

# MANAGING MIXED SPECIES HARDWOOD STANDS FOR MAXIMUM FINANCIAL RETURNS

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ABSTRACT – Mixed yellow-poplar-oak stands were simulated on a computer using forward recursive dynamic programming to find financially optimal stand density, composition, and rotation length. Percent composition of oak was varied, and the marginal value of each thinning was determined as well as the marginal value of different types of thinning strategies. Improvement thinning and thinning from above were found to be the most valuable strategies, with the first thinning increasing soil expectation value by 11 to 19%, with subsequent thinnings improving SEV by less than 5%.

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## Introduction

Yellow-poplar-oak stands are among the most productive and valuable in the central and southern Appalachians and plateau regions of the eastern United States. This coverteype is dominated by fast-growing yellow poplar (*Liriodendron tulipifera*), and slower growing, but more valuable (in terms of per unit volume) white oak (*Quercus alba*) and northern red oak (*Quercus rubra*). Financial optimization of this forest type involves questions not only of rotation length and total stand density, but management of the composition of the forest over the rotation. General characteristics and management of the yellow-poplar-oak coverteype were presented by Beck and Della-Bianca (1981). The approach used in this research is based on computer simulation using dynamic programming to find optimal financial regimes.

## Methods

Forward-recursive dynamic programming (DP) has been widely applied to solving optimal stand density and rotation length questions (Amidon and Akin 1968, Kao and Brodie 1980, Arthaud and Klemperer 1988, Pelkki 1998). The basic formulation of dynamic programming permits an efficient search of all stand states as defined in a network. Forward recursive dynamic programming starts at some set of initial conditions and moves forward (in time) towards final harvest.

Mathematically, dynamic programming starts with an objective function (1) which sums

$$f_N(Y_N) = \sum_{i=0}^N r_i(T_i) \quad (1)$$

where:  $Y_N$  = state vector describing stand state at age N

$r_i()$  = transformation function that returns value of management  
action T taken at time i

the present value of all harvesting actions from the initial condition to the final harvest.

To move the stand between stages (the intervals at which management activities can take place) a transformation function (2), “grows” the stand from one stage to the next.

$$Y_i + G_i(Y) = X_{i+1} \text{ (for } i = 0, 1, 2, 3, \dots, N-1) \quad (2)$$

where  $G_i()$  = the growth transformation function or vector that grows  
a state in stage n to a state in stage n+1

$X_i$  = a state that has been grown but has not had any management action ( $T_i$ ) upon it.

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Table 1.—Stand tables with number of trees per acre by 1-inch diameter class by species for low-oak, medium-oak, and high-oak composition forests at age 20 years.

Diameter Class	Number of trees per acre at age 20 by initial oak composition class								
	Low oak			Medium oak			High oak		
	White oak	Red oak	Yellow-poplar	White oak	Red poplar	Yellow-oak	White oak	Red oak	Yellow-poplar
3	32	32	110	63	64	73	94	96	37
4	21	32	134	42	64	89	63	96	45
5	9	19	94	19	38	62	57	57	31
6	4	8	52	8	16	34	24	24	17
7	1	4	24	2	7	16	11	11	8
8	1	11		2	7	3	3	4	
9		4			2			1	
10		2			1			1	
11		1			1				

Connecting the states in a stage before management to their “after management” condition is equation 3, which applies the silvicultural activities ( $T_i$ ) to the stand. Ending conditions are specified by equations 4 and 5 and simply states that the final management action is a clearfelling (even-aged management).

$$X_i - T_i = Y_i \quad (3)$$

$$X_N - T_N = 0 \quad (4)$$

$$Y_N = 0 \quad (5)$$

The dynamic programming network is a sequential decision making process that searches a network of all admissible defined stand states, stage by stage until an ending condition is reached. Linking the stages together is a recursive equation (6) that

$$f_N(Y_N) = \max_{(Y_{i-1}, T_i)} [r_i(X_i, T_i) + f_{i-1}(Y_{i-1})] \quad (6)$$

is based on the principle of optimality (Dykstra 1984), which states that “given the current state of the system, an optimal policy for the remaining stages is independent of any policy adopted in previous stages.” In other words, states are defined precisely enough so that the previous actions taken to arrive at a particular state do not affect future management decisions.

The practical application of dynamic programming requires that state neighborhoods (Kao and Brodie 1984, Arthaud and Klemperer 1988, Pelkki 1997) be used to change continuous state variables (number of trees per acre and net cubic foot volume per acre) to discrete state intervals. In this study, state neighborhoods were defined by basal area (ft.<sup>2</sup>/acre), net volume (ft.<sup>3</sup>/acre), and percent oak composition, determined by all oak basal area divided by total stand basal area. Intervals for the state variables were:  $\pm 20$  ft.<sup>2</sup> basal area per acre,  $\pm 20$  ft.<sup>3</sup> volume per acre, and  $\pm 2.5\%$  oak composition. The growth interval between stages was set at 5 years.

The objective function optimizes net present worth ( $r_i()$  – NPW function), and the stopping condition was found by maximizing soil expectation value (the present value of an infinite number of rotations). Thus, once SEV declined, the DP algorithm stopped.

The growth transformation function was provided by the northeast variant of TWIGS, an individual-tree growth model (Miner et. al. 1988, Yaussy and Gale 1992, Yaussy 1993). Initial, 20-year old stand diameter distributions for site index 80 (base age = 50, base species = red oak) were derived from published data (Schnur 1937, Beck and Della-Bianca 1981) and are presented in table 1. Tree grade distributions were derived from the Forest Service permanent plot data from the Eastwide database and

**Table 2.—Percent of stems by potential tree grade.**

Species	G1	G2	G3	BG
Yellow-poplar	30	22	26	22
Red oak	22	31	26	21
White oak	21	26	22	31

**Table 3.—Definition of harvest actions as executed by NESTER DP software.**

Harvest Action	Abbreviation	Definition
Do nothing (no action)	N	No removal of stems beyond normal mortality from growth model.
Clearfelling	CC	All stems > 1" DBH are removed from stand. Merchantable stems (> 5") have positive value; pre-merchantable stems are removed with cost.
Thinning from above	TA	Stems in the stand list are removed from largest to smallest diameter until target basal area to be removed is reached.
Thinning from below	TB	Stems in the stand list are removed from smallest to largest diameter until target basal area to be removed is reached.
Thinning from above and below	TAB	Target basal area to be removed is determined. 50% of this target is removed in a thinning from below, 50% is removed in a thinning from above.
Improvement thinning	TI	Stems are removed until the target basal area is reached. First, below-grade trees are removed from largest to smallest diameter. Then trees are removed from smallest to largest diameter in grade three and grade two quality classes. Then non-grade trees are removed in ascending diameter. Finally, grade 1 trees are removed in ascending diameter.
High-grading	HG	Stems are removed until the target basal area is reached. Sawtimber trees are removed in descending grade (1 to 3), and harvested by descending diameter within each grade. Once all grade sawtimber is removed, the remaining trees are harvested by descending diameter.

are presented in table 2. Applying the TWIGS program to these initial conditions resulted in growth and yield projections that were consistent with other data (McGee and Della-Bianca 1967, Beck and Della-Bianca 1972 and 1975, Schlaegel et al 1969, Knoebel et. al. 1986) and suitable for internal comparisons.

Management actions ( $T_i$ ) possible in the DP simulation included thinning from above (TA), thinning from below (TB), thinning from above and below (TaB), improvement thinning (TI), high-grading (HG), do nothing (N) and clearfelling (CC). Table 3 provides a brief definition of how each harvest treatment was modeled by the DP software. Thinning was permitted in 10% increments, from 10% to 50% of the stand's initial basal area. In each thinning operation, oaks could be strongly favored (all removals first in non-oak), slightly favored (two-thirds of removals in non-oaks), or not favored (thinning done regardless of species). Timber stumpage prices were obtained from Timber Mart-South and Kentucky State Timber price reports and are presented in table 4. A real price increase of 1% per year for sawtimber and 0.5% per year for pulpwood was assumed, and a real 5% discount rate was used to convert future returns to present dollars. A fixed cost of \$80 per acre was included for all intermediate and final harvests, and penalty was charged in the removing of pre-merchantable (dbh < 5 in.) stems. Annual management costs (taxes, interest charges, administration costs) were assumed to be equal to annual revenues (hunting and recreation leases). For a detailed description of the management parameters, see (Kirillova 2001).

**Table 4.—Initial stumpage prices by species group and product class.**

Species	G1 (\$ / MBF)	G2 (\$ / MBF)	G3 (\$ / MBF)	BG (\$ / MBF)	Pulp (\$/Mcf)
Yellow-poplar	160	110	60	30	240
Red oak	300	215	120	60	110
White oak	250	180	100	50	110

**Table 5.—Optimal management regimes for high-, medium-, and low-oak composition stands under dynamic programming simulations with unrestricted thinnings.**

Oak Composition	SEV (\$/ac)	Rotation Length	Year	Thinning regime Strategy	Oak Preference	Percent BA removed
High	\$ 207.7	70	30	TI	High	50
			40	TA	None	35
			45	TI	High	35
			55	TA	None	40
Medium	199.7	75	30	TI	High	50
			45	TA	None	25
			55	TI	High	45
			60	TA	None	30
Low	192.8	90	30	TI	None	20
			45	TA	None	45
			55	TI	None	30
			60	TA	None	30
			70	TI	Moderate	45
			80	TA	None	45

DP simulations were run to determine the value of thinning strategies, the number of allowed thinnings and the effects of thinning on the production of high-quality oak stems in stands with varying levels of oak composition.

## Results

Without any intermediate harvests, stands with low-oak composition had rotation ages of 45 years and a soil expectation value (SEV) of \$158.8 per acre. Medium-oak composition stands had a rotation length of 45 years and an SEV of \$158.3 per acre. The high-oak composition stand had a rotation age of 40 years and an SEV of \$172.2 per acre under the “no thinning” options.

### Optimal rotation schemes with thinning

Simulations were run without restricting either number of thinnings or the thinning strategy employed. Table 5 provides the optimal rotation value and harvest schedule for stands with high-, medium-, and low-oak compositions. Greater oak composition, results in greater SEV and shorter rotations. The general pattern of the thinning regimes is two or three sequences of improvement thinnings (TI) followed by thinnings from above (TA) without any special preference to retaining oak in the stand. In only one instance, low-oak composition and the third improvement thinning, was there slight preference given to retaining oak

### Value of thinning strategies

Five DP simulations were run under each of the tree levels of oak composition allowing only one thinning strategy but not restricting the number or intensity of the thinnings. Table 6 provides the SEV and rotation age for each oak composition level allowing one particular strategy.

**Table 6.—Value of individual thinning strategies on high-, medium, and low-oak composition stands.**

Oak Composition	Thinning strategy	SEV	Rotation Age
High	TI	201.2	50
	TA	181.7	90
	HG	176.0	55
	No thinning	172.2	40
Medium	TI	188.4	65
	TA	184.3	85
	HG	175.2	75
	TaB	161.6	65
	No thinning	158.3	45
Low	TA	178.6	95
	TI	175.6	55
	HG	170.6	90
	TaB	162.9	60
	No thinning	158.8	45

**Table 7.—Effect of number of allowed thinnings of any type on high-, medium-, and low-oak composition stands.**

Oak Composition	Number of thinnings	Thinning strategies used	SEV (\$/ac)	Rotation Length
High	0	—	172.2	40
	1	TI	201.2	50
	2	TI	201.2	50
	3	TI	201.2	50
	4	TI, TA	207.7	70
Medium	0	—	158.3	45
	1	TI	184.3	50
	2	TI	188.3	65
	3	TI, TA	192.8	65
	4	TI, TA	199.7	75
Low	0	—	158.8	45
	1	TI	175.6	55
	2	TI, TA	179.9	75
	3	TI, TA	188.9	75
	4	TI, TA	191.1	80
	5	TI, TA	191.9	85
	6	TI, TA	192.8	90

For high- and medium-oak composition stands, improvement thinning is the single most valuable strategy, increasing SEV by 17% and 19%, respectively, while increasing rotation length by 10 and 20 years, respectively. The most valuable single thinning strategy for stands with low-oak composition was thinning from above, which increased SEV by 12% and rotation length by 50 years.

The second most valuable thinning strategy for high- and medium-oak stands is thinning from above, increasing SEV by 6% and 16%, respectively and rotation length by 50 and 40 years, respectively. For low-oak composition stands, the second most valuable thinning strategy is improvement thinning, which increased SEV by 11% and rotation length by 10 years.

After improvement thinning and thinning from above, high grading (HG) improves SEV in stands with all levels of oak composition. Increases in SEV for high-, medium- and low-oak stands are 2%, 10%, and 7%, respectively, while rotation length increases by 15, 30, and 45 years, respectively.

Thinning from above and below (TaB) improves SEV in medium- and low-oak composition forests. SEV gains are low, 2% for medium-oak and 3% for low-oak stands, while increasing rotation length by 20 years and 15 years, respectively. Thinning from above and below did not provide financial gains in high-oak stands.

Finally, the strategy of thinning from below (TB) did not result in any financial improvements in any of the stands.

### **Value of number of thinnings**

Additional DP runs were made constraining the number of thinnings, but not restricting the thinning strategy employed. Table 7 provides the results of this sensitivity analysis.

For all levels of oak composition, the thinning strategy used when only one thinning was allowed was improvement thinning. Improvement thinnings increased SEV by 11% to 19%, while increasing rotation length by 5 to 10 years.

For high-oak composition forests, increasing the number of allowed thinnings to 2 or 3 has no effect on the optimal rotation scheme. Increasing the number of thinnings to four increases SEV by an additional 3% and rotation length by another 20 years, with improvement thinnings and thinnings from above employed in sequence as shown in table 5. Allowing more than 4 thinnings did not improve SEV for high-oak stands.

In medium-oak composition forests, a second improvement thinning increased SEV by an additional 2% over a single improvement thinning, and adds an additional 15 years to the rotation length (Table 6). Allowing three thinnings permits thinning from above to be employed, increasing SEV by another 2% without increasing rotation length. Finally, allowing four thinnings produces the management sequence for medium-oak shown in table 5, with a 4% increase in SEV and a ten year addition to the rotation length.

A second thinning in low-oak composition stands increases SEV by 2% while increasing rotation length by 20 years. Allowing 3, 4, 5, and finally 6 thinnings permits sequences of and improvement thinning followed by thinning from above, culminating with the low-oak rotation scheme shown in table 6.

### **Number of thinnings and output of quality timber**

The effect of a single improvement thinning in high- and medium-oak stands is demonstrated by figures 1 and 2. A single improvement thinning results in a dramatic increase in the production of sawtimber volume from grade 1 and grade 2 trees. In high-oak stands, a single improvement thinning causes the percent of total sawtimber volume produced from grade 1 trees to increase from 8% to 32% and the production of sawtimber from grade 1 and grade 2 trees increases from 50% to 90% of total sawtimber production. In medium-oak stands, a single improvement thinning increased percent volume production from grade 1 trees to increase from 10% to 27%, and volume production from grade 1 and grade 2 trees increased from 47% to 85%.

In low-oak stands, the effects of the first improvement thinning are less dramatic (Figure 3). Production of volume from grade 1 trees increases from 7% to 15%, and volume from grade 1 and grade 2 trees increases from 41% to 53%. The number of thinnings that maximizes volume production from grade 1 trees involves an improvement thinning followed by a thinning from above (23% of volume from grade 1 trees). Three thinnings (two improvement thinnings and one thinning from above) results in the maximum percent of volume production from grade 1 and grade 2 trees (80%) in low-oak stands.

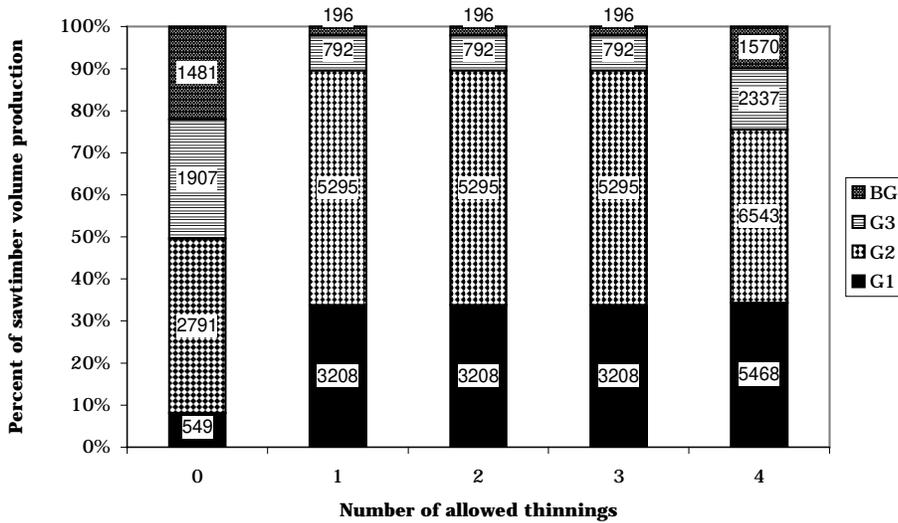


Figure 1.—Volume production for high-oak stands by number of allowed thinnings.

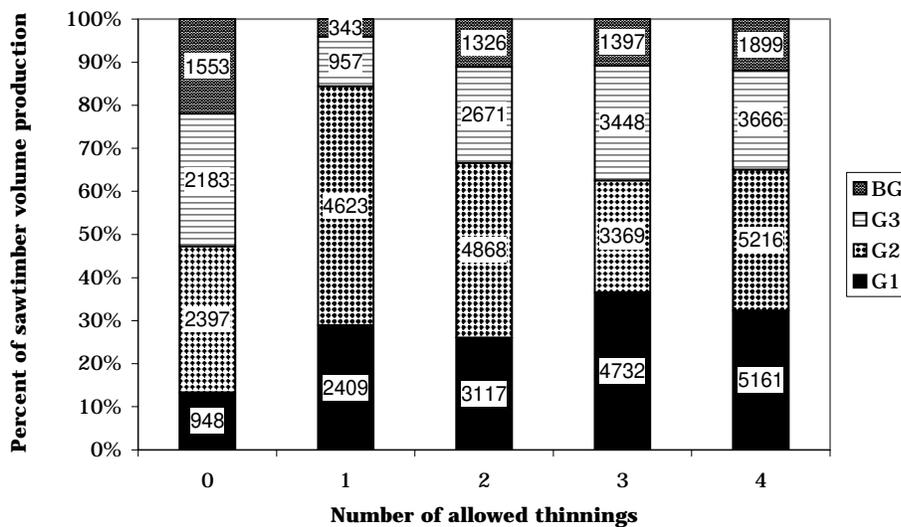


Figure 2. —Volume production for medium-oak stands by number of allowed thinnings.

### Discussion

Clearly, the simulation results point to the potential gains from improvement thinning. Improvement thinning removes the lowest quality trees first, favoring future growth on the most valuable stems. In stands where oak is a dominant component of the forest, the price premiums for grade 1 and grade 2 oak trees (table 4) provide economic incentives for focusing volume production heavily on high-quality oak stems. However, even in stands dominated by yellow-poplar, which has a lesser price premium for high-quality stems (table 4), improvement thinning is still the best choice if only one thinning is to be employed.

All thinnings increase rotation age, but improvement thinning typically adds 5 or 10 years to the rotation while thinning from above adds from 20 to 50 years to the rotation. This can be explained by the strategies themselves. Improvement thinning removes the poorest quality stems (below grade) from largest to smallest, and if additional stems must be removed, stems are removed smallest to largest within each grade class, starting with grade 3 and moving towards grade 1. Thus, improvement thinning is similar to thinning from below, but favoring high-value stems, and as such, it will not lengthen the time the dominant stems in a stand reach financial maturity. Thinning from above, however, removes the largest diameter stems, regardless of quality, in an effort to release intermediate stems and capture economic returns early in the rotation. The residual stems are smaller and require a longer rotation to reach financial maturity.

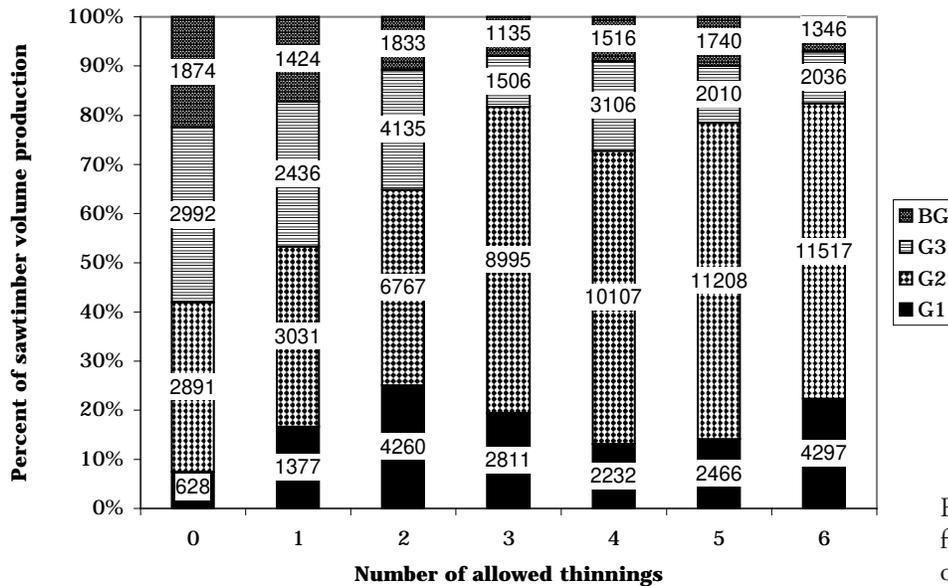


Figure 3.—Volume production for low-oak stands by number of allowed thinnings.

The advantages to this computer simulation approach lie in the ability to rapidly simulate a variety of management regimes quickly and conduct sensitivity (with and without) analyses. Weaknesses to this approach are that the input stands are hypothetical and presumed “typical” 20-year old yellow-poplar-oak stands. Also, the growth and yield data presented by this model should be applied with caution. While the yields were compared and found reasonable with other published data, the individual-tree models upon which they are based are not well suited for making long-term growth projections of more than 30-40 years. The outputs can be compared internally for judging the value of different actions. However, this is a caution that should be applied whenever modeling is used to make silvicultural recommendations.

When applying these results, it should be noted that the optimal simulations included 4-6 thinning entries, but the majority of economic gains were achieved with three or fewer thinnings. Considering that oak was retained in all simulations until the final regeneration felling, it possible, but by no means guaranteed, that successful natural regeneration of yellow-poplar-oak stand will ensue.

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