Assessing the Risk of Cumulative Burned Acreage Using the Poisson Probability Model[1]

Marc R.Wiitala[2]

Abstract

Resource managers are frequently concerned that the area burned by wildfire over time will impede achievement of land management objectives. Methods that use the Poisson probability model to quantify that risk are described. The methods require a concise statement of an adverse wildfire outcome and information on fire frequencies and sizes. An example is presented that illustrates use of the risk assessment procedure to quantify the trade-off between burned acreage risk and the cost of a fuels treatment project that reduces risk.

Forest wildfire can pose a significant risk to achieving forest land management objectives. Water quality in municipal watersheds can be threatened by wildfire events. Anadromous fisheries can be endangered by the stream sedimentation resulting from runoff when rainstorms follow a large fire or even a series of smaller fires. Maintaining minimum acceptable levels of a wildlife habitat can also be at risk to the cumulative effects of wildfire. Sustained timber supplies are also at risk to wildfire events. For these reasons resource managers are concerned with the uncertainty posed by wildfire. Their concern centers on both the threat to meeting land management objectives and the costs of wildfire risk management.

Accurately assessing the uncertainty posed by wildfire is frequently quite difficult, particularly if the outcomes of concern involve collections of random events. When adequate data are available, quantitative techniques can help in estimating the probabilities of wildfire outcomes involving joint random events, as has been shown for adverse fire movement (Wiitala and Carlton 1994). These procedures will help reduce the biases that can creep into purely subjective assessments of these probabilities (Cleaves 1994).

This paper describes how probability theory can be used to assess the risk posed by wildfire to achieving resource management objectives. The procedures combine fire size and frequency data with the Poisson probability model to calculate the chance that burned area will exceed some threshold for a given area and period. A hypothetical example is presented that illustrates use of the risk assessment procedure to quantify the trade-off between burned acreage risk and the cost of a fuels treatment project that reduces risk.

Wildfire Risk

The forest wildfire environment is characterized by much randomness. Specific wildfire outcomes are governed by the laws of chance. Resulting cumulative effects can jeopardize achieving resource management objectives. While management opportunities exist to alter the odds of this game of chance, avoiding the game is not possible. Resource managers will always face some level of wildfire risk as defined by the chance of an undesirable wildfire outcome, and not just the chance or probability of a wildfire ignition.

The definition of an undesirable wildfire outcome must be stated in concrete terms relative to the risk of not achieving a resource management objective. For example, a resource objective might be to maintain at least a minimum level of a

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[2] Operations Research Analyst, Pacific Southwest Research Station, USDA Forest Service, 1221 SW Yamhill Street, Suite 200, Portland, OR, 97205 email: mwiitala/r6pnw_portland@fs.fed.us

forest habitat judged necessary to sustaining a threatened species. The undesirable outcome could then be defined as a single or set of wildfires causing a habitat loss that violates the minimum acceptable level. Time and magnitude are important dimensions of this definition of an undesirable outcome. With respect to time, the more quickly desired vegetation can be recovered by an ecosystem, the shorter will be the period over which to evaluate risk. For a watershed in which an anadromous fishery is at risk to sedimentation after a wildfire, the risk period is short because a regrowth of vegetation in areas burned may only take a few years. Yet, the risk period would be long where recovery of lost old-growth forest habitat may require decades.

The difference in area between an existing or projected habitat level and a minimum desired level is one yardstick against which to measure the risk posed by wildfire. This difference can be considered an insurance "reserve" or maximum acceptable loss. The collection of random wildfire losses eliminating this reserve defines the undesirable outcome. From a risk assessment standpoint, the question naturally follows "What is the probability of losing the habitat reserve to wildfire?" From a resource management viewpoint, the question is "What resource management or protection actions can be taken to keep the risk within tolerable levels?"

Methods

For most areas of the United States, substantial statistics have been collected on wildfire (Fullman and Brink 1975). The existence of these statistics opens opportunities for analytic approaches to risk assessment. Probability theory and statistics on wildfire frequency and size can be used to calculate the risk of not achieving specific resource management objectives.

Poisson Probability Law and Fire Occurrence

A Poisson random process is characterized by the counting of individual events over space and time (Sundararajan 1991). Many physical random phenomena are found to follow, at least approximately, a Poisson random process (Parzen 1960). Mandallaz and Ye (1997) found the Poisson probability model to characterize well the forest wildfire ignition process.

The simple Poisson model is proposed in this paper as a practical approach to modeling the randomness of wildfire occurrence even if the model is only approximate. Accordingly, where wildfires are estimated to occur in a given area with average frequency μ per unit of time, the probability of observing k fires over time t is given as:

$$\Pr(k;\mu t) = \frac{e^{-ut}(\mu t)^k}{k!} \qquad [1]$$

in which μt, is called the rate of the Poisson process.

To illustrate the model's use, consider an area that receives on average five fires per year. The chance of receiving seven fires in a given year is:

$$\Pr(7;5) = \frac{e^{-5}(5)^7}{7!} = 0.104 \qquad [2]$$

The probability of a compound event, such as receiving seven or more fires in a year, can also be computed. This would most easily be accomplished by calculating probabilities for each outcome in the range zero to six fires. Summing these results would give the probability of receiving six or fewer fires (0.762), and subtracting that value from one would give the probability of receiving seven or more fires in a year (0.238).

Poisson Probability Law and Fire Size

Modeling wildfire frequency with the Poisson probability distribution is a major step toward addressing the risk of adverse wildfire outcomes. Wildfire is a necessary but not sufficient condition of the risk equation. Actual risk arises in regard to the area burned and not necessarily the number of wildfires. Therefore, addressing variability in fire size is as important as addressing variability in wildfire numbers.

Substantial amounts of effort have gone into the search for probability models that characterize variability in wildfire size. An excellent overview of these efforts can be found in Alvarado-Celestino (1992). Variability in wildfire data is represented best by theoretical models that accurately characterize extreme values arising from infrequent but very large fires, such as the Pareto and Weibull continuous probability distributions. Combining one of these models for fire size variability with the Poisson process for fire frequency would result in a compound Poisson probability model presenting significant mathematical and computational challenges. Furthermore, this approach would require sophisticated statistical techniques to identify and estimate the parameters of a probability model best characterizing a particular fire size data set (Alvarado-Celestino 1992).

A more intuitive, mathematically tractable, and less labor intensive tact is offered. It is based on the premise that any continuous distribution used to represent the variability in wildfire size can be approximated by a discrete distribution with a suitable size and number of classes. From historical data an empirical distribution would be constructed from estimates of mean and relative frequency for each fire size class (Pitman 1993). According to the empirical distribution, a fire would have a chance of burning the mean area for a class equal to the estimated relative frequency for the class. Lacking adequate historical data, professional judgment may be required to make these estimates.

Modeling fire size variability with an empirical probability model permits using a valuable theoretical attribute of the Poisson process used to model fire frequency. A Poisson process can be mathematically partitioned into independent Poisson component processes when events of the general process randomly take on attributes of the components (Ross 1989). When this happens, the rate μ of the general Poisson process is apportioned to the component processes on the basis of the relative frequency of events in these component processes. For wildfire occurrence, this mathematical result allows apportioning total fire occurrence among several fire size classes, each following an independent Poisson process. The result opens a door to a practical method for computing the risk of an undesired wildfire outcome, that is, the probability total burned area will exceed some level.

Computation Methods

Computing the probability of interest requires determining wildfire outcomes, estimating individual and joint event probabilities, and aggregating results. For instance, we can hypothesize an area at risk to more than 2,000 burned hectares in a given year. In the area, fires arise in three size classes with frequencies of 6, 2, and 0.5 per annum. Mean fire sizes for these classes are 10, 250, and 1,000 hectares, respectively. Given these three independent fire generating random processes, an adverse outcome would be the joint occurrence of seven, four, and one fires in the respective size classes. This outcome would result in the burning of 2,070 hectares. The probabilities of the individual outcomes from the three fire size classes are given, respectively, in the following equations:

$$\Pr(7;6) = \frac{e^{-6}(6)^7}{7!} = 0.14 \qquad [3]$$

$$\Pr(4;2) = \frac{e^{-2}(2)^{4}}{4!} = 0.09 \qquad [4]$$

$$\Pr(1;0.5) = \frac{e^{-0.5}(0.5)^{1}}{1!} = 0.30 \qquad [5]$$

The probability of the joint occurrence of these three outcomes is very unlikely. Its value is less than 0.004 as computed by the product of the probabilities of the individual independent outcomes. Because of the small chance of occurrence, this joint fire outcome poses little risk. However, many other combinations of fires in the three size classes will yield adverse outcomes. When aggregated, their total probability could pose a significant risk.

One way to computationally evaluate total risk is by enumerating, calculating, and aggregating probabilities for all joint fire occurrence outcomes that do not exceed some unacceptable level of burned hectares. Subtracting the resultant from 1.0 gives the desired risk estimate. For the three-class example, total risk is computed as:

$$1-\sum_{k=0}^{2}\sum_{j=0}^{8-4k}\sum_{i=0}^{200-100k-25j}\frac{e^{-0.5}(0.5)^{k}}{k!}.\frac{e^{-2}(2)^{j}}{j!}.\frac{e^{-7}(7)^{i}}{i!}=0.134 \qquad [6]$$

The upper limits on the summation signs are constructed from the threshold level of 2,000 hectares and fire sizes of 10, 250, and 1,000 hectares, respectively, for *i, j,* and *k*. The consequence of indexing through the summation signs is to enumerate and evaluate all combinations of fire events resulting in combined burned area less than or equal to the threshold. For example, when index *k* is 1 to signify one 1,000 hectare event in the combination, index *j*, signifying the number of 250 hectare events, can index 0 through 4 without the cumulative acres of events in these two fire classes exceeding 2,000 hectares. A similar process of indexing holds for *i* when combinations of fires in the *k* and *j* class do not exhaust the 2,000 hectare threshold.

Applying the Risk Assessment Techniques

Development of these quantitative methods suggested potential applications in wildfire risk assessment. The hypothetical data used in developing these methods are further considered in looking at the application of the risk assessment method. Attention focuses first on the creation of a risk profile for a resource management area. Afterwards, the method is used to look at trading off risk and cost when a management activity is proposed for mitigating potential fire behavior.

Creating a Risk Profile for a Project Area

Determining a minimum acceptable level of burned hectares consistent with meeting resource management objectives is often fraught with uncertainty. For example, in a watershed, depending on the intensities of wildfires and on other uncertain variables, the threshold separating acceptable and unacceptable outcomes may be unclear and might fall within a range of burned hectares. This uncertainty begs for more information on risk than a single probability estimate.

The solution to the problem is to conduct sensitivity analysis by calculating risk estimates over a range of burned area thresholds. This is illustrated for a 20,000 hectare watershed whose water quality is at risk to any combination of fire events burning more than 2,000 hectares over a 3-year period. The watershed is estimated from historical data to have the following fire size class and frequency statistics for the 3-year period:

Size class	Mean fire size (hectares)	Mean fire frequency
1	10	12.00
2	250	3.00
3	1,000	0.75

From these statistics a risk profile for both periods is created for thresholds ranging from 1,000 to 3,000 hectares in steps of 500. Applying the computational methods developed earlier, the probability of burning a greater area than each hectare threshold gives the following risk profile:

Burned area threshold (hectares)	Probability of exceeding threshold
1,000	0.694
1,500	0.497
2,000	0.304
2,500	0.177
3,000	0.092

Probability estimates for the range of thresholds provide much more information than the single probability estimate for the 2,000 hectare threshold. The sensitivity analysis for the 3-year period clearly shows the risk of more serious outcomes decreasing rapidly. On the other hand, burning more than 1,000 hectares in 3 years is quite likely.

Trading Mitigation Cost against Risk Reduction

The risk of unfavorable wildfire outcomes can often be reduced in an area by resource management and protection activities. Fire size can be decreased by manipulating vegetation that fuels fire in ways that reduce fire behavior. Increased fire prevention efforts can reduce fire frequency. Additional initial attack fire suppression resources will contain fires more quickly and at smaller sizes. The benefit of these risk mitigation expenditures is not always fully measurable in dollars. Under these circumstances examining the improvement in the wildfire risk profile for an area may provide additional support for expenditures.

Trading mitigation cost for risk reduction is illustrated by expanding on the previous watershed example. A program for treating fire fuels in the watershed is proposed at annual cost of $10,000 for purposes of mitigating fire behavior increasing fire suppression effectiveness. As a result most fires will be contained more quickly at smaller sizes. Although the average number of fires in each class is not expected to change over the 3-year period, the average size of fires in the three size classes is projected to decrease after treatment as follows:

Size class	Mean fire frequency	Mean fire size before	Mean fire size after
1	12.00	10	3
2	3.00	250	100
3	0.75	1,000	500

From the projected after treatment fire sizes, a new risk profile is calculated with the following results:

Burned area threshold (hectares)	Probability of exceeding threshold before	Probability of exceeding threshold after
1,000	0.694	0.240
1,500	0.497	0.066
2,000	0.304	0.014
2,500	0.177	0.002
3,000	0.092	0.000

Although this example is hypothetical, it does exemplify the kind of information to be expected from use of the risk assessment methods outlined in this paper. In this instance, by reducing fire size to about one-half, the fuels treatment program nearly eliminates the risk of adverse outcomes. A manager would have to decide whether this substantial risk reduction is worth the $10,000 annual cost of the fuels treatment program.

This watershed example also illustrates another important point about risk assessment. If a change in average hectares burned is used as a proxy for a change in risk, as is often done, this can be misleading. The expected hectares burned in the 3-year period declined from 1,620 to just 701, whereas the probability of burning more than 2,000 hectares declined dramatically from 0.304 to 0.014.

Discussion

A critical aspect of every risk assessment is determining the probabilities of potential outcomes. When opportunities exist, using formal methods from probability theory can increase the quality of risk assessments.

This paper has provided one example of how the Poisson model and the mathematical methods of probability theory can be used to address a particular class of risk assessment problem in resource management. The quantitative technique described can improve on the probability estimates made by other means.

The technique does require input data on which to make probability calculations for the risk assessment, such as fire frequency and size used in the example. Most often these inputs must be estimated from sample data or acquired through expert judgment. To the extent estimates of the input data are precise, reliable, and unbiased, the resulting probability calculations will be of high quality.

On another note, reduction of the continuous distribution for fire size to a discrete distribution with few size classes may also create bias in the risk calculations. As threshold values approach the mean of a size class, probability estimates will tend to underestimate the true probabilities. This bias can be mitigated by choosing more size classes for the empirical distribution.

This paper did not address several issues and procedures for acquiring good data on fire frequencies and sizes upon which to make probability calculations. One that should be highlighted centers on the degree to which historic frequency and size data will represent conditions in the future. Ecosystems and fire suppression policies change over time. To the extent this occurs, the deeper the reach into the historic data pool and/or the greater the reach of the risk assessment into the future, the less reliable will be the probability calculations. A similar issue arises when data from a larger geographic area are used for estimating risk in a smaller area. These issues are beyond the scope of the current paper. Nevertheless, they warrant future research. Potential users of the risk assessment techniques described in this paper are cautioned to consider these matters.

Caution also should be exercised in the use of the methods described in this paper to assess the risk of adverse outcomes involving both magnitude and duration of a condition. If concern, for example, centers on assessing the risk of sustained habitat loss exceeding some duration, use of more sophisticated probability methods or stochastic simulation techniques will be required.

The methods outlined in this paper need not be confined to assessing burned acreage risk. Other random phenomena in resource management may follow Poisson processes with frequency and size attributes that make threshold risk assessment meaningful.

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