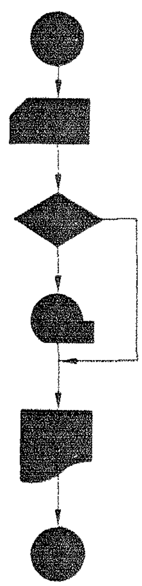
THE NORTHEASTERN FOREST-INVENTORY DATA-PROCESSING SYSTEM. X. INFORMATION FOR PROGRAMMERS SUBSYSTEM OUTPUT.



by
Robert C. Peters and
Robert W. Wilson Jr.

U. S. FOREST SERVICE RESEARCH PAPER NE-78
1967
NORTHEASTERN FOREST EXPERIMENT STATION, UPPER DARBY, PA.
FOREST SERVICE, U.S. DEPARTMENT OF AGRICULTURE
RICHARD D. LANE, DIRECTOR

About the Authors

ROBERT C. PETERS obtained his Bachelor's degree from the University of California in 1960 and his Master's at Yale University in 1961. He joined the Forest Service in 1961 as a research forester, and was assigned to the Station's biometrics unit from 1961 until 1965, when the unit was discontinued. Mr. Peters played a key role in the development of the data-processing system reported here.

ROBERT W. WILSON, JR. took his Bachelor's degree at The Pennsylvania State University in 1947 and his Master's and Ph.D. degrees at Yale University in 1948 and 1965, respectively. He joined the U. S. Forest Service in 1948 and has worked in various research capacities for the Northeastern Forest Experiment Station. From 1961 to 1965 he was in charge of the Station's biometrics unit at New Haven, Conn. He is assigned at present to the Forest Insect and Disease Laboratory at West Haven, Conn.

THE NORTHEASTERN FOREST-INVENTORY DATA-PROCESSING SYSTEM. X. INFORMATION FOR PROGRAMMERS SUBSYSTEM OUTPUT.

■

Contents

PREFACE

THIS paper is the tenth in a series of ten papers prepared to to describe the forest-inventory data-processing system of the Northeastern Forest Experiment Station. This system was devised for using modern, large-scale, high-speed computers in processing forest-inventory data. The series will comprise the following papers:

I. Introduction.
II. Description of subsystem EDIT.
III. Operation of subsystem EDIT.
IV. Information for programmers — subsystem EDIT.
V. Description of subsystem TABLE.
VI. Operation of subsystem TABLE.
VII. Information for programmers—subsystem TABLE.
VIII. Description of subsystem OUTPUT.
IX. Operation of subsystem OUTPUT.
X. Information for programmers — subsystem OUTPUT.

X-A. INTRODUCTION

ONE of the major projects of the U. S. Forest Service is a nationwide forest survey, which is designed to obtain useful and timely information about the timber resources of the United States. In the course of the surveys, which are made mainly on a state-by-state basis, great masses of detailed data are collected about timber volumes, growth, timber cut, and other characteristics of the timber resource.

In recent years the volume of information obtained from forest-survey field plots has increased greatly. The task of compiling and analyzing this mass of data with mechanical computing machines was both cumbersome and time-consuming.

A solution to this problem was seen in the development of the high-speed electronic computers. The Northeastern Forest Experiment Station, which was responsible for conducting the forest survey of the heavily forested Northeastern States, investigated the possibilities and devised the Northeastern Forest-Inventory Data-Processing System.

This paper gives information for programmers about part of this system. Program OUTPUT is designed for use in conjunction with program TABLE (see part V of this series). Its sole function is to produce and print fully labelled tables of statistics for sampled populations from tabular summaries of the samples that were previously produced in program TABLE.

A general description of the program and detailed instructions for its use in solving data-processing problems are given in parts VIII and IX. In the following chapters will be found selected programming information that will be useful if the programs must be modified for any reason. The program write-ups and information on the program source decks may be obtained from the Northeastern Forest Experiment Station, 6816 Market Street, Upper Darby, Pennsylvania 19082.

The program is written in the standard IBM FORTRAN IV language, and is operative at the Yale University Computer Center on an IBM 7094/7040 direct coupled system under the IBSYS DCS operating system with IBJOB processor.[1] It will operate with little or no modification on other comparable systems. The main requirements for a machine on which to operate the standard version of the program are a 32K word core, a minimum of 36 bits per word, binary arithmetic capability, and 5 tape drives or equivalent input/output devices.

X-B. MODIFICATION OF DIMENSIONED SPACE

The standard version of program OUTPUT carries restrictions on both the dimensions and the overall size of problem that can be handled in a single processing run. These restrictions are a result of the manner in which dimensioned space has been allocated (table 1) and the total space available in a given operating system. The program has been written so that all modifications of dimensioned space can be made in the subprogram called MAINO. No other parts of the program need be touched for this purpose. The use of dimensioned space and the means of changing dimensions are discussed in detail below.

[1] Mention of a particular product should not be construed as an endorsement by the Forest Service or the U. S. Department of Agriculture.

Table 1. — **Summary of dimensioned space restrictions, and associated program variables and arrays**

Item	*Restriction*	*Variable*	*Arrays*
Maximum number of input tables	40	NTBLE	NTAB, TOTAL, TOTVAR, LTABLE
Maximum number of cells in all output tables	15,000	NDIMEN	AIMP

Number of Input Tables

In the standard version of program OUTPUT, up to 40 input tables per data set may be contained in the input data. This is consistent with the number that may be specified in the control deck for program TABLE (see item 321). To change this maximum, the following steps (and only these) must be taken:

1. In the subprogram called MAINO, the variable named NTBLE must be set equal to the desired maximum value.
2. In the subprogram called MAINO, the DIMENSION statement must be changed so that the first dimension of the array NTAB equals the new value of NTBLE.
3. In the subprogram called MAINO, the DIMENSION statement must be changed so that the dimension of the arrays LTABLE, TOTAL, and TOTVAR equals the new value of NTBLE.

Number of Cells in All Output Tables

In the standard version of program OUTPUT, up to 15,000 locations are available for compiling output tables. To change this maximum, the following steps (and only these) must be taken:

1. In the subprogram called MAINO, the variable named NDIMEN must be set equal to the desired maximum value.
2. In the subprogram called MAINO, the DIMENSION statement must be changed so that the dimension of the array AIMP equals the new value of NDIMEN.

If the space required for compilation of the output tables specified in the output table selection card (item 412) exceeds the dimensioned space, a message will be printed during the reading of the control deck and processing will halt. The space required can be computed as follows:

$$K \left[\sum_{i=1}^{n} (r_i + 1) \times (c_i + 1) \right]$$

K = A multiplier, the value of which depends upon the sampling option (columns 19-21, item 411) and the sums option (columns 16-18, item 411) chosen; as follows:

Sampling Option	*Values of K*	
	Sums	*No sums*
1 or 2	3	2
3 or 4	6	4
5 or 6	8	6

n = The total number of output tables requested in the output table selection card (item 412).

r_i = The number of rows in the ith output table, as specified in columns 24-25 of an output table definition card (item 321 in part VI-B).

c_i = The number of columns in the ith output table, as specified in columns 27-28 of an output table definition card (item 321 in part VI-B).

X-C. PROGRAMMING FEATURES

The following items will be of interest to programmers who plan to modify the standard version of program OUTPUT for use on other computers or under other operating systems.

Tape Assignments

In the standard version of the program the FORTRAN logical tape assignments are as follows:

Unit	*Use*
3	Scratch tape.
5	Monitor input for program deck and control deck.
6	Monitor print for job summary.
12	Output printed tables of population statistics.
19	Input of sample summaries written in the binary mode.

These tape assignments can be changed to fit local conditions by loading appropriate file routines with the program. See your systems representative or the section entitled FORTRAN files in the IBM IBJOB processor manual, file number 7090-27.

Use of Sense Switches and Sense Lights

No sense switches are used in the program. All sense switches are set at normal monitor settings.

No sense lights are used.

Use of Program Halts

There are no halts in program OUTPUT.

Subprogram Names and Functions

The following tabulation lists all subprograms in program OUTPUT:

Subprogram Name	*Function*
MAINO	Main calling program.
RESULT	Major logic and calling sequence which reads and checks all control cards and sets up processing logic.
READ	Reads the binary input tape containing the sample summary tables.
ADDRES	Calculates all table addresses in array AIMP and stores them in array NTAB.
STO	Does initial weighting and summing of sample summary tables to population tables.
CALCUL	Calculates final output values for the tables of population statistics.
SETHED	Sets up the title and row and column labels for each output table, table by table.
OUTPUT	Prints the tables of population statistics with title and labels, table by table.
ERROR	Calculates sampling errors for each output table, table by table.

Important Arrays and Variables

The following are the principal arrays and variables used in program OUTPUT:

Array	*Dimension*	*Description*
AIMP	NDIMEN	Storage for all tables.
NTAB	LTBLE x 13	Indexing information for array AIMP, where LTBLE is the number of input tables for a sample and the second dimension locations are used as follows: 1 = The number of rows in the ith input table.

		2 = The number of columns in the ith input table.
		3 = The name of the ith input table.
		4 = The beginning location in AIMP of the means for the ith input table.
		5 = The beginning location in AIMP of the variances for the ith input table.
		6 = The beginning location in AIMP of the covariances for the ith input table.
		7 = The beginning location in AIMP of the temporary storage for the calculation of final means for the ith input table.
		8 = The beginning location in AIMP of the temporary storage for the calculation of final variances for the ith input table.
		9 = The beginning location in AIMP of the temporary storage for the calculation of final covariances for the ith input table.
		10 = The beginning location in AIMP of the population group sums for the ith input table.
		11 = The beginning location in AIMP of the population group variances for the ith input table.
		12 = The location in the AIMP temporary storage for calculation of final means, of the grand mean for the ith input table.
		13 = The location in the AIMP temporary storage for calculation of final variances, of the grand variance for the ith input table.
TOTAL	LTBLE	Storage for independent estimates of table grand means read from control deck, item 424 or 432.
TOTVAR	LTBLE	Storage for independent estimates of table grand variances read from control deck, item 424 or 432.
LTABLE	LTBLE	Storage for information read from the table selection card, item 412 in the control deck.
TITLE, T	12	Storage array for descriptive population title that is read in (item 421).
IDENT	6	Storage array for sample (data set) identification fields read from control deck (item 422).
IDPIO	6	Storage array for the sample (data set) identification fields read from the binary input tape.
CHEAD	51 x 4	Storage array for the column headings of an output table. Space is provided for a heading of four alphameric words (second dimension) for each of 51 columns, including row totals (first dimension).

RHEAD	51 x 4	Storage array for the row tags for an output table. Space is provided for a tag of four alphameric words (second dimension) for each of 51 rows, including column totals (first dimension).
TAB	12	Working storage.
ST	12	Working storage.
QT	12	Working storage.
VAR	2	Working storage.
RE	3	Working storage.
SST	3	Working storage.
NPSTEP	1	Total number of tables read from data input tape for a sample (data set).
PLOT IPLOT	1	Total number of sampling units used to make the data set tables read from data input tape.
AREA	1	Population expansion factor, read from control deck (item 423).
ALLPI	1	Sum of sample weights, read from control deck (item 423).
POINT	1	Sample weight, read from control deck (item 431).
ADJUST	1	Sample adjustment factor, read from control deck (item 431).
NUNIT	1	Number of samples to be processed for the population (item 423).

X-D. SUMMARY OF ESTIMATING PROCEDURES

In this chapter the six processing options available in program OUTPUT are presented in detail.

Vector notation is used to make the presentation of computing procedures more compact and easier to read. An input vector, $\overset{I}{Y}$, is a one-dimensional array representing a sampling unit attribute. An output vector, $\overset{O}{Y}$, represents an output table (in general, a two-dimensional array or matrix) summarizing the sampling unit attribute. A final output vestor, $\overset{F}{Y}$, represents an estimate of the population attribute corresponding to the sampling unit attribute. Elements of these vectors are represented by $\overset{I}{y}_i, \overset{O}{y}_i$, or $\overset{F}{y}_i$.

It must not be inferred from what follows that the arithmetic is the arithmetic of vectors or matrices; although, in general, it is correct vector arithmetic as shown. What is implied is simply the sequential and independent application of the indicated operation to each pair of equivalent elements from the two vectors. In this sense, the procedures will generalize to the case of matrices; otherwise they will not.

Other notational conventions adopted here are the use of a bar over an attribute symbol ($\bar{Y}$) to symbolize the arithmetic mean of an attribute; and the use of a dot replacing a subscript ($y_{.k}$) to indicate the sum over all members of the set represented by the subscript.

OPTION 1.—Process as a 100-percent Sample of the Population

Compute: $\overset{F}{Y}_{..}$

Given: Sets of $\overset{O}{Y}_{j.}$; and ns

Where:

- j = Subscript for the jth set of sampling units
- k = Subscript for the kth sampling unit
- $\overset{F}{Y}_{..}$ = A final attribute vector (output table) for a survey unit, containing the sum over the entire population of sampling units of the sampling-unit attribute vectors, $\overset{O}{Y}_{jk}$
- $\overset{O}{Y}_{j.}$ = An attribute vector (output table) that contains the sum over a set of sampling units of the sampling-unit attribute vectors $\overset{O}{Y}_{jk}$, which represent a summary of a sampling-unit attribute input vector
- ns = The number of sets of sampling units

Procedure: None.

Output:

$$\overset{F}{Y}_{..} = \sum_{j=i}^{ns} \overset{O}{Y}_{j.}$$

OPTION 2.—Process as a Single Random Sample of the Population

Compute: $\overset{F}{Y}_{..}$, $V\overset{F}{Y}_{..}$, $ST\overset{F}{Y}_{..}$, and $SE\overset{F}{Y}_{..}$,

Given: $\overset{O}{Y}_{..}$, $V\overset{O}{Y}_{..}$, and wt

Where:

- j = Subscript for the jth sample stratum or set of sampling units; hence, not applicable in this case of a single set in the survey unit

k = Subscript for the kth sampling unit

$\overset{F}{Y}..$ = A final attribute vector (output table) for a survey unit, containing an estimate of the sum over the entire population of sampling units of the sampling-unit attribute vectors $\overset{O}{Y}_{jk}$

$V\overset{F}{Y}..$ = The variance of $\overset{F}{Y}..$

$ST\overset{F}{Y}..$ = The standard error of $\overset{F}{Y}..$

$SE\overset{F}{Y}..$ = The sampling error (in percent) of $\overset{F}{Y}..$

$\overset{O}{Y}..$ = An attribute vector (output table) that contains the arithmetic mean of the sampling-unit attribute vectors, $\overset{O}{Y}_{jk}$, which represent a summary of a sampling-unit attribute input vector

$V\overset{O}{Y}..$ = The variance of $\overset{O}{Y}..$

wt = The total number of sampling units in the survey-unit population

Output: $\overset{F}{Y}.. = \overset{O}{Y}..\ wt$

$$V\overset{F}{Y}.. = V\overset{O}{Y}..\ wt^2$$

$$ST\overset{F}{Y}.. = \sqrt{V\overset{F}{Y}..}$$

$$SE\overset{F}{L}.. = \frac{ST\overset{F}{Y}..}{\overset{F}{Y}..}\ 100$$

OPTION 3.—Process as Random Samples from Known Populations Strata

Compute: $\overset{F}{Y}..$, $V\overset{F}{Y}..$, $ST\overset{F}{Y}..$, and $SE\overset{F}{Y}..$

Given: Sets of $\overset{O}{Y}_{j}.$, $V\overset{O}{Y}_{j}.$, n_j ; and ns, wt

Where:

j = Subscript for the jth sample stratum or set of sampling units

k = Subscript for the kth sampling unit

$\overset{F}{Y}..$ = A final attribute vector (output table) for a survey unit, containing an estimate of the sum over the entire population of sampling units of the sampling-unit attribute vectors, $\overset{O}{Y}_{jk}$

$\overset{F}{VY}_{..}$ = The variance of $\overset{F}{Y}_{..}$

$\overset{F}{STY}_{..}$ = The standard error of $\overset{F}{Y}_{..}$

$\overset{F}{SEY}_{..}$ = The sampling error (in percent) of $\overset{F}{Y}_{..}$

$\overset{O}{Y}_{j.}$ = An attribute vector (output table) that contains the arithmetic mean for a stratum of the sampling-unit attribute vectors, $\overset{O}{Y}_{jk}$, which represent a summary of a sampling-unit attribute input vector

$\overset{O}{VY}_{j.}$ = The variance of $\overset{O}{Y}_{j.}$

n_j = The known size of stratum or weight to be applied to the stratum

ns = The number of sample strata in a survey unit

wt = Total number of sampling units in the survey-unit population

Procedure:

$$\overset{F}{Y}_{..} = \frac{1}{n.} \sum_{j=1}^{ns} n_j \; \overset{O}{Y}_{j.}$$

$$\overset{F}{VY}_{..} = \frac{1}{n.^2} \sum_{j=1}^{ns} n_j^2 \; \overset{O}{VY}_{j.}$$

Output:

$$\overset{F}{Y}_{..} = \overset{F}{Y}_{..} \; wt$$

$$\overset{F}{VY}_{..} = \overset{F}{VY}_{..} \; wt^2$$

$$\overset{F}{STY}_{..} = \sqrt{\overset{F}{VY}_{..}}$$

$$\overset{F}{SEY}_{..} = \frac{\overset{F}{STY}_{..}}{\overset{F}{Y}_{..}} \; 100$$

OPTION 4.—Process as Random Samples from Population Strata Estimated from a Primary Random Sample

Compute: $\overset{F}{Y}_{..}$, $\overset{F}{VY}_{..}$, $\overset{F}{STY}_{..}$, and $\overset{F}{SEY}_{..}$

Given: Sets of $\overset{O}{Y}_{j.}$, $\overset{O}{VY}_{j.}$, n_j, and ns, wt

Where:

j = Subscript for the jth sample stratum or set of sampling units

k	=	Subscript for the kth sampling unit
$\overset{F}{Y}$	=	A final attribute vector (output table) for a survey unit, containing an estimate of the sum over the entire population of sampling units of the sampling attribute vectors, $\overset{O}{Y}_{jk}$
$V\overset{F}{Y}..$	=	The variance of $\overset{F}{Y}..$
$ST\overset{F}{Y}..$	=	The standard error of $\overset{F}{Y}..$
$SE\overset{F}{Y}..$	=	The sampling error (in percent) of $\overset{F}{Y}..$
$\overset{O}{\bar{Y}}_{j.}$	=	An attribute vector (output table) that contains the arithmetic mean for a stratum of the sampling-unit attribute vectors, $\overset{O}{Y}_{jk}$, which represent a summary of a sampling-unit attribute input vector
$V\overset{O}{\bar{Y}}_{j.}$	=	The variance of $\overset{O}{\bar{Y}}_{j.}$
n_j	=	The number of sampling units in the first (photo) sample of a sample stratum
ns	=	Number of sample strata in a survey unit
wt	=	Total number of sampling units in a survey unit population

Procedure:

$$\overset{F}{\bar{Y}}.. = \frac{1}{n.} \sum_{j=1}^{ns} n_j \overset{O}{\bar{Y}}_{j.}$$

$$V\overset{F}{\bar{Y}}.. = \left[\frac{1}{n.^2 - n.} \sum_{j=1}^{ns} \left[(n_j^2 - n_j)\, V\overset{O}{\bar{Y}}_{j.} + n_j \overset{O^2}{\bar{Y}}_{j.}\right]\right] - \left[\frac{1.0}{n. - 1} \overset{F}{\bar{Y}}{}^2..\right]$$

Output:

$$\overset{F}{Y}.. = \overset{F}{\bar{Y}}..\ wt$$

$$V\overset{F}{Y}.. = V\overset{F}{\bar{Y}}..\ wt^2$$

$$ST\overset{F}{Y} = \sqrt{V\overset{F}{Y}..}$$

$$SE\overset{F}{Y}.. = \frac{ST\overset{F}{Y}..}{\overset{F}{Y}..}\ 100$$

OPTION 5.—OPTION 4 Modified to Obtain Stratum Ratios for Application to Independent Estimates of Stratum Means and Variances

Compute: $\overset{F}{Y}..$, $V\overset{F}{Y}..$, $ST\overset{F}{Y}..$ and $SE\overset{F}{Y}..$

Given: Sets of $\overset{O}{Y}_{j.}$, $V\overset{O}{Y}_{j.}$, $C\overset{O}{V}_{j.}$, $\bar{t}_{.j.}$, $vt_{.j.}$, n_j; and ns, wt

Where:

i	=	Subscript for the ith element of a vector
j	=	Subscript for the jth sample stratum or set of sampling units
k	=	Subscript for the kth sampling unit
$\overset{F}{Y}..$	=	A final attribute vector (output table) for a survey unit, containing an estimate of the sum over the entire population of sampling units of the sampling-unit attribute vectors, $\overset{O}{Y}_{jk}$
$V\overset{F}{Y}..$	=	The variance of $\overset{F}{Y}..$
$ST\overset{F}{Y}..$	=	The standard error of $\overset{F}{Y}..$
$SE\overset{F}{Y}..$	=	The sampling error (in percent) of $\overset{F}{Y}..$
$\overset{O}{Y}_{j.}$	=	An attribute vector (output table) that contains the arithmetic mean for a stratum of the sampling-unit attribute vectors, $\overset{O}{Y}_{jk}$, which represent a summary of a sampling-unit attribute input vector
$V\overset{O}{Y}_{j.}$	=	The variance of $\overset{O}{Y}_{j.}$
$C\overset{O}{V}_{j.}$	=	The mean covariance for a stratum of sampling-unit attribute vectors $\overset{O}{Y}_{jk}$, and the sums (totals) of elements in these vectors $\overset{O}{y}_{.jk}$
$\bar{t}_{.j.}$	=	An independent estimate of $\overset{O}{y}_{.j.}$, the arithmetic mean over the entire population of sampling units in a sample stratum, of the sums of the elements of the sampling-unit attribute vectors $\overset{O}{Y}_{jk}$
$v\bar{t}_{.j.}$	=	The variance of $\bar{t}_{.j.}$
n_j	=	The number of sampling units in the first (photo) sample of a sample stratum
ns	=	Number of sample strata in a survey-unit population
wt	=	Total number of sampling units in a survey-unit population

Procedure:

$$R_{j.} = \frac{\overset{O}{Y}_{j.}}{\overset{O}{\bar{y}}_{.j.}} \; ; \quad \overset{F}{Y}_{j.} = R_{j.} \; \bar{t}_{.j.} ; \quad \overset{F}{Y}_{..} = \frac{1}{n.} \sum_{j=1}^{ns} n_j \overset{F}{Y}_{j.}$$

$$VR_{j.} = R^2_{j.} \left[\frac{\overset{O}{VY}_{j.}}{\overset{O}{Y}{}^2_{j.}} + \frac{\overset{O}{v\bar{y}}_{.j.}}{\overset{O}{\bar{y}}{}^2_{.j.}} - 2 \frac{\overset{O}{CV}_{j.}}{\overset{O}{Y}_{j.} \; \overset{O}{\bar{y}}_{.j.}} \right]$$

$$\overset{F}{VY}_{j.} = \overset{F}{Y}{}^2_{j.} \left[\frac{VR_{j.}}{R^2_{j.}} + \frac{v\bar{t}_{.j.}}{\bar{t}^2_{j}} \right]$$

$$\overset{F}{VY}_{..} = \left[\frac{1}{n^2. - n.} \sum_{j=1}^{ns} \left[(n^2_j - n_j) \; \overset{F}{VY}_{j.} + n_j \; \overset{F}{Y}{}^2_{j.} \right] \right] - \left[\frac{1.0}{n. - 1} \overset{F}{Y}{}^2_{..} \right]$$

Output:

$$\overset{F}{Y}_{..} = \overset{F}{Y}_{..} \; wt$$

$$\overset{F}{VY}_{..} = \overset{F}{VY}_{..} \; wt^2$$

$$\overset{F}{STY} = \sqrt{\overset{F}{VY}_{..}}$$

$$SEY_{..} = \frac{\overset{F}{STY}_{..}}{\overset{F}{Y}} \; 100$$

OPTION 6.—OPTION 4 Modified to Obtain Population Ratios for Application to Independent Estimates of Population and Variances

Compute: $\overset{F}{Y}_{..}$, $\overset{F}{VY}_{..}$, $\overset{F}{STY}_{..}$, and $\overset{F}{SEY}_{..}$

Given: Sets of $\overset{O}{Y}_{j.}$, $\overset{O}{VY}_{j}$, $\overset{O}{CV}_{j.}$, n_j; and $\bar{t}_{..}$, $v\bar{t}_{..}$, ns, wt

Where:

i	= Subscript for the ith element of a vector
j	= Subscript for the jth sample stratum or set of sampling units
k	= Subscript for the kth sampling unit
$\overset{F}{Y}_{..}$	= A final attribute vector (output table) for a survey unit, containing an estimate of the sum over the entire popula-

tion of sampling units of the sampling-unit attribute vectors, $\overset{O}{Y}_{jk}$

$\overset{F}{VY}..$ = The variance of $\overset{F}{Y}..$

$\overset{F}{STY}..$ = The standard error of $\overset{F}{Y}..$

$\overset{F}{SEY}..$ = The sampling error (in percent) of $\overset{F}{Y}..$

$\overset{O}{\bar{Y}}_{j.}$ = An attribute vector (output table) that contains the arithmetic mean for a stratum of the sampling-unit attribute vectors, $\overset{O}{Y}_{jk}$, which represent a summary of a sampling-unit attribute input vector

$\overset{O}{\overline{VY}}_{j.}$ = The variance of $\overset{O}{\bar{Y}}_{j.}$

$\overset{O}{\overline{CV}}_{j.}$ = The mean covariance for a stratum of sampling-unit attribute vectors, $\overset{O}{Y}_{jk}$, and the sums (totals) of elements in these vectors $\overset{O}{y}_{.jk}$

n_j = The number of sampling units in the first (photo) sample of a sample stratum

$\bar{t}...$ = An independent estimate of $\overset{F}{y}...$, the arithmetic mean over the entire population of sampling units, of the sums of the elements of the sampling-unit attribute vectors, $\overset{O}{Y}_{jk}$

$v\bar{t}...$ = The variance of $t...$

ns = Number of sample strata in a survey unit.

wt = Total number of sampling units in the survey-unit population

Procedure:

$$\overset{F}{\bar{Y}}.. = \frac{1}{n.} \sum_{j=1}^{ns} n_j \overset{O}{\bar{Y}}_{j.}; \qquad R. = \frac{\overset{F}{\bar{Y}}..}{\overset{F}{\bar{y}}...}$$

$$\overset{F}{\overline{VY}}.. = \left[\frac{1}{n.^2 - n.} \sum_{j=1}^{ns} [(n^2_j - n_j) \overset{O}{\overline{VY}}_j + n_j \overset{O}{\bar{Y}}{}^2_{j.}] \right] - \left[\frac{1.0}{n. - 1} \overset{F}{\bar{Y}}{}^2.. \right]$$

$$\overset{F}{\overline{CV}}_{..} = \left[\frac{1}{n_{.}^{2} - n_{.}} \sum_{j=1}^{ns} \left[(n_j^2 - n_j)\, \overset{o}{\overline{CV}}_{j.} + n_j\, \overset{o}{\overline{Y}}_{j.}\, \overset{o}{\overline{y}}_{.j.} \right] \right] -$$

$$\left[\frac{1.0}{n_{.} - 1}\, \overset{F}{\overline{Y}}_{..}\, \overset{F}{\overline{y}}_{...} \right]$$

$$VR_{.} = R^2_{.} \left[\frac{\overset{F}{\overline{VY}}_{..}}{\overset{F}{\overline{Y}}{}^{2}_{..}} + \frac{\overset{F}{\overline{vy}}_{...}}{\overset{F}{\overline{y}}{}^{2}_{...}} - 2\, \frac{\overset{F}{\overline{CV}}_{..}}{\overset{F}{\overline{Y}}_{..}\, \overset{F}{\overline{y}}_{...}} \right]$$

$$\overset{F}{\overline{Y}}_{..} = R_{.}\, \overline{t}_{...}$$

$$\overset{F}{\overline{VY}}_{..} = \overset{F}{\overline{Y}}{}^{2}_{..} \left[\frac{VR_{.}}{R^2_{.}} + \frac{v\overline{t}_{...}}{\overline{t}^{2}_{...}} \right]$$

Output:

$$\overset{F}{Y}_{..} = \overset{F}{\overline{Y}}_{..}\, \mathbf{wt}$$

$$\overset{F}{VY}_{..} = \overset{F}{\overline{VY}}_{..}\, \mathbf{wt^2}$$

$$\overset{F}{STY} = \sqrt{\overset{F}{VY}_{..}}$$

$$\overset{F}{SEY}_{..} = \frac{\overset{F}{STY}_{..}}{\overset{F}{Y}_{..}}\ 100$$