by A. Jeff Martin

A Computer Program for Analyzing PERT Networks

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ABSTRACT

A computer program designed to help the planner in analyzing PERT networks is described. The program, written in FORTRAN IV, is short and easy to use. Inputs are minimal and no lengthy set of control codes or cards are needed. The use of the program is demonstrated by means of an example problem for a National Forest timber sale. A source listing of the program, the input instructions, output from the example problem, and an explanation of the output tables are presented in the report.

A Computer Program for Analyzing PERT Networks

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Headquarters of the Northeastern Forest Experiment Station are in Upper Darby, Pa. Field laboratories and research units are maintained at:

- Amherst, Massachusetts, in cooperation with the University of Massachusetts.
- Beltsville, Maryland.
- Berea, Kentucky, in cooperation with Berea College.
- Burlington, Vermont, in cooperation with the University of Vermont.
- Delaware, Ohio.
- Durham, New Hampshire, in cooperation with the University of New Hampshire.
- Hamden, Connecticut, in cooperation with Yale University.
- Kingston, Pennsylvania.
- Morgantown, West Virginia, in cooperation with West Virginia University, Morgantown.
- Orono, Maine, in cooperation with the University of Maine, Orono.
- Parsons, West Virginia.
- Pennington, New Jersey.
- Princeton, West Virginia.
- Syracuse, New York, in cooperation with the State University of New York College of Environmental Sciences and Forestry at Syracuse University, Syracuse.
- Warren, Pennsylvania.

REGARDLESS of whether you are managing forest lands, operating a wood-using firm, or conducting forestry research, it is probably safe to say that planning is one of your most difficult tasks. And because it is difficult, it is all too often postponed from day to day, week after week, until too late for effective planning.

Since planning is a vital aspect of all managerial and research activities, it should receive at least as much attention as other aspects—in some cases, perhaps, more. To be effective, planning must be systematic and timely, and reflect as many relevant details as is possible. And the planning effort should be directly related to the difficulty and complexity of the problem. Therefore, as problems get larger and planning becomes more involved, our need for aids to simplify the process increases.

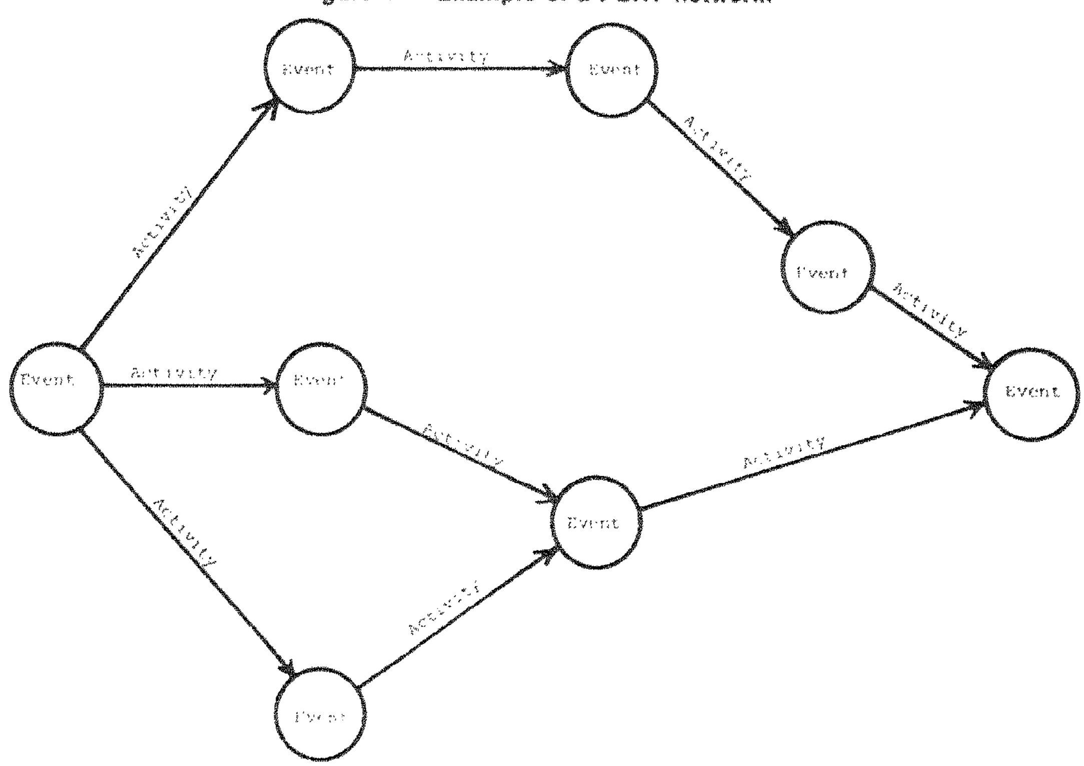
One technique that is very easy to use and can simplify the organization of details associated with planning is PERT (Program Evaluation and Review Technique). PERT was developed by the U. S. Navy in 1958 for the Polaris Submarine Program. A similar technique called CPM (Critical Path Method), which is actually an extension of PERT, was developed by the DuPont Company at about the same time (Miller 1963).

PERT AND WHAT IT CAN DO

Basically, PERT consists of a network of events and activities that depicts a project (fig. 1). The network is a "picture" of all the work that is necessary to reach an objective. The network and its analysis are designed to evaluate progress toward attaining the goal by examining the time needed to complete each event in relation to the time allotted for the entire project (Evarts 1964).

Since PERT is designed to deal only with the element of time (CPM deals with time and costs), it must be considered a complement to

Figure 1.— Example of a PERT network.



other methods of planning and control. However, it is an extremely valuable planning tool that has seen a wide variety of uses: plant expansion or location, new product development, advertising, marketing, road construction, bridge and dam building, preparation of bids, and many others.

Although PERT is not widely used in forestry and forest products industries, some uses of it have been explored. Ramsing (1966, 1967) illustrated a practical use of CPM in the construction of logging roads. Mater (1967) found that PERT was useful in reducing the costs of sawmill modernization. And Davis (1968) explained how PERT could be applied to resource management, using a slash burn on a Douglas-fir clearcut as an example. There are other illustrations, but in general, PERT hasn't been as widely accepted in the management of natural resources as perhaps it should. For example, PERT could be valuable in planning timber sales or coordinating the many individual studies associated with a complex research problem.

I think one of the reasons why the use of PERT has been limited is that although it was designed to simplify planning, when it is applied to a real-world situation, the result is usually a rather large network that may cause the user to throw up his hands. This is because what appear to be rather simple secondary computations in the textbook become quite tedious with the larger problems. Unfortunately, the PERT application may end there without the user's receiving the full benefit of a complete network analysis.

It was to avoid this outcome that, when faced with a rather large network for a research program dealing with forest residues, I wrote the computer program described on the following pages. The program, written in FORTRAN IV, is short and has few input requirements. There are no restrictions on the number of activities leading to or from an event, and the only limitation in the present version is that the network can have no more than 500 activities. This could be easily expanded by changing only two statements in the program. I hope the program is simple enough to be used for even the very large and complicated networks.

Unlike most canned programs that may be available for PERT analyses, the program described in this report requires no lengthy set of control cards or operating codes. And, since a source deck is employed, the user can easily switch computer centers with no time lost in learning a new system. Having a source deck also makes it easy for the user to change parts of the program, such as the format for input or output, if he wants to.

Although certain aspects of PERT are explained here, this is not intended to be a thorough treatment of the technique. Readers not familiar with PERT will probably want to consult another reference such as Evarts' introductory text (1964).

A PERT NETWORK FOR A NATIONAL FOREST TIMBER SALE

The easiest way to demonstrate how one would prepare the necessary information to use the computer program to analyze a PERT network is to follow through an example. So with apologies to the National Forest System for making their job appear much easier than it really is, let's consider the events and activities leading to the award of a timber sale contract 15 months (60 weeks) from now (fig. 2). Although the example is oversimplified and excludes sale administration and postharvest activities, it should suffice for illustrating the use of the program.

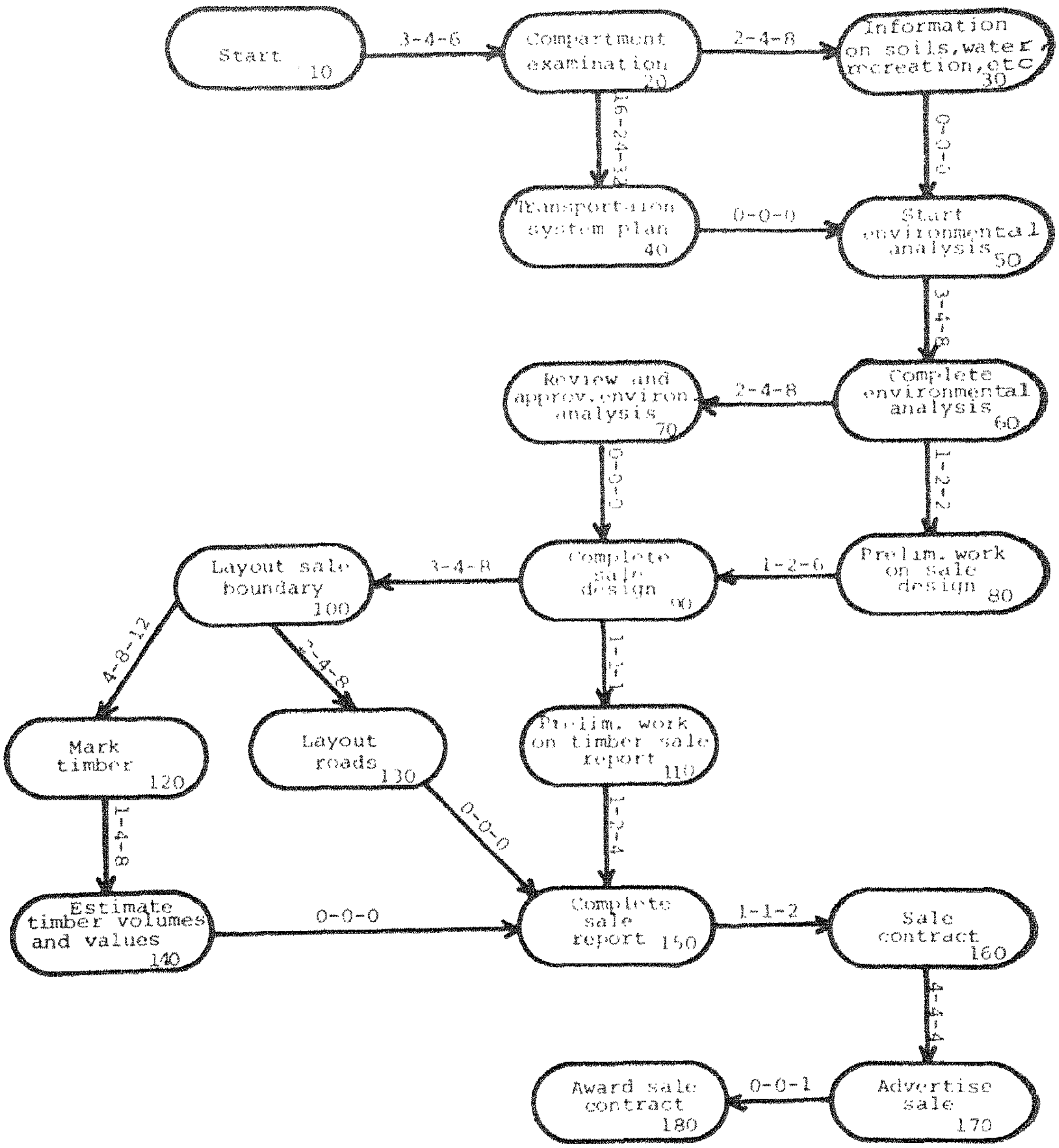
Events, unless otherwise noted, signify the completion of a particular part of the overall project. Activities are things that must be accomplished in order for the events to occur.

The numbers in each event are simply sequential code numbers to permit easy identification of and references to a particular event. Numbers along each activity line are time estimates (in weeks) for completing that activity; the first number being the most optimistic, the second the most likely, and the third the pessimistic. Activities are identified by the numbers of both their successor and predecessor events.

So now if we take the activity times, plus the scheduled completion time of 60 weeks, we have all the information needed to analyze the network and determine the:

- Expected time to complete each event
- Earliest expected time to complete each event

Figure 2.—A simplified PERT network for awarding a National Forest Timber Sale Contract.



- Latest allowable time to complete each event if the timber sale contract is to be awarded at the scheduled time
- Slack times for each activity
- Critical path—that path through the network that will require the most time
- Probability of having all of the events completed so that the sale contract can be awarded within the time allotted.

PROGRAM INPUTS

In general

Card Type 1.—A title, composed of any alphanumeric characters, punched in columns 1 - 80.

Card Type 2.—The scheduled completion time for the project, if one has been determined, in columns 1 - 5 (format is F5.1); and the time units involved (days, weeks, months, etc.) punched in columns 8 - 13. If for some reason the project has no specified completion time, columns 1 - 5 should be left blank; the program will assign a scheduled completion time equal to the earliest expected time to complete the last event.

Card Type 3.—The activity cards. One card is required for each activity (each "connecting" line in the network), *starting with the last event* in the network. On each activity card, we must first identify the activity by using the event numbers that succeed and precede the activity:

Columns 1 - 5. (Format I5) - the successor event identification number (must be a numerical code).

Columns 11 - 15. (Format I5) - the predecessor event identification number (must be a numerical code).

Then the time estimates for the activity must be punched on the card as follows:

Columns 21 - 25. (Format F5.1) - TO - the optimistic time estimate.

Columns 31 - 35. (Format F5-1) - TM - the most likely time estimate.

Columns 41 - 45. (Format F5.1) - TP - the pessimistic time estimate.

Activity cards should be prepared sequentially in reverse order (starting with the last event). This means that the listing of successor event numbers should proceed from the last event in exact reverse numerical order until the first event in the network is reached. The predecessor event numbers will not normally be in exact reverse order; however, this is not required, even for those activities that have the same sucessor event.

In some cases, the user may have only one time estimate available for some or all of the activities. If this occurs, enter the single time estimate in the "most likely" field (cols. 31 - 35). If the entire network consists of single time estimates for each activity, it won't be possible to generate the probability of completion; however, the critical path will still be identified.

Card Type 4. - This is simply a blank card to be placed after the last activity card in the input deck. It is used to signal the end of the job.

For the example

Figure 3 shows how the input cards were prepared for the timber sale network shown in figure 2.

OUTPUT FROM THE TIMBER SALE PROBLEM AND WHAT IT MEANS

After the information from the timber sale network was properly entered on a coding sheet, the data cards were punched and processed with the source program that is presented in the Appendix. The output shown in figure 4 was obtained.

The first table displays information about each activity in the network; the first five columns are simply a duplication of the input data. The column headed "TE" is the expected time that each activity will take. It is a weighted average of the optimistic, most likely, and pessimistic time estimates, calculated with the formula:

$$TE = \frac{TO + 4TM + TP}{6}$$

The rationale for this and other formulas can be found in various texts (for example, Evarts 1964).

The remaining columns in this table are defined by Evarts (1964) as follows:

EET = earliest expected time. This is the time when an event can be expected to be completed. EET for any event is the total of all the expected times (TE) of all the activities that precede it:

Figure 3.—Input data for the timber sale contract example.

Line	Cols 1–10	Cols 11–15	Cols 16–25	Cols 26–35	Cols 36–45
1	EXAMPLE PROBLEM (NATIONAL FOREST TIMBER SALE)				
2	60 WEEKS				
3	180	170	0	0	1
4	170	160	4	4	4
5	160	150	1	1	2
6	150	140	0	0	0
7	150	130	0	0	0
8	150	110	1	2	4
9	140	120	1	4	8
10	130	100	2	4	8
11	120	100	4	8	12
12	110	90	1	1	1
13	100	90	3	4	8
14	90	80	1	2	6
15	90	70	0	0	0
16	80	60	1	2	2
17	70	60	2	4	8
18	60	50	3	4	8
19	50	40	0	0	0
20	50	30	0	0	0
21	40	20	16	24	32
22	30	20	2	4	8
23	20	10	3	4	6
24					

Figure 4.—Output from the timber sale contract example.

PERT NETWORK ANALYSIS

EXAMPLE PROBLEM (NATIONAL FOREST TIMBER SALE)

SUCCESSOR EVENT	PREDECESSOR EVENT	TO	TM	TP	TE	EET	LAT	SLACK
		(WEEKS)						
180	170	0.0	0.0	1.0	0.2	59.1	60.0	0.9
170	160	4.0	4.0	4.0	4.0	58.9	59.8	0.9
160	150	1.0	1.0	2.0	1.2	54.9	55.8	0.9
150	110	1.0	2.0	4.0	2.2	40.2	54.6	14.4
150	130	0.0	0.0	0.0	0.0	45.8	54.6	8.8
150	140	0.0	0.0	0.0	0.0	53.7	54.6	0.9
140	120	1.0	4.0	8.0	4.2	53.7	54.6	0.9
130	100	2.0	4.0	8.0	4.3	45.8	54.6	8.8
120	100	4.0	8.0	12.0	8.0	49.5	50.4	0.9
110	90	1.0	1.0	1.0	1.0	38.0	52.4	14.4
100	90	3.0	4.0	8.0	4.5	41.5	42.4	0.9
90	70	0.0	0.0	0.0	0.0	37.0	37.9	0.9
90	80	1.0	2.0	6.0	2.5	37.0	37.9	0.9
80	60	1.0	2.0	2.0	1.8	34.5	35.4	0.9
70	60	2.0	4.0	8.0	4.3	37.0	37.9	0.9
60	50	3.0	4.0	8.0	4.5	32.7	33.6	0.9
50	30	0.0	0.0	0.0	0.0	8.5	29.1	20.6
50	40	0.0	0.0	0.0	0.0	28.2	29.1	0.9
40	20	16.0	24.0	32.0	24.0	28.2	29.1	0.9
30	20	2.0	4.0	8.0	4.3	8.5	29.1	20.6
20	10	3.0	4.0	6.0	4.2	4.2	5.1	0.9

PERT NETWORK ANALYSIS

EXAMPLE PROBLEM (NATIONAL FOREST TIMBER SALE)

CRITICAL PATH ACTIVITIES

SUCCESSOR EVENT	PREDECESSOR EVENT	TO	TP	DS
180	170	0.0	1.0	1.0
170	160	4.0	4.0	0.0
160	150	1.0	2.0	1.0
150	140	0.0	0.0	0.0
140	120	1.0	8.0	49.0
120	100	4.0	12.0	64.0
100	90	3.0	8.0	25.0
90	70	0.0	0.0	0.0
90	80	1.0	6.0	25.0
80	60	1.0	2.0	1.0
70	60	2.0	8.0	36.0
60	50	3.0	8.0	25.0
50	40	0.0	0.0	0.0
40	20	16.0	32.0	256.0
20	10	3.0	6.0	9.0

STANDARD NORMAL DEVIATE = 0.243

EET (successor) = EET (predecessor) + TE (activity)
For those events that have more than one activity arrow leading to them, more than one EET is calculated. When this happens, the program selects the largest one and uses it in succeeding calculations.

LAT = latest allowable time. This is the time by which an event must be achieved if the project is to be completed on schedule. LAT for any event is calculated by subtracting from the scheduled length of the projects, the length of the longest path backward from the end of the network to the event in question:
LAT (predecessor) = LAT (successor) - TE (activity) For events that have two or more succeeding activities, more than one LAT is calculated. When this happens, the program selects and uses the smallest one.

SLACK = The latest allowable time minus the earliest expected time. The slack of a particular path is the difference between the time scheduled for the entire project and that needed for the path. All events that lie on the same path in the network have identical slack times. The *critical path*, starting with the first event and ending with the last, consists of those activities having the identical slack figure which has the smallest algebraic value.

The second table displays information about those activities that make up the critical path. DS is simply the difference between TP and TO, squared. The standard normal deviate is calculated with the following formula:

$$SND = \frac{SLACK^*}{\sqrt{\Sigma DS/36}}$$

Where: SND = the standard normal deviate
SLACK* = the slack time for a single activity on the critical path

By using the computed standard normal deviate in conjunction with a table showing the cumulative standard normal distribution, it is possible to estimate the probability of completing a project on schedule. This approach compares that part of the area under the normal curve . . . represents the time allowed by the schedule to the total area under the curve. For our example problem, the standard normal deviate of 0.243 indicates that the probability of awarding the timber sale contract by the scheduled time is 0.60.

Often, after the first analysis of a PERT network, management may feel that the probability of completing the project on time is too low. Typically, the first response would be to reexamine activities, since the scheduled completion time will normally be fixed. PERT, by identifying the critical path, directs attention where it will do the most good. If it is possible to shorten the time for some activities by adding or reassigning personnel, working overtime, or by some other means, it will be most beneficial to do so on the critical path.

DISCUSSION

PERT is a proven aid to the complex planning process; yet it does require many repetitious calculations for the larger networks. However, the computer program that has been described makes short work of this tiresome aspect.

To demonstrate how easy it is to use the program, a rather large network (not shown) consisting of 100 events and 150 activities was analyzed. Transfer of information from network to coding forms required 45 minutes and keypunching (including verification) required another 25 minutes. The problem was processed (compiled and executed) in 1.45 seconds on an IBM 370 computer at a cost of $0.44. Thus a problem that would cause us to wave a white flag at the thought of hand calculations was efficiently solved in slightly over 1 hour at a very low cost.

The PERT network and its analysis should be thought of as a dynamic planning aid, and not a static point in the planning process. One of the advantages in having a computer program handy is that if we should need to reexamine a network in light of some new information, we can do so quite readily. Actually this is frequently done (or should be) with PERT networks because very often when networks are drawn, someone on the staff may spot oversights or decide too many events have been used, or that initial time estimates are inaccurate, or that activity arrows are improperly placed. Also, after the passage of time, it is usually advisable to update the network and analyze it again—to see what effects completed activities have on the overall project. With the computer program, any number of changes or proposed amendments can be evaluated in a very short time—usually by changing only a few cards in the data deck.

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APPENDIX

Source Listing of the Computer Program for PERT Analyses

```
      DOUBLE PRECISION TE,TTE,GTTE,TL,STL,SLACK,SMIN,SCT
      DIMENSION IS(500),IP(500),TE(500),TTE(500),GTTE(500),T(
     1STL(500),SLACK(500),TITLE(20),TO(500),TM(500),TP(500),(
      DATA TE,TTE,GTTE,TL,STL,SLACK/3000*0.0D0/,N/0/,SDS/0.0/
      READ(5,500) (TITLE(J),J=1,20),SCT,(UNITS(J),J=1,2)
  500 FORMAT(20A4/F5.1,2X,2A3)
      DO 10 I=1,500
      READ(5,510) IS(I),IP(I),TO(I),TM(I),TP(I)
  510 FORMAT(I5,5X,I5,3(5X,F5.1))
      IF(IS(I).EQ.0) GO TO 20
      N=N+1
      IF(TO(I).EQ.0.0) TO(I)=TM(I)
      IF(TP(I).EQ.0.0) TP(I)=TM(I)
      TE(I)=(TO(I)+(4.0*TM(I))+TP(I))/6.0
      TE(I)=TE(I)*10.0
      IR=TE(I)
      REM=TE(I)-(FLOAT(IR))
      IF(REM.GE.0.5) IR=IR+1
   10 TE(I)=(FLOAT(IR))/10.0
   20 I=N
   30 K=IS(I)
      L=IP(I)
      TTE(I)=TE(I)+GTTE(L)
      IF(IS(I).EQ.IS(I+1)) GO TO 40
      GTTE(K)=TTE(I)
      GO TO 50
   40 IF(TTE(I).LT.GTTE(K)) GO TO 50
      GTTE(K)=TTE(I)
   50 I=I-1
      IF(I.GT.0) GO TO 30
      IF(SCT.GT.0.0) GO TO 60
      SCT=GTTE(K)
   60 DO 90 I=1,N
      K=IS(I)
      L=IP(I)
      IF(I.GT.1) GO TO 70
      TL(K)=SCT
   70 TL(L)=TL(K)-TE(I)
      IF(STL(L).EQ.0.0) GO TO 80
      IF(TL(L).LT.STL(L)) GO TO 80
      TL(L)=STL(L)
      GO TO 90
   80 STL(L)=TL(L)
   90 SLACK(I)=TL(K)-TTE(I)
      WRITE(6,600) (TITLE(J),J=1,20),(UNITS(J),J=1,2)
  600 FORMAT(1H1,36X,21HPERT NETWORK ANALYSIS//12X,20A4///1X,
     1R PREDECESSOR/2X,5HEVENT,7X,5HEVENT,6X,2HTO,9X,2HTM,9X,
     2E,9X,3HEET,8X,3HLAT,7X,5HSLACK//25X,31(1H-),1H(,2A3,1H)
```

```
      DO 100 I=1,N
      K=IS(I)
100   WRITE(6,610) IS(I),IP(I),TO(I),TM(I),TP(I),TE(I),TTE(I),TL(K),
     1SLACK(I)
610   FORMAT(1H ,1X,I5,7X,I5,4X,7(F6.1,5X))
      SMIN=SLACK(1)
      DO 110 I=2,N
      IF(SLACK(I).LT.SMIN) SMIN=SLACK(I)
110   CONTINUE
      WRITE(6,620) (TITLE(J),J=1,20)
620   FORMAT(1H1,36X,21HPERT NETWORK ANALYSIS//12X,20A4///1X,24HCRITICAL
     1 PATH ACTIVITIES//1X,21HSUCCESSOR PREDECESSOR/2X,5HEVENT,8X,5HEVEN
     2T,6X,2HTO,9X,2HTP,9X,2HDS///)
      DO 120 I=1,N
      IF(SLACK(I).NE.SMIN) GO TO 120
      DS=(TP(I)-TO(I))**2
      SDS=SDS+DS
      WRITE(6,630) IS(I),IP(I),TO(I),TP(I),DS
120   CONTINUE
630   FORMAT(1H0,1X,I5,7X,I5,4X,3(F6.1,5X))
      IF(SDS.EQ.0.0) GO TO 130
      K=IS(1)
      S=(SCT-GTTE(K))/(SQRT(SDS/36.0))
      WRITE(6,640) S
640   FORMAT(1H0,26HSTANDARD NORMAL DEVIATE = ,F7.3)
      GO TO 140
130   WRITE(6,650)
650   FORMAT(1H0,36HSTANDARD NORMAL DEVIATE IS UNDEFINED)
140   WRITE(6,660)
660   FORMAT(1H1)
      STOP
      END
```

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