A Dynamic Programming Approach to Determining Optimal Forest Wildfire Initial Attack Responses[1]

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Abstract

A mathematical optimization model, based on the operations research technique of deterministic dynamic programming, is offered as a method to search quickly through available options to find the economically efficient set of initial attack resources to suppress a wildfire. Considerations in selecting initial attack resources for an efficient initial response include cost of transportation and use, line construction productivity, response times, resource complementaries, size of fire upon discovery, rate of fire spread, fire damage, mop-up cost, and fire benefits. The dispatch optimization model has several applications, such as developing pre planned and real-time dispatches and evaluating the economic efficiency of new initial attack technologies.

For many years a quick and strong initial response was deemed paramount to successfully suppressing a forest fire. To achieve this objective, the criterion of closest forces was used to select the suppression resources deemed necessary for the initial attack effort. Much of the early supporting research focused on automating the process of finding closest forces (Mees 1978). Little attention was given to the cost of the initial attack response and the economic trade-offs that could be made with costs and resources losses associated with fire size.

Over the past two decades the cost of maintaining an initial attack organization of sufficient size to make quick and strong initial attack responses increased steadily and significantly. Facing ever tightening budgets, dispatchers became more interested in balancing the cost of the initial suppression response against the benefit of reducing fire size. In response, researchers began to explore formal methods to help dispatchers select the most efficient initial attack suppression response. Even before the concern with economic efficiency, Parks (1964) presented an analytic solution for finding the economically optimal suppression effort. When the efficiency issue became more pressing, Parlar and Vickson (1982) revisited Parks' model. They offered an alternative solution using optimization techniques from control theory. Both models focused on the most efficient size and timing of the general suppression effort. Neither model considered the importance to the dispatch decision of differences among individual suppression resources in response times, line building rates, and other significant fire fighting traits. Because these considerations were and remain important aspects of the dispatch decision, neither model became operational.

In subsequent research developments, resource differences important to the dispatch decision were considered. A foray in this area was initiated by Wiitala (1986) in pioneering the use of dynamic programming to find cost-effective dispatches. A similar approach was taken by Kourtz (1989) for dispatching water bombers and for delivering crews by helicopter. Although Wiitala (1986) attempted to account for all initial attack suppression costs, the dynamic programming algorithms by Kourtz (1989) minimized only transportation cost. Kourtz (1989) did not formally consider resource loss or other types of suppression related costs nor the economics of the strength of the attack.

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In determining an appropriate suppression response, the efficiency-minded dispatcher considers all costs and losses. A balance must be struck between the cost of the suppression effort and those costs and losses associated with fire size. However, finding the cost efficient dispatch is a task made difficult by the availability of numerous initial attack resources dispersed over many locations that exhibit a wide range of characteristics. For each available suppression resource the dispatcher must know its location, transportation cost, line building cost, line construction rate, and response time. Also important are line building interactions between resources and any constraints affecting the intensity and duration of a resource's performance. Even with a modest 30 different resources from which to choose a dispatch, the number of combinations that can feasibly contain a fire can easily range into the millions. With so may choices, identifying the most efficient combination to dispatch to a fire is nearly impossible without assistance of computers and operations research techniques.

This paper presents an operations research technique that can help the efficiency-minded dispatcher quickly identify an appropriate initial attack response. This objective is accomplished in two steps. The first step sets forth a general mathematical formulation of the wildfire dispatch optimization problem. The second step reformulates the general mathematical optimization problem to permit using the technique of deterministic dynamic programming to achieve an economically efficient dispatch.

Model Formulation

With many resources available to respond to a fire, the number of possible dispatches may be very large. Every dispatch will have its own fireline building trajectory, containment time, and cost profile. This profile will depend on the types, arrival times, and production rates of dispatched resources. Assuming resources will be sent immediately and will build fireline until the fire is contained, the cost of containment, J, can be formally stated as:

$$J = \sum_{i \in X} x_i \left(C0_i + C1_i (t - b_i) \right) + C2\left(a(t)\right) + C4(t) \quad [1]$$

in which:

X = the set of initial attack units available to respond to a fire,

x_i = a binary variable (0,1), where a 1 indicates the i^{th} resource from the set, X, of available initial attack resources is included in the dispatch,

t = the length of time the fire has been burning,

b_i = time from fire initiation at which the i^{th} suppression resource begins line building,

a(t) = a function relating area burned to t,

$C0_i$ = the fixed cost of transporting the i^{th} fire fighting resource to and from the fire,

$C1_i$= the cost per time unit of using the i^{th} suppression resource at the fire,

C2 = resource loss per unit of burned area,

C3 = mop-up costs per unit of burned area,

C4 = the cost per unit of time of other activities not directly related to fire line construction.

For the case of continuous line building resources, the mathematical problem of finding the most efficient dispatch and containment time, t, to a wildfire is defined as:

$$\text{mininmize} \quad J = \sum_{i=1}^{n} x_i \left(C0_i + C1_i (t - b_i) \right) + C2\left(a(t)\right) + C3\left(a(t)\right) + C4(t) \quad [2]$$

subject to

$$\sum_{i=1}^{n} x_i L_i (t - b_i) \geq p(t), \quad [3]$$

$$x_i = 0, \text{ for } b_i \geq t, \quad [4]$$

$$x_i = 0 \text{ or } 1, \text{ for } b_i < t, \quad [5]$$

$$t > \min b_i. \quad [6]$$

in which

p(t) = a function describing the relationship between fireline required for containment and burning time, t

L_i = the amount of fire line that can be produced per unit of time by the i^{th} suppression resource.

Constraint (3) requires constructed fireline to exceed fireline needs for containment, while constraint (4) ensures only those suppression units with arrival times less than containment time will be considered for a solution. The binary decision variable, x_i, in constraint (5) requires all suppression resources not violating constraint (4) to be considered for dispatch. Constraint (6) requires containment time to be greater than the earliest resource arrival time.

Depending on the types of resources available for dispatch, several additional constraints and modifications are necessary to address important interrelationships between resources. As an example, for an airtanker's delivery of retardant to contribute to the line building effort, the presence of a ground crew is required either upon arrival of airtankers or shortly afterwards. Also, to make a contribution to the suppression effort, water tenders require the presence of engines.

The mathematical model defined by equations (1) through (6) describes, in the general case, a nonlinear mixed-integer programming problem. The integer nature of the problem arises because many initial attack suppression units are indivisible. Nonlinearity may arise with respect to area related costs and resource loss. These values will increase over time at changing rates related primarily to variable growth in burned area.

Suitably structured, the dispatch problem might be solved by available commercial software packages capable of nonlinear optimization. However, when the dimensions of the problem grow, the optimization methods employed by these packages may not find a solution or may require inordinate amounts of time to find a solution. Neither outcome is acceptable in a dispatch environment.

For some optimization problems the technique of dynamic programming (Bellman 1957) provides a more computationally efficient alternative to other optimization techniques. Fortunately, the objective function stated in equation (2) can be mathematically reformulated to the equivalent:

$$\min_{j \in T} \left\{ \min_{i \in X} \left\{ x_i \left(C0_i + C1_i (t_j - b_i) \right) \right\} + C2\left(a(t_j)\right) + C3\left(a(t_j)\right) + C4(t_j) \right\} \quad [7]$$

to take advantage of dynamic programming. This reformulation separates the global optimization problem into two procedural steps. In step one dynamic programming is used to find the minimum cost combination of resources from the set of available resources, X, for each containment time t_j in the set T of possible containment times, considering only suppression costs (the expression comprising the interior brackets of equation (7)). This first step is subject to the same constraints stated in equations (3) through (6). Step two identifies the containment time in T that minimizes overall cost. Overall cost includes not only suppression costs, but also time, $C3(t_j)$, and fire size, $C2(a(t_j))$, related costs, and resource loss.

Dynamic Programming Algorithm

Once a fire is detected and location established, initial attack arrival times for all suppression resources can be established. Those resources that can deliver line production to the fire before a specified containment time are viable candidates for the initial response. The time between a unit's arrival and fire containment determines the amount of fire line that can be constructed. This time also determines total cost of line construction that, when added to dispatch and retrieval costs, yields a lump sum cost for using each resource. After stating the containment time objective, determining the most cost effective suppression response is just a matter of finding the combination of resources that will deliver at lowest cost the fireline required for containment.

This reformulation of the dispatch problem allows it to be viewed as a special case of the "distribution of effort" problem (Wagner 1975) with a strong analogy to a discrete capital budgeting problem. The objective for each containment time is to distribute in a least cost way the estimated fireline needs (budget) among the potential sources of fireline-the suppression resources (investments). This fireline allocation problem is easily solved by dynamic programming because the problem can be decomposed into a series of interrelated subproblems, whose easily obtained individual solutions can be composed to solve the larger problem (Nemhauser 1966). In the parlance of dynamic programming, each suppression resource represents a stage, i, in a sequential decision process. At any stage, the state of the system is given by the amount of fire line, 1, available to be allocated. The dynamic programming recursive formula describing minimum cost at each stage of the solution process is:

$$f_i(1) = \min\left(x_i C_i + f_{i-1}(1 - x_i L_i)\right), \qquad [8]$$

in which Li is now defined as the total amount of line the ith unit can contribute to the suppression effort, Ci is the cost of using the ith resource, and xi takes on the value 0 or 1 to indicate the decision to exclude or include a resource in the dispatch.

A forward recursive algorithm programmed in FORTRAN is used to solve the dynamic programming problem. For each containment time, the algorithm finds the optimum dispatch over a range of fireline needs, not just the one of interest. Several performance enhancing techniques are used to find a solution. The optimum dispatch (recursive solution) for any level of fireline need at any stage is encoded as 0's and l's in a character array retained in computer memory. This avoids the computational bottleneck of having to use large amounts of external storage typical of many dynamic programming algorithms. The array encoding process also facilitates quick decoding of the solution.

An equally important means of enhancing computational performance is accomplished by deleting from the decision rule character array redundant rules that will be unnecessarily evaluated at each stage of the solution process. Redundant decision rules arise at each state of the solution process when a given rule holds true over a range of a state variable (in this case, the amount of fireline to be supplied by potential resources).

The FORTRAN program also serves to process the initial attack resource data file for the specific data inputs required by the dynamic programming algorithm. Management of data inputs and optimizer outputs is currently handled by a system of linked and compiled spreadsheets. To further facilitate use of the optimization model, efforts are currently underway to embed the dynamic programming algorithm in a windows computer environment.

Table 1-*Estimates of fire size and needed fireline for alternative containment times.*

Containment time	Fire size	Fireline
hours	hectares	meter
1.5	1.4	483
2.0	2.1	604
2.5	3.1	724
3.5	5.7	966
4.0	6.9	1,086
4.5	8.5	1,207
5.0	10.5	1,328
5.5	12.5	1,448
6.0	14.6	1,569
6.5	17.0	1,690

Model Application

Application of the optimization model is illustrated for a hypothetical fire occurring on the Mt. Hood National Forest near Portland, Oregon. The data for the example is taken from a preplanned dispatch. Before applying the model, the dispatcher must determine fire characteristics for a set of fire management scenarios, prepare a list of information on available initial attack resources, and estimate per acre resource loss and mop-up cost.

Fire Management Scenarios

A fire management scenario is an estimate of fireline needed to contain a fire at some future time. In this example the fire exhibits a forward rate of spread of 1.67 meters per minute in medium logging slash. With additional information on the fire environment, fireline and fire size estimates were made for 13 potential containment times *(table 1)*.

Initial Attack Resources

There are a number of initial attack resources available to respond to the fire *(table 2)*. The optimization model uses the attributes of these resources to impose various restrictions on resource use and account for interactions between resources in calculating how much fireline a resource can produce for a particular containment time.

The type code is used primarily for purposes of determining how the model calculates a resource's contribution to fireline production *(table 2)*. For example, a positive resource type code indicates a continuous fireline building resource, like a crew. A zero code, like that used for airtankers, signifies the production of fireline in discrete increments. The type codes for engines, their crews, and supporting water tenders permit addressing several issues of fireline productivity related to the availability of water. Fireline production of an engine unit is divided between the engine and its crew. This allows accounting for the reduction in total fireline building rate to that of a hand crew if the engine exhausts its water supply before fire containment. The water tender code tells the model to compute a tender's potential contribution to fireline based on the restoration of fireline production to all engines with the same group code which will exhaust their water supplies before containment.

The group number not only associates a water tender with the engines it will tend, it also forces the dynamic programming optimizer to select resources sequentially within the group. For engines and tenders, this feature ensures a water tender will not be selected without first selecting the engines it tends.

The resource availability variable serves two purposes. A designation of 0 or 1 indicates an initial attack resource is unavailable or available, respectively, for dispatching. If a value of 2 is indicated, this will force the dynamic programming optimizer to include a resource in all dispatches for which it can arrive before the containment time objective. This latter feature allows a dispatcher flexibility to deal

Table 2-*Initial attack resources and key attributes.*

Description	Type	Group	Availability	Response	Production	Transport	Use
				min	m/hr	$/trip	$/hr
Patrol #180	3	2	1	30	10	91.63	26.25
Engine 93 200	3	3	1	40	10	227.43	44.45
Engine 47 1000	3	3	1	60	40	246.65	59.45
Tender 48 2000	4	3	1	90	0	213.85	37.70
Engine 64 200	3	5	1	75	10	301.88	44.45
Engine 24 600	3	5	1	88	40	338.07	59.45
Tender 65 1000	4	5	1	110	0	238.98	37.70
Helitack #16	1	7	1	50	60	1,896.50	135.00
Smokejumpers #18	1	8	1	94	101	1,653.33	170.00
BD crew #110	1	9	1	77	60	979.00	175.00
Fire crew #2 10	1	10	1	120	60	1,242.50	175.00
Engine 31 200	3	12	1	130	10	447.70	42.71
Hotshot crew #120	1	14	1	135	200	3,388.00	440.00
Dozer	1	15	1	120	400	299.00	89.20
Airtanker#1.1	0	18	1	55	200	7,200.00	0.00
Airtanker#1.2	0	18	1	95	200	4,000.00	0.00
Airtanker#2.1	0	19	1	65	200	7,200.00	0.00
Airtanker#2.2	0	19	1	105	200	4,000.00	0.00

with issues other than economic efficiency that might influence the initial attack response, for example, when a dozer cannot be used because of steep terrain.

Minimum Cost Suppression Response

The dynamic programming optimization model determines the minimum cost suppression response to provide the needed fireline for containing a fire at a specified future time. For each containment time *(table 1)* this is achieved when the optimization model identifies which of the suppression resources *(table 2)* could reach the fire before a specific containment time objective. It then computes the amount of fireline and cost resulting from the use of each resource.

For example, ten resources can contribute to containment of the fire at 1.5 hours *(table 3)*. From these the dynamic programming algorithm selects the minimum cost dispatch to achieve the 483 meters of fireline required at the 1.5 hour containment time. Both the resources available to respond and the minimum cost response for each containment time are then determined *(table 4)*.

A dollar amount associated with the minimum cost response for each containment time can be obtained *(table 4)*. These values can be plotted to show that the least cost containment time is 4 hours at a cost of $2,186 *(fig. 1)*. The associated response is primarily an engine response including use of the dozer. The two earliest containment times require use of expensive retardant delivery by airtankers because ground resources could not provide sufficient production to meet the containment needs in these short time frames *(table 4)*. The costs are nearly seven times that of the most cost effective ground resource response.

Minimizing All Costs and Resource Loss

Although a containment time of 4 hours is the least cost suppression response, this time is not necessarily the most efficient when considering other costs and losses. Because the area of a fire increases at an increasing rate over time, mop-up costs and resource loss can begin to weigh heavily at the greater containment times, depending on their importance on a per hectare basis.

The most efficient suppression response is found by identifying the containment time with the least total cost after adding mop-up costs and resource loss to suppression costs. This is illustrated using the current example, where resource loss and mop-up costs are estimated at $750 and $200 per hectare, respectively. The most efficient response is now observed to be that for the 2.5 hour containment time *(fig. 2)*.

Table 3-*Suppression resources available to respond to a 1.5 hour containment time.*

Description	Fireline	Cost
	meters	dollars
Patrol 80 #1	12.1	107.88
Engine 93 200	10.0	256.19
Engine 47 1000	22.1	276.37
Engine 64 200	4.0	312.99
Engine 24 600	2.0	440.05
Helitack #16	46.3	1,397.75
Smokejumpers #18	26.5	1,698.66
BD Crew #110	28.2	1060.67
Airtanker #1.1	201.2	7,200.00
Airtanker #2.1	201.2	7,200.00

Table 4-*Least suppression cost dispatches by containment time.*

Description	Containment time (hours)											
	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0
Patrol 80 #1	X[1]	X	X	X	X	X	---[1]	X	---	---	X	X
Engine 93 200	X	X	X	X	X	X	X	X	X	X	X	X
Engine 471000	X	X	X	X	X	X	X	X	X	X	X	X
Tender 48 2000	X	X	X	X	X	X	X	X	X	X	X	X
Engine 64 200	---	X	X	---	---	---	X	---	---	---	---	---
Engine 24 600	---	---	X	---	---	---	X	---	---	---	---	---
Tender 65 1000	---	---	---	---	---	---	X	---	---	---	---	---
Helitack #1	X	X	X	---	---	---	---	---	---	---	---	---
Smokejumpers #18	---	---	X	X	---	---	---	---	X	X	X	X
BD crew #110	---	---	X	X	X	---	---	X	---	---	---	---
Fire crew #210		---	---	---	---	---	---	---	---	---	---	---
Engine 31200			---	X	---	X	X	---	---	---	---	---
Hotshot crew #120			---	---	---	---	---	---	---	---	---	---
Dozer			X	X	X	X	X	X	X	X	X	X
Airtanker #1.1	X	X	---	---	---	---	---	---	---	---	---	---
Airtanker #1.2		X	---	---	---	---	---	---	---	---	---	---
Airtanker #2.1	X	---	---	---	---	---	---	---	---	---	---	---
Airtanker #2.2		---	---	---	---	---	---	---	---	---	---	---
Suppression cost ($)	16,437	13,932	7,131	4,751	3,797	2,186	2,952	3,548	4,168	4,324	4,656	4,819

[1]*X indicates a resource was selected for dispatch; --- indicates just consideration for dispatch.*

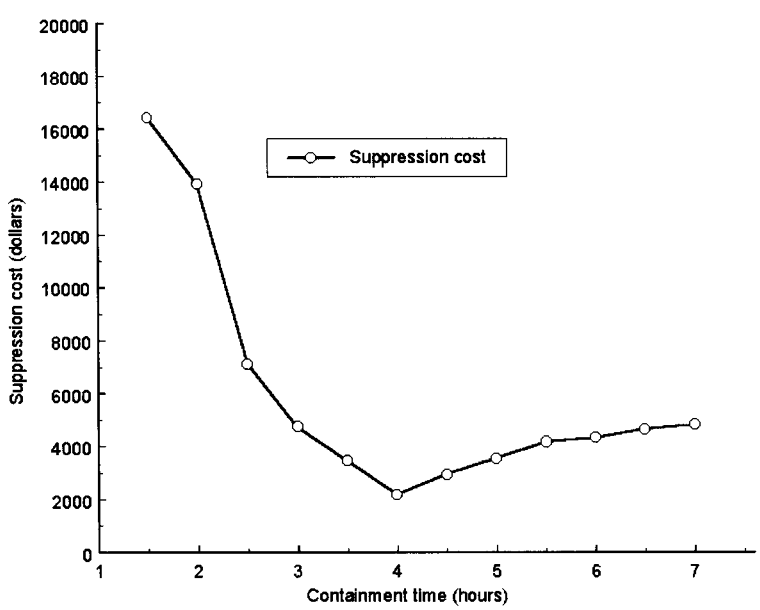
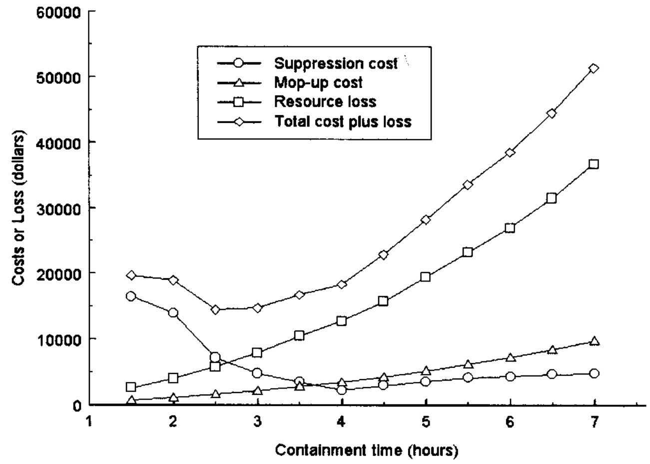


Figure 1

Minimum suppression cost by containment time.

Figure 2
Costs and resource loss by containment time.



The most efficient dispatch can be very sensitive to the size of per hectare resource loss and mop-up costs. In this example, higher per hectare loss and cost values would favor the more rapid, expensive, and aggressive attack requiring the use of retardant carrying airtankers. A rapidly spreading fire, all other things equal, also favors an aggressive attack.

Discussion

The dynamic programming model presented in this paper offers a powerful computational tool for finding the most efficient dispatch of available suppression resources to a fire. It also can provide a significant amount of information on economic trade-offs of alternative suppression strategies. This information can be very important to dispatchers when faced with the uncertainty of additional fires and a need to allocate resources between fires.

On the technological side, one virtue of the dynamic programming dispatch optimization model is computational speed. This is particularly important in an environment where timely results are critical. The efficiency of the algorithm is demonstrated by the Mt. Hood National Forest dispatch example where the least cost response of 26 suppression units to 13 containment times took less than 2 seconds to solve on a 200 MHZ micro computer.

For expository purposes, this paper has focused on the use of dynamic programming in a dispatch environment. The algorithm, as contained in the software package, IASELECT (Wiitala 1989), has also been used to address problems in fire planning, ranging from fuels management risk assessment (Wordell 1991) to the evaluation of suppression technologies (Schlobohm 1996).

Opportunities exist for additional research and development to further refine the dynamic programming algorithm presented in this paper. One possibility, internalizing the equations for mop-up costs, resource loss, and fire perimeter growth, would allow the algorithm to search for both the optimal response and containment time.

A more challenging research endeavor would be to formulate a dynamic programming algorithm to solve the continuous time dispatch problem as described in equations (1) through (6). This would ensure the optimum dispatch was not overlooked, which can result when considering a limited number of containment times.

References

Bellman, Richard. 1957. **Dynamic programming.** Princeton: Princeton University Press; 339 p.

Bratten, Frederick W.; Davis, James B.; Flatman, George T.; Keith, Jerold W.; Rapp, Stanley R.; Storey, Theodore G. 1981. **FOCUS: a fire management planning system-final report.** Gen Tech. Rep. PSW-49. Berkeley, California: Pacific Southwest Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 34 p.

Kourtz, Peter. 1989. **Two dynamic programming algorithms for forest fire resource dispatching.** Canadian Journal of Forest Research 19: 106-112.

Mees, Romain M. 1978. **Computing arrival times of firefighting resources for initial attack.** Gen. Tech. Rep. PSW-27. Berkeley, CA: Pacific Southwest Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 21 p.

Nemhauser, Gary L. 1966. **Introduction to dynamic programming.** New York: John Wiley and Sons, Inc.; 256 p.

Parks, George M. 1964. **Development and application of a model for application of forest fires.** Management Science 10(4): 760-766.

Parlar, Mahmut; Vickson, R.G. 1982. **Optimal forest fire control: an extension of Park's model.** Forest Science 28(2): 345-355.

Schlobohm, Paul M. 1996. **Use a comparison model to guide technology decisions.** Fire Management Notes 56(l):12-14.

Wagner, Harvey M. 1975**. Principles of operations research**. 2d ed. New Jersey: Prentice-Hall, Inc.; 1039 p.

Wiitala, Marc R. 1986. **Optimum fire suppression responses: a dynamic programming solution**. Unpublished draft supplied by author.

Wiitala, Marc R. 1989. **IASELECT: Initial attack resource selector user's manual.** Unpublished draft supplied by author.

Wordell, Thomas L. 1991. **A fire protection analysis for the Beaver Creek Watershed**. Technical fire management project report. Washington Institute; 36 p.