An Integrated System for Static and Dynamic Risk Evaluation at a National Level

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Abstract
An integrated approach for forest fire risk assessment over a wide geographical area is presented in this paper. Such an assessment is finalized to support decisions regarding long-term planning and pre-operational phases. Risk assessment is carried out both in dynamical and in static situations. The information used for risk assessment is relevant to hazard, and to vulnerability and cost of the exposed elements. Hazard assessment is carried out, in the dynamic case, by use of a cascade of two models, the former tracking the moisture content of the available dead fine fuel, and the second providing an estimate of the potential spread and of the linear intensity of a fire possibly ignited in the cell. The two models are driven by meteorological information (real-time and forecast) and by territorial information stored in a GIS database. Instead, in the static case, hazard assessment is based on a deep analysis of the fires historically occurred in each cell.

Introduction
Since ancient times a great number of wildfires have burnt Mediterranean forests. In fact, during the centuries, the necessity to sustain the growing population has altered the original equilibrium of natural habitat. A large amount of the forested areas was destroyed aiming at creating new spaces for settlements, pasturage and agriculture. Later on, the industrial age and the consequent large urban settlements dramatically transformed the territory. In this connection, woodlands were the places where Mediterranean populations found the materials and the energy (wood, timber and lignite) needed by the industry. Nowadays, in the middle of the petrol age, forests have lost their importance, but also as a consequence of the urban sprawl, the population living near or inside a vegetated/forested area has grown. On the counterpart, the rural population has decreased and, by now, a growing population without any rural or agricultural knowledge lives and visits woodlands, perceiving them only as leisure-holiday areas.

In this connection, the presence comes out of two different land uses, separated by a borderline, which divides the natural environments from the anthropic settlements. Therefore, it should not be surprising to recognize that the maximum

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1 An abbreviated version of this paper was presented at the second international symposium on fire economics, policy, and planning: a global view, 19–22 April, 2004, Córdoba, Spain.
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frequency of forest fire ignitions arises along such borderline, namely in the so-called Wildland Urban Interfaces (WUI). In fact, where the peripheral urban zones enter in contact with productive agricultural surfaces or abandoned areas, a situation of serious hazard arises, for instance related to the agricultural practices that generally require the ignition of fires in proximity of zones rich of biomass and near inhabited areas. Actually, in some regions the frequency of fire occurrences, and the effects that they have on natural habitat and human activities make wildfires represent the most remarkable natural risk also for the complexity of the organization needed to manage such a risk and to effectively fight an active forest fire.

Some preliminary notations can be made about terminology; in fact, in the literature, the terms risk and hazard are considered as nearly equivalent. Instead, in this paper, the term hazard will be used to designate the physical distribution of the conditions favoring a potential ignition, irrespective of the presence over the territory of an exposed element. The latter term will be used to designate the set of entities, which can suffer from an external stress (in this case, a fire) and suffer damage. Exposed elements are, of course, related to human lives, settlements, industrial activities, infrastructure, and also wildlife areas. To analyze the impact of an external stress over an exposed element, the vulnerability of the element has to be taken into account. Then, the risk suffered by an exposed element can be evaluated on the basis of the existing hazard, of the vulnerability of the element, and of some measure of the value of the element itself.

At least in Mediterranean areas, the ignition of a fire in a vegetated area is, almost in the totality of the cases, imputable to human activities (either as a voluntary action or as an involuntary consequence of the human activities), and, therefore, not properly recognizable as a natural event. However, the propagation of an ignited fire is heavily influenced by the characteristics (topographical, vegetational, etc.) of the interested territory, as well as by the meteorological conditions and the conditions of the fuel (especially as regard its moisture content). Actually, it is clear that it is improper to think of forecasting the ignitions, whereas, it seems to be sensible to assess and forecast the danger that a (somehow) lighted fire may find favorable conditions for its propagation and, therefore, the consequence of such a fire on the territorial system (exposed elements).

The purpose of this paper is to provide a first development of an integrated system for support in decisions regarding long term planning and pre-operational phases aiming at reducing the impact of wildfires over the territory. To this end, hazard assessment is first considered, both as regards the static and the dynamic cases. Then, a possible way to evaluate the risk relevant to a single territorial unit is introduced, and, on this basis, two optimization problems are defined in order to formalize the above mentioned decisional processes.

The hazard assessment

As happens in connection with other kinds of natural risks, forest fire hazard assessment can take place starting within three different conceptual frameworks. In the first one, which can be denoted as static hazard assessment, the evaluation and the distribution of the hazard over the territory is carried out on the basis of topographic information and land use (including vegetational cover), climate, possibly considered for the different seasons, and average fuel (i.e., vegetation) conditions, again referred to the various seasons. Static hazard assessment has also to
be based on the analysis of the data available from historical series corresponding to forest fires occurred in the considered region.

The purpose of such an assessment could be that of preserve the forest from the fire and to plan the sizing and the location of the different kinds of resources (men, trucks, engines, aircrafts, infrastructures, etc.) necessary to manage forest fire risk over a wide territory. Another objective of such an analysis could be that of obtaining indications about land use and urban planning, over a small-medium regional area. Actually, the design and the allocation in protected or natural areas of infrastructures dedicated to forest fires emergency constitutes a serious issue, owing to the necessity of limiting the impact over wildlife, as well as the construction and maintenance costs. The management of extinguishing ground operations require a wide range of facilities and a perfect coordination among the subjects involved in the emergency phase. In addition, it is necessary to ensure a continuous action of the emergency resources and thus it is fundamental to design an overall system able of proving continuous water supply when and where it is necessary.

Within the second framework, which will be denoted as dynamic hazard assessment, it is assumed that real-time information is available, and that the hazard assessment is carried out with reference of a certain time horizon (say 2-3 days) for which reliable meteorological forecasts are available. Along with forecast information, the real-time information used for dynamic hazard assessment may come from different sources: present weather conditions, ground-measured data relevant to vegetational conditions, data coming from satellite or airborne sensors (again, mainly referring to vegetational conditions). With a slight abuse of terminology, in the following the term “real-time information” will be intended to include also the meteorological forecasts for the short time horizon over which forest fire hazard is assessed.

The main advantage of a dynamic hazard assessment is that of identifying, within the considered territory, the areas affected by the highest hazard, and the time intervals within the considered time horizon in which this hazard takes place. The purpose of dynamic hazard assessment is that of getting reliable information useful to take a number and a variety of pre-operational actions that can reduce the impact of potentially lighted fire over the considered territory, within the considered time horizon. Such actions may include, for instance, re-locating the available resources over the territory, recalling day-off resources to service, alerting local authorities or emergency managers, issuing prohibitions of some dangerous agricultural practices (such as stubble burning), and patrolling the areas affected by the highest hazard.

Finally, the third framework is that corresponding to a situation in which some active fires have been detected, and there is the problem of selecting the best actions to fight such fires, taking into account the information corresponding to the distribution of hazard over the considered territory. In this case, a hazard assessment relevant to each detected fire has to be carried out. On this basis, an operational decision procedure can be applied in order to support the decision makers in taking decisions about the actions to undertake in order to contrast effectively or to extinguish the detected fires.

Summing up, for any of the above-mentioned three frameworks, an information-processing phase, aiming at performing some hazard assessment has to be considered, followed by a decision-oriented phase, whose objective is the selection of the best actions to undertake, on the basis of the available information. What
distinguishes the three frameworks, which can be denoted as information processing/decision levels, is the time horizon characterizing their operations and the available information. Clearly, information generated at any of the levels is passed to the subsequent one.

For the sake of completeness, a fourth level can be added to the above outlined conceptual scheme, referring to information processing and decision making after having extinguished a fire. Actually, the actions relevant to such a level are of a considerable importance in the management of forest fire risk, but on the whole, their discussion is beyond the scope of the present paper.

![Figure 1—Conceptual scheme of wildfire hazard assessment. Each block in solid lines represents a procedure (or a module of the system), whereas dotted blocks represent input or output information, and lines represent information flow (including, of course, information relevant to actions to be undertaken).](image-url)

In this paper only the first two frameworks are considered, whereas the real time hazard assessment and the post-event phase are not considered. Namely, static and dynamic hazard assessment procedures relevant to planning and to pre-operational phases are discussed; moreover, problems and policies relevant to the mitigation of the expected or forecasted risk, for a certain area, and for a determined set of exposed elements.

**The dynamic hazard assessment**

It is convenient, for ease of presentation, to present this module before the static hazard assessment module. The structure of the considered module can be represented as in Figure 2. The structure depicted in this figure shows that the overall module may be decomposed into two models, namely the fuel moisture model and the potential fire spread model. The function of the fuel moisture model is that of representing the dynamic behaviour of the distribution, over the territory, of the variable expressing the water content of the fuel that is mostly interested by the ignition process. The second model is used to quantitatively describe the behaviour of a lighted fire, leaving out of consideration any possible extinguishing action. Such a
model is not used to obtain a forecast of the propagation process of a given fire, but only to evaluate the potential riskiness after a possible ignition.

![Diagram](image.png)

**Figure 2**— A schematic representation of the structure of the dynamic forest fire hazard assessment module.

The information that can be used by the two sub-models represented in Figure 2 is partly static and partly dynamic. Namely, static information is relevant to topographic and territorial data (orography, land use, road network, etc.), which can be obtained by a Geographical Information System (GIS), and to the vegetational cover of the considered areas. The latter kind of information may be considered as static, even as regards biomass density. In fact, the seasonal biomass dynamics is much slower than the dynamics of the two sub-models depicted in Figure 2, so that one can reasonably think of considering the average vegetational load for each season of interest.

On the other hand, dynamic information may be of several different kinds. First of all, there may be a network of ground sensors (rain gauges, anemometers, solar radiation sensors, etc.) capable of providing real-time measurements of variables whose relevance is apparent for the evaluation of forest fire hazard. Other sensors may be used to acquire information related to fuel moisture. Finally, a source of dynamic information is also provided by the outputs of a meteorological model, assuming that the forecasts over a horizon of suitable length [say, 48-72 hours] can be considered sufficiently reliable.

The above functions are discrete in time and space. Actually, it is reasonable to choose the time discretization interval and the space discretization grid as those corresponding to the outcomes of the meteorological model. The variables that are determined to assess the forest fire hazard on each cell of the space grid are the rate of spread and the linear intensity that a fire could assume (in case of a successful ignition). The rate of spread is obtained through the application of a model, which is not used to evaluate the dynamics of a given fire but to evaluate the physical characteristics that a fire could attain, in each cell, on the basis of the variables locally the possibility of a successful ignition and fire propagation.

At this moment, a first implementation of the system has been realized, which is based only on the information provided by the outputs of a meteorological Limited Area Model (LAM), and on a GIS database. As regards the LAM, the information used consists of: the cumulate rainfall [m] in each time interval of three hours, the air temperature [K], the dew point temperature [K], the wind speed [m s⁻¹], and the wind direction [rad]. Besides, a Digital Elevation Model (DEM) has been utilized to define
the average value of the aspect angle [deg], the slope [%], and the elevation [m] of each grid cell.

The vegetation information available to the system is organized in a vectorial map of the whole area. The morphological data are stored in a GIS database and are: the fuel loads [kg m⁻²], the average height of the plants [m], and the crown height [m]. Moreover, the physiological characteristics of the fuels, that is the average seasonal tissue moisture content [%] of live fuel are included in the GIS database, as well as the average seasonal Higher Heating Value (HHV) [kJ kg⁻¹], both for dead and live fuels.

**The fuel moisture model**

The basic structure of the proposed fuel moisture model closely resembles the one corresponding to Byram’s equation (Byram, 1963). However, the justification of the structure is somehow different and, more important, the dependence of the model parameters on meteorological variables is represented in an effective and operational way. Namely, such a dependence will be structured on the basis of some simple semi-physical considerations, making use of a set of calibration parameters that can be determined in order to fit the model performances with experimental observations.

A first remark that has to be done in connection with the proposed fuel moisture model is that only the dynamics of the dead fine fuel is modeled. Instead, the live fuel moisture is considered practically time-invariant, and provided by values corresponding to the specific vegetational cover and to the considered season.

The dynamics of the dead fine fuel moisture is represented by using, for each cell k over the considered region, a specific model, which does not interact with the models of the other cells, as no fire propagation is represented.

Then, let \( u_k^o(t) \) represent the dead fine fuel moisture at cell k, at time instant t. It is assumed that the evolution of the above quantity is governed by the differential equation

\[
\frac{du_k^o(t)}{dt} = K_1 \text{step}(t) - K_2 u_k^o(t)
\]

(1)

where \( \text{step}(t) \) is the unit step function⁴. In fact, the solution of (1) has an asymptotic behaviour determined only by the ratio \((K_1/K_2)\), namely

\[
u_k^o(t) = \frac{K_2 u_k^o(0) - K_1}{K_2} e^{-K_1 t} \text{step}(t) + \frac{K_1}{K_2} \text{step}(t)
\]

(2)

Of course, the asymptotic value \((K_1/K_2)\) is independent from the initial state \(u_k(0)\), and the transient behaviour is decaying (increasing) if \(u_k(0) > (K_1/K_2)\) \(u_k(0) < (K_1/K_2)\). Observe that the “time constant” characterizing the speed at which the transient term in the r.h.s. of (2) vanishes, is given by \(1/K_2\).

Actually, note that the solution (2) of eq. (1) is correct only in the assumption of time-invariance of coefficients \(K_1\) and \(K_2\), which however, as it will be discussed

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⁴ Function \(\text{step}[x]\) is defined as equal to 1 if \(x\geq0\) and equal to 0 otherwise.
below, must be considered time-varying, as their values depend on a set of meteorological variables. Thus, the use of solution (2) is correct only whenever the dynamics of model (1) (which is characterized by the time constant \(1/K_2\)) is considerably slower than meteorological dynamics (determining the variation of \(K_1\) and \(K_2\)). However, discretization of (2) is in any case allowed, even when \(K_1\) and \(K_2\) are significantly time-varying. For this reason, hereafter the dependence of such coefficients on time (and on cell \(k\)) will be explicitly reminded by the notation.

It is assumed that coefficients \(K_{1,k}(t)\) and \(K_{2,k}(t)\) are functions of the meteorological variables \(p_k(t), w_k(t), \rho_k(t), \tau_k(t)\), that is the cumulated rain \(p_k(t)\) [m], the wind intensity \(w_k(t)\) [ms\(^{-1}\), rad], the relative humidity \(\rho_k(t)\) [%], and the air temperature \(\tau_k\) [K], respectively. In the proposed model, instead of trying to model such a dependence through thermodynamic considerations, a semi-physical structure is assumed by expressing the asymptotic value \((K_{1,k}(t)/K_{2,k}(t))\) as a function of the meteorological variables in the following way:

\[
\frac{K_{1,k}(t)}{K_{2,k}(t)} = e^{\alpha_1 + \alpha_2 \tau_k(t)} \text{ if } p_k(t) \leq p^* \tag{3}
\]

\[
\frac{K_{1,k}(t)}{K_{2,k}(t)} = \beta_1 \text{ if } p_k(t) > p^* \tag{4}
\]

where \(\alpha_i (i=1,\ldots,3), \beta_1\) are constants having suitable dimensions and \(p^*[m]\) is a threshold value for the cumulated rain. Note that (3) holds in absence of significant rainfall (in the last time interval), whereas (4) holds whenever such a rainfall cannot be neglected. Of course, the constant values must be selected so that

\[
\beta_1 > e^{\alpha_1 + \alpha_2 \tau_k(t)} \tag{5}
\]

for any possible value of \(\rho_k(t)\), and \(\tau_k(t)\). Note that the dependence of the r.h.s. of (4) on \(\rho_k(t)\) can be justified by observing that, the higher the value of \(\rho_k(t)\), the higher the asymptotic value of \(u_k^*(t)\). Besides, the fact that the r.h.s. of (4) is independent of \(\rho_k(t)\) can be justified by the assumption that the asymptotic values of the fuel moisture is independent of the rainfall intensity (whenever such an intensity exceeds a certain threshold). Finally, the fuel moisture is uncorrelated with temperature and humidity in case of rain, as the rain brings the fuel moisture condition at the fiber saturation point, which is greater than 35% (Cheney, 1981). The assumed values of parameters \(\beta_1\) and \(\alpha_i (i=1,\ldots,3)\), are reported in Tab 1.

As regards the dependency of \(K_{2,k}(t)\) from meteorological variables, recalling that \(1/K_{2,k}(t)\) is the time constant that (in time-invariant meteorological conditions) characterizes the transient behaviour represented in (2), the following structure can be proposed:

\[
K_{2,k}(t) = \alpha_4 e^{u_k^*(t) - u_k^*(t) - \alpha_5 \tau_k(t)} \text{ if } p_k(t) \leq p^* \tag{6}
\]

\[
K_{2,k}(t) = e^{\beta_2 \max(p_k(t) - p^*, 0)} \tag{6}
\]

where further constants having suitable dimensions have been introduced. For the sake of simplicity, only the case of \(\tau_k > 0\) is considered. The structure of the r.h.s. of (6) may be justified as follows. First of all, note that \((1/K_{2,k}(t))\) represents the time
constant of the transient term in (2). Then, observe that the first term in the r.h.s. of (6) applies in absence of significant rainfall and thus \( K_{2,k}(t) \) represents the speed of the drying (resp., moistening) process when \( u_k^0(t) > e^{\alpha_2+\alpha_3\gamma(t)} \) (resp., \( u_k^0(t) < e^{\alpha_2+\alpha_3\gamma(t)} \)). Clearly, the dependence proposed in (6) makes so that, in absence of a significant rainfall, high values of temperature and wind intensity favor drying and hamper moistening. Instead, the second term in the r.h.s. of (6) applies in presence of significant rainfall, and assumes a linear dependence of the moistening speed on the rainfall intensity. Of course, from (3), (4) it turns out that

\[
K_{1,k}(t) = K_{2,k}(t) \left[ \text{step}\left[p^* - p_k(t)\right] e^{\alpha_2+\alpha_3\gamma(t)} + \left[1 - \text{step}\left[p^* - p_k(t)\right]\right] \beta_1 \right] 
\]

where \( T \) is the length of the discretization interval (3 hours), and the time variable \( t \) is (now) an integer number.

Table 1—List of parameter values for the fuel moisture model

<table>
<thead>
<tr>
<th>Fuel moisture parameter</th>
<th>Model parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td></td>
<td>24.210</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td></td>
<td>34.940</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td></td>
<td>0.100</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td></td>
<td>0.2317</td>
</tr>
<tr>
<td>( \alpha_5 )</td>
<td></td>
<td>7.5 x 10^{-3}</td>
</tr>
<tr>
<td>( \alpha_6 )</td>
<td></td>
<td>17.5 x 10^{-6}</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td></td>
<td>40.000</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td></td>
<td>0.100</td>
</tr>
<tr>
<td>( p^* )</td>
<td></td>
<td>0.001</td>
</tr>
</tbody>
</table>

Obviously, the behavior of the fuel moisture model is deeply affected by the value of parameters \( \alpha_i \) (i = 1,..,6), and \( \beta_i \) (i = 1, 2), and \( p^* \). Actually, a preliminary calibration of such parameters, for a case study relevant to the whole Italian territory and to the summer season, has led to the values reported in Table 1. Essentially, such a calibration has been conducted on the basis empirical evidence over a wide set of test cases (relevant to actually detected fires).

**The potential spread model**

The purpose of this model is that of providing a measure of the hazard, for each cell in the considered area. The dynamic information that such a model uses is that relevant to forecast meteorological variables, and that provided by the fuel moisture model. In the same time, the propagation model makes use of topographical information, and of information related to vegetation (kind and density per m²), again referred to the considered cells. The information concerning density and kind, for each cell, of dead and live fuel, is considered as static, as only seasonal variations are
considered, simply by taking different values of the relevant parameters for the various seasons.

The development of the potential spread model will follow the same basic lines first proposed by Drouet (1974), as regards the definition of a forest fire propagation model, but introducing some important novelties as regards the procedures to evaluate the forest fire hazard.

It is convenient to recall again that, in the present paper, we are not properly interested in a fire propagation model; instead, we are interested in determining a quantitative evolution of the hazardousness over the whole considered region. Such a hazardousness is related to the behavior of a potential fire after an accidental or deliberate ignition. The first information on which the potential spread model is built is represented by the nominal rate of spread $v_{0,k}$, which is a quantity referring to standard conditions as regards the temperature and the average live fuel moisture, in absence of wind, within a perfectly flat terrain, and with perfectly dry dead fuel. Obviously, $v_{0,k}$ depends on cell index $k$, as it depends on the kind of fuel (i.e. particle size, bulk density, moisture, and chemical composition of the fuel) and on the vegetation density (biomass per square meter) of live and dead fuel. Besides, such a value has to be specified in connection to the various seasonal conditions, as they determine the average moisture of live fuel. Clearly, the determination of $v_{0,k}$ needs a great amount of experimental tests and a deep knowledge on the vegetation covering over the territory. On this basis, the potential rate of spread, which takes into account the influence of the meteorological variables, can be defined and determined as follows

$$v_{k}(t) = v_{0,k} W_{k}(t) S_{k} D_{k}(t)$$

where

- $Z_{k}(t)$ is a (multiplicative) correction [dimensionless] due to air temperature, at time $t$ and in cell $k$, with respect to the standard temperature ($0^\circ$C) assumed as the reference one;
- $W_{k}(t)$ is a (multiplicative) correction [dimensionless] due to wind speed on flat terrain, at time $t$ and in cell $k$;
- $N_{k}(t)$ is a normalization term [dimensionless] which takes into account the influence of topography on coefficient $W_{k}(t)$;
- $S_{k}$ is a (multiplicative) correction [dimensionless] due to the slope of the cell $k$;
- $D_{k}(t)$ is a (multiplicative) correction [dimensionless] due to the dead fine fuel moisture, at time $t$ and in cell $k$.

The way such terms depend on real-time information can be modelled as follows. Term $Z_{k}(t)$ in (13) can be assumed to be given by (Drouet, 1974)

$$Z_{k}(t) = e^{\gamma_{1} \chi_{k}(t) (1+\frac{\chi_{k}(t)}{\gamma_{2}})}$$

where $\gamma_{1}, \gamma_{2}, \chi_{k}(t)$ are parameters having suitable dimensions, and $\chi_{k}(t)$ is the total cloud cover (dimensionless) of cell $k$ at time $t$, ($\chi_{k}(t) \in [0, 1]$), which can be directly measured, or forecast by meteorological models.
As regards term $W_k(t)$, the following expression can be used (Drouet, 1974)

$$W_k(t) = \left(1 + \beta_1 \left[ \frac{w_k(t)}{\delta_3} + \tanh \left( \frac{w_k(t)}{\delta_4} - \delta_5 \right) \right] \right) \left[ 1 - \frac{w_k(t)}{\delta_5} \right]$$ (11)

where parameters $\delta_i (i=1,..5)$ have suitable dimensions and are fixed as in Tab. 2.

Wind speed has the most remarkable influence on fire behaviour when the angle $\theta_k(t)$ between wind direction and cell aspect is null. Obviously, in case of flat terrain or weak slope steepness, the influence of wind speed on the rate of spread is independent of the cell aspect. On the other hand, in case of significant slope steepness the angle between the wind direction and the cell aspect heavily influences the (potential) fire behaviour, in that, when $\theta_k(t)$ is about $\pi$, wind speed influence over the rate of spread has to be negligible. Then, in order to represent the influence of topography over the effect on wind speed (represented via the correcting factor $W_k(t)$), term $N_k(t)$ has been introduced in eq. (9), given by

$$N_k(t) = 1 + \frac{2 \arctg s_k}{\pi} \left( W_k(t) - 1 \right) \frac{2 \arctg s_k}{\pi} e^{-\frac{(\theta_k(t) - \pi)^2}{2\varepsilon^2}}$$ (12)

where:

$\varepsilon$ is a parameter having suitable dimensions, whose value, reported in Tab. 2, is purposely chosen in order to allow a dependence on $\theta_k(t)$ like the one that will be described in the following;

$s_k$ is the slope steepness [dimensionless].

It is important to observe that slope steepness has a twofold effect on fire propagation. In fact, besides to conditioning the wind effect on propagation, as above discussed, slope steepness has also a direct effect, as the flames of a fire burning upslope are positioned closer to the fuels ahead of the fire. This dries and preheats the fuels at a greater rate than if they were on flat terrain. Thus, it is necessary to introduce term $S_k$ in equation (9), representing the slope contribution to the rate of spread, and structured as follows

$$S_k = 1 + \lambda_1 \left( \frac{2 \arctg s_k}{\pi} \right)$$ (13)

where $\lambda_1$ is a dimensionless parameter. In Tab. 2 the value selected for such parameter is reported.

As already pointed out, only dead fine fuel moisture dynamics is taken into account, whereas moisture of live fuel is determined only (as a static parameter) on the basis of seasonal average values for the kind of vegetation characteristic of the considered cell. That is to say, whereas dead fine fuel moisture is represented via a variable $u_k^0(t)$, live fuel moisture is represented by a constant $u_k^1$.

Then, the influence of the dead fine fuel moisture can be taken into account by the introduction in (9) of the dimensionless term $D_k(t)$ defined as

$$D_k(t) = e^{-\frac{(u_k^0(t))^2}{\phi}}$$ (14)
where $\phi$ is a dimensionless parameter (whose value has been fixed as reported in Tab. 2).

### Table 2—List of parameter values for the potential fire spread model

<table>
<thead>
<tr>
<th>Potential spread parameter</th>
<th>Model parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\nu$</td>
<td>0.500</td>
</tr>
<tr>
<td></td>
<td>$\gamma_1$</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>$\gamma_2$</td>
<td>1.400</td>
</tr>
<tr>
<td></td>
<td>$\phi$</td>
<td>24.040</td>
</tr>
<tr>
<td></td>
<td>$\lambda_1$</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>$\delta_1$</td>
<td>1.500</td>
</tr>
<tr>
<td></td>
<td>$\delta_2$</td>
<td>0.848</td>
</tr>
<tr>
<td></td>
<td>$\delta_3$</td>
<td>16 x 10$^3$</td>
</tr>
<tr>
<td></td>
<td>$\delta_4$</td>
<td>1.250</td>
</tr>
<tr>
<td></td>
<td>$\delta_5$</td>
<td>25 x 10$^4$</td>
</tr>
</tbody>
</table>

Having so clarified the way to compute the potential rate of spread $v_k(t)$, which provides a quantification of the quickness characterizing the (potential) spreading of a fire, it is necessary to quantify also the intensity of the phenomenon, which is the ultimate measure of the hazard. To this end, Byram’s equation (1959) can be used to determine the (potential) hazard $H_k(t)$, that is the fire linear intensity [kWm$^{-1}$], namely

$$H_k(t) = v_k(t) \sum_{i=0}^{1} LHV_k^i \cdot d_k^i$$

(15)

where

$$d_k^0, \ ( d_k^1 ) \ [\text{kg m}^{-2}]$$

is the density of dead fuel (live fuel), for the considered season in cell $k$;

$LHV_k^0 (t), \ ( LHV_k^1 )$ is the Lower Heating Value [kJ kg$^{-1}$] of the dead fine fuel, (live fuel) in $k$ at time $t$, given by:

$$LHV_k^0 (t) = HHV_k^0 \left[ 1 - \frac{u_k^0 (t)}{100} \right] - Q \frac{u_k^0 (t)}{100}$$

(16)

$$LHV_k^1 = HHV_k^1 \left[ 1 - \frac{u_k^1}{100} \right] - Q \frac{u_k^1}{100}$$

(17)

where

$HHV_k^0, \ ( HHV_k^1 )$ is the Higher Heating Value [kJ kg$^{-1}$] of the dead fine fuel, (live fuel) based on the prevailing species composition in cell $k$, whereas $Q$ is the latent heating value [kJ kg$^{-1}$].

### The static hazard assessment

As it has been pointed out in the introduction, in the Mediterranean regions the ignition of a wildfire is in almost the totally of cases imputable to human intervention either as a voluntary action or as an involuntary consequence of some activity. Thus, although a high potential hazard could affect significant portions of the territory, not all the areas characterized by high hazard actually burn.
Then, it is essential, in order to evaluate the static hazard over a considered territory, to have access and make use of the information related to a large amount of fires occurred over a wide area (for instance, the entire Italian national territory) and for a period of time long enough to be statistically representative of the average climate conditions for the considered regions (e.g., 10 years).

For instance, referring to the Italian territory, according to Forest Service’s Daily Journal of Detected Fires (Corpo Forestale dello Stato, 2002), a set of data relevant to Y years have been collected. For each fire, the following information is available:

- the geographical coordinates of the ignition;
- the burnt area (woodlands and non forested areas);
- the number and the typology of means that took part in the intervention;
- the duration of the intervention;
- the meteorological local conditions relevant to the intervention time interval.
- the topography and the vegetation of the case study area.

Then, for each occurred fire, a hazard index is defined as follows:

\[
\tilde{h} = \left( \kappa_1 A_1 + \kappa_2 A_2 \right) \frac{\sum_{m=1}^{M} \lambda_m x_m \tau_m}{\tau_{\text{tot}}} \quad (18)
\]

where:

- \( \tilde{h} \) \([\text{m}^2] \) is the hazard index;
- \( \kappa_1, \kappa_2 \) \([\text{dimensionless}] \) are weighting coefficients;
- \( A_1, A_2 \) \([\text{m}^2] \) is the reported burnt area of woodlands and non-forested lands respectively;
- \( \lambda_m, m=1,\ldots,M \) \([\text{dimensionless}] \) is a coefficient relevant to each different typology of means belonging to class \( m \) (trucks, engines, aircrafts, helicopters) whose value represents a measure of the effectiveness of that class of means as regards the extinguishing action;
- \( x_m, m=1,\ldots,M \) \([\text{dimensionless}] \) is the number of means belonging to class \( m \), which took part in the extinguishing actions;
- \( \tau_m, m=1,\ldots,M \) \([\text{s}] \) is the duration of the intervention, as regards means belonging to class \( m \);
- \( \tau_{\text{tot}} \) \([\text{s}] \) is the total duration of the intervention.

The meaning of \( \tilde{h} \) is apparent, as it is an index useful to measure the hazard of the detected fires, which takes into account the kind of wildfire, and also the number and the typologies of actions that can take place in the extinguishes action.

Then, a set of hazard class \( \sigma=1,\ldots,\Sigma \) can be defined, by introducing suitable threshold values for index \( \tilde{h} \). On this basis, for each cell \( k \) =1,\ldots,K, the static hazard \( H_k \) can be defined as a function of the number of occurrences of wildfires of the various classes, in cell \( k \), within the considered time horizon \( Y \).
\[ H_k = \sum_{y=1}^{Y} \sum_{\sigma=1}^{\sigma} \psi_{\sigma} N_{ky}^{\sigma} \quad (19) \]

where:

- \( H_k \) [dimensionless] is the static hazard relevant to cell \( k \);
- \( \psi_{\sigma} \) [dimensionless] is a weighting coefficient relevant to wildfires of class \( \sigma \);
- \( N_{ky}^{\sigma} \) [dimensionless] represents the number of wildfires of class \( \sigma \) occurred in cell \( k \) during the year \( y \).

### The risk assessment

The forest fire risk assessment and the subsequent mitigation are strictly related with the typology of the considered exposed elements. Such elements are defined both by their physical characteristics, and by their functionality inside a more complex territorial system.

In the proposed methodology, it is assumed that the territorial system is described and modeled by using a limited number or class of exposed elements, which, at different spatial and time scale, must be preserved and protected by the community from a potential forest fire, in order to avoid their loss, damage, or temporary unavailability. Thus, data relevant to land-use, infrastructures, and urban zones, can be identified and stored in a GIS database, and a number \( n \) of classes of objects can be defined, in order to represent the various classes of exposed elements in each cell \( k \).

The remarkable number of exposed elements identifiable on the territory and the heterogeneity of their physical characteristics or their use, impose a radical simplification of the system aiming at classifying within homogeneous classes the greater number of elements without a great loss of information. To this end, seven general classes of areal and linear exposed elements have been defined, as reported in the Table 3. Besides, considering urban settlements, one can observe that in these zones the fuel load is generally negligible and thus the cell integrally occupied by such exposed elements are characterized by null hazard. For this reason, in order to obtain a meaningful evaluation of the risk affecting such settlements, a buffer zone has to be defined, whose depth is a function of the typology or class of the settlement. The presence of a fire in such a buffer zone can determine a potential loss of functionality of the settlement or, in the worst case, can yield a real physical threaten.

<table>
<thead>
<tr>
<th>n</th>
<th>Typology</th>
<th>Class description</th>
<th>Buffer zone depth [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Areal</td>
<td>Urban settlements</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>Areal</td>
<td>Production settlements</td>
<td>500</td>
</tr>
<tr>
<td>3</td>
<td>Areal</td>
<td>Cultivated areas</td>
<td>200</td>
</tr>
<tr>
<td>4</td>
<td>Areal/Linear</td>
<td>Highway network and service areas</td>
<td>300</td>
</tr>
<tr>
<td>5</td>
<td>Linear</td>
<td>Traffic network</td>
<td>300</td>
</tr>
<tr>
<td>6</td>
<td>Areal/Linear</td>
<td>Railway network and accessory areas</td>
<td>300</td>
</tr>
<tr>
<td>7</td>
<td>Areal/Linear</td>
<td>Electrical network and accessory areas</td>
<td>300</td>
</tr>
</tbody>
</table>
In urban settlements residential and commercial activities are concentrated; the
presence of a wildfire in these areas or in their proximity represents an extremely
dangerous event. The industrial settlements are characterized by a prevailing
presence of industrial or manufacturing installations or by the presence of
commercial centers of great dimensions. They are generally located in the peripheral
zones of the inhabited centers, sometimes in direct contact with the forested areas or
with shrublands or cultivations. The cultivated areas and the neighboring ones are the
zones where it is more frequent the ignition of a fire that often can degenerate in a
severe forest fire. The traffic and railway networks constitute an important linear
element of discontinuity. These networks are often interested by the phenomenon of
forest fires that, among the various natural catastrophes, is the one that most
frequently affects their normal functioning. Besides, the production of electricity is
often interrupted by wildfires; the inefficiency is serious if the fires are wide,
prolonged and interests main electric lines. The high-tension lines interruptions could
provide severe consequences on the regular power supply or in the functionality of
the network (overloads). Besides, the interruption of the lines of high tension is
necessary to guarantee the safety of the operators in the extinguishing phases,
particularly when airplanes are used.

The “value” or cost of cell k, Ck, can be defined as follows

\[ C_k = \sum_{n=1}^{7} \xi_{nk} c_n \]  

where

- \( C_k \) [€] estimated cost of cell k;
- \( \xi_{nk} \in \{0,1\} \) is a coefficient that expresses the absence/presence of objects of class n
  within cell k;
- \( c_n \) [€] cost of exposed element n (expressed in euro), assumed as a measure of the
  value of the existing objects, i.e., the economic expense that is necessary to rebuild
  or restore the considered element.

In addition, a vulnerability function has to be introduced, with the purpose of
modeling the relationship among hazard and the physical/functional characteristic of
cell k (i.e., implicitly, of the exposed elements that are present in cell k). The
analysis of the fire propagation dynamics is crucial as regards the vulnerability
evaluation. First, it is apparent that wildfire vulnerability cannot be defined merely as
a relationship between the stress solicitation and the effects on the exposed element.
Actually, forest fire dynamics is comparable to extinguishing dynamics and,
therefore the physical vulnerability of the elements must be considered mainly as a
function of the available extinguishing resources. Thus, vulnerability is assumed to
be a function of the intervention efficiency, defined in terms of available resources
(weighted by their distance to the considered cell) that can cope with a fire in case of
an emergency or, in pre-operational phase, can patrol the cell in order to avoid an
ignition. In addition, the vulnerability of a cell k is usually dependent on the physical
(geographical) characteristics of the cell and on the functional relationships between
the existent facilities or structures that, in case of an external stress of given
magnitude, can intervene on that cell.

The resources can be partitioned in two sets; the first one includes the mobile
resources that are relocated in a certain area for a long period of time (season). These
resources are generally scarce but their dynamics, slower then the dynamic of an active fire and the high costs required for their displacement suggest a limited movement/repositioning (at least seasonal) in the area affected by the highest value of risk. On the other hand, the other set of resources is relevant to the (fixed) infrastructures that should be utilized in case of emergency; this set includes the water supply points, and the Forest Service firefighters’ stations, where the mobile resources are located for medium/long interval of time.

Specifically, it is assumed that the vulnerability can be defined as the product of two independent terms. The first one, $\tilde{V}_k$, that is the nominal vulnerability of cell $k$, is assumed as a (given) parameter whose value is a function of the physical characteristics of cell $k$, and the existent infrastructures able to serve cell $k$, namely

$$\tilde{V}_k = 1 + \left( \chi_1 d_k + \chi_2 s_k + \chi_3 o_k + \chi_4 e_k \right)$$  \hspace{1cm} (21)

where

- $\tilde{V}_k \in (0, 1)$ [dimensionless] is the nominal vulnerability of cell $k$;
- $d_k$ [m$^{-1}$] accessibility of cell $k$ for landforce intervention, expressed in terms of meters of road network per m$^2$ of cell;
- $s_k$ [m] distance between the cell $k$ and the nearest available water supply point;
- $o_k$ [m] index of the roughness of the cell, expressed as follows
  $$o_k = b_k \Delta z_k \quad k=1,..,K$$  \hspace{1cm} (22)
  where $b_k$ represents the number of main river branches (valley) in cell $k$, and $\Delta z_k$ represents the average slope [%] of cell $k$;
- $e_k \in [0, 1]$ is $= 1$ if in cell $k$ there are signaled at least one obstacle for the air navigation (electric networks, cableways, etc.);
- $\chi_i, i=1,..,4$ are suitable parameters.

Finally, the value of $V_k$, that is the actual vulnerability of cell $k$, is given by the product between the nominal vulnerability of cell $k$, $\tilde{V}_k$, and a term introduced in order to weight the promptness of the available resources $m_h$ located in cell $h$ and ready to intervene on cell $k$

$$V_k = \tilde{V}_k \frac{1}{1 + \theta e^{-z_k}}$$  \hspace{1cm} (23)

where

- $V_k \in (0, 1)$ [dimensionless] is the vulnerability of cell $k$;
- $z_k = \min \left\{ \frac{1}{m_h} d_{kh} \right\}$ [m] is a term which takes into account the number of the nearest resources $m_h$ placed in cell $h$ and distant $d_{kh}$ from cell $k$;
- $\theta$ is a suitable parameter.
Once the vulnerability, the cost, and the hazard (both static and dynamic) have been defined and evaluated, a suitable way of defining the risk associated to a given cell is that of simply taking the product of such quantities.

\[ R_k = H_k \times C_k \times V_k \]  \hspace{1cm} (24)

**Forest fire risk mitigation via mathematical programming approach**

**The planning phase**

From the above section, it appears that territorial planning must be finalized not at the total elimination of the risk, but at its mitigation. The mitigation of the risk on a territorial system can be reached according to two different approaches:

a) reduction of the hazard \( H_k \), with direct intervention on the physical characteristics of the available fuel in cell \( k \), i.e., with actions of forest planning toward to the decrease of the available caloric power on the considered cell, or maintaining sufficient moisture fuel values by using opportune techniques able to irrigate the considered area;

b) reduction of the vulnerability \( V_k \), by means of structural or functional interventions on the exposed elements, i.e., by the construction of new infrastructures of water supply, Fire Brigade barracks, etc.

Both approaches generally require a considerable economic expense. In this paper, only the second typology of intervention is considered. Thus, the problem is formalized as that of optimally assigning the total available (monetary) resources among the whole set of cells, aiming at creating new infrastructures or reallocating the existing ones, in order to minimize the (potential) damage suffered by the territorial system. The problem is formalized with the objective of minimizing the maximum risk over the set of considered cells. In addition, construction costs for each new resource must be taken into account, as well as the necessity of penalizing the nominal assignment of extinguishing resources located at a certain cell \( j \) to a cell \( k \) too distant from that cell.

Then, the problem can be formalized as:

\[
\begin{align*}
\min & \quad y \\
\text{s.t.} & \quad y \geq I_k \times V_k \times C_k \quad k = 1, \ldots, K \\
& \quad (20) \div (24) \\
& \quad \sum_{j=1}^{J} c_j \cdot m_j \leq \bar{C} \\
\end{align*}
\]  \hspace{1cm} (26)

where

- \( y \) is the cost function;
- \( c_j \) is the cost of construction of a new infrastructure in location \( j \);
- \( m_j \) is the number of available resources in cell \( j \);
is the total amount of (economical) resources available for the planning phase in the whole target area.

**The pre-operational phase**

In a pre-operational phase, the problem to be faced is the optimal partition of the total available mobile resources (i.e., aircrafts, initial-attack teams, etc.) among the set of cells (say, Z cells) having \( R_k > 0 \). Such a problem must be formalized taking into account the objective of minimizing the weighted sum of the differences between the (hypothetical) request resources and the actual assigned resources for each cell of interest. In addition, transportation costs from a location \( j \) to another \( h \) must be taken into account, as well as the objective of penalizing the assignment of resources located at a certain location \( j \) to a cell \( k \) too distant from that location.

The problem is modelled assuming that the resources are continuous, for instance represented by the amount of power available to extinguish forest fires, i.e., \( m^3/s \) of water disposable on the fire front considering the time needed for the refueling, the scooping and the mission between the fire and the water supply zone. Besides, the current amount of available power in all locations \( j \), \( \tilde{U}_j \), should be provided for the problem formalization.

On the other hand, the service demand \( D_k \) [kW] is represented by the sum over the whole time horizon of the (forecasted) linear intensity \( H_k(t) \) [kW m\(^{-1}\)] times the side length of cell \( k \), \( \Delta_k \) [m]

\[
D_k = \sum_{t=0}^{T-1} H_k(t) \Delta_k \quad (27)
\]

The decision variables of the problem are

\( q_{jh} \) [kW] resources located at location \( j \) that should be moved to location \( h \), \( j=1,...,K; h=1,...,K; j \neq h; \)

\( u_{jk} \) [kW] resources located in \( j \), assigned to satisfy predicted demand of cell \( k \), \( j=1,...,K; k=1,...,K; \)

the problem can be stated as

\[
\min \sum_{k=1}^{K} \left( \sum_{n=1}^{N} \sum_{k=1}^{K} c_{kn} V_k \max \left( D_k - \sum_{j=1}^{J} u_{jk}, 0 \right) \right) + \alpha \sum_{j=1}^{J} \sum_{h=1}^{K} d_{jh} q_{jh} + \beta \sum_{j=1}^{J} \sum_{k=1}^{K} d_{jk} u_{jk} \quad (28)
\]

where

\( \alpha, \beta \) are suitable weighting parameters

s.t.

\[
\sum_{k=1}^{K} u_{jk} = \tilde{U}_j + \sum_{j=1}^{J} q_{jh} - \sum_{j=1}^{J} q_{hj} \quad (29)
\]

which represents the resource conservation constraint.
Of course, once the optimum solution of such a problem is found, it is necessary to convert this solution in the terms of the actual resources, which are indeed discrete.

### Conclusions and further research directions

An approach has been presented in this paper for the structural and operational design of a decision support system aiming at forest fire risk management. Both static hazard and dynamic hazard have been considered and risk assessment is based on the estimate of such hazards as well as on the evaluation of the vulnerability and of the cost associated to each cell. On the basis of the risk distribution estimate over the considered territory, two problems have been formalized. The first one is related to the planning of monetary resource assignment in order to reduce the overall vulnerability on the territory. The second one is finalized to optimally reallocating the available extinguishing resources over the territory, in a pre-operational phase, i.e., whenever an incoming situation of high fire risk is forecasted.

Present research activity is oriented towards more proper definition of the vulnerability and risk of a cell, as well as a more suitable definition of the static hazard of the single cell. Experimental implementations of the two decisional procedures presented in the previous section are presently carried out.

### Acknowledgement

The activities reported in the paper are presently carried out with reference to the case study relevant to the Italian Civil Protection and have been funded by the Gruppo Nazionale per la Difesa dalle catastrofi Idrogeologiche GNDCI, U.O. n. 3.28, Special project n. 4 Structural and operational design of a decision support system based on a national geographical information system and aiming at forest fire risk management.

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