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Correlation and Prediction of Snow Water Equivalent from Snow Sensors

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Since 1982, under an agreement between the California Department of Water Resources and the USDA Forest Service, snow sensors have been installed and operated in Forest Service-administered wilderness areas in the Sierra Nevada of California. The sensors are to be removed by 2005 because of the premise that sufficient data will have been collected to allow "correlation" and, by implication, prediction of wilderness snow data by nonwilderness sensors that are typically at a lower elevation. Because analysis of snow water equivalent (SWE) data from these wilderness sensors would not be possible until just before they are due to be removed, "surrogate pairs" of high- and low-elevation snow sensors were selected to determine whether correlation and prediction might be achieved. Surrogate pairs of sensors with between 5 and 15 years of concurrent data were selected, and correlation and regression were used to examine the statistical feasibility of SWE prediction after "removal" of the wilderness sensors. Of the 10 pairs analyzed, two pairs achieved a correlation coefficient of 0.95 or greater. Four more had a correlation of 0.94 for the accumulation period after the snow season was split into accumulation and melt periods. Standard errors of estimate for the better fits ranged from 15 to 25 percent of the mean April 1 snow water equivalent at the high-elevation sensor. With the best sensor pairs, standard errors of 10 percent were achieved. If this prediction error is acceptable to water supply forecasters, sensor operation through 2005 in the wilderness may produce predictive relationships that are useful after the wilderness sensors are removed.

Retrieval Terms: snow sensor, water supply forecasting, snow water equivalent prediction, snow pillow

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In Brief...

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In 1982 the USDA Forest Service agreed to allow the California Department of Water Resources (DWR) to temporarily install fifteen snow sensors in some wilderness areas administered by the Forest Service (FS), in the Sierra Nevada. The intent of the temporary installation was to correlate and, by implication, allow development of prediction equations that would allow water supply forecasters to make predictions of snow water equivalent (SWE) at the wilderness sites on the basis of data collected by sensors in nonwilderness areas. Removal is mandated because, according to the FS, installations such as snow sensors do not comply with the uses of wilderness intended by the Wilderness Act (PL 88-572).

This analysis assesses the feasibility of predicting the SWE at high-elevation snow sensors from low-elevation sensors using least squares regression. Because of the short record of most wilderness sensors, ten pairs of sensors with longer records were selected as surrogates for the wilderness-nonwilderness pairs. Of the ten pairs, only two pairs met the initial criterion of a correlation coefficient of at least 0.95. Six of the ten had a correlation coefficient of less than 0.75. Record smoothing and deletion of unchanging values yielded little improvement. Improved results were obtained by dividing the records for the ten

pairs into accumulation and ablation periods. April 1 was used as the division date. Six pairs then yielded correlation coefficients of 0.94 or better for the accumulation period, but only one pair reached that level during ablation.

The selection of the ten sensor pairs was based on minimizing horizontal and elevational difference and eliminating sensors that had frequent technical problems or short records. Colocation within a drainage basin was also a goal. Elevational difference proved to be an important attribute for pairing, and distance and basin were less important. This analysis underscored the difficulty in *a priori* selection of pairs. Future analyses should define pairs by calculating correlation coefficients for between five and 15 sensors within 500 m elevation and a large distance such as 75 km. The 500-m difference in elevation may minimize the co-occurrence of rain at the low sensor and snow at the high sensor.

The lowest standard errors (SEs) of the regressions for the sensor pairs ranged from 15 to 25 percent of the high-elevation mean April 1 SWE. Other sensor pairs yielded SEs of 30 to 55 percent of the mean April 1 SWE. Although improved sensor selection and more sophisticated prediction methods would probably yield better pairings and lower SEs, even the best SEs obtained in this analysis were 10 percent of the high-elevation sensor's long-term April 1 mean SWE. Confidence intervals (95 percent) around predictions based on these "best" equations would be at least ± 20 percent of the April 1 mean SWE. If this prediction error is acceptable to water supply forecasters, then data collection through 2000 or 2005 in the wilderness areas may produce predictive relationships that are useful after the wilderness sensors are removed.

Introduction

In 1982 the USDA Forest Service (FS) agreed to allow the California Department of Water Resources (DWR) to temporarily install fifteen snow sensors¹ in some wilderness areas administered by the Forest Service, in the Sierra Nevada. The intent of the temporary installation of sensors in wilderness was to allow "correlation" (defined below) between wilderness and nonwilderness sensors. If such a correlation could be established, then the FS believed that forecasters could predict future water supply on the basis of data collected at sensors in the nonwilderness areas. Under the terms of a Special-Use Permit signed by the FS and the California Department of Water Resources (DWR) in December 1982, each sensor was to be allowed a 10-year correlation period. If correlation is not successfully achieved after 10 years, a 5-year extension may be allowed. Removal is mandated because, according to the FS, installations such as snow sensors do not comply with the uses of wilderness intended by the Wilderness Act (PL 88-572). In that the last sensors were installed in 1990, they all must be removed by 2005 or sooner under the current agreement.

The Environmental Assessment (EA) that was prepared by the FS recognized the need to include both wet and dry April-to-July runoff years (AJRO) in the correlation period by stipulating that AJROs from both the upper and lower deciles should be represented. Two wilderness sensors that were installed before the 1983 season do have upper and lower decile seasons because of the wet 1983 and the dry 1,987 seasons. In the history of the basins of concern, there is less than a 35 percent chance that both upper and lower decile AJROs will occur during a 10-year period. The chance increases to 60 percent during a 15-year period.

The EA set a standard of 0.95 as a successful correlation level. The only reference to analytical methods to be used during this temporary installation and correlation exercise was stated in the Special-Use Permit: "Standard statistical techniques acceptable to the FS will be used in making this determination." However, when researchers at the Pacific Southwest Research Station and DWR began development of a statistical methodology for predicting snow water equivalent (SWE) with sensors in 1988 (Brandow and Azuma 1988), they recognized two problems in the Permit between the DWR and FS. First, achieving an "acceptable" correlation of 0.95 did not necessarily mean that a low-elevation (nonwilderness) record could successfully predict the SWE at a wilderness sensor. Second, time periods covered by existing wilderness sensor records were too short to test whether an acceptable level of correlation could be achieved.

¹The snow sensor network provides water supply forecasters in California with current snow water equivalent information at 110 sites in California's northern coastal and inland mountains. The sensors supply part of the information needed to predict annual runoff to the state's reservoirs and rivers. Fifteen sensors are located in USDA Forest Service-administered wilderness areas. Each sensor consists of a thin stainless steel tank with a surface area of 1.8 m² that is covered with a few centimeters of soil, a buried radio transmitter, and a solar panel and antenna mounted on a 6-m tall mast.

Correlation versus Prediction

The correlation coefficient "r" is a measure of the linear relationship between two variables, such as SWE from snow sensors X and Y (Haan 1977):

$$r_{x,y} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}} \quad (1)$$

where x_i and y_i are the observed SWEs, \bar{x} and \bar{y} are the means of the observed SWEs, and n is the number of daily SWE pairs of observations.

Although correlation is a measure of a linear relationship, regression is a more suitable technique for predicting how one variable changes given a change in another variable. Regression fits are often evaluated by R^2 , the coefficient of determination, which is a measure of the ability of the regression line to explain variations of the dependent variable (Haan 1977):

$$R^2 = \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^n (y_i - \bar{y})^2} \quad (2)$$

where \hat{y}_i are the estimated values.

The pertinent practical question is how accurately can a daily SWE at a high-elevation, wilderness sensor be predicted on the basis of the SWE at a low-elevation, nonwilderness sensor? Evaluation of the goodness of fit of the regression is typically based on the standard error of estimate of the regression equation (SE) and on the coefficient of determination, R^2 (Haan 1977):

$$SE = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2}} \quad (3)$$

Because of the concern for error around the prediction, this report will focus on SE, but will report R^2 and the correlation coefficient, r , as well.

Length of Record

The second issue relates to the length of record of the wilderness sensors. Most of these sensors were not installed until after 1982, and some were not completed until 1990. By the time sufficient record is obtained and analyses performed, the 10-year deadline will have passed. A solution to this problem is to select "surrogate sensor pairs" of high- and low-elevation sensors that currently have adequate records and perform the analysis on them (Brandow and Azuma 1988). Brandow and

Azuma selected nine pairs of sensors on the basis of proximity, elevational difference, degree of correlation of their co-located snow courses, and representation from the Trinity basin and the north, central, and south Sierra Nevada. Correlations of 0.95 were attained with some surrogate sensor pairs, but the SEs ranged from 10 to 30 percent of the peak predicted SWE. The authors concluded that this magnitude of error would adversely affect the accuracy of the forecast of the current water supply, especially in some of the southern Sierra Nevada basins with extensive wilderness (Brandow and Azuma 1988).

It is presumed that the results from the analysis of the surrogate pairs apply to the wilderness-nonwilderness sensors. The only difference between surrogate pairs and wilderness-nonwilderness pairs is the wilderness designation of the high-elevation wilderness sensor. If good predictive relationships were obtained using surrogate pairs, good results might be achievable with the 10 or 15 years of data that would be available from the no wilderness-wilderness sensors before the wilderness sensors were removed from FS-administered sites. Conversely, if poor predictions were obtained, it would indicate that the use of predicted data for the wilderness sites would not be appropriate.

This paper reports on the feasibility of predicting SWE at high elevations from SWE obtained by sensors at low elevations in the Sierra Nevada using least squares regression and provides guidelines for selection of no wilderness-wilderness sensor pairs.

Correlation and Regression Methodology and Results

Characteristics of Data from Snow Sensors

A snow sensor measures the weight and hence the SWE of the snowpack via the pressure in the fluid-filled tanks. A pressure transducer produces a voltage that is measured, converted to a digital value, and transmitted to a central archiving site at daily or more frequent intervals. Various technical and physical problems contribute to errors in the data stream (see Brandow and Azuma 1988; McGurk 1986; and Suits 1985 for further discussion). The two major types of errors that affect SWE prediction are missing data and short-term fluctuations (chatter or flutter). Data maybe missing for long periods because of sensor leaks or major system failures, or for short periods because of temporary telemetry problems. In either case, the missing data hinder the development of a prediction equation or prevent prediction for varying periods.

Chatter refers to the approximately 1- to 5-cm variations in SWE that occur between daily readings and that cannot be

attributed to either precipitation or melt. An example of chatter can be seen on April 17 for the 1986 plot at the Central Sierra Snow Laboratory (CSSL) (*fig. 1*). The 5-cm spike in the sensor trace is not echoed by an independent measurement from an isotopic profiling snow gauge (Kattelmann and others 1983), and less than 1 cm of precipitation was recorded at CSSL during that time. Smoothing algorithms can dampen these oscillations, but chatter nevertheless adds noise to the data. Inherent accuracy of the transducers is in the same range as the chatter and varies on the basis of transducer manufacturer and the range measured by the transducer.

The typical seasonal plot of SWE vs. time from a snow sensor in the mid-to-high elevation zone of the Sierra Nevada has an accumulation period, a peak SWE around 1 April, and an ablation (melt) period that ends when the snow is gone (*fig. 1*). The steep portions are storm (rising SWE) or melt (falling SWE) events, and the horizontal intervals are clear weather periods with little melt. The sensor's advantages include its ability to respond to new snow or melt within hours and to report these data at regular intervals or upon demand. Midwinter rains on the snowpack are not generally detectable except at nonwilderness sites that also have storage precipitation gauges. The disadvantages of sensors are their complexity, their need for seasonal maintenance, their inaccessibility (both geographic and position under several meters of snow), and the data reduction and archiving requirements of the telemetered SWEs. Sensors are, in general, reasonably accurate, efficient tools to repeatedly measure SWE nondestructively in remote locations (Kattelmann and others 1983; McGurk 1986). The profiling snow gauge SWEs in *figure 1*, in contrast, are probably accurate to within ± 0.5 cm but require a nuclear source and detector, plus line power and an operator (Kattelmann and others 1983).

Selection of Surrogate Sensor Pairs

The surrogate sensor pair technique used by Brandow and Azuma (1988) was also used in this analysis. Length of record, geographic proximity, and difference in elevation between members of the pair were the basis of the pairings. A list of 32 sensors that were deemed to be the most reliable and accurate with respect to on-site SWE sampling was supplied by DWR (Hart 1989). Length of record was a primary criterion, and then proximity and elevational difference were considered, resulting in a selection of seventeen sensors that were sorted into a list of ten pairs (*table 1*). All but three of the sensors were in operation by 1971, so most of the pairs had a potential 15 years of record. Most of the pairs are in the same river basin and are less than 30 km apart (*fig. 2*). The mean elevational difference between the chosen sensor pairs is 586 m, but ranges from 215 m to 1,460 m. Seven of the ten pairs are between 300 m and 675 m apart in elevation. The ten pairs ranged from 8 to 34 km apart, and had a mean proximity of 24.3 km.

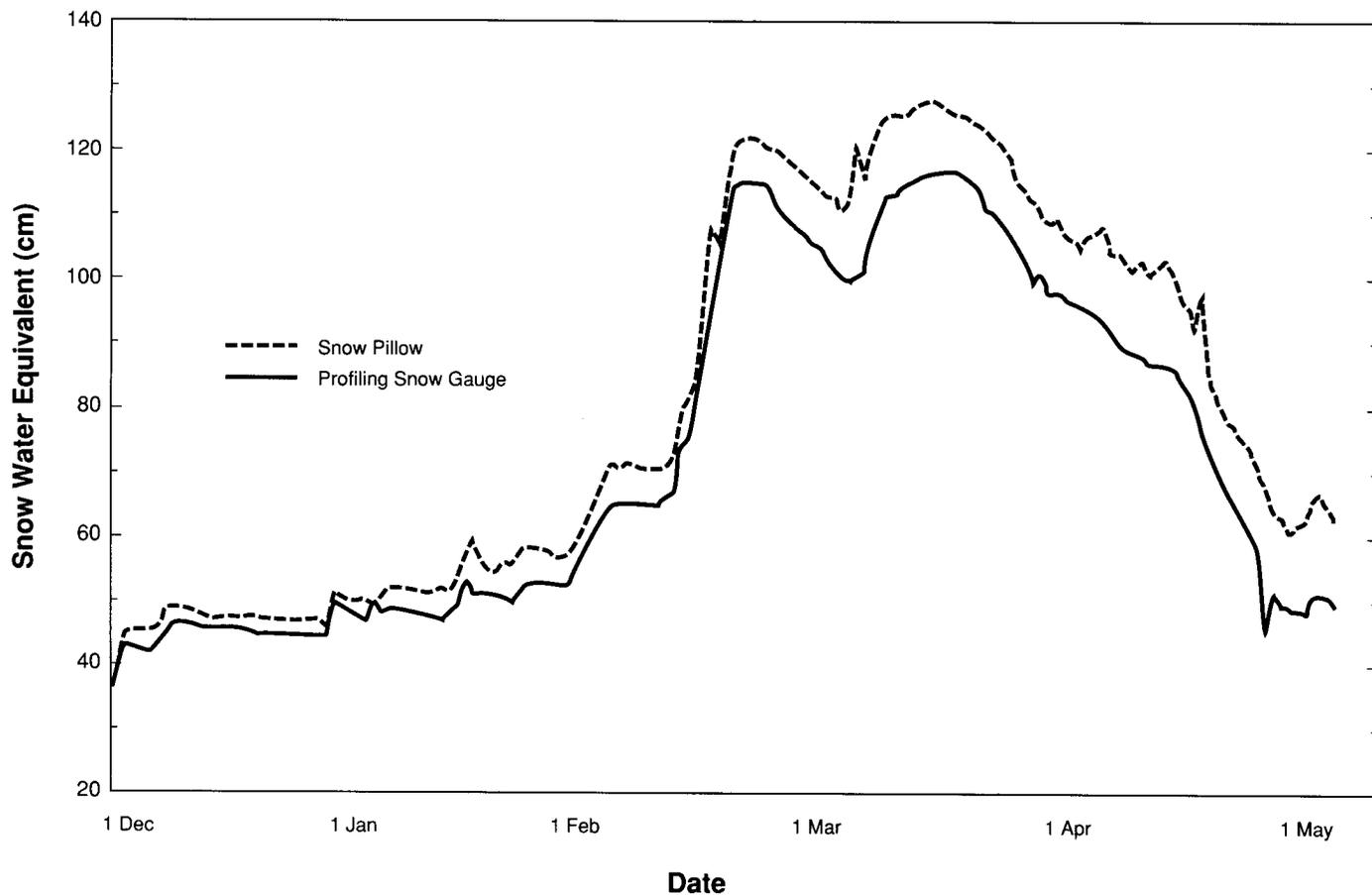


Figure 1-Snow water equivalent in water year 1986 from a snow sensor and an isotopic profiling snow gauge at the Central Sierra Snow Laboratory near Soda Springs, California.

Table 1-Elevation differences, geographic proximity, and number of years with at least thirty data values for the ten snow sensor pairs used in a surrogate pair analysis

Sensor name	Elevation	Years	Sensor name	Elevation	Years	Elev. difference	Proximity
	<i>m</i>	<i>no.</i>		<i>m</i>	<i>no.</i>	<i>m</i>	<i>km</i>
Blue Canyon	1615	12	Central Sierra Snow Laboratory	2100	10	485	23
Robbs Saddle	1800	14	Schneiders	2665	15	865	34
Bloods Creek	2195	8	Mud Lake	2410	15	215	18
Poison Ridge	2100	7	Green Mountain	2410	15	310	25
Graveyard	2100	5	Green Mountain	2410	15	310	8
Poison Ridge	2100	7	Kaiser Pass	2775	16	675	31
Green Mountain	2410	15	Mammoth Pass	2895	12	485	20
West Woodchuck	2680	17	State Lakes	3140	26	460	29
Giant Forest	2030	16	Upper Tyndall	3490	18	1460	32
Quaking Aspen	2195	6	Pascoes	2790	15	595	22

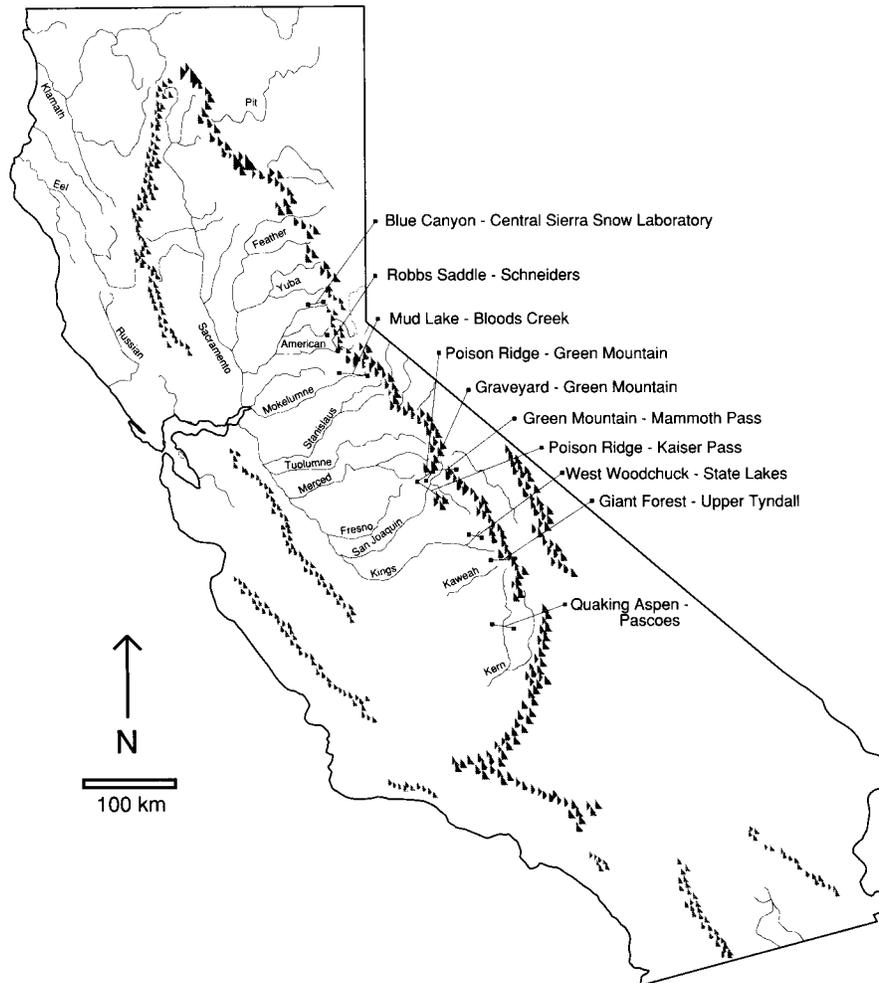


Figure 2-Location of surrogate sensor pairs in California's Sierra Nevada.

Processing of Sensor Records

Sensor data were supplied by DWR on magnetic tape. Records of instantaneous daily SWE depth started within a year of the sensor's construction (table 1) and concluded with the 1986 water year. Some of the records have days, weeks, or entire years missing because of equipment malfunctions. Each record was screened for anomalies by automatic search routines and by eye. Daily values that were negative, extremely large, or showed large deviations from prior and post values were coded as missing values. The resultant data file was termed the "complete" or "all-data" file. Simple linear regression was performed on the daily SWE depths of each pair of sensors (Minitab 1985). The high- and low-elevation sensors were the dependent and independent variables, respectively, and the number of matching dates (sample size) was recorded, as well as the SE and the R^2 (table 2).

Record Reduction

The typical snow sensor record has frequent non-storm periods during which the SWEs remain largely unchanged.

Because there are so many unchanging values, the regression may weight those values more than the changing values. To overcome this problem, a reduction process was used to delete consecutive points with identical values. This data reduction also would reduce the serial correlation that is typical of time series data; this type of day-to-day "persistence" violates the independence assumption typically made with regression. As a result, the sample sizes decreased by a mean value of 18 percent with a range between 9 and 46 percent. Regression was then performed with the "reduced data set" file (table 2). The procedure reduced the R^2 insignificantly and increased the standard errors between 0.9 and 2.3 cm.

Record Smoothing

A three-value moving average algorithm was applied to the all-data file to reduce the magnitude of the chatter (table 3). This type of smoothing is usually chosen for its simplicity. A longer moving average was not used because of the desire to retain the timing of small (5-10 cm) changes associated with storms of a day's duration or less. The smoothed daily SWE for day "i", k'_i , was the average of k_{i-1} , k_i , and k_{i+1} (Bloomfield 1976). The

Table 2—Regression results of daily snow water equivalent (SWE) values for the surrogate snow sensor pairs for the complete and reduced data sets

Sensor pair and data set	Sample size	Standard error	Percent of April 1 SWE ¹	Coefficient of determination (R ²)
		<i>cm</i>		<i>pct</i>
Blue Canyon–Snow Lab				
- all data	624	23.2	27	50
- reduced data	620	23.3	27	50
- reduced accum.	519	21.8	25	52
- reduced ablation	103	16.3	19	61
Robbs Saddle–Schneiders				
- all data	2146	34.4	39	58
- reduced data	1959	35.3	40	57
- reduced accum.	1285	19.1	22	81
- reduced ablation	579	46.0	52	40
Bloods Creek–Mud Lake				
- all data	1133	25.0	22	83
- reduced data	1031	25.5	22	82
- reduced accum.	628	10.4	9	96
- reduced ablation	372	22.3	20	81
Poison Ridge–Green Mountain				
- all data	508	43.8	56	51
- reduced data	471	44.9	57	49
- reduced accum.	261	15.8	20	88
- reduced ablation	212	58.7	75	40
Graveyard–Green Mountain				
- all data	334	43.4	56	55
- reduced data	284	44.3	57	49
- reduced accum.	158	16.4	21	88
- reduced ablation	127	54.6	70	36
Poison Ridge–Kaiser Pass				
- all data	635	36.1	45	60
- reduced data	608	36.0	45	60
- reduced accum.	324	13.0	16	94
- reduced ablation	275	39.6	50	57
Green Mountain–Mammoth Pass				
- all data	972	40.0	37	41
- reduced data	838	41.1	38	37
- reduced accum.	534	28.5	26	63
- reduced ablation	306	35.3	33	27
West Woodchuck–State Lakes				
- all data	3216	11.1	15	91
- reduced data	1734	13.3	18	88
- reduced accum.	995	5.8	8	97
- reduced ablation	682	18.2	25	82
Giant Forest–Upper Tyndall				
- all data	2857	28.5	40	46
- reduced data	1499	31.3	44	37
- reduced accum.	922	22.2	32	54
- reduced ablation	536	33.7	48	33
Quaking Aspen–Pascoes				
- all data	772	13.6	22	91
- reduced data	565	15.0	24	90
- reduced accum.	343	6.3	10	97
- reduced ablation	195	20.0	32	88

¹Percentage is the SE (equation 3) divided by the long-term mean of the April 1 SWE of the high-elevation sensor, multiplied by 100.

Table 3—Regression results of daily snow water equivalent (SWE) values for the surrogate snow sensor pairs for the smoothed and reduced data

Sensor pair and data set	Sample size	Standard error	Percent of April 1 SWE ¹	Coefficient of determination (R ²)
		<i>cm</i>		<i>pct</i>
Blue Canyon-Snow Lab				
-all smooth	623	23.2	27	50
-reduced smooth	620	23.1	27	50
-red. smooth accum.	518	21.7	25	52
-red. smooth ablation	104	16.2	19	62
Robbs Saddle-Schneiders				
-all smooth	2145	34.3	39	58
-reduced smooth	1960	35.3	40	57
-red. smooth accum.	1379	18.6	21	82
-red. smooth ablation	590	45.8	52	40
Bloods Creek-Mud Lake				
-all smooth	1131	24.9	22	83
-reduced smooth	1040	25.4	22	82
-red. smooth accum.	665	10.1	9	97
-red. smooth ablation	381	22.3	20	81
Poison Ridge-Green Mountain				
-all smooth	507	43.7	56	51
-reduced smooth	484	44.4	57	49
-red. smooth accum.	267	15.7	20	88
-red. smooth ablation	220	58.0	74	40
Graveyard-Green Mountain				
-all smooth	333	43.4	55	54
-reduced smooth	297	43.8	56	50
-red. smooth accum.	165	16.1	21	88
-red. smooth ablation	133	54.1	69	38
Poison Ridge-Kaiser Pass				
-all smooth	634	36.1	45	60
-reduced smooth	617	36.0	45	60
-red. smooth accum.	342	12.6	16	94
-red. smooth ablation	277	39.2	49	57
Green Mountain-Mammoth Pass				
-all smooth	971	40.0	37	41
-reduced smooth	873	40.7	38	38
-red. smooth accum.	568	28.0	26	64
-red. smooth ablation	309	34.7	32	27
West Woodchuck-State Lakes				
-all smooth	3215	11.0	15	91
-reduced smooth	2314	12.6	17	89
-red. smooth accum.	1546	5.2	7	97
-red. smooth ablation	778	18.5	25	81
Giant Forest-Upper Tyndall				
-all smooth	2855	28.5	41	46
-reduced smooth	2125	30.0	43	42
-red. smooth accum.	1466	20.9	30	59
-red. smooth ablation	672	34.5	49	35
Quaking Aspen-Pascoes				
-all smooth	770	13.6	21	91
-reduced smooth	618	14.5	23	90
-red. smooth accum.	416	5.8	9	98
-red. smooth ablation	206	19.6	31	89

¹Percentage is the SE (equation 3) divided by the long-term mean of the April 1 SWE of the high-elevation sensor, multiplied by 100.

subsequent calculation for k_{i+1} used k_i , not k'_i , in the calculation. When missing values were encountered as the k_{i-1} , or k_{i+1} , value, the k_i value was retained. When k_i was missing, the missing value code was retained in the smoothed file.

Reduction was also applied to the smoothed files, but smoothing decreased the efficiency of the reduction process by spreading out chatter to surrounding values. Reduction of the smoothed files reduced the size of the all-data file by an average of 12 percent and ranged from less than 1 to 28 percent. Smoothing increased the standard errors from 0 to 1.6 cm, and by 0.7 cm on the average. The greatest increases in standard error were matched with the largest decrease in sample size. Although the sums of squared differences decreased after smoothing and reduction, the sample size decreased more. Larger standard errors reflect the position of the sample size as a divisor in the calculation of SE (equation 3), and indicate that regression yielded a poorer fit after reduction and smoothing.

Record Division-Accumulation and Ablation

When SWEs for high- and low-elevation sensors are plotted, an hysteresis curve results: the accumulation curve has a different path than the ablation curve (fig. 3). The maximum SWE at the Quaking Aspen and Pascoes sensors occurred close to April 1 in 1981, which supports a common assumption made by snow hydrologists in California. There was a rapid loss of SWE at the lower elevation site in April and early May. The curve becomes essentially vertical when the lower site reaches 0 cm SWE and the high-elevation site is still snow covered.

The ablation curve differs from the accumulation curve because of an elevation-influenced melt process. Although higher elevations receive slightly more intense solar radiation, the air temperatures are low enough to retard melting. This process results in the snow at lower sites melting out before that at the higher sites, even in cases in which the two sites get nearly an equivalent amount of seasonal precipitation (fig. 3).

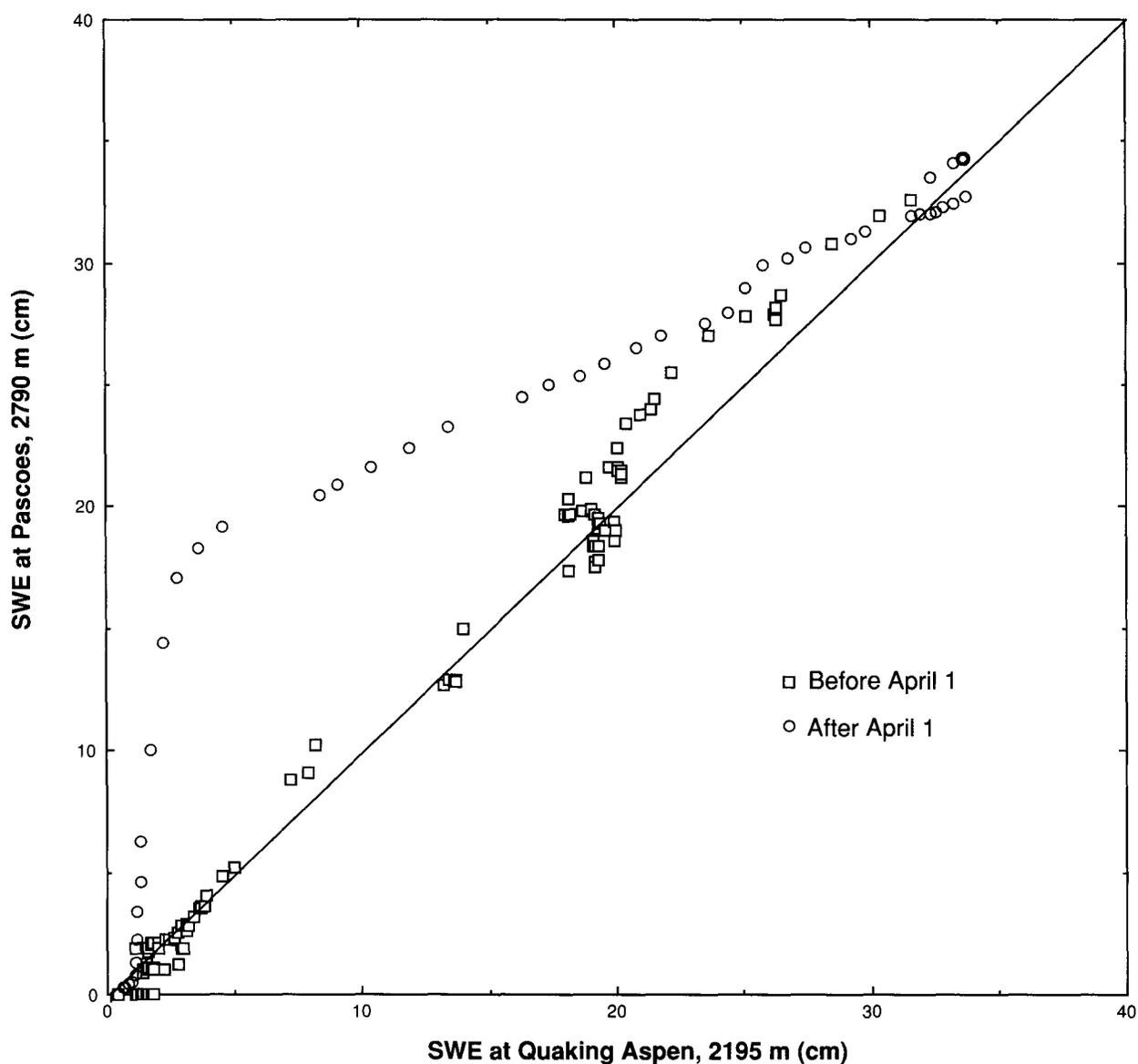


Figure 3-Hysteresis curve for snow water equivalent (SWE) at Quaking Aspen vs. Pascoes for water year 1981.

Although each year has a different time of peak SWE, April 1 was used as the division date for all years and sites. To incorporate the hysteresis effect, the multi-year records for each site were split into accumulation and ablation files, and regression equations were fit to the split files. The split typically yielded a noticeable improvement in the SE and R^2 for the accumulation season (tables 2 and 3). Except for one case, there was a corresponding minor worsening in the SE and R^2 for the ablation period. Before splitting the files, only three of the 10 sensor pairs had R^2 in excess of 80 percent (correlation of 0.89). After splitting, seven of the ten pairs had an accumulation R^2 in excess of 80 percent. Two of the three remaining sensor pairs could be corrected to achieve a better-than-85-percent R^2 by eliminating questionable data and very low snowfall years with little or no snow at the lower sensor.

Meltout at Low-Elevation Sensors

A regression equation for prediction of high-elevation SWE is useful only when there is snow at the low-elevation site. When

the low-elevation SWE is zero, the predicted high-elevation SWE is the y-intercept of a linear regression equation. In that the y-intercept is a constant, it is poorly suited to predict a varying high-elevation SWE as snow disappears at the low-elevation site. The time of meltout at low-elevation sensors varies from early May to early June, and the SWE at Kaiser Pass varied from 30 to nearly 160 cm over a 5-year period (fig. 4). The rate at which snow at Kaiser Pass melted was relatively linear, and the slope was relatively consistent from year to year. A plot of a 5-year period revealed that the high-elevation SWE at the time of meltout of the snow at the low-elevation sensors might be refined by adjusting it on the basis of classification of the water year SWE into low, normal, or high years (fig. 4). A constant-value decay rate might then be applied to predict meltout at high-elevation sensors.

A multiple regression equation was developed with the all-data file using the SWE value of the high-elevation sensor when the low-elevation sensor melted out as the dependent variable. Independent variables included the SWE of the low-elevation

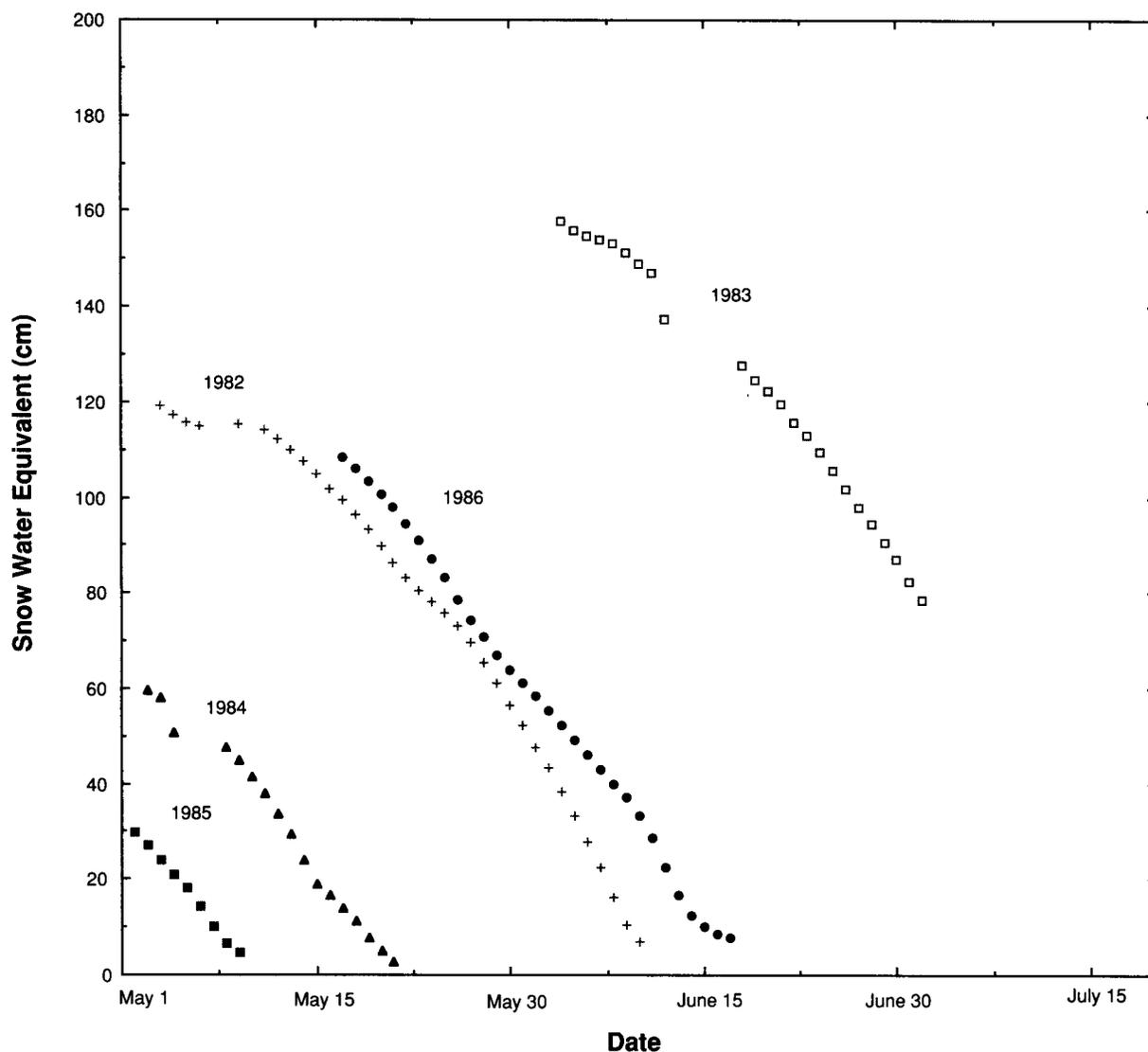


Figure 4-Ablation at Kaiser Pass after meltout at Poison Ridge for water years 1982-1986.

sensor on April 1, the number of days from April 1 to meltout (lag) at the low-elevation site, and dummy variables representing the water year magnitude:

$$\text{SWE}_{\text{High}} = A + B(\text{SWE}_{\text{Low, April}}) + C(\text{Meltout lag}_{\text{Low}}) + D(\text{High/Med. Peak SWE}) + E(\text{Low Peak SWE}) + \text{error} \quad (4)$$

where high and low refer to the high- and low-elevation sensors, respectively, and A through E are least-squares coefficients. A low-SWE class had an April 1 SWE of less than 80 percent of the long-term April 1 mean SWE. A medium-SWE class had an April 1 SWE of 80-120 percent of the long-term April 1 mean SWE, and a high-SWE class had an April 1 SWE greater than 120 percent of the long-term April 1 mean SWE. Three pairs of sensors were used to evaluate this technique: Robbs Saddle - Schneiders, West Woodchuck - State Lakes, and Giant Forest - Upper Tyndall. Of the three equations, the best had an adjusted R^2 of 36 percent, and an SE of 23.9 cm, indicating a weak relationship between high-elevation SWE and the independent variables.

Another potential way of incorporating water year magnitude would be to include a second variable in the regression equation that would represent the year effect. Although this might have improved the least squares fit in the historical data set, no information would exist for this variable when the equation was being used in real time as a year progressed and water supply forecasts were being made.

Multiple Regression

Although it is overly simplistic to assume that the high-elevation SWE is solely a function of low-elevation SWE, reliable information exists only for the low-elevation SWE. High-elevation SWE also varies in response to the magnitude of the water year, the number of events with rain-snow elevations between the two sensors, and other less quantifiable factors. Two cases were evaluated by multiple regression to determine whether a pair of low-elevation sensors would produce a better predictive relationship than a single sensor. The "all-data" files for Green Mountain and Poison Ridge were used to predict SWE at Kaiser Pass, and Robbs Saddle and Mud Lake were used to predict SWE at Schneiders. In both cases, the multiple regression had higher R^2 and lower SE than the simple regression. However, analysis of the sums of squares showed that most of the improvement was due to selection of a low-elevation site that was a better predictor of the high-elevation site than the sensor that was originally selected.

Discussion

Although the theme of this paper is prediction of SWE for a high-elevation sensor that has been "removed," no prediction can completely compensate for the discontinuation of a sensor's data. When a sensor is discontinued, one sample of a time series of interest is gone. The variation and potential extremes are thereafter lost from the record. This loss may or may not be important to the process of predicting the seasonal water supply or flood probability.

Current and Future Uses

The primary current use of the snow sensors is as an input to updates of the monthly water supply forecasts for California's water users. DWR's Cooperative Snow Surveys pool the data from both the sensors (for the updates) and the snow courses for the monthly forecasts) to obtain basin indices. The indices are then used in a multiple regression model that was derived with the full complement of courses. The regression model also contains variables for expected future seasonal precipitation, the prior season's water year magnitude, and other terms. If a basin snow index is based on ten or more courses or sensors, dropping a single sensor would have a small but relatively unimportant effect on the forecast of the AJRO. If a whole group of high-elevation sensors was excluded, however, a rather serious bias could result. Because higher elevations typically receive more snowfall, the bias would likely result in an underprediction of the AJRO. The bias could be compensated for by recalculating the forecasting equations, but the loss in information from the high-elevation areas of the basin would weaken the predictive relationship.

California's water management agencies and irrigation districts base their water allocation and purchasing plans on the water supply forecasts. The economic consequences of the forecasts are extensive: irrigated acreages, crop selections, and million-dollar purchases of supplementary water are based on the water supply forecasts. Pacific Gas & Electric and Southern California Edison use the data to optimize hydroelectric production and meet water quality and flow requirements. The sites for the recently-installed sensors were chosen because of the perceived need to have data for areas that are currently underrepresented by sensors. The trend of discontinuing measurements at selected snow courses will leave some basins with few sample sites. Sensors, once installed, provide a cost-effective and safe alternative to monthly visits to snow courses by snow surveyors.

Although the current forecasting system pools the data from many sensors to yield a basin index, it is likely that future techniques will use more modern, distributed models. The current lumped-basin regression technique was developed before current computer and telemetry capabilities. As demands

for more frequent and more accurate forecasts increase, DWR will likely shift to more sophisticated tools. These improved models will require better data than simple monthly snow course basin indices. The daily temperature and SWE values from each of the sensors in a basin will be crucial components for the new models. The elimination of a group of sensors at the top of a number of California's basins may limit the implementation of the next generation of prediction techniques.

Regression Problems

Regression is the obvious tool for predicting a high-elevation SWE from a low-elevation sensor's SWE, but some of the technique's rules are violated in this situation. An assumption of regression is that the independent variable (low-elevation SWE) has no or very low error compared to the dependent variable. In this case, both sensors' SWEs have approximately the same error magnitude. This violation of theory may result in a weaker relationship and larger error bands (standard error) around the predicted values.

Another theoretical problem with the independent variable is termed independence or serial correlation. A wide range of the independent variable's values should be possible in successive samples. In this application, however, an SWE from today is probably not more than a few centimeters different than one from yesterday or tomorrow. In spite of the violation of regression theory due to serial correlation, the regression coefficients are almost unbiased. The SE, however, is almost certain to be underestimated, so actual SEs may be even larger than those listed in this study.

Correlation versus Regression

The Environmental Assessment signed by the Forest Service defined 0.95 as a successful correlation between two sensors. This is equivalent to an R^2 of 90 percent, but neither statistic is the best measure of how good the relationship will be for water supply forecasting. The SE is a more appropriate measure of accuracy in terms of forecasting in that it provides a method for estimation of how the predicted SWE may vary from the actual SWE. If the SE is relatively large compared to the long-term April 1 SWE, the uncertainty of the prediction may be too great for it to be useful. In general, SE decreases as R^2 increases, so the two measures of the relationship are inversely related (*tables 2 and 3*). For these data, an SE of 17-21 cm generally corresponds to an R^2 of between 80 and 90 percent, and is generally between 19 and 25 percent of the April 1 SWE for a site.

The typical SE in this study is much greater than the 2-3 percent transducer error. The SE is also frequently greater than the difference between the high- and low-elevation sensors (*fig. 3*). For Quaking Aspen and Pascoes, the SE for the accumulation period is 6.3 cm, and the SE for the ablation period is 20.0 cm (*table 2*). In 1981, the accumulation portion of the curve is less than 5 cm from the 1:1 line (*fig. 3*). For the ablation portion of the curve, the difference between the two sensors is less than 15

cm in all cases. The regression results are based on 5 years of data, however, and other years show a more complicated pattern (*fig. 5*). The example from water year 1984 is illustrative of why the SE is large for the regression equation: the 30 cm depth at Quaking Aspen has four different depths at Pascoes at various times, ranging from a SWE of 25 to 50 cm. During January, the SWE at Quaking Aspen declined by more than 12 cm while the Pascoes SWE remained fairly constant. This period produced an "offset," which was due to the reasonably common midwinter dry spell, during which warmer temperatures at the lower site allowed melting. Little melt occurred at the higher site. This pattern was repeated near the peak SWE, and there are indications that an event with a rain-snow line between the two sensors may have occurred.

Confidence Intervals

SEs are also used in the calculation of confidence intervals. A 95 percent confidence interval around a new predicted high-elevation SWE near the mean of the low-elevation sensor is the SWE plus or minus approximately twice the value of the SE. For example, the Poison Ridge - Kaiser Pass pair has an SE of 36.1 cm and a mean low-elevation SWE of 22.4 cm. If a confidence interval for that mean value were predicted for a following year, the high-elevation SWE would be $72.9 \text{ cm} \pm 72.2 \text{ cm}$. This wide interval results when the SE is over 45 percent of the long-term April 1 SWE, illustrating the seriousness of large SEs. The interval would become even wider as SWE values diverged from the mean SWE. A range of that width is no better than a random selection of a value limited only by the April 1 mean SWE, indicating that the prediction equation was essentially useless.

In contrast to the near "worst case" example shown above, the West Woodchuck - State Lakes pair has an SE of 11.1 cm and a mean low-elevation SWE of 39.9 cm. The SE is 15 percent of the high-elevation sensor April 1 mean SWE, and the 95 percent confidence interval for a value near the mean is $36.8 \text{ cm} \pm 22.2 \text{ cm}$. By splitting the data into accumulation and ablation files or by better pairings, SEs might be reduced to 10 percent of the April 1 mean SWE. This could result in confidence intervals that were plus and minus 25 percent of the predicted value.

Record Reduction and Smoothing

The reduction algorithm had between a one and 46 percent effect on sensor record size and a negligible effect on SE and R^2 . If a sensor produced a record with little chatter, then a larger number of points were dropped (e.g., West Woodchuck - State Lakes, *table 2*). Although this procedure was, in part, designed to reduce serial correlation problems, its application did not result in marked changes in SE or R^2 . A more dramatic effect might have occurred if the criteria had been set differently. Instead of dropping a value if it was identical to a prior value, a wider filter such as "SWE plus-or-minus 0.5 cm" might have been used. A band filter might have better accomplished the goal of removing points of unchanging SWE during non-storm, non-

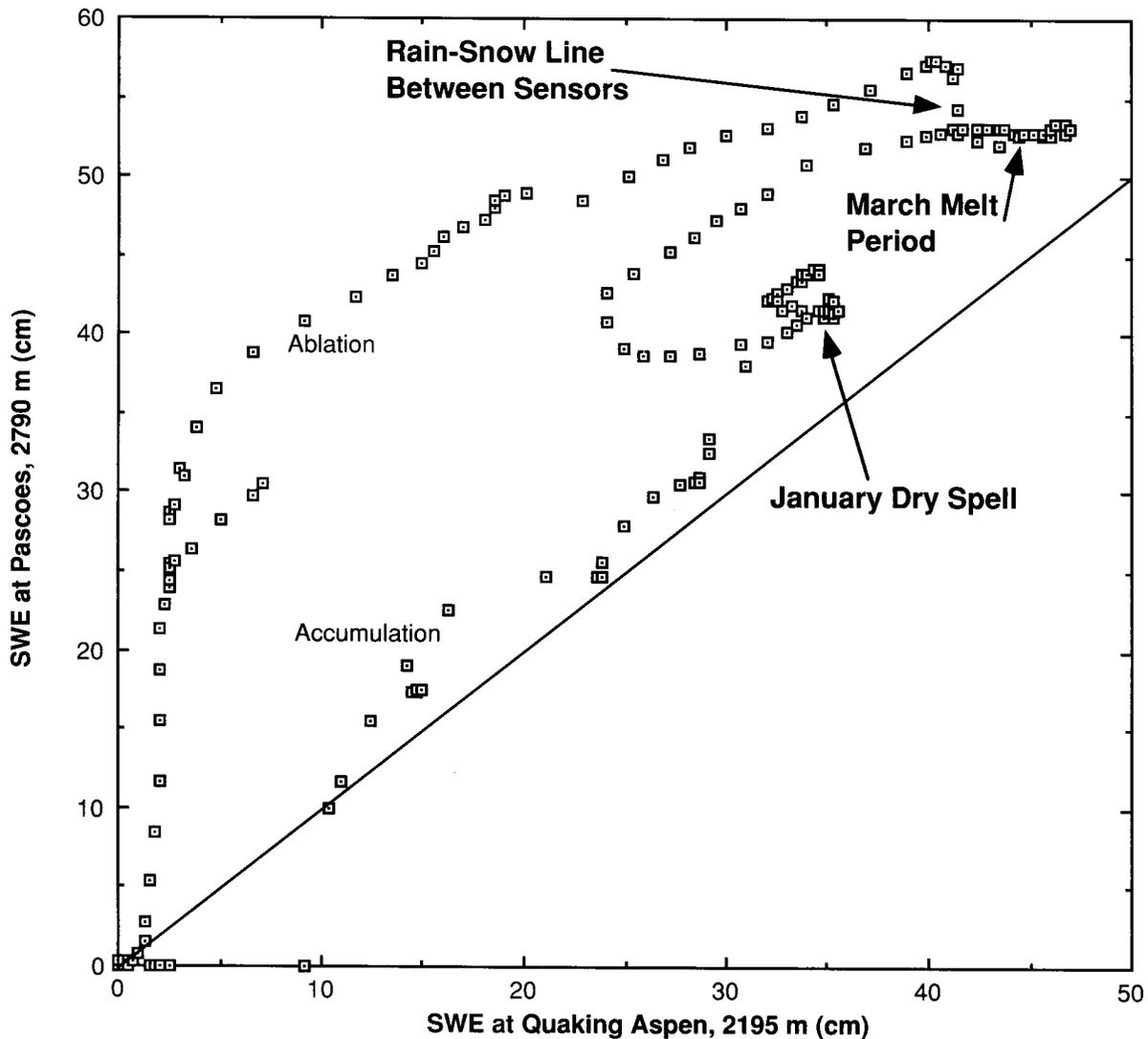


Figure 5-Hysteresis curve for snow water equivalents (SWE) at Quaking Aspen and Pascoes snow sensors for water year 1984.

melt periods such as occurred during much of December 1984 (fig. 1).

The smoothing process was devised to treat the minor deviations that result from transducer chatter and other errors of electronic or temperature-related origin. These errors typically cause small spikes in one record *without* accompanying spikes in the paired sensor record. More complex smoothing or wider moving averages would delay sensor response to real storm events. The smoothing algorithm reduced the magnitude of the spikes, but it transformed a single spike into a three-value blip. Smoothing had a negligible effect on the SE and R^2 (tables 2 and 3).

Smoothing reduced the efficiency of the reduction process. For the West Woodchuck - State Lakes pair, the reduced file had 1734 points as compared to 2314 points for the smoothed and reduced file. The smoothing process spread the spike to neighboring points, which then escaped deletion by the reduction algorithm. The optimal combination would probably be a band filter reduction scheme followed by a smoothing algorithm.

Accumulation and Ablation Division

In almost all cases, splitting the record at April 1 improved the regression fit for the accumulation phase of the reduced record (tables 2 and 3). The ablation phase then showed a poorer linear regression fit, typified by larger SEs and lower R^2 . Six of the ten pairs had an R^2 of 88 percent or higher for the accumulation period, and the mean SE as a percent of the April 1 SWE was 14 percent.

It is not surprising that a linear relationship works less well during ablation than during accumulation in light of figures 3 and 5. A severe problem with fitting a linear equation is caused by the string of declining SWEs at the high-elevation sensor while the low-elevation sensor has already reached an SWE of zero. An approach to the problem would have been to subdivide the ablation phase into a before-and-after meltout of snow at the low-elevation sensor. A linear equation might have been used for the pre-meltout portion, and a temperature index melt equation might have been applied to the high-elevation sensor's SWE

for the post-meltout period. The prediction of high-elevation SWE proved problematic, however, in that the prediction attempt described (equation 4) performed poorly at the three sites that were evaluated. The time of meltout is affected by factors other than water year class and time of meltout at the low-elevation sensor. For example, water years 1982 and 1986 had similar peak SWEs at both sites, but Poison Ridge melted out more than two weeks later in 1986 than in 1982 (fig. 4). Inclusion of historical data such as average daily temperatures might improve the predictive relationship, but these data were not used because of their unavailability during real-time prediction.

Multiple Regression

For the two cases examined, the addition of a second low-elevation sensor produced a better relationship than when only the original low-elevation sensor was used to predict the high-elevation SWE. In the Kaiser Pass case, Poison Ridge is clearly a poor predictor in that the SE is 36.1 cm (table 4). The inclusion of Green Mountain halved the sample size, decreased the SE by 7.3 cm, and increased the R² from 60 to 74 percent. The results were nearly as good, however, with Green Mountain as the only independent variable. The improvements were even more dramatic for the Schneiders cases in that the SE dropped from 34.4 cm (paired with Robbs Saddle) to 12.3 cm (paired with Robbs Saddle and Mud Lake). Robbs Saddle was paired with Schneiders because of its westerly position in the American River basin, but the sensors are elevationally and horizontally distant (865 m and 34 km), resulting in a weak relationship. Mud Lake is 255 m lower than Schneiders and 12.3 km away, so even though it is in the Mokelumne rather than the Cosumnes basin, it is a better predictor of Schneiders than Robbs Saddle. The SE and R² for the Mud Lake - Schneiders pairing are 12.6 cm and 94 percent, as opposed to 34.4 cm and 58 percent for the Robbs Saddle - Schneiders pairing (table 4).

The inclusion of a second sensor adds another type of theoretical regression violation called multicollinearity. Independent variables should be measures of different processes that

Table 4-Comparison of regression results from simple and multiple regressions for two high-elevation sensors

Sensor pairs	Sample size	Standard error	Coefficient of determination (R ²)
		<i>cm</i>	<i>pct</i>
Kaiser from Poison Ridge	635	36.1	60
Kaiser from Green Mountain	966	29.5	74
Kaiser from Poison Ridge and Green Mountain	382	28.8	74
Schneiders from Robbs Saddle	2146	34.4	58
Schneiders from Mud Lake	2997	12.6	94
Schneiders from Robbs Saddle	2105	12.3	95

are related to the variable of interest. The SWEs at two sensors are likely to vary in a parallel manner, and much of the information in one record is contained in the other. If the addition of a second sensor improves the prediction capability, such as with both Kaiser and Schneiders, it may mean that the initial pairing should be dropped in favor of the new pairing.

Rain-Snow Line

Storm-to-storm variation of the rain-snow elevation is a characteristic of the Sierra Nevada. Much of a season's snow is deposited on both the upper and lower sensors by large, frontal storms that move west-to-east across the Sierra Nevada. Each storm has a different elevational boundary between rain and snow, and occasionally the lower-elevation sensor may receive rain while the higher-elevation sensor receives snow. This phenomenon complicates the prediction of SWE at high-elevation sensors, but no easy solutions are available. The obvious recourse is to pick as an independent variable a sensor that is close to the sensor of interest in elevation and geographic proximity. However, this solution is unrealistic because all of the California snow zone has only 110 sensors, so finding reliable, nearby sensors is very difficult. Indeed, because of the high cost of installation and operation, most sensors are separated by several hundred meters elevation and 20-30 km. At these distances and elevational differences, it is very likely that the rain-snow line will fall between any sensor pair for several storms during a season. Five of the ten pairs analyzed here are 485 m or more apart in elevation, and these five have R² values of 60 percent or less. The three pairs with an R² in excess of 0.80 averaged 425 m in elevational difference. A maximum difference of 500 m might be a good general cutoff value for sensor pairs.

It might be expected that nearby sensors would be likely to receive concurrent precipitation and of a similar type. However, these ten pairs did not show a good relationship between distance and R² or SE (tables 1 and 2). The ten pairs analyzed herein ranged from 8 to 34 km apart, and averaged 24.3 km in distance. Some distant pairs (West Woodchuck and State Lakes, 29 km) had a good predictive relationship, while adjacent pairs (Graveyard and Green Mountain, 8 km) had a poor predictive relationship.

The distance relationship is obviously confounded by elevational and siting influences. Sensors that are 20 km apart but at a similar elevation (Bloods Creek and Mud Lake) may be more likely to form a good pair than sensors that are 10 km apart but at very different elevations. Sensors that are low enough to fall in the transient snow zone (Blue Canyon) are also unlikely to provide a good match to a higher sensor in the continuous snow zone (Central Sierra Snow Laboratory) in spite of a moderate elevational difference and reasonable proximity. No clear "cutoff" distance can be recommended on the basis of this analysis.

Conclusions

The goal of this analysis was to assess the feasibility of predicting the SWE at high-elevation snow sensors from low-elevation sensors using least squares regression. The need for the prediction arises from the USDA Forest Service policy that calls for removal of sensors sited in wilderness after a correlation period. The initial criterion for an acceptable correlation was a correlation coefficient of 0.95. Because of the short record of most wilderness sensors, this analysis selected ten pairs of sensors with longer records as surrogates for the wilderness pairs.

Of the ten pairs, only two pairs met the initial criterion. Six of the ten had a correlation coefficient of less than 0.75. Record smoothing and deletion of unchanging values were done but with no improvement in results.

Improved results were obtained by dividing the records for the ten pairs into accumulation and ablation periods. April 1 was used as the division date. Six pairs then yielded correlation coefficients of 0.94 for the accumulation period, but only one pair reached that level in the ablation phase. Attempts to improve the SE and R^2 of the ablation relationship by including a classification of water year magnitude and time of meltout at low sites failed.

Although the original criterion used correlation as a measure of acceptability, the standard error of estimate of the regression equation is a better measure. The lowest SEs of the regressions for the sensor pairs ranged from 15 to 25 percent of the high-elevation mean April 1 SWE. Other sensor pairs yielded SEs of 30 to 55 percent of the mean April 1 SWE. This error is 10-20 times larger than the sensor transducer error, and could have an adverse effect when used as an input to the water supply forecast models.

The selection of the ten sensor pairs was based on minimizing horizontal and elevational difference. The pool of potential sensors was additionally constrained by eliminating sensors that had frequent technical problems or short records. Colocation within a drainage basin was also a goal. The analysis suggested that elevational difference is an important attribute for pairing, but distance and basin were less important. Future analyses could define pairs by calculating correlation coefficients for all sensors within 500 m elevation and a large distance such as 75 km. The 500-m difference in elevation may minimize the co-occurrence of rain at the low sensor and snow at the high sensor.

Adding a second low-elevation SWE and performing multiple regression improved the prediction capability in both cases examined. In one case the SE decreased by 20 percent but was still 25 percent of the site's April 1 mean SWE. The second case provided dramatic improvement in SE, but the attempt revealed that the initial pairing was actually a poor one. In both cases simple regression with the new sensor yielded results almost as good as with the multiple equation. This result underscored the difficulty in *a priori* selection of pairs. Greatly improved results might have been obtained for each of the ten high-elevation sensors considered in this analysis if between five and 15 low-elevation sensors had been screened by a correlation matrix process.

Although improved sensor selection and alternative prediction methods may yield better pairings and lower SEs, even the best SEs obtained in this analysis were 10 percent of the high-elevation sensor's long-term April 1 mean SWE. Confidence intervals (95 percent) around predictions based on these "best" equations would be at least ± 20 percent of the April 1 mean SWE. If this prediction error is acceptable to water supply forecasters, then data collection through 2000 or 2005 in the wilderness areas and determination of the best wilderness-nonwilderness pairs may produce predictive relationships that are useful after the wilderness sensors are removed.

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