are entered into a computer data file. No assumptions about the distribution type of the parent population need to be made. The computer then selects random particles from the parent distribution with replacement to create subsamples to which particles are added until a specified mass (e.g., m_{ss} = 50 kg) is exceeded. The subsample mass needs to be large enough to avoid bias against or towards large particles in the sample (sample size for bias avoidance: Section 5.4.2.1, Eq. 5.52, and Section 5.4.3.1, Eq. 5.60 and Fig. 5.20).

Sampling with no replication

The smallest subsample size (q = 2) collected from the parent population consists of two subsamples, each with a sample mass of $m_{ss} \geq 50$ kg and a total sample mass of 2 $m_{ss} \geq$ 100 kg. The largest sample size might comprise 30 subsamples (q = 30) with a mass of 30 $m_{ss} \geq 1{,}500$ kg. The particle sizes of all percentiles of concern D_p are computed for each subsample, for example the seven percentiles D_5, D_{16}, D_{25}, D_{50}, D_{75}, D_{84}, and D_{95}. The smallest subsample comprises two values for each percentile D_p, whereas the largest subsample comprises 30 values for each D_p. Although Hogan et al. (1993) used mm-units,

these guidelines recommend that the analyses be performed in units of ϕ, because percentiles in ϕ approach normality better than percentiles in mm.

The next step in the two-stage procedure is to compute the sample standard deviation s_{pq} for the q percentile values D_p, assuming that the q values for the percentile particle size D_p approximate a normal distribution. Either Eq. 5.53 or a preprogrammed function in a spreadsheet program may be used to compute the sample standard deviation. The absolute error $e_{\pm Dp}$ (in mm) around a percentile D_{pq} is computed from:

$$e_{\pm Dp;q} = \frac{t_{1-\alpha/2;q-1}}{\sqrt{q}} \cdot \quad s_{pq} \tag{5.56}$$

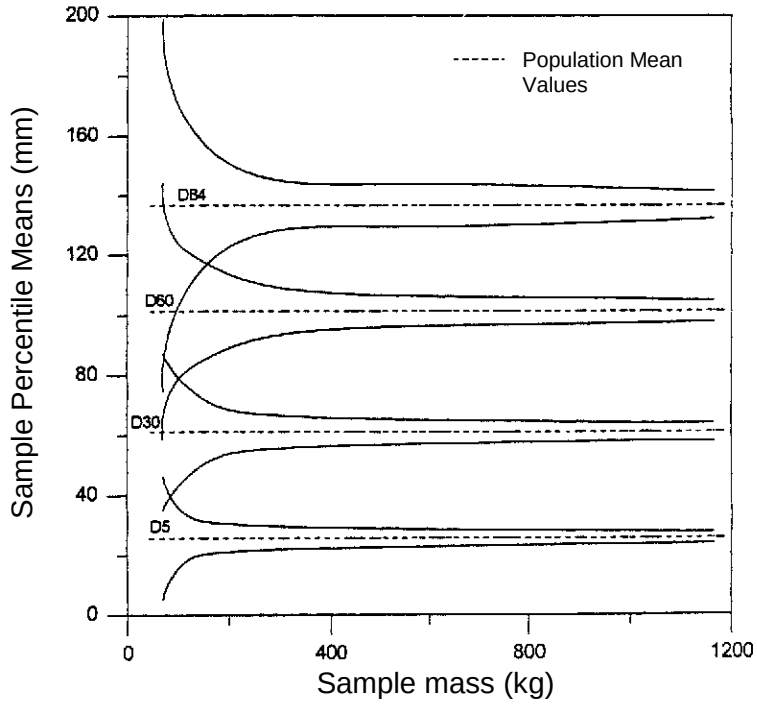
which is the general sample-size equation Eq. 5.55 or 5.2 solved for the error term. Table 5.2 provides values of Student's t. Alternatively, the absolute error $e_{\pm Dp}$ added or subtracted from the *population* percentile value $D_{p\mu}$ could be computed. If the study requires a result in terms a percent error, the percent error $e_{\%Dp}$ around a percentile D_p is computed from

$$e_{\%Dp} = \frac{e_{\pm Dp}}{D_{p\mu}} \cdot \quad 100 \tag{5.57}$$

Best-fit regression function for visualizing the data trend

For all percentiles of concern, the error computed for each sample size (Eq. 5.56) is plotted against that sample size. Data plotted from these computations may scatter considerably (due to the lack of sample replications, Section 5.4.2.1). An example of such scatter can be observed in Fig. 6.18 (Section 6.4.3.1). In order to visualize the trend of the data, a best-fit regression function is fitted through the points (Fig. 5.17). Knowing that the trend of the curves describes a decrease of sampling error e with $1/\sqrt{q}$, the regression function may have the form of $e = a \cdot \quad q^{-0.5}$.

The resulting graphs for positive, as well as negative errors, approach the x-axis asymptotically from both sides ("trumpet curve") (Fig, 5.17). Graphs as these can be established for all percentiles of concern. Graphical visualization of the relationship between sample size and error is useful when determining where to make the compromise between tolerable error, sample size, and expendable effort and costs. Note, however, that the smoothed graphs imply an unduly high precision of the computed relationship between sample size and precision. A further caveat of this methodology is that two-stage approach used for the computations is not designed to determine errors around low and high percentiles, which may not approach normality for low sample sizes. Thus, the true precision may differ from the computed precision. However, the computations are relatively easy and may suit as a first approximation of sampling precision.



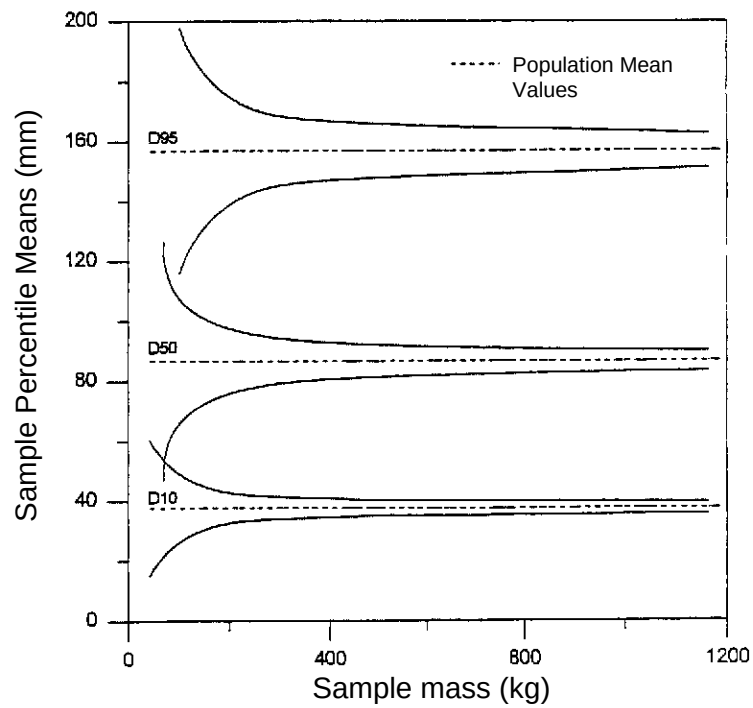


Fig. 5.17: Results of two-stage computer sampling with no replications for a bed-material sample from the Cache la Poudre River, Colorado. Relation between absolute error (in mm) around percentile particle sizes of the D_5, D_{30}, D_{60}, and D_{84} (top), and D_{10}, D_{50}, and D_{95} (bottom) added and subtracted from the population percentile particle-size and sample mass. The error curves have been smoothed by fitting a regression function (from Hogan et al. 1993).

Error-curves

The error is at a maximum when sample size is small (or mass in this case) and becomes smaller as sample size increases. At some point, a further increase in sample size contributes only insignificantly to a further decrease in sampling error (see also Figs. 5.10 and 5.11 in Section 5.2.3.4). The absolute error around a percentile in units of mm for a given sample size is smallest for small percentiles and increases for larger percentiles (Fig. 5.17). However, a more interesting result from the study by Hogan et al. (1993) is that the percent error for a specified sample size is not symmetrically distributed around paired percentiles. For a fixed sample size, the percent error is largest around the D_5, strongly decreases towards the D_{10}, is lowest for the D_{84} and increases slightly for the D_{95}. This result can be expected for the bed-material from the South Fork Cache la Poudre which is a coarse gravel or cobble distribution slightly skewed towards fines. This finding is also in agreement with the results obtained by Rice and Church (1996b) for their bootstrap analysis of a large sample from the Mamquam River (Section 5.2.3.4).

For the bed-material sample analyzed in Fig. 5.17, a sample mass of 200 kg determines the D_{10} particle size to within ± 5 mm of the population D_{10} particle size of 38 mm. More than 500 kg are needed to define the D_{50} to the same absolute precision of ± 5 mm. The increase in the absolute error for higher percentiles for a given sample size (or mass) is a result of using mm-units for the analysis. It is recommended that the analysis be performed in ϕ-units if the underlying distribution approaches normality in ϕ-units. The error would then be highest for low and high percentiles and lowest around central percentiles. The distribution of errors around low and high percentiles for a specified sample size is discussed in Section 5.4.4.

Replicate sampling

Scatter in the data points can be reduced if precision is computed repeatedly for different sets of samples of the same subsample size, and if the mean precision for a given subsample size is plotted. The more sets of subsamples collected and analyzed, the higher the precision of the relationship between sample size and certainty of the result.

Replicate sampling reduces the scatter in the plots of sampling precision versus sample size. The number of replicates needed to produce smooth error curves increases as the sorting of the parent distribution becomes poorer, and as subsamples with smaller mass are taken. Rice and Church (1996b) recommend that about 200 replicates be performed for each sample size. Ferguson and Paola (1997) even used 500 replicates. Because this might exceed the capacity of user-developed spreadsheets, the use of commercially available bootstrapping programs may be required (e.g., Resampling Stats, available as an add-in program to Microsoft Excel, Section 5.2.3.4).

For 200 replications, each sample size q is represented by 200 replicates r_1, r_2, ... r_{200}. Each of the 200 replicates has a slightly different composition of particle sizes. Thus, the particle size of the D_{50} and all other percentiles is slightly different for each of the 200 replicates constituting the sample size q. The variability is reduced when the D_{50} particle

size associated with the sample size q is taken as the arithmetic mean of the 200 individually computed D_{50} particle sizes ($D_{50,q,200}$). Likewise, the D_{84} particle size computed for the sample size q = 3 is the arithmetic mean of all the D_{84} values obtained from 200 repetitions with a sample size 3 ($D_{84,3,200}$).

The actual two-stage sampling procedure is the same as described in Section 5.4.2.1, with the exception that each particle-size percentile represents the arithmetic mean of 200 replicates. A diagram explaining the resampling procedure for two-stage sampling is provided in Fig. 5.18, for a preset subsample mass of 50 kg, two subsample sizes q = 2 with m_2=100 kg, and q = 3 and m_3=150 kg, and with D_{16} as the percentile of concern.

5.4.3 Analytical computation of sample mass (Ferguson and Paola 1997)

Sample size necessary to obtain a specified precision is influenced by a variety of factors (Section 5.1), but volumetric sample-size equations discussed thus far have not addressed many of those factors. The empirical recommendations that determine sample mass as a function of D_{max} were developed for various sampling goals and physical settings. Thus, sample-mass requirements vary widely between different equations (Section 5.4.1). None of the D_{max}-based sample-mass recommendations provides information on percentile errors. The two-stage approach (Section 5.4.2.1) can be used to indicate the error around the sample mean or median for a specific set of subsamples. A computerized two-stage re-sampling approach provides a surrogate for percentile errors (Section 5.4.2.2). A bootstrap approach that re-samples a large parent distribution repeatedly (e.g., 200 times) can reliably quantify percentile errors (Section 5.2.3.4) once a large sample is collected.

However, a methodology is needed that allows the user to make a reliable estimate of the sample mass required for a tolerable error around a specified percentile for a given stream setting before the sample is collected, and to compute the sampling precision for a collected sample. With this task in mind, Ferguson and Paola (1997) developed sample-mass equations with the following properties: the equations (1) allow the user to compute the sample mass necessary for avoiding bias; (2) are suitable for computing the relationship between sample-size and error for any percentile(s) of concern, and (3) can be applied to bed-material of any standard deviation or sorting coefficient. However, a pilot study is needed to estimate the bed material D_{50} and the standard deviation (sorting). A drawback of the approach is that the computations are based on an assumed normal distribution, and results are correct only if an underlying normal distribution in ϕ-units can be assumed for the deposit, which strictly speaking is rarely the case.

The sample-mass equations determined by Ferguson and Paola (1997) were derived from three large computer-generated particle-size populations with standard deviations or sorting coefficients of 0.5, 1.0, and 1.5 σ. The samples were generated based on an underlying lognormal distribution of particle mass per size class for particle sizes in mm-units (equivalent to a normal distribution in terms of ϕ-units). Random samples with replacement were drawn by computer from these parent populations until samples of

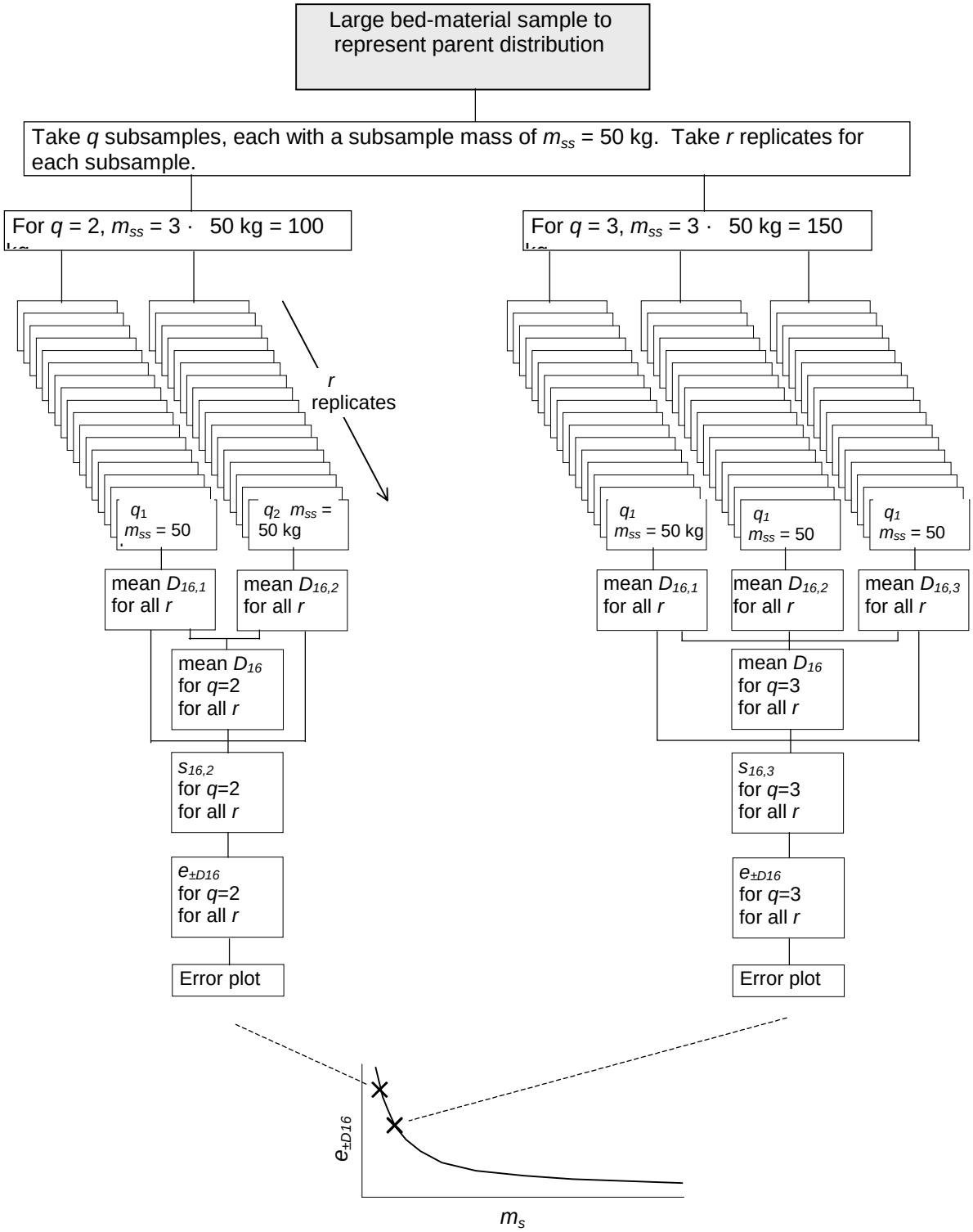


Fig. 5.18: Diagram explaining a resampling procedure with replicates for the example of a preset subsample mass of 50 kg, with subsample sizes q = 2 and m_{q2}=100 kg, and q = 3 and m_{q3}=150 kg, and with D_{16} being the percentile of concern. $s_{16,2}$ and $s_{16,3}$ are the standard deviation of the D_{16} particle sizes of the 2 or 3 subsamples, respectively.

specified volumes were reached. Each sample size was represented by 500 replicate samples to ensure a high precision of the computed relationship between sample size and error.

Computation of relative sample volume and absolute sample mass

In order to develop graphs that are applicable to all particle sizes, Ferguson and Paola (1997) expressed sample size as the ratio of the total sample volume V and the volume of the D_{50} particle size V_{50}. A spherical or ellipsoidal D_{50} particle of 32 mm, for example has a volume of $\pi/6\ D_{50}^3 = 17.16\ cm^3$. A relative sample volume of $V/V_{50} = 1$ corresponds to a volume of 17.16 cm^3, and a sample mass of $V \cdot \rho_s = 17.16\ cm^3 \cdot 2.65\ g/cm^3 = 45$ g or 0.045 kg, where ρ_s is the particle density. Similarly, relative sample volumes (V/V_{50}) of 10, 100, 1,000, and 10,000 correspond to absolute sample masses of 0.45, 4.5, 45, and 455 kg, respectively. For a D_{50} particle of 16 mm, the corresponding sample masses are 0.006, 0.057, 0.57, 5.7, and 57 kg, respectively (see Figs. 5.19 a-c, 5.21 a-c and 5.22 a-c). Thus, to compute sample mass in absolute terms, the D_{50} particle size needs to be known.

Estimation of the D_{50} particle size from one other percentile and the distribution sorting

If the only percentile known from a distribution is the D_{84}, for example, then the user can determine the respective D_{50} particle size if the distribution sorting is known, and if a normal distribution in terms of ϕ-units can be assumed. The D_{50} particle size can then be determined graphically (Fig. 2.19 in Section 2.1.5.4) or analytically. Fig. 2.19 can be used to identify the D_{50} if the distribution sorting s_I is close to the values of 0.5, 1, or 1.5. The curve with the appropriate sorting coefficient is shifted to the right or left until the curve passes through the one known percentile value, e.g., $D_{84} = -6.5\ \phi$. The D_{50} particle size can then be read from the shifted curve. The ϕ_{50} percentile particle size can be estimated analytically if the sample standard deviation and one other percentile size is known (Gilbert 1987):

$$\phi_{50} = \phi_p + (Z_p \cdot s) \qquad \text{for } \phi_p > \phi_{50} \tag{5.58}$$

or

$$\phi_{50} = \phi_p - (Z_p \cdot s) \qquad \text{for } \phi_p < \phi_{50} \tag{5.59}$$

where ϕ_p is the particle size of the known percentile, and Z_p indicates the distance between the percentile p and the median (i.e., ϕ_{50}) in terms of standard deviation. Z_p can be obtained from standard statistics tables (e.g., Gilbert 1987, p. 254, Table A1). Values for Z_p for frequently used percentiles are provided in Tables 5.9 and 5.1.

Table 5.9: Values for Z_p for various percentiles (See Table 5.1 for more values)

Percentiles:	50	65 35	75 25	84 16	90 10	95 5	97.5 2.5	99 1
Z_p	0	0.385	0.675	0.995	1.282	1.645	1.96	2.327

Example 5.15:
From a previous study it is known that the D_{75} = 68 mm, and the sample sorting s = 1.67. Convert the D_{75} percentile size into ϕ-units: ϕ_{75} = -3.3219 log(68) = -6.09 ϕ. Compute ϕ_{50} using Eq. 5.58: ϕ_{50} = - 6.09 + (0.675 · 1.67) = - 6.09 +1.13 = -4.96 ϕ. Converting back to mm yields: $D_{50} = 2^{4.96}$ = 31.1 mm

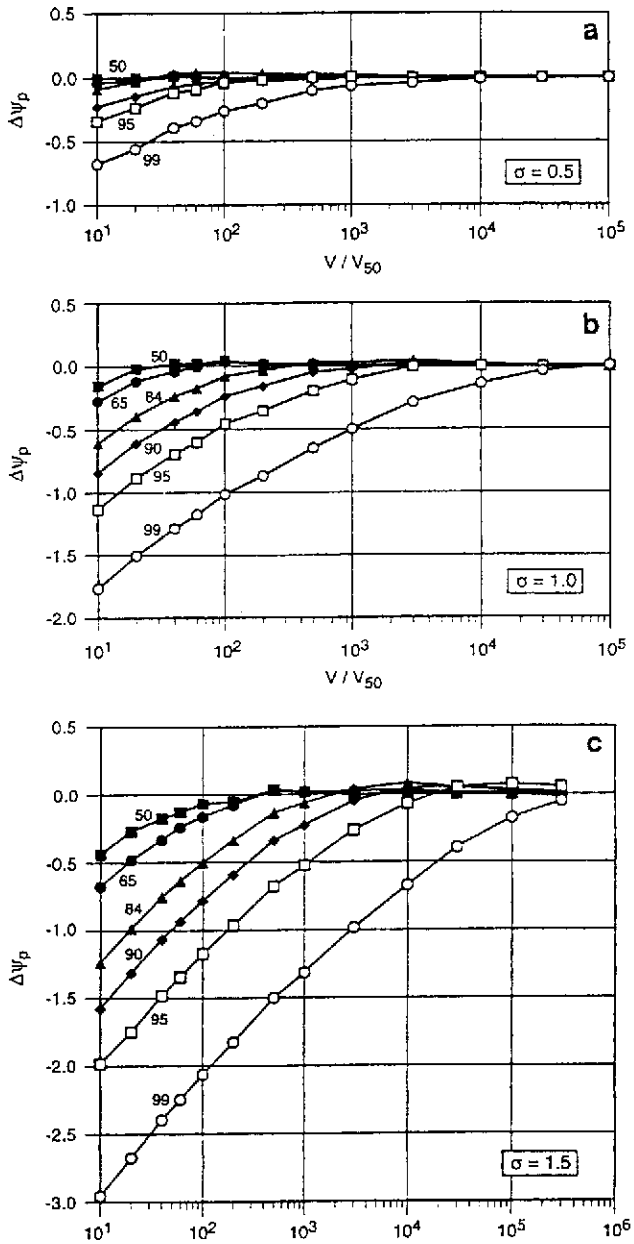
5.4.3.1 Sample mass for bias avoidance

The analysis by Ferguson and Paola (1997) indicated that small samples are systematically biased towards the fine fraction (Fig. 5.19 a-c). This becomes evident as the number of large particles is relatively small in a given sample volume. Thus, there is a less than average chance for large particles to be included in a small sample. Consequently, the sample particle-size distribution is finer than the population size distribution. Bias due to the chance *presence* of an overly large particle in an individual sample is not addressed in this computation, because particles larger than the parent distribution cannot be drawn from the parent population by the computer. But the occurrence of bias in an individual sample may introduce a pronounced error into the resulting particle-size distribution (Sect. 5.4.1.4).

Figs. 5.19 a-c indicate that bias is more pronounced for poorly sorted rather than for well sorted sediment. Fig. 5.15 also shows that the relative sample mass required for avoiding bias for the D_{95} particle size is approximately two orders of magnitude larger than the sample mass for avoiding bias in the D_{50}. On the basis of these results, Ferguson and Paola (1997) propose a dimensionless equation for determining the bias-avoiding sample volume V_b . V_b is a relative sample volume scaled by the volume of the D_{50} particle V_{50}:

$$\log\left(\frac{V_b}{V_{50}}\right) = 1.3 + \log(\sigma) + 0.9\,\sigma \cdot Z_p \qquad (5.60)$$

Z_p describes the distance between the percentile p and the median in terms of the standard deviation of a normal distribution (Table 5.9). Eq. 5.60 can be used for any percentile. The percentile for which bias is avoided is specified through the selection of an appropriate value of Z_p. To apply to the D_{84}, Z_p is selected as 0.995 from Table 5.9 or Table 5.1.



D_{50} (mm)							Sample mass (kg)
11.3:	0.020	0.20	2.0	20.0	200	2,002	
16:	0.057	0.57	5.7	56.8	568	5,683	
22.6:	0.160	1.60	16.0	160.0	1,600	16,017	
32:	0.455	4.55	45.5	454.7	4,547	45,467	
45:	1.264	12.64	126.4	1,264.4	12,644	126,439	
64:	3.637	36.37	363.7	3,637.3	36,373	363,734	
	10	100	1,000	10,000	100,000	1,000,000	
	V/V_{50}						

Fig. 5.19 a-c: Relation between bias in terms of ϕ-units ($\Delta\psi_p$) and relative sample volume V/V_{50} for given percentiles between D_{50} and D_{99}, and standard deviation of $\sigma = 0.5\ \phi$ (a), $\sigma = 1.0\ \phi$ (b), and $\sigma = 1.5\ \phi$ (c). Numbers on the curves indicate percentiles. Bias is the difference (continued on next page)

between the sample mean particle size of a given percentile in ϕ-units, averaged over 500 replicate samples, and the particle size of that percentile in the population. A negative difference indicates the sample percentile is finer than the population percentile. Sample size is a multiple of the volume V_{50} of the population median particle-diameter. (Reprinted from Ferguson and Paola (1997), by permission of John Wiley and Sons, Ltd.). Discrete values of sample mass (kg) are given for various D_{50} particle sizes (mm) at the bottom of the plot. The numbers on the bottom of the plots indicate sample mass values in kg for 10^1, 10^2, 10^3, 10^4, 10^5, and 10^6 V/V_{50} for various D_{50} particle sizes.

<u>Example 5.16:</u>
Assume the bed-material sample in ϕ-units approaches a normal distribution and has a standard deviation of s =1.5 ϕ. Relative sample volume V_b/V_{50} for avoiding bias in the estimation of the D_{84} particle size is computed from Eq. 5.60:

$$\frac{V_b}{V_{50}} = 10^{(1.3 + \log(1.5) + 0.9 \cdot 1.5 \cdot 0.995)}$$

$$= 10^{(1.3+0.176+1.343)} = 10^{2.819} = 659$$

For a D_{84} particle size of 90 mm, and a sorting of s =1.5 ϕ, D_{50} is 32 mm (see Fig. 2.19 or Eq. 5.58). Sample volume without pore space is

$$V_b = 659 \cdot (\pi/6) \cdot D_{50}^3 = 659 \cdot 17.16 \text{ cm}^3 = 11{,}307 \text{ cm}^3$$

Multiplication by particle density ρ_s =2.65 g/cm^3 provides sample mass

$$m_b = 11{,}307 \cdot 2.65 = 29{,}963 \text{ g} = 30 \text{ kg}$$

Dividing by an assumed bulk density of 1,500 kg/m^3 for shoveled gravel gives the sample volume of 0.02 m^3 which is about 2 household buckets of 10 liters each.

Relative sample volume in terms of V/V_{50} for bias avoidance was computed with Eq. 5.60 and plotted against sediment standard deviation σ for various percentiles between D_{50} and D_{99} in Fig. 5.20. Fig. 5.20 indicates that a relative sample volume of V/V_{50} = 30 is required for avoiding bias in the D_{50} particle size in a distribution with a standard deviation of 1.5 σ. The numbers on the side of the plot present the absolute sample mass in kg for relative sample volumes of 10, 100, 1,000, etc. If the D_{50} particle size of the deposit was 32 mm, the column under 32 mm is used to interpolate between 4.5 and 45 kg. A relative sample volume of 659 V/V_{50} is approximately $6.6 \cdot 4.5 \text{ kg} \approx 30$ kg.

Sample mass for bias avoidance can become very large for high percentiles in poorly sorted river beds. A relative sample volume of 35,000 V/V_{50} is needed to avoid bias around the D_{95} in a poorly sorted distribution with $s = 2\ \phi$. If the distribution has a D_{50} particle size of 64 mm, an absolute sample mass of 3.5 · 3640 kg = 12.7 metric tons is needed.

The widely used sample-mass requirements by Church et al. (1987) suggest that the mass of a particle of the D_{max} size should comprise 0.1, 1, and 10% of the sample mass (Section 5.4.1.1). For comparison, these criteria are also plotted in Fig. 5.20. Fig. 5.20 shows that even the10% criterion suffices to prevent bias in all but the 99th percentile.

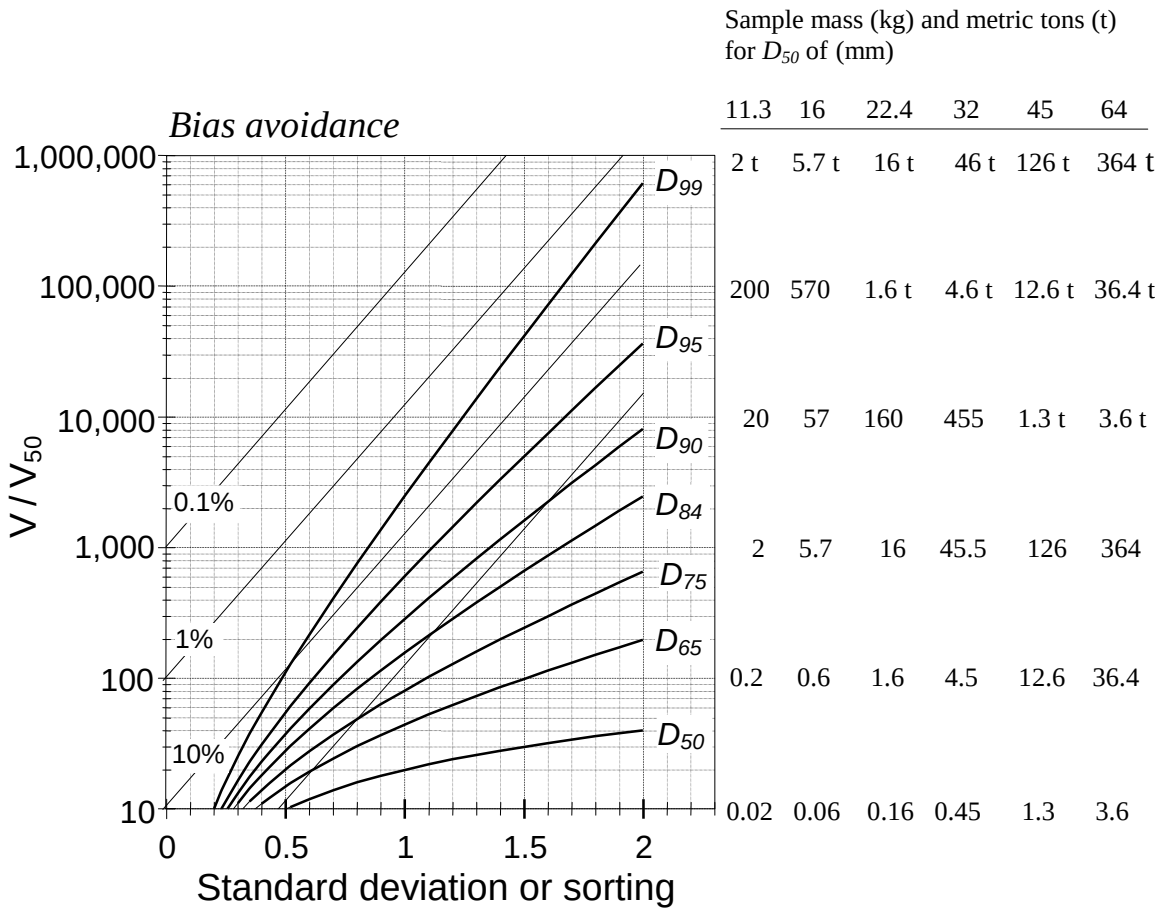


Fig. 5.20: Relation between relative sample volume (V/V_{50}) for bias avoidance and sediment sorting for various percentiles between D_{50} and D_{99}. Sample sizes for paired percentiles (e.g., D_{10} and D_{90}) are identical. The numbers on the right side of the plot indicate sample-mass values in kg and metric tons for 10^2, 10^3, 10^4, 10^5, 10^6, and 10^7 V/V_{50} for various particle sizes of D_{50}. (Modified from Ferguson and Paola (1997), by permission of John Wiley and Sons, Ltd.).

5.4.3.2 Sample mass for specified acceptable error

Sampling precision can be quantified by means of the percentile standard error s_p in ϕ-units between replicate samples. In unbiased samples, s_p is computed from Eq. 5.53. The percentile standard error s_p relates to the absolute error $e_{\pm\phi p}$ around a percentile in ϕ-units by

$$e_{\pm\phi p} = Z_{1-\alpha/2} \cdot \quad s_p \tag{5.61}$$

where $Z_{1-\alpha/2}$ is 1.96 for a 95% confidence limit (Table 5.1). Thus, a percentile standard error s_p of ±0.15 ϕ-units is equivalent to an absolute error $e_{\pm\phi p}$ of almost ± 0.3 ϕ-units, which, in turn, corresponds to a percentage error in mm-units $e_{\%Dp}$ of -18 to +23%, and a percentage standard error in mm-units $s_{p,\%D}$ of -9 to +12% (Fig. 5.8 in Section 5.2.3.4).

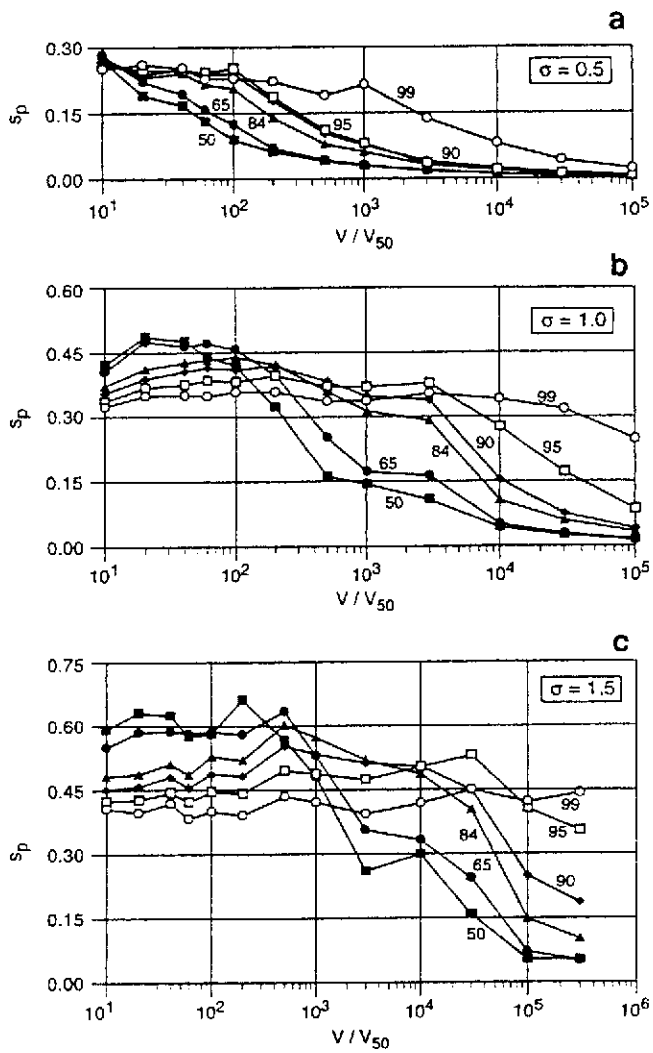
The results of the bootstrap procedure by Ferguson and Paola (1997) in Fig. 5.21 a-c illustrate a similar trend to the results by Rice and Church (1996b; Figs. 5.10 and 5.11) and by Hogan et al. (1993; Fig. 5.17). The error decreases with sample size or sample volume as a function of $1/\sqrt{n}$ or $1/\sqrt{V}$, respectively. For volume-based sampling, it appears that the error decreases only after a threshold sample volume has been exceeded, but this phenomenon may be due to the logarithmic scale of sample size along the *x*-axis.

Results by Ferguson and Paola (1997) clearly show the relationship between standard deviation, sample mass, and sampling error. Sample mass for a specified standard error is orders of magnitude larger for poorly sorted sediment than for well-sorted sediment. Sample mass for a specified error is also larger for the D_{95} percentile size than for the D_{50}. On the basis of this analysis, Ferguson and Paola (1997) developed a dimensionless equation that facilitates computing sample volume V_g necessary to obtain a specified percentile standard error s_p when sampling a population with a standard deviation σ:

$$\log\left(\frac{V_g}{V_{50}}\right) = 1.4 + 4.2 \log(\sigma) + 0.9\,\sigma \cdot \quad Z_p \;\; - 2 \log(s_p) \tag{5.62}$$

where Z_p is the *p*th percentile variate of the unit normal distribution (Tables 5.9 and 5.1). For a preset percentile standard error s_p = 0.15 ϕ-units, the last term in Eq. 5.62 yields the numerical value of -1.65 and simplifies to

$$\log\left(\frac{V_g}{V_{50}}\right) = 3.0 + 4.2 \log(\sigma) + 0.9\,\sigma \cdot \quad Z_p \tag{5.63}$$



D_{50} (mm)	10	100	1,000	10,000	100,000	1,000,000
11.3:	0.020	0.20	2.0	20.0	200	2,002
16:	0.057	0.57	5.7	56.8	568	5,683
22.6:	0.160	1.60	16.0	160.0	1,600	16,017
32:	0.455	4.55	45.5	454.7	4,547	45,467
45:	1.264	12.64	126.4	1,264.4	12,644	126,439
64:	3.637	36.37	363.7	3,637.3	36,373	363,734

Sample mass (kg)

V/V_{50}

Fig. 5.21 a-c: Results from the bootstrap analysis: Relation between percentile standard error s_p and relative sample volume V/V_{50} for given percentiles between D_{50} and D_{99}, and population standard deviation of $\sigma = 0.5\ \phi$ (a), $\sigma = 1.0\ \phi$ (b), and $\sigma = 1.5\ \phi$ (c). Numbers on curves indicate percentiles. Note that the bottom plot (c) covers one more log cycle of sample size than plots a and b (Reprinted from Ferguson and Paola 1997, by permission of John Wiley and Sons, Ltd.). The numbers on the bottom of the plots indicate sample-mass values in kg for 10^1, 10^2, 10^3, 10^4, 10^5, and 10^6 V/V_{50} for various sizes of D_{50}.

If the error around the D_{50} is of concern, the last term of Eq. 5.63 can be omitted since the numerical value of Z_p becomes 0.

Example 5.17:
For a deposit that can be assumed to approach a normal distribution and has a sorting of s = 1.2, the relative sample volume V_g/V_{50} required for estimating the D_{95} particle size to within a standard error of ±0.15 ϕ-units is (Eq. 5.62) is

$$\frac{V_g}{V_{50}} = 10^{(1.4 + 4.2 \log(1.2) + 0.9 \cdot 1.2 \cdot 1.645 - 2 \log(0.15))}$$
$$= 10^{(1.4+0.33+1.78-(-1.65))} = 10^{5.16} = 144{,}544.$$

If the bed material D_{95} particle size is 200 mm, the D_{50} particle size can be computed from Fig. 2.19, or Eqs. 5.58 and 5.59 and is 50.9 mm. The absolute sample volume V_s can then be computed from

$$V_s = 144{,}544 \cdot (\pi/6) \cdot D_{50}^3 = 144{,}544 \cdot 69.05 \text{ cm}^3$$
$$= 9{,}980{,}494 \text{ cm}^3 \approx 10 \text{ m}^3.$$

Multiplication by particle density ρ_s =2650 kg/m^3 provides the sample mass

$$m_s = 10 \text{ m}^3 \cdot 2650 \text{ kg/m}^3 = 26{,}500 \text{ kg} = 26.5 \text{ metric tons.}$$

Dividing by a bulk density of 1.5 kg/m^3 for shoveled gravel, sample bulk volume is 17.7 m^3 (approximately the volume of a small office).

Relative sample volume in terms of V/V_{50} for sample precision of ±0.1, ±0.15, and ±0.2 ϕ standard errors was computed with Eq. 5.63 and plotted versus the sediment sorting for various percentiles between D_{50} and D_{99} in Figs. 5.22 a -c. The graphs indicate that relative sample volume, and thus sample mass, strongly increases with sediment sorting and with an increase in the percentile size being addressed. Because the parent distribution was Gaussian in terms of ϕ-units, sample mass for a preset error and sorting are symmetrically distributed around the mean, and thus identical for paired percentiles such as the D_{10} and the D_{90}.

Fig. 5.22 is used similar to Fig. 5.20. The first step is to select the plot with the appropriate error (plot a, b, or c). If, for example, the task is to estimate the sample size necessary to remain below an absolute error of ±0.2 ϕ around the D_{75} in a gravel bed with a standard deviation of s = 1.5 ϕ, select Fig. 5.22 a. A relative sample volume of V/V_{50} = 12,000 is obtained from the graph for D_{75} in Fig. 5.22 a. If the bed-material D_{50} particle size is 64 mm, the absolute sample mass may be read on the right side of the plot as 1.2 ·

36.4 t = 43.5 metric tons. This example is somewhat extreme because the accurate determination of the D_{95} subsurface particle-size in gravel-bed streams is usually not the task of volumetric sampling. Sample mass is orders of magnitude smaller if the D_{50} is the percentile of interest, and if the bed-material is better sorted. A relative sample volume of V/V_{50} = 2,500 suffices for estimating the D_{50} to within an absolute error of ±0.2 ϕ if the bed material sorting is 1ϕ. For a D_{50} particle size of 22.6 mm, sample mass on the right side of the plot can be read as 2.5 · 16 kg = 40 kg.

The user may be frequently surprised by the large sample sizes necessary for volumetric samples in coarse gravel and cobble-bed streams. Sample masses larger than a few 100 kg are usually not feasible to collect in mountain streams. It may become necessary to reduce the tolerable error for the study, or to restrict precision requirements to central percentiles.

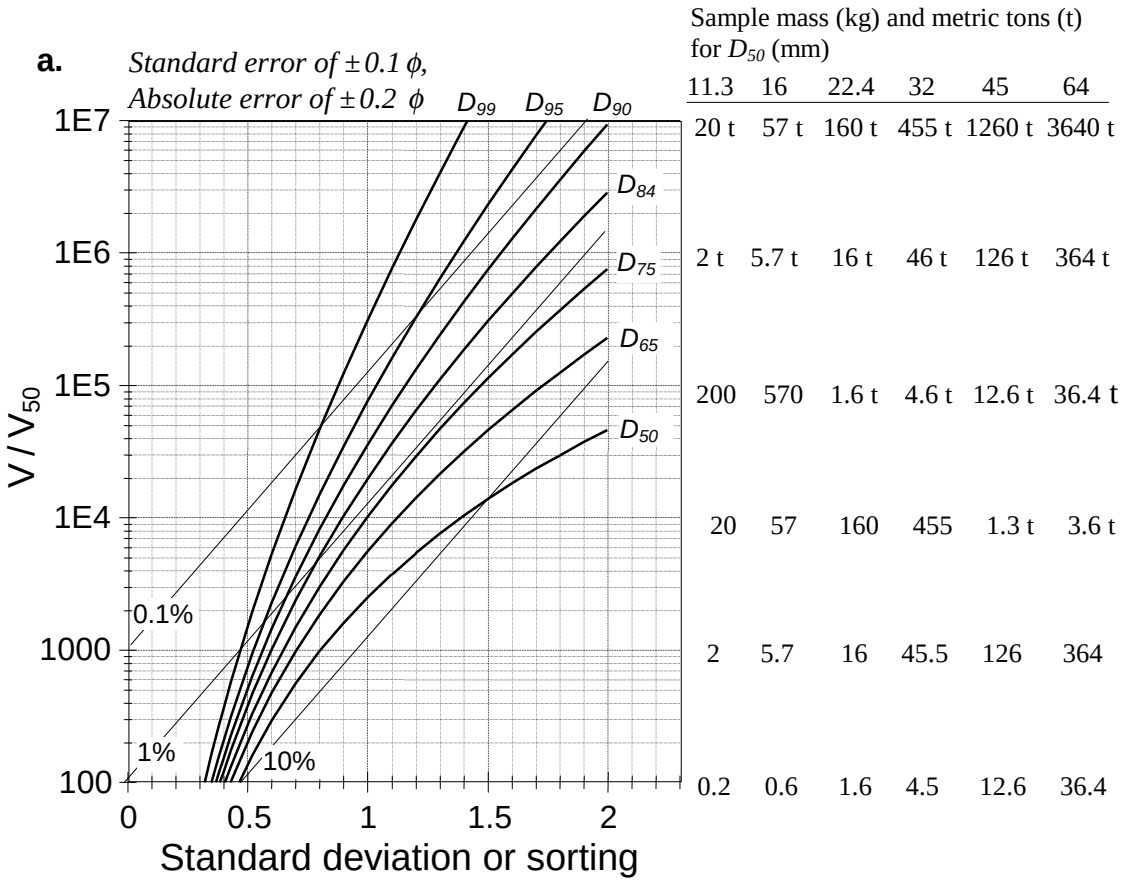
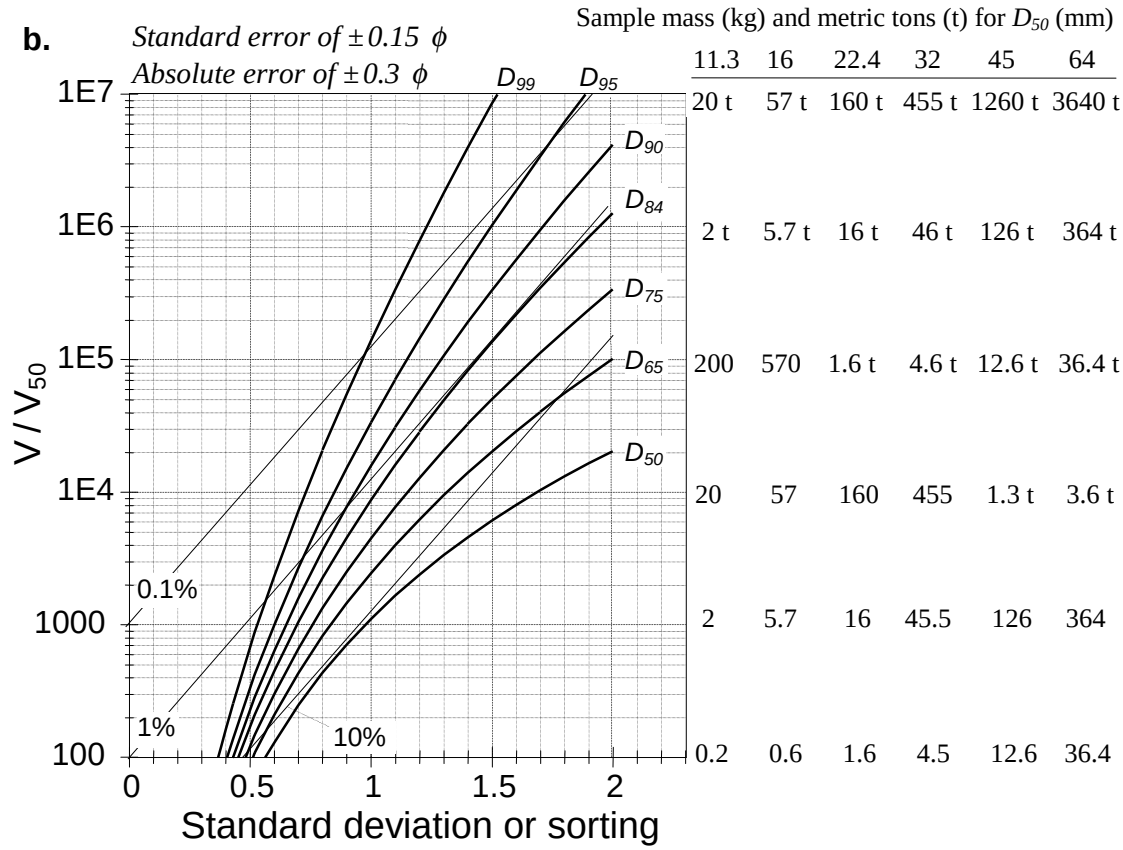


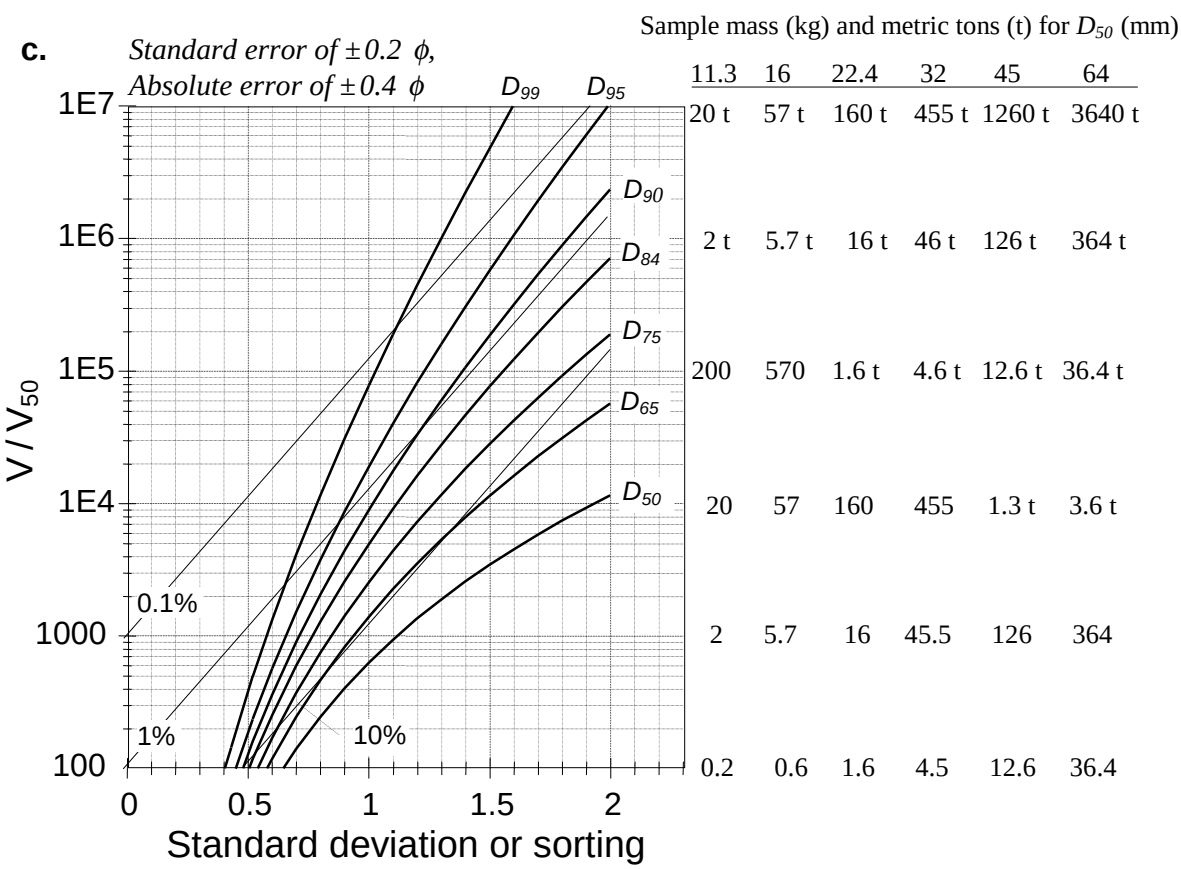
Fig. 5.22 a-c: Relative sample volume V/V_{50} as a function of sediment sorting for various percentiles between D_{50} and D_{99} for specified standard errors of ± 0.1 ϕ-units (b), ± 0.15 ϕ-units (c), and of ± 0.2 ϕ-units (d). Absolute sample mass is a multiple of the volume V_{50} of the median-sized particle. The 0.1, 1, and 10% sample-mass criteria by Church et al. (1987) are included for comparison. (Modified from Ferguson and Paola (1997), by permission of John Wiley and Sons, Ltd.). The numbers on the right side of the plots indicate sample mass values in kg and metric tons for 10^2, 10^3, 10^4, 10^5 , 10^6, and 10^7 V/V_{50} or various sizes of D_{50}.

(continued on next page)

b.


Standard error of ±0.15 ϕ
Absolute error of ±0.3 ϕ
Sample mass (kg) and metric tons (t) for D50 (mm)
11.3 16 22.4 32 45 64
20 t 57 t 160 t 455 t 1260 t 3640 t
2 t 5.7 t 16 t 46 t 126 t 364 t
200 570 1.6 t 4.6 t 12.6 t 36.4 t
20 57 160 455 1.3 t 3.6 t
2 5.7 16 45.5 126 364
0.2 0.6 1.6 4.5 12.6 36.4
D99
D95
D90
D84
D75
D65
D50
0.1%
1%
10%
1E7
1E6
1E5
1E4
1000
100
V / V50
0
0.5
1
1.5
2
Standard deviation or sorting

c.


Standard error of ±0.2 ϕ,
Absolute error of ±0.4 ϕ
Sample mass (kg) and metric tons (t) for D50 (mm)
11.3 16 22.4 32 45 64
20 t 57 t 160 t 455 t 1260 t 3640 t
2 t 5.7 t 16 t 46 t 126 t 364 t
200 570 1.6 t 4.6 t 12.6 t 36.4 t
20 57 160 455 1.3 t 3.6 t
2 5.7 16 45.5 126 364
0.2 0.6 1.6 4.5 12.6 36.4
D99
D95
D90
D84
D75
D65
D50
0.1%
1%
10%
1E7
1E6
1E5
1E4
1000
100
V / V50
0
0.5
1
1.5
2
Standard deviation or sorting

The widely used sample-mass criteria by Church et al. (1987), i.e., the mass of a particle of the D_{max} size comprises 0.1, 1, and 10% of the sample mass (Fig. 5.14, Section 5.4.1.1) are plotted in Fig. 5.22 for comparison. The 1% criterion (D_{max} <1% of sample mass) is sufficient to determine the D_{84} and all smaller percentiles with an absolute error of ±0.3 ϕ-units in bed material with a sorting coefficient between 1 and 2. In fact, the sample requirement could be an order of magnitude or two less than the 1% criterion for determining the D_{50} particle size to within an acceptable absolute error of ±0.3 ϕ-units. However, the 0.1 % criterion needs to be applied for large percentiles > D_{84}, or if more stringent error criteria are used.

5.4.4 Comparison of error curves for low, central, and higher percentiles

The general shape of the error curve, i.e., the relationship between precision and sample size $e = f\,(1/\sqrt{n})$ is similar for all percentiles, irrespective of the manner in which the error was computed, and irrespective of any assumptions made about the parent distribution. However, sampling error is not automatically smallest for the smallest percentile (e.g., the D_5), but is controlled by the way in which the sampling error was computed. The error can be lowest for either the D_5, D_{50}, or the D_{95} within a specified gravel population depending on whether the error was computed:

- as absolute or percent error,
- in terms of mm or ϕ-units, and
- from an assumed symmetrical, or asymmetrical underlying size distribution.

Consequently, comparisons of errors around different percentiles need to specify exactly how the error was computed and which assumptions were made about the underlying distributions.

5.4.4.1 Symmetrical parent distributions

Absolute error in units of ϕ and mm

In symmetrical, unskewed parent distributions, absolute errors in ϕ-units for a given sample size are paired around the mean. Thus, error curves are identical for the ϕ_5 and ϕ_{95} percentiles, and the ϕ_{16} and ϕ_{84} percentiles. The errors are highest for the distribution tails, i.e., the ϕ_5 and ϕ_{95} percentiles, and lowest for the ϕ_{50}. The error curves for a theoretical Gaussian distribution provided by Rice and Church (1996b) in Fig. 5.10 are an example for the systematical distribution of errors. If the same degree of precision is desired for each percentile, a smaller sample size suffices to determine the error around the mean or some central percentile than for the fine or the coarse tail. If the error analysis is performed in mm-units, the absolute mm-errors are highest around high percentiles (e.g., D_{95}), and lowest around small percentiles (D_5). The error curves by Hogan et al. (1993) in Fig. 5.17 are an example. The relative positions of error curves are sketched for particle sizes in ϕ- and mm-units, absolute and relative errors, for symmetrical, unskewed, as well as for asymmetrical, skewed distributions in Fig. 5.23.

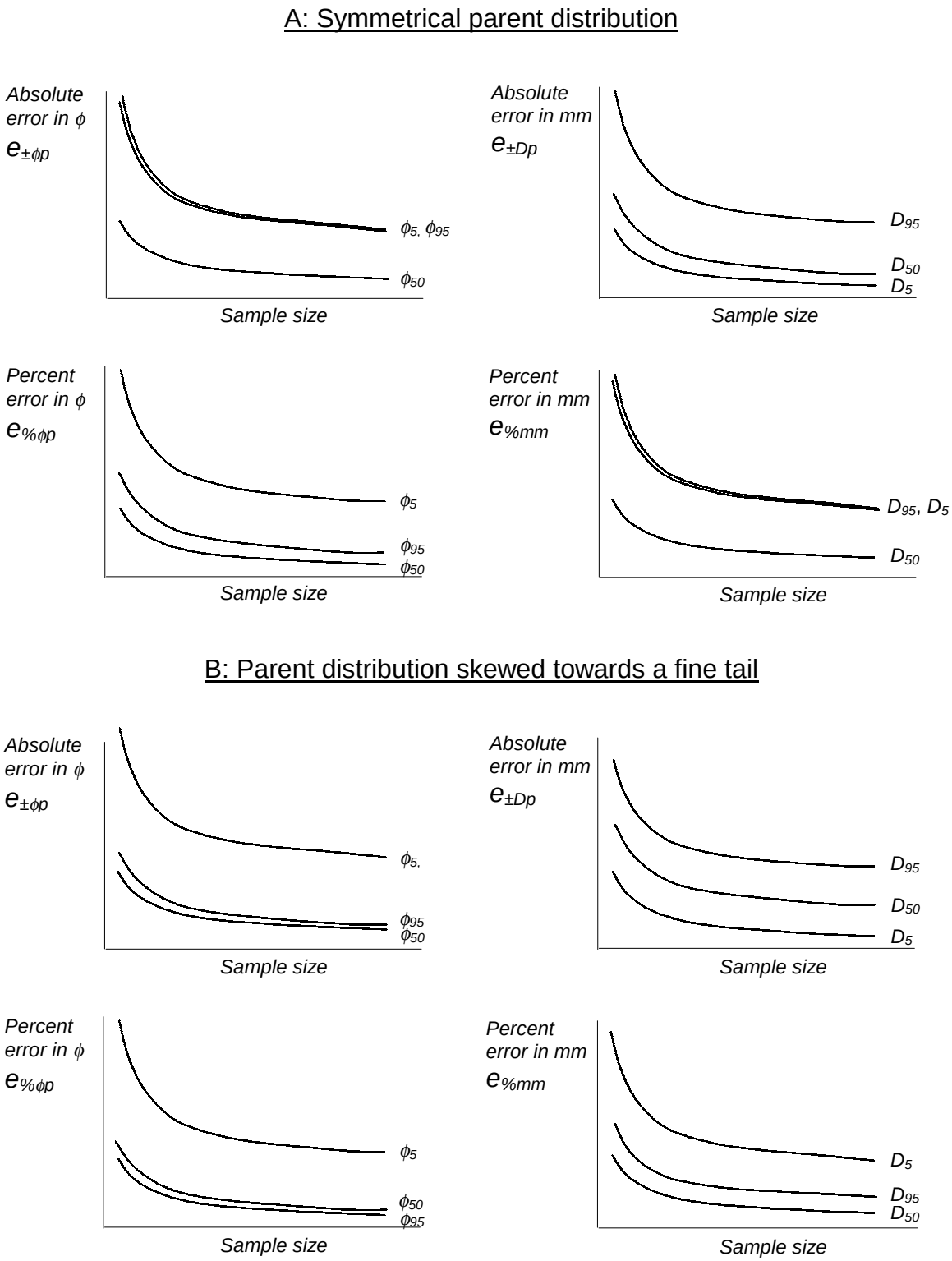


Fig. 5.23: Comparison of error curves around the D_5, D_{50}, and D_{95} percentile for different computations of error (absolute, percent, ϕ -units and mm). All computations are for the same gravel deposit. A normal and symmetrical parent distribution was assumed in (A), and an asymmetrical distribution positively skewed towards a tail of fines was assumed in (B).

Percent error in units of ϕ and mm

The percent error around percentiles in ϕ-units is lowest for the ϕ_{50}, higher for the ϕ_{95}, and significantly higher for the ϕ_5. The relative error around percentiles in mm-units is paired, with the lowest error for the D_{50}, and equally high errors for the D_5 and D_{95} (Fig. 5.23).

5.4.4.2 Asymmetrical parent distributions skewed towards a fine tail

Absolute error in units of ϕ and mm

Particle-size distributions, even when analyzed in ϕ-units, are rarely symmetrical in coarse gravel-bed streams, but are often positively skewed towards a tail of fine particles. The position of the error curves for the ϕ_5, ϕ_{50}, and ϕ_{95} is different for symmetrical and asymmetrical parent distributions.

For asymmetrical parent distributions that are skewed towards a fine tail, absolute errors around percentiles in ϕ-units for a given sample size are highest around the ϕ_5, lowest for the ϕ_{50}, and only slightly higher around the ϕ_{95} than around the ϕ_{50}. The error curves provided by Rice and Church (1996b) for the bootstrap analysis in Fig. 5.11 are an example. If the same degree of precision is desired for each percentile, nearly the same sample size that suffices to determine the D_{50} of the distribution is sufficient for the D_{95} as well. However, a huge sample mass is required to estimate the D_5 to within the same precision. An error analysis in mm-units results in absolute mm-errors being highest around the D_{95}, and lowest around the D_5. See the error curves by Hogan et al. (1993) in Figs. 5.17 a and b for an example. Fig. 5.23 b compares error curves for absolute and relative errors in ϕ and mm for skewed distributions. The position of the ϕ_5 and ϕ_{95} curves are switched if the distributions are negatively skewed towards a tail of coarse particles (e.g., beds comprising mostly sand and a few larger gravel particles).

Percent error for units in ϕ and mm

The percent error around percentiles in ϕ-units is approximately equally low for the D_{50} and the D_{95}, and highest for the D_5. The relative error around percentiles in mm-units is lowest for the D_{50}, higher for the D_{95}, and highest for the D_5 (Fig. 5.23 b).