Analysis of Area Burned by Wildfires Through the Partitioning of a Probability Model[1]

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Abstract

An analysis of forest fires by using a partitioned probability distribution is presented. Area burned during a fire is fitted to a probability model. This model is partitioned into small, medium, and large fires. Conditional expected values are computed for each partition. Two cases are presented: the two-parameter Weibull and the Truncated Shifted Pareto probability models. The methodology allows a comparison of area burned and cost for small, medium, and large fires among different attack strategies. The partitioned functions may also be used as a basis to refocus fire management optimization with a multiple objective formulation.

Introduction

Fire management decisions are frequently made on the basis of many uncertain and highly variable factors, such as fire behavior, weather forecasts, fire effects, and performance of initial attack force. Although most of the modeling of these components have been deterministic, fire managers probably balance their judgments about the uncertain factors against their preferences for possible consequences or outcomes, deviating from the optimal solution provided by a decision-making model. All of these deviations from an optimum solution implicitly recognize the effect of large fires in deciding what level of effort or resources to allocate in fire management. Despite that forest fires are highly stochastic and complex events, most of the decision models that are uses are based on techniques that use the expected value of the damage function, and a few others are deterministic models.

Fire management decisions obviously are influenced by the concern about the occurrence of large fires. Large fires with low probability of occurrence have a large impact in natural, social, and economic systems. However, most fires are extinguished in the initial stages and remain small. These smaller fires have a large probability of occurrence, but the resulting damage is practically negligible on an individual basis. Because of these probabilities, the expected value of a damage function misrepresents both extremes. Also, the use of expected value does not adequately represent the consequences associated with different fire management policies.

Traditional decision-making and budgeting in forest fire management have been based on the postulate that resources must be allocated in proportion to the resource value to be protected. However, it has been mentioned in several forums that the high costs of fire control are not justified when a strict economic analysis is made because the optimization criterion consists largely of costs (Gale 1977). Moreover, actions that are seen to reduce the risk of the occurrence of large wildfires justify high expenditures made on those large fires, as well as those made on smaller fires.

This paper presents a methodology to incorporate different damage levels in fire management decision-making by analytically incorporating large fires. The methodology partitions a probability model to describe area burned, to address different damage levels and their probability of occurrence. The assumption is

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that partitioning the probability distribution of fire damage will enable comparisons of the cost of different initial attack strategies for different damage levels.

The probability distribution of fire damage will be partitioned to segregate the small fires that have a large probability of occurrence but cause little damage, the large fires that cause extensive damage but have low probability of occurrence, and the intermediate events. By using this approach, more information about fire damage variability is accounted for in the decision process and large fires are analytically included.

Modeling the partition containing the large fires is emphasized because they are responsible for most of the damage caused to the forest. A great amount of resources are devoted to fighting or preventing large fires and on restoration. The few large fires, as opposed to the many small fires, are of interest to the public and have more influence on the policies of a fire organization.

Partition of a Probability Distribution

Show (1921) stated that large fires have been misrepresented in the current fire size classification systems because they have been pooled into a single fire class. He pointed out the need of subdividing the fire classes similarly to a geometric progression according to the fire size. In this paper, we propose that partitioning the distribution will reflect more accurately the fire occurrence distribution. The distribution function fitted to fire sizes is partitioned in three ranges. They represent three damage levels: the small fires with high probability of occurrence; the medium damage fires; and the large fires with a low probability of occurrence but high consequences.

For illustration purposes of the partition methodology, we used the two-parameter Weibull and the Truncated Shifted Pareto (TSP) distributions (Alvarado 1992). Those two distributions were fitted to the fire occurrence records from 1961 to 1988 for the Provincial Forest Service of Alberta, Canada; *table 1* includes the parameter estimates for those two distributions. The data was also separated by resources used in the initial attack: fires in which only man power was reported, those in which air attack was used, and those fires in which man and other equipment different than aircrafts were used.

The number of partitions into which the probability axis is divided depends on the nature of the problem and concerns of the decision-makers. In cases where risk is involved, three partitions are usually made: one is the range for high frequency events with low damage; one for intermediate events; and a third one consisting of the events that represent large losses with low probability of occurrence.

Table 1- *Parameter estimates for the two-parameter Weibull and the Truncated Shifted Pareto distributions.*

Initial attack strategy	Parameter estimates			
		Two-parameter Weibull distribution		
	c	ase[1]	a	ase[1]
All fires	0.3263	0.0014	1.718	0.0382
Manpower only	0.3442	0.0193	1.3028	0.0364
Air attack	0.3044	0.0024	2.4165	0.11
Ground attack	0.3344	0.0036	2.4496	0.1349
		Truncated Shifted Pareto distribution		
	a	ase	b	ase
All fires	0.0605	0.0012	2.1585	0.0197
Manpower only	0.0553	0.0013	2.0033	0.0243
Air attack	0.0727	0.0029	2.2821	0.0404
Ground attack	0.0769	0.0041	2.3241	0.0551

[1] Asymptotic standard error of the estimate.

There is no consensus in the literature on the level at which events should be considered as extremes. In some instances the cut-off is made for practical reasons on the probability axis. These are referred to as the "events that exceed a quantile." Examples are sea wave extremes that exceed the level 1:10,000 (Dekkers and Haan 1987) or those with large return periods, e.g., rain depths with a return period of more than 100 years (Foufoula-Georgiou 1989). Large forest fires are usually referred to in terms of area burned. The variance may also be used if the normal distribution is used as a risk function, e.g., one or two times the variance. Another approach is to use past experience regarding damage levels, e.g. fire size classes, emergency state declarations, or others.

The partitions mapped on the probability axis can represent three damage levels *(fig. 1)*. The 1 to 1-α_i range include the events with large probability of occurrence but cause the lowest damage (0 to β_{ij} interval on the damage axis). The events bounded by α_i and α_{ij} are those events causing an intermediate damage. Extreme value theory and risk management focus on events known as *low probability/high consequence* events. These are the events in the lower partition of the probability axis that cause the most damage *(fig. 1)*.

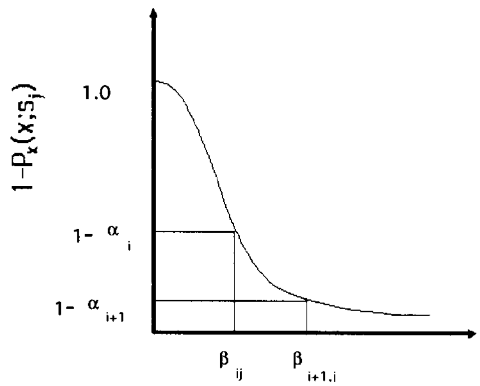


Figure 1

Mapping of the partition of the probability axis onto the damage axis (Asbeck and Haimes 1984).

The partitions used in this study are based on the fire size classes defined by Canada's Province of Alberta (Alberta Forest Service 1985). The first partition includes the fire size classes A and B, which includes the fires that burned from 0.1 to 4.0 hectares. For the second and third partitions we presented two cases. For the second partition, the upper limit depends on the lower limit of the large fire level. It consists either of those in size classes C and D, i.e., from 4 to 200 hectares; or it includes C and D and part of E classes, with the lower limit larger than 4 hectares and the upper limit set to exclude the upper 1 percent of the fires. The third partition encompasses the large fires. Two cases of large fires are studied: fires in the size class E, i.e., larger than 200 hectares, and fires in the upper one percentile of the observed distribution.

Once the partition values are defined either in terms of probability or damage, they are mapped onto both the probability and damage axis. Asbeck and Haimes (1984) and Karlsson and Haimes (1988b) consider that the problem is to find a β_{ij} for each partition point α_i for $i=1......,n$, and values of s_j for $j=1,...,q$, such that $P(\beta_{ij}; s_j)= \alpha_i$. Then, the existence of a unique inverse $P_x^{-1}(x;s_j)$ is guaranteed from the following standard probability assumptions: the $p_x(x;s)$ is nonnegative and:

$$\int_{\infty}^{-\infty} p_x(x;s)dx = 1 \qquad [1]$$

with the probability of $(a<X<b)$ given by:

$$P(a < X < b) = \int_a^b p_x(x;s)dx \qquad [2]$$

The probability and damage axes are partitioned into a set of n ranges, three in this paper. The partition points on the probability axis are denoted by α_i. For each partition point and variable s_j, the three initial attack strategies as defined in the study, there exists a unique damage or loss β_{ij} such that:

$$P_X(\beta_{ij};s_j)=\alpha_i \quad [3]$$

Because the inverse $P_X^{-1}(x;s_j)$ is assumed to exist, thence:

$$\beta_{ij}=P_x^{-1}(\alpha_i;s_j) \quad [4]$$

If the interest is to represent the large fires without being restricted to a parent distribution, in addition to partitioning of a distribution function, other methodologies can be used. One of them is to select an upper threshold so high in the parent distribution such that the exceeding observations converge to one of the extreme value models. For the case of wildfires, the fire size observations are highly concentrated on the first fire classes. Thus, distribution of fires in the upper size class, or those in an upper percentile may converge to an extreme value model (Alvarado 1992). This model is the limiting convergence of the right tail of the parent probability model, and will include only the large fires.

Another approach is to create the proposed functions with a censoring approach. The idea is that the right tail exists, but it is beyond the local fire fighting capabilities. The reasoning for this approach assumes that the fire organization's efforts did not maintain the size of these fires within the domain of the censored distribution. Regardless of the approach, the fire size distribution can be included in an optimization problem as a set of functions, both the unrestricted distribution and the restricted partitions of it.

Conditional Expected Values

The computation of the conditional expected values is a question of defining the probabilities that the random variable X falls within each of the selected partitions (Houghton 1988, Karlsson and Haimes 1988a). The β_{ij}, specified in equation 4, are used for the definition of the conditional expectations. They are formally expressed by:

$$f_i(s_j) = E\{x \mid P_x(x;s), x \in [\beta_{ij}, \beta_{i+1,j}]\} \quad [5]$$

Represented in an integral form, the conditional mean is expressed by:

$$f_i(s_j) = \frac{\int_{\beta_{ij}}^{\beta_{i+1,j}} x p_X(x;s_j)dx}{\int_{\beta_{ij}}^{\beta_{i+1,j}} p_x(x;s_j)dx}, i = 1,\ldots,q \quad [6]$$

Let θ_i, denote the denominator of equation 6, then from using the assumption from equation 2:

$$\theta_i = \int_{\beta_{ij}}^{\beta_{i+1,j}} p_x(x;s)dx = P_x(\beta_{i+1,j};s) - P_x(\beta_{ij};s) \quad [7]$$

Equation 7 can be stated in terms of the probability partition points *(fig. 1)* as:

$$\theta_i = \alpha_{i+1} - \alpha_i \quad [8]$$

Substitution of the equation 8 into 6 gives the following general equation to compute the conditional expected values:

$$f_i(s_j) = \frac{1}{\alpha_{i+1} - \alpha_i} \int_{\beta_{ij}}^{\beta_{i+1,j}} x p_X(x; s_j) dx \qquad [9]$$

The conditional expectation of the fire size distribution, given that the fire is in that partition, is in some sense an incomplete first moment (Raiffa and Schlaifer 1961) or an expected mean for a certain range (Houghton 1988). Notice in equation 9 that if the partition points occur at 1 and 0, i.e., the integration is over the entire distribution range, then, the equation becomes the unconditional expected value.

In the case of the first partition, the conditional expected value depends only on the upper partition point $\hat{\alpha}_i$ that represents the cumulative probability that determines the upper bound of the low damage range. Equivalently, the partition point α_i on the probability axis corresponds to the partition points $\beta_{ij,}$ on the damage axis *(fig. 1)*. The conditional expected value of the second partition, which defines the intermediate damage, depends on two partition points, α_i and α_{I+1}. The third partition corresponds to α_{I+1} to 1.

An analytical solution to equation 9 generally does not exist. Karlsson and Haimes (1988a, 1988b) developed a methodology for solving the equation for certain distributions and parameter values. They found it easy to solve for the exponential case. They derive near-closed-form expressions for the normal and lognormal distributions. As for the Weibull, a closed form may not exist, depending on the values of the shape parameter.

Houghton (1988) presents a closed form solution for the TSP distribution. Although he derives it by partitioning on the *x*-axis and calls it *mean over a range*, the result is a closed form of the integral 9. Houghton derives the formula using the complementary cumulative distribution *G(x)=1-F(x)*. In the formula over the range, $x_d=\beta_{ij}$ to $x_e=\beta_{i+1,j}$ are equivalent to the partitions on the probability axis $G_e=\alpha_{ij}$ to $G_d=_{i+1,j}$. The mean over a range is defined by:

$$\bar{x}_{d,e} = \frac{1}{G_d - G_e} \int_{x_d}^{x_e} x f(x) dx \qquad [10]$$

Because the inverse (equation 4) is guaranteed to exist from the probability assumptions, let $y=G(x)$, $x=G^{-1}(y)$ and $dy=f(x)dx$, then:

$$\bar{x}_{d,e} = \frac{1}{G_d - G_e} \int_{G_e}^{G_d} G^{-1}(y) dy \qquad [11]$$

$$\bar{x}_{d,e} = \frac{1}{G_d - G_e} \int_{G_e}^{G_d} \Big[a \Big\{ [T^u (1 - T^u) G]^{-b} - 1 \Big\} + x_c \Big] dG \qquad [12]$$

which gives the equation presented by Houghton (1988) to compute the expected value over a range:

$$\bar{x}\beta_{ij}\beta_{i+1,j} = x_c - a + \frac{a\{[T^u + (1-T^u)\alpha_{i+1,j}]^{1-b} - [T^u + (1-T^u)\alpha_{ij}]^{1-b}\}}{(1-T^u)(1-b)(\alpha_{i+1,j} - \alpha_{ij})} \qquad [13]$$

Equation 13 can be expressed in terms of the α's and β's partition points as:

$$\bar{x}_{d,e} = x_c - a + \frac{a\Big\{[T^u + (1-T^u)G_d]^{1-b} - [T^u + (1-T^u)G_e]^{1-b}\Big\}}{(1-T^u)(1-b)(G_d - G_e)} \qquad [14]$$

For the Weibull model, Karlsson and Haimes (1988a) found that a closed form exists only if the inverse of the shape parameter, *c*, is a positive integer. If it is the case, the following equation applies:

$$f_i = \frac{a}{\alpha_{i+1} - \alpha_i} \Big\{ (1-\alpha_i) \Big[\Big(1n \frac{1}{1-\alpha_i} \Big)^{1/c}$$

$$+ \sum_{k=1}^{1/c} \Big[(1n \frac{1}{1-\alpha_i})^{k-1} \prod_{j=k}^{1/c} j \Big] \Big]$$

$$- (1-\alpha_{i+1}) \Big[\Big(1n \frac{1}{1-\alpha_{i+1}} \Big)^{1/c}$$

$$+ \sum_{k=1}^{1/c} \Big[(1n \frac{1}{1-\alpha_{i+1}})^{k-1} \prod_{j+k}^{1/c} j \Big] \Big] \Big\}$$

For the case of $1/c$ not integer, which is the case most of the time, a numerical integration technique must be used to solve equation 9. The expression $1/c$ is an integer only in cases where the Weibull's *c* parameter is equal to or less than one. It must be recalled that the desirable properties of the Weibull distribution do not hold for shape parameter values less than one.

Numerical integration methods are used to evaluate definite integrals that can not be evaluated analytically. Basically, the numerical integration methods are rules to integrate a function over a small number of intervals by subdividing the interval *[a,b]* into *n* equal parts of length *h=(a-b)/n* (Press and others 1986).

Table 2-*Conditional means for the first two partitions of the two-parameter Weibull and the Truncated Shifted Pareto distributions.*

Fire classes/ size range, ha	Partition points		Conditional means[1]	
	α_I	α_{i+1}	Weibull[2]	TSP[3]
All Fires- sample mean: 179.36				
Entire distribution[4]	1.0000	0	11.1868	156.14000
A-13/0-4.0	1.0000	0.1453	0.5395	0.49590
C-D / 4.0-200	0.1453	0.0223	58.3031	29.18050
C-99 pct/4.0-874	0.1453	.0100	74.5375	68.26610
D+/200+	0.0223	0		3096.58000
>99pct/>874	0.0100	0		6701.10000
Manpower only- sample mean: 72.65				
Entire distribution[4]	1.0000	0	6.9489	56.41000
A-13/0-4.0	1.0000	0.1299	0.5540	0.41950
C-D / 4.0-200	0.1299	0.0162	46.3264	24.87200
C-99 pct/4.0-441	0.1299	0.0100	50.6339	38.95090
D+/200+	0.0162	0		1639.35000
>99pct/>441	0.0100	0		2564.08000
Air attack: sample mean: 385.60				
Entire distribution[4]	1.0000	0	20.9786	398.44000
A-15/0-4.0	1.0000	0.1588	0.5112	0.61907
C-D / 4.0-200	0.1588	0.0332	75.9436	29.51370
C-99 pct/4.0-3185	0.1588	0.0100	133.8069	119.87100
D+/200+	0.0332	0		4957.43000
>99pct/>3185	0.0100	0		16166.60000
Ground attack: sample mean: 206.74				
Entire distribution[4]	1.0000	0	14.5222	546.31000
A-13/0-4.0	1.0000	0.1784	0.5699	0.59165
C-D / 4.0-200	0.1784	0.0254	53.5775	43.0875
C-99 pct/4.0-757	0.1784	0.0100	67.4529	132.18900
D+/200+	0.0223	0		8645.00000
>99 pct/>757	0.0100	0		21698.70000

[1] Parameters estimated with the maximum likelihood method.
[2] Means approximated with the Newton-Cotes 8 panel rule.
[3] Means computed from the exact closed form equation.
[4] Unconditional mean.

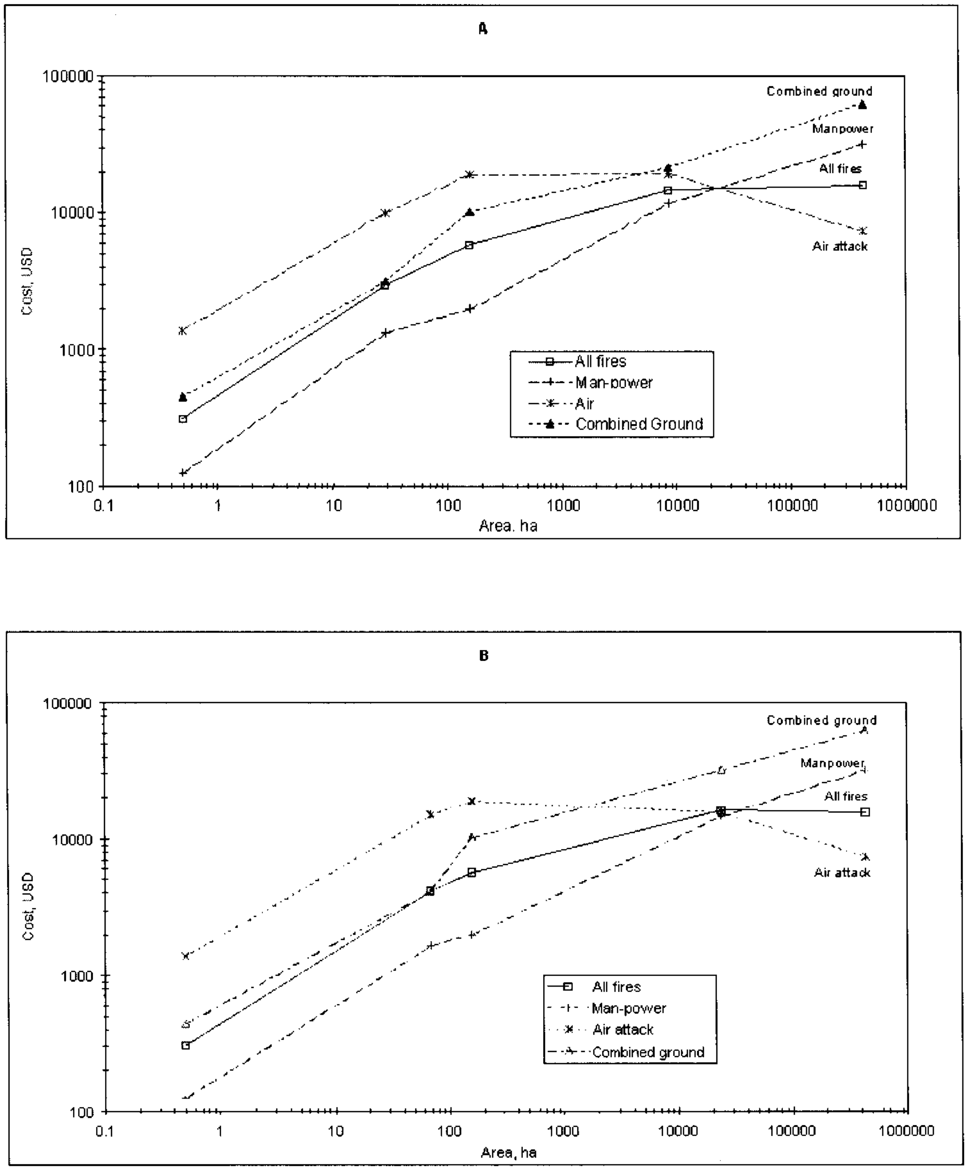


Figure 2

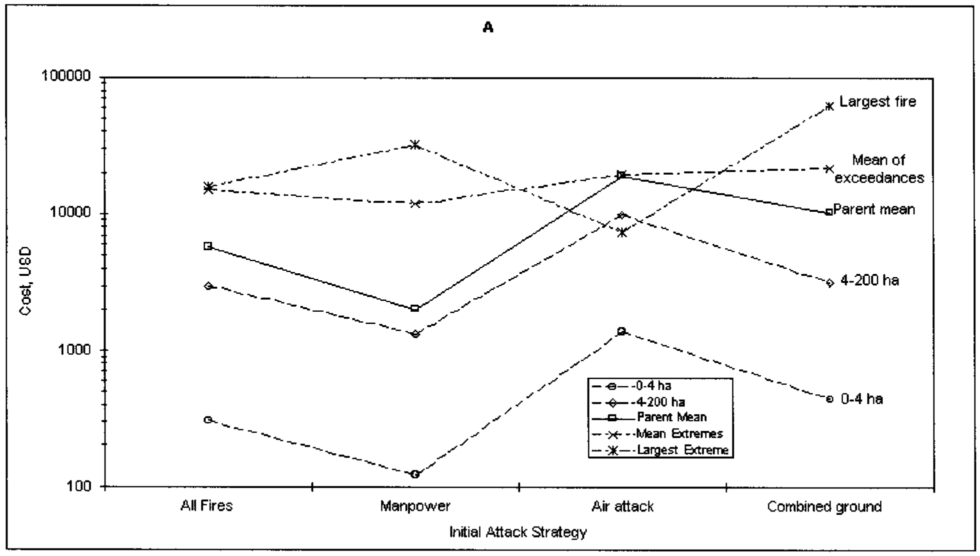
Relation between cost and expected area burned of different partitions by initial attack strategy. A) Lower threshold for the upper partition set at 200 hectares. B) Lower threshold for the upper partition set at 99 percent of the distribution.

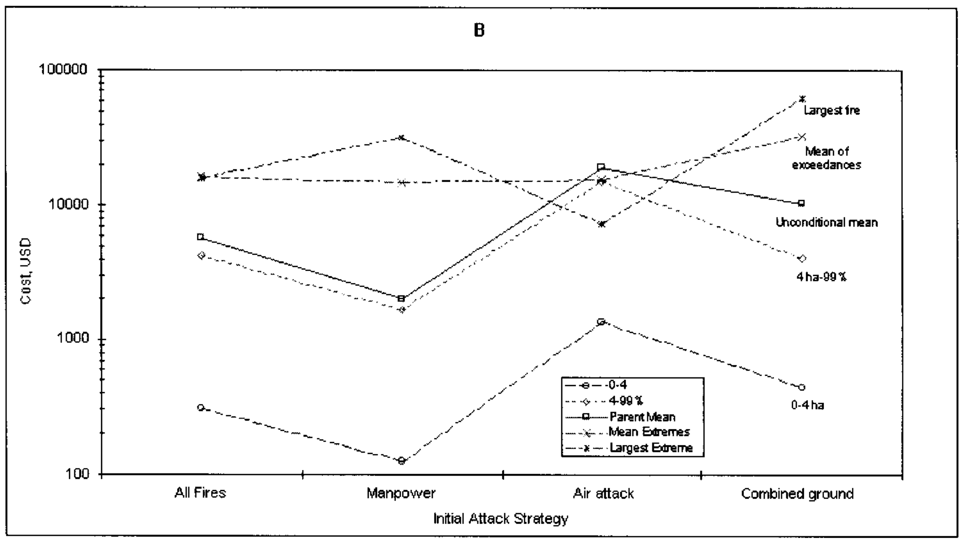
There are a variety of methods for evaluation of integrals, among them Simpson's rule, Newton-Cotes formulas, Romberg's integration, and Gaussian quadratures.

Numerical integration algorithms are commonly available in public domain software and in most of the commercial mathematical software. The method used in the study was an adaptive recursive Newton-Cotes 8 panel rule, implemented in most mathematical software. The advantage of using this rule over Simpson's, which is the most commonly used, is that it avoids numerical overflow when evaluating functions with soft singularities. That is the case of the Weibull distribution that becomes unbounded at the origin if the shape parameter, c, gets close to zero. In that case, the integration of the first partition is unbounded if integrated from zero. The problem was avoided in this study by shifting the lower limit slightly away from zero (e.g., to 0.05).

In cases where the Weibull's shape parameter is close to zero, it may be preferable to use one of the open or semi-open type numerical integration rules (Abramowitz and Stegun 1972) for which several algorithms are also available (Press and others 1986). The conditional expected values were computed for the first two partitions of the two-parameter Weibull and the TSP distribution *(table 2)*. For the third partition, only the TSP was calculated (Alvarado 1992, Alvarado and others 1998).

Figure 3
Expected cost by initial attack strategy of the expected area burned for several partitions of the fire size distribution. A.) Lower threshold for the upper partition set at 200 hectares. B) Lower threshold for the upper partition set at 99 percent of the distribution.





The conditional means computed for the first partition of the TSP differ only slightly from that of the Weibull's. However, the discrepancies are larger for the partition included in the fire classes C and D. The same situation occurs for the second partition when the upper limit is 99 percent of the distribution. That occurs because as it was seen in a preliminary analysis from probability plots, the TSP better fits the upper part of the distribution. The opposite situation was observed for the Weibull's plots, which presents a concave pattern over all the prediction range

The conditional means for the Weibull for the initial attack strategies are larger than the global or unconditional means. That does not seem logical because if larger fires are included, then the distribution mean should increase; but, this did not happen in this study because of the short-tailed attribute of the Weibull distribution. A monotonic increasing trend is seen in both the conditional and unconditional expected means of the TSP distribution. Again, that is because of TSP being a long-tailed distribution. The better fit over the entire distribution and better representation of the conditional means over the larger fire sizes supports the selection of the TSP over the Weibull to represent the entire fire size distribution.

Application of Conditional Means to Fire Cost Analysis

We present the relation between the expected cost for initial attack strategies and the expected area burned for the entire probability distribution. We present the conditional expected values and the expected area burned for the fires exceeding a threshold. The cost function was developed by Alvarado (1992) on the basis of previous work by González-Cabán and others 1987.

The costs for the unconditional expected area burned were calculated as well as the two lower partitions of the fire size distribution, and the expected value of fires exceeding a threshold *(figs. 2, 3).* It can be seen from these plots that the expected cost for small and medium fires is greater when using air attack and lower when using manpower only. The cost for the expected area of the all-fire data set resembles the ground attack strategy. Obviously, the cost of the expected area does not represent adequately the cost of manpower nor air attack. It is obvious that the global expected cost over predicts the small fires and completely misrepresents the cost of large fires.

When fires get larger, the parallel relation existing for the cost of small and medium fires does not hold. It can be observed that for large fires cost patterns are different from those observed for small fires. For large fires, it becomes more expensive to attack with either manpower only or ground equipment than to use air attack. That relation is reasonable because the amount of manpower and ground equipment units allocated to a large fire conflagration need to be excessively large to provide efficient fire fighting. The cost of maintaining such a large force also grows exponentially.

In contrast, the number of air attack units is more limited than the other fire fighting resources; therefore, the air cost does not increase as fast. In fact, the cost may decrease as fires grow larger (Gale 1977). There may be several reasons for this. The weather conditions that generated the large fire situation may likely have changed; subsequently, the fire may behave less threatening than at the active-growing phase. Usually, there is also an expenditure ceiling, which means that economic resources are not infinite. Additionally, the efficiency of ground attack crews and equipment may greatly improve when there is air support.

The difference of setting the threshold for the third partition at 200 hectares or 99 percent of the distribution is also observed in the cost of the expected area burned. When the third partition includes only the upper one percentile of the distribution, the expected costs of large fires of the manpower and air strategies, as well as the pooled all-fire data, are very similar. It seems that more partitions on the probability distributions would allow an equilibrium point between the cost of the different strategies, where there is cost indifference for pairs of initial attack strategy.

One immediate result of having several functions that describe different damage levels and costs for different initial attack strategies is that the optimization can be solved through multiple objective techniques (Chankong and Haimes 1983a, Karlsson and Haimes 1988b, Keeney and Raiffa 1976, Waller and Covello 1984).

The important role of large forest fires in fire management decision-making has never been neglected by fire managers. However, to date they have always been excluded from most of the analytical models. The solution to this problem has always been to resort to present empirical and subjective answers.

In conclusion, we propose to find a distribution function that better approximates the distribution of fire sizes. This distribution is then partitioned into ranges that represent different fire damage levels. The purpose of using this distribution is to represent the small, medium-sized fires. The partition that includes the large fires is approached from the perspective that they exceed a specified threshold. These thresholds may be given in actual area burned or may include fires in an upper percentile.

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