STAND DIFFERENTIATION ABILITY

in Northern Hardwoods

by Adrian M. Gilbert

U. S. FOREST SERVICE RESEARCH PAPER NE-37
1965
NORTHEASTERN FOREST EXPERIMENT STATION, UPPER DARBY, PA.
FOREST SERVICE, U. S. DEPARTMENT OF AGRICULTURE
RALPH W. MARQUIS, DIRECTOR

The Author . . .

ADRIAN M. GILBERT, formerly chief of the Division of Forest Economics, Recreation, and Wildlife Habitat Research at the Northeastern Forest Experiment Station, was recently promoted to director of the U. S. Forest Service Division of Programs and Special Projects, Washington, D.C. A graduate of the University of Michigan in 1943, he did graduate study at the Swiss Federal Institute of Technology at Zurich, and received his Ph.D. degree in 1962 at the New York State University College of Forestry at Syracuse. This paper is based upon a dissertation presented to the faculty of the College of Forestry at Syracuse in 1962.

STAND DIFFERENTIATION ABILITY

in Northern Hardwoods

Contents

Importance of Stand Behavior

THERE ARE GAPS in our knowledge of forest stand behavior. In particular, we know little about differentiation in even-aged stands. We have observed for years that, within a stand, trees differentiate into diameter and crown classes. But few attempts have been made to measure this differentiation in even-aged stands, or to explain it.

There is an increasing interest in stand behavior. In New England, where this study was conducted, many public and private owners are placing their northern hardwood forests under management. Stumpage returns have reached levels high enough to induce some owners to practice siliculture on their managed lands.

The opportunities for research are increasing too. Each added piece of knowledge facilitates new studies and produces new knowledge. We are learning how to break down the northern hardwood forest type into classes suitable for efficient study. Site studies will enable us to stratify areas by estimates of productivity. Tree-grade and growth studies will enable us to sort trees and stands into value classes. And the application of statistics to silvicultural research enables us to design better studies with more confidence in our results.

There are two ways in which data on stand behavior in northern hardwood stands could be used. First, the choice of treatments to apply to stands, and their timing, depends in part upon the way we expect a stand to react. For example, a species with a tendency to stagnate into one crown class would have to be thinned early if large timber were desired.

Second, stand-behavior data could also be used to make silvicultural research more efficient. If we knew how much differentiation normally exists within a stand, we could make better estimates of sample sizes and could devise better tests of cleanings and thinnings. The data would be more sensitive, and the results easier to interpret.

Review of Literature

A Danish forester, L. A. Hauch, in a book he had written with A. Oppermann (1898, p. 101), speculated about the ability of trees in a stand to divide into classes. He called this ability *Spredningsevne* (literal translation: *spreading ability*).[1] The existence of these classes was evident to all foresters. But species seemed to differ in this ability. And this ability to divide into classes seemed to vary with site.

In 1910, Hauch clarified his theory:

> "Spreading ability expresses the stratification into size classes. One could possibly define spreading ability as the ability of a given species, in the same environment and under the same treatment, to develop a large or small number of individuals in the upper and middle height classes from a given number of stems. Thus, a species possesses a larger spreading ability as the number of stems in these height classes is less. It is possible that the difference between the tallest and shortest in a dense stand of a species with a small spreading ability could be as great as that of a species with a large spreading ability but the number of suppressed is less. The middle and upper crown classes are more strongly represented in a species with a weak spreading ability. I cannot relinquish the opinion that there is a correlation between spreading ability and the tendency of a species to develop straight stems and steady growth." [2]

In other words, Hauch claimed that not only did the variances differ in forest stands, but that the distributions were not normal. The way they were skewed depended upon the species.

Lönnroth (1926) studied the structure and development of even-aged Scotch pine stands on three site types in Finland. In addition to determining the average values for a number of stand characteristics, he calculated the variances. Lönnroth was one of the first to test Hauch's hypotheses. He found differences in variation among the site types in height and diameter. At that time many of the statistical tests of significance either had not been devised or were not often applied to biological data.

Lutz (1932) related site quality to stand density in both hardwood and softwood stands. Once the stands had closed and competition had begun, good sites usually had fewer stems per unit area than poor. Presumably the inherent capacity for rapid growth is expressed more fully on the better sites. "These vigor-

---

[1] Translated as *deviation capacity* by Svend O. Heiberg.
[2] Translated by Adrian M. Gilbert.

ous seedlings develop rapidly and soon overcome their neighbors; dominance is expressed early," he said.

Lutz reasoned that, on the poorer sites, growth was slower and an early expression of dominance was lacking. This led to a more or less uniform rate of development, and "differentiation into crown classes is delayed." He also said that "competition is not as eliminative" on poorer sites.

In American forestry terminology, expression of dominance usually refers to the tendency of a species to hold a large or small number of trees in the dominant plus co-dominant crown class. This is the way Lutz used the term.

Deen (1933) studied the expression of dominance in white pine in New England. He pointed out that the distribution of numbers of trees by size classes should follow the normal curve. "The more regular the stand," he said, "the more the trees are crowded in relatively few size classes and the weaker the expression of dominance."

Deen measured the standard deviation of diameter as an index of the expression of dominance. He showed a strong correlation between the standard deviation of diameter and other stand features such as density, average radial growth of dominants, standard deviation of height, and standard deviation of crown spread.

Deen defined expression of dominance as "the division into diameter and crown classes resulting from an unequal rate of growth in the trees of a pure, evenaged stand." His concept was almost as broad as Hauch's, but expression of dominance persists solely as a description of crown-class differentiation.

Deen discussed factors that might influence the expression of dominance — inherent characteristics, seed size and origin, site, within-plot age variation, density, weeviling, and silvicultural treatment. He attempted to set a standard of acceptable expression of dominance, which was described as a stand with such differentiation of crown classes that the dominant trees could maintain the largest rate of growth consistent with the development of well-formed crop trees. He found that on the better sites white pine had an acceptable expression of dominance.

Kurth (1946) studied the structure and quality of young evenaged beech stands in Switzerland. He found that the variance of height increased with age. However, the coefficient of variation of height was higher in sapling stands than in young pole stands. Kurth concluded that the stratification into quality classes was not clearly influenced by macrosite. In these young stands the immediate environment dominates the development.

Concept of Stand Differentiation Ability

What emerges here is a concept of *stand* development that attempts to explain the differentiation that is found in pure even-aged stands. I believe that a large part of the difficulty in finding a good expression for the concept lies in the vagueness of what a forester means by a stand. It is easier to tell what a stand is *not* than what it *is*. A stand is not an area classification. Nor is it merely a number of trees growing in proximity. A stand is not a unit of forest management, as a compartment, block, or working circle.

The SAF Forestry Terminology (1958) defines a stand as: "An aggregation of trees or other growth occupying a specific area and sufficiently uniform in composition (species), age, arrangement, and condition to be distinguishable from the forest or other growth on adjoining areas."

Leibundgut has a similar definition.[3]

> "Under stand is understood a part of the forest that differs on the basis of its species composition, its age, its structure, or any other important characteristic; *and therefore is the subject of separate silvicultural planning and handling.*"

I define a stand as a dynamic community of trees characterized by the way it reacts to natural or manmade stimuli. The development of the trees in a stand reflects: (1) each tree's inherent characteristics (genotype); (2) its reaction, as an individual stem, to its environment (phenotype); and (3) the reaction of the trees to each other — a special kind of environmental response.

It should be noted that this composite reaction of trees to each other and to environment or stimuli (treatment) is not the sum of the individual stem reactions. It may be more or it may be less. It is this within-stand-interaction and the fact that the interaction differs from one stand to another that distinguishes a forest stand from an orchard or any other group of trees growing in proximity.

Apple trees in an orchard vary from each other. These differences among apple trees that are due to inherited characteristics are genotypic: those due to reaction of the genotype with environment are phenotypic. It is these kinds of individual variation that have been studied intensively by the geneticists.

---

[3]From lecture notes of Hans Leibundgut, Professor of Silviculture at the Swiss Federal Institute of Technology, Zurich, Switzerland. Italics are the author's.

Trees in a stand express genotypic and phenotypic variation. They also express the reaction of one tree to another. This is the most difficult kind of phenotypic variation to assess because of the peculiar mix of individual interaction and stand interaction effects. Yet, stand reaction is the basis of much silvicultural practice—cleanings, thinnings, and selection-system cuttings.

It is the necessity to assess this all-embracing stand behavior that has led, in part, to difficulty in fitting words to a concept. It is also true that the peculiarities of a stand as a unit of silvicultural treatment have not been generally understood.

Now, when we return to the phenomenon of stand behavior, this ability of a stand to differentiate into crown classes, into diameter classes, and into quality classes, and to vary this ability with species, with age, and with site, we see that a term to describe this idea should include the word, *stand.* For this reason, I propose *stand differentiation ability* (SDA) for this concept. In silviculture, the stand is the unit of differentiation (Toumey and Korstian, 1947).

Conditions of the Study

A study was conducted in the Green Mountains of Vermont, the Taconic Range in Massachusetts-New York, and a small portion of the White Mountains of New Hampshire, to:

1. Test the hypothesis that several northern hardwood species vary in stand differentiation ability (SDA), as measured by diameter, height, crown classes, and quality classes.
2. Test the hypothesis that SDA within and among species varies with age and site.

The study was limited to three species, sugar maple *(Acer saccharum* Marsh.), yellow birch *(Betula alleghaniensis* Britton), and paper birch *(Betula papyrifera* Marsh.). Sugar maple and yellow birch are two of the most prevalent and most valuable hardwood species in northern New England. Paper birch is less common, but highly valued. From observation, it was thought that the three species would provide a range in stand differentiation that would be well suited for study.

The study was limited to natural stands in ages up to 50 years. This facilitated species comparisons. After 50 years, some hardwood stands (paper birch) begin to break up, since the species matures relatively early. Other species, such as sugar maple, are very long-lived. If the species were not comparable by age classes, an extra source of variation would be introduced. Also, as stands age, there is greater variation in density.

Pure stands of each species were sought, to reduce the variation due to species mixture. *Pure* was defined as at least 75 percent of the number of stems or basal area of the major species in the main crown canopy. Normally, there is so much variation in species composition that an extremely large number of samples would be needed just to reduce the variation due to composition. Stem count was used as the basis of purity of stocking in the younger stands with average diameters less than 2 inches; basal area was used as the criterion of purity in stands with an average diameter of more than 2 inches. Basal area is not a meaningful measure of stocking in very young stands.

The stands were on as uniform sites as can be found in this mountainous, glaciated terrain. Aspect, slope percent, rockiness, and drainage were used as important criteria of site in the search for uniformity of conditions.

The stands were even-aged, with a maximum range of ages within the stand of 6 years. Since one hypothesis was that stand differentiation varies within and among species by age, variation in age within a given stand reduces the sensitivity of the test.

Each stand was well-stocked. Within the bounds of age and site differences, the crowns were fully closed and showed no evidences of exceptional mortality.

Stands with sprout clumps were admitted, as long as at least 50 percent of the stems were single. While sprouts have a large initial surge in growth, this advantage disappears after about 10 years in these hardwood stands. A check of the data used in this study revealed no apparent influence of sprouts upon the analysis.

The stands were undisturbed: there were no signs of past cutting, grazing, or fire during the life of the stand. In a few cases, stands with slight mouse or rabbit damage were admitted.

The botanical range of all three species is extensive; for example, paper birch extends across the continent. The study area included such a small part of the natural range of the species that clinal or ecotypic differences are unlikely. On the other hand, the study area was well within the commercial range of the three species, and the results of the study should be applicable on similar sites (climatic, physiographic, and edaphic) in the Northeast.

Methods and Materials

The Areas

The ecological area under study has been defined by Braun (1950) as the New England Section of the Northern Appalachian Highland Division. Here the hardwood forests are found on slopes up to 2,400 feet in elevation. The entire area has been glaciated, and most of the soils are tills. The White Mountains are composed of non-calcareous crystalline gneisses and metamorphic rock. The Green Mountains are composed of schists and some gneisses. A few of the soils are calcareous. Most are podzols or brown podzolics (U.S. Dept. Agr., 1938).

Total precipitation averages 40 inches a year; snowfall 80 to 100 inches a year. The average temperature is about 70° F. in July and about 20° F. in January (U. S. Dept. Agr., 1941). Since most weather data were collected at lower elevations, the averages probably differ somewhat in the mountainous area. In the mountains, one would expect slightly higher precipitation, lower January temperatures, and a shorter frost-free period. But the general climatic picture would not differ from that reported and summarized.

Most of the area had been cut over at least once (Gilbert and Jensen, 1958). The even-aged stands followed clear-cutting for charcoal or fuelwood. Some areas had been burned over.

Other stands had seeded in on old fields as the mountain farms were abandoned. Land-clearing reached its peak in northern New England during the middle of the nineteenth century. Since then, thousands of acres have reverted to forest cover, and the trend continues today.

The Species

The northern hardwood forest is a complex of types found in the colder zones of eastern North America. The major type in the northern hardwood complex is the climax birch-beech-maple type (SAF, 1954). There are numerous associate species, including some conifers.

Sugar maple and yellow birch are two long-lived species that are found in this climax type (Godman, 1957; Gilbert, 1960). Sugar maple also has pioneer characteristics. About half the sugar maple plots in this study were located on abandoned farm land; only one yellow birch plot was found on abandoned land. Paper birch is a shorter-lived species that is rarely found in the climax type (Hutnik and Cunningham, 1961). Of the 15 paper birch plots, 10 were on abandoned land.

Data Collection

The data consisted of measurements on a series of plots, located on a variety of conditions within the scope of the study. Data were taken from 43 plots and 1,175 trees:

Species	*No. trees*	*No. plots*
Sugar maple	15	395
Yellow birch	13	344
Paper birch	15	436
	43	1,175

The plots, all rectangular, varied in area from 3 to 90 milacres, depending upon the average size of trees. Plots were kept small to minimize within-plot variation, but it was desired to measure at least 15 trees of the key species on each plot.

The diameter of all trees larger than 0.50 inches in diameter breast height was measured to the nearest 0.1 inch. Tree heights were measured with either a pole or a Blume-Leiss hypsometer.

All trees were tallied by crown class, using four classes (SAF, 1958) — dominant, co-dominant, intermediate, and overtopped. There was a complete tally of heights for 29 of the 43 plots.

All stems of the key species on each plot were classified by quality into five classes:

1. *Prime tree.* —Admits trees with straight boles; single stem; bole well-pruned for age; lean less than 5 percent; no rotten cull; crown well-shaped on at least 3 sides; branching habit conducive to self-pruning and good crown development.

2. *Good tree.*— Admits crook or sweep up to 20 percent; not more than 2 crooks; lean up to 20 percent; up to 5 percent rotten cull; may be main stem of clump; may contain up to 25 percent of bole in persistent branches; crown well-shaped on at least 2 sides; branching habit conducive to development of relatively clean bole.

3. *Fair tree.* — Admits crook or sweep up to 35 percent, up to 4 crooks; lean up to 35 percent; 15 percent rotten cull; any member of sprout clump; up to one-half bole in persistent branches; acute or coarse branching; may be forked but will produce one 16-foot log below fork.

4. *Poor tree.* — Admits up to 50 percent sweep or crook, any number of crooks; lean up to 50 percent; up to 35 percent rotten cull; may have persistent branches all along stem; may be forked but will produce one 8-foot log below fork.

5. *Cull tree.* — All others.

For most trees, the principal reason for quality classification was recorded (fig. 1). The quality classification was subjective. Since one person classified all the trees, there was greater consistency in application than if several had classified. The weakness of this kind of classification is that a stem in a milieu of poorer-looking stems might be classified differently than if surrounded by equally good or better stems.

Stand age was determined by averaging five increment cores taken at breast height. Five years were added to the average core reading as the estimated time to reach breast height.

Data Analysis

Methodological problems. — In this study we were concerned with stand development as measured in distributions of individual stems. This posed problems in methodology that could not and still cannot be completely resolved.

Before the stand closes, there are differences among the stems. We define the period before the stand closes as that in which there is little or no visible effect of competition for crown space among stems. The stems have not begun to lose their lower branches, and the stands show little or no differentiation into crown classes. Early in the development of the stand, the stems may compete with minor vegetation. Just as the stand closes, however, this minor vegetation usually disappears.

During this period before the stand closes, the observed differences in measurable features are due largely, if not entirely, to the responses of stems as individuals to inherited characteristics and to microsite. What could be measured here is the response of individual stems (dependent variable) to inheritance and microenvironment (independent variables) and to the interaction of these variables.

After the stand closes, individual stem responses are a function not only of inheritance and microsite, but also of competition (Toumey and Korstian, 1947).

Now, stand responses are a function not only of the same factors that influence individual stems, but also of the individual stem responses — and all the interactions. Thus, in measuring stand responses we usually take the dependent variable, the individual stem response, and use it as an independent variable. In a sense what we are doing is ignoring what we do not know about individual stem response.

Statistical tests. — If we are cognizant of these difficulties of studying stand behavior, we can design experiments that are not

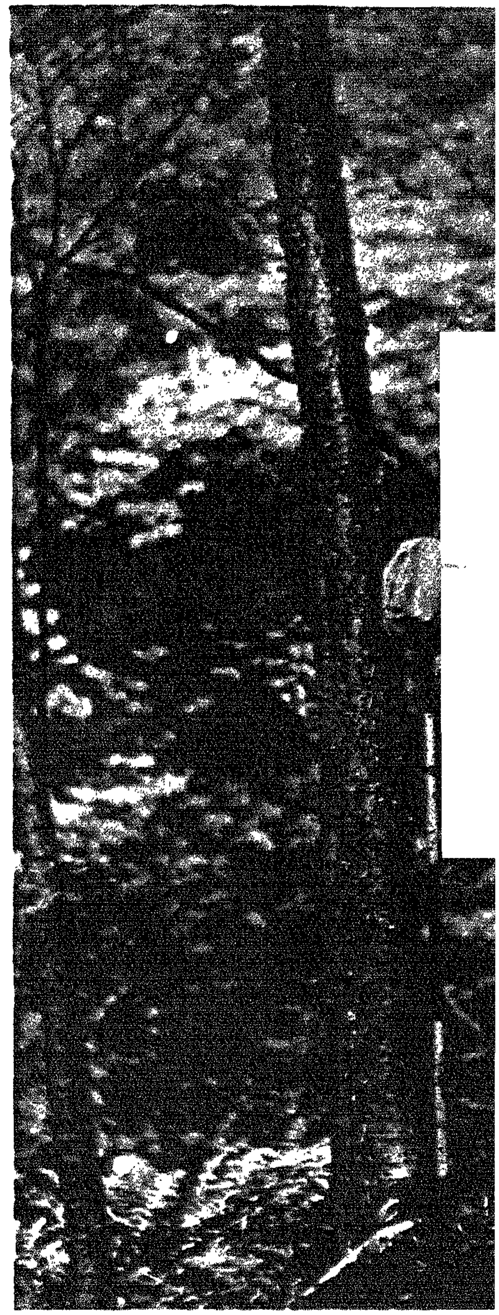

Figure 1. — A prime-quality yellow birch tree, in Mt. Mansfield State Forest, Vermont.

too contrived. And, we can be humble about the conclusions we reach. Since it is difficult to design studies with enough power, we run the risk of accepting false hypotheses — the *beta* error (type II error).

In this study, whenever the null hypothesis was tested by an appropriate statistical test, the probability of rejecting a true hypothesis was set at 10 percent rather than 5 percent.

Differentiation in Stand Diameter

Analysis

Standard deviation of diameter. — In this study, standard deviation of diameter at breast height was treated as a continuous variable. The regression equation was $Y = bX$. While it is true that the distribution of the standard deviation of diameter may not be normal at each X, this would have little effect upon the tests of significance. These tests are not very sensitive to non-normality. To test if the standard deviation of diameter of each species is related to age, the significance of the regression coefficient, b, was tested by the *t* test, according to Snedecor's formula for ratio regression.

Results

Standard deviation of diameter. — Stands of these three species differentiate more in diameter as they increase in age. The standard deviation of diameter, by which the differentiation was measured, increases at least 0.04 inch a year for all species (fig. 2). This is the same trend that Meyer (1930) found by using stand diameter as the independent variable.

Since the *t* tests revealed significant regression coefficients, "b", this relationship between age and standard deviation of diameter is statistically significant (table 1).

Species comparisons. — There remained the hypothesis that the species differ in differentiation of diameter. The null hypothesis of equal variance among all species was rejected (table 9). However, sugar maple and yellow birch had variances about the regression line that might not differ significantly.

Subsequent *t* tests showed that, though the variances in the standard deviation of diameter for sugar maple and yellow birch did not differ significantly, the slopes of the regression lines did differ enough to be significant.

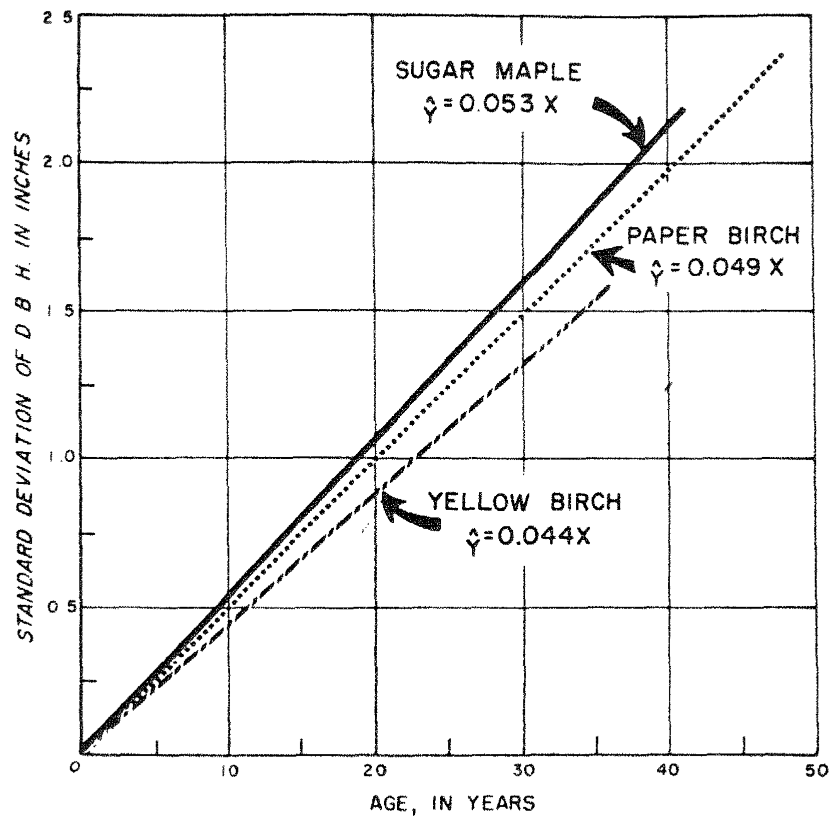

Figure 2. — Species compared for standard deviation of stand diameter in relation to age.

So far, we have learned that the three species do show differentiation in diameter, which increases with age, and that the differentiation is not the same for all species. We should know something about actual diameter increases to learn if this differentiation is large or small in relation to diameter increment.

Coefficient of variation. — The coefficient of variation is a ratio that could provide a more meaningful comparison of differentiation in diameter among species. It is the ratio of the standard deviation to the mean. Since the plots were of different ages, provision must be made for variation due to age in making species comparisons by the coefficient of variation.

Estimates of the standard deviation of diameter for various ages were available from the regressions discussed in the previous section.

Similar estimates for mean diameter at various ages were determined by ratio regression (variance was not homogeneous), with the regression line going through the origin. The equation was of the form Y = bX (fig. 3). It was found that sugar maple had the least increase in average diameter with age (table 2). Tests showed that the regression lines for the three species differ significantly (table 9).

With the two sets of regression data, one for standard deviation and one for mean diameter, the coefficient of variation at various ages can be determined. At the mean age for each species, coefficients of variation for each species were:

Species	*Mean age (years)*	*Mean s (inches)*	*Mean d.b.h. (inches)*	*Coefficients of variation (percent)*
Sugar maple	26.3	1.40	2.25	62.6
Yellow birch	26.5	1.17	2.77	42.2
Paper birch	29.1	1.45	4.50	32.2

An average coefficient of variation (CV) for all ages can also be computed for each species:

CV = (Regression coefficient b of standard deviation of d.b.h. over age) / (Regression coefficient b of average d.b.h. over age) × 100.

Note that on the basis of standard deviation alone, an absolute measure, yellow birch had the least variation in diameter (b = 0.044). However, with the coefficient-of-variation ranking, a measure of variation relative to the mean diameter, paper birch had the lowest ratio (32 percent).

Table 1.—Analysis of data for standard deviation of stand diameter

Species	Number of plots	Range of ages, years	Regression equation[1]	90-percent confidence interval[2]	*t* value
Sugar maple	15	17-41	Y = 0.053X	0.040 - 0.065	7.24***
Yellow birch	13	17-36	Y = 0.044X	0.038 - 0.050	13.72***
Paper birch	15	8-48	Y = 0.049X	0.034 - 0.064	5.67***

[1]Y = standard deviation of d.b.h.; X = age.
[2]Standard deviation/year.
*** = Very highly significant.

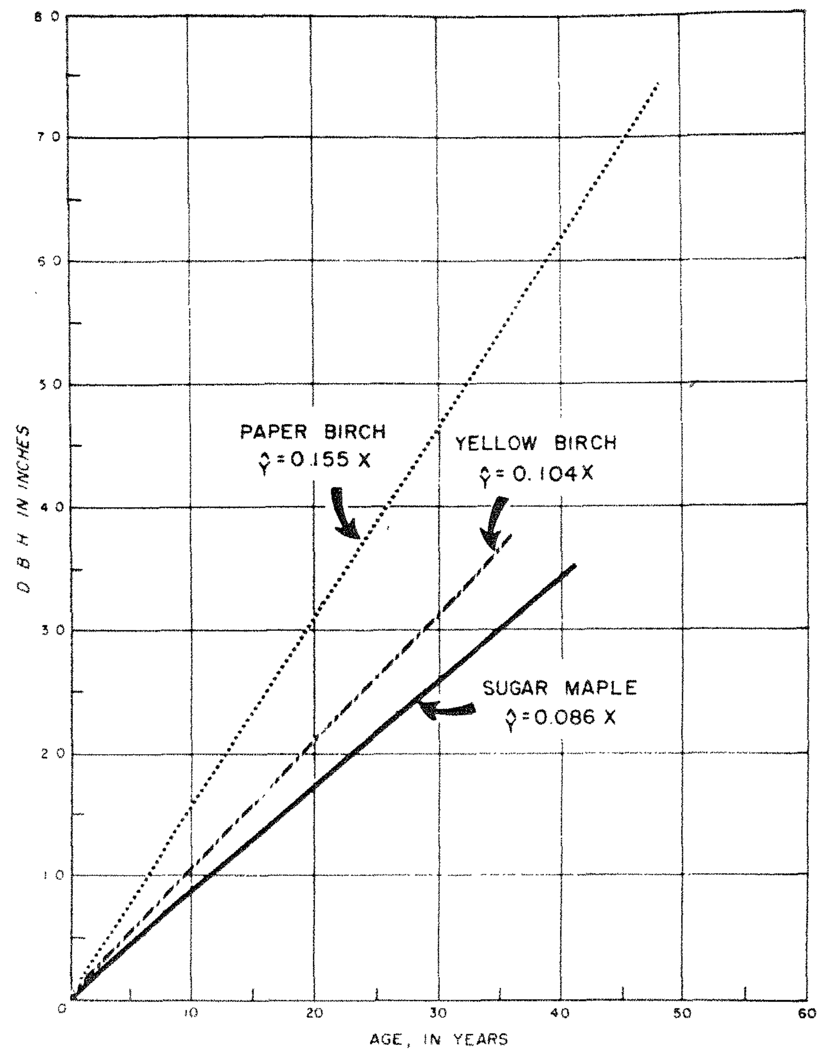

Figure 3. — Average stand diameter in relation to age; species comparisons.

Hauch (1905) had stated that *Spredningsevne* and shade tolerance were not directly related. Sugar maple is a very tolerant species, and it did have the largest standard deviation. But yellow birch, intermediate in shade tolerance, had the least differentiation in diameter. Paper birch, an intolerant species, ranked next to sugar maple in diameter differentiation. However, when ranked by co-

Table 2.—Analysis of data for regression of average diameter breast height over age

Species	Number of plots	Range of ages, years	Regression equation[1]	90-percent confidence interval[2]	t value
Sugar maple	15	17-41	$Y = 0.086X$	0.074 - 0.098	12.19***
Yellow birch	13	17-36	$Y = 0.104X$	0.093 - 0.115	16.78***
Paper birch	15	8-48	$Y = 0.155X$	0.133 - 0.176	12.76***

[1]Y = average d.b.h.; X = age.
[2]Average d.b.h./years.

efficient of variation, shade tolerance and standard deviation of d.b.h. coincided. We still lack the knowledge to sort out these interactions.

In tests of statistical significance, the species do differ. Yet we do not know if these differences, in themselves, are large enough to influence response to silvicultural treatment. Interpretation must wait until the results of the other tests are considered.

Differentiation in Total Height

Analysis

Differentiation in total height of stands was analyzed by methods similar to that used for stand diameter. Total height was treated as a continuous variable and related to age. The regression equations were of the form $Y = bX$. Data for the height analysis were available for fewer plots than for diameter.

Species	*No. plots*	*No. trees*
Sugar maple	11	297
Yellow birch	7	185
Paper birch	11	368

However, these data were sufficient to fit a regression line with one independent variable and to make tests for significance of the regression coefficient, b.

Results

The only significant regression coefficient of standard deviation of stand height over age was for sugar maple:

Species	*No. plots*	*Range of ages (years)*	*t value*
Sugar maple	11	17.41	3.38*
Yellow birch	7	20.30	1.22
Paper birch	11	23.48	1.41

These data indicate that within the range of ages tested, for both yellow and paper birch, SDA in height was not significantly dependent upon age. For paper birch, the regression coefficient was negative, indicating an inverse relationship. In other words, as paper birch stands grow older there is a tendency for less variation in height.

For the stands included in this analysis, factors other than age may have a greater effect upon differentiation in total height. Only for sugar maple was there a significant correlation between standard deviation of height and age.

The paper birch data showed an interesting trend that indicated a reduction in height differentiation with increase in age. This contrasts with sugar maple and yellow birch, where the tendency was for increased differentiation with age. It is possible that the analysis of distribution into crown classes will clarify the analysis of height relationships.

Differentiation into Crown Classes

The Choice of Test

In this analysis, we were interested in differences in the ability to differentiate into crown classes. In other words, we were dealing with a discrete variable.

The null hypotheses were that: (1) species do not differ in the ability to maintain stems in the upper crown classes; and (2) within and among species, SDA does not vary with site and age.

We were interested in counts, not in normal distributions. Since crown classes, site classes, and age classes are discrete variables, the chi-square test for contingency tables was appropriate.

The Choice of Classes

The variables — site and age — were broken into enough classes to permit the testing of the hypotheses and still have biological meaning. For testing the differentiation into crown class and quality class, age was divided into two classes — 0 to 26 years, and 27 years and older. At about 25 years, most stands of these hardwood species are at the transition from cleanings to thinnings;

there is greater confidence in the selection of crop trees. The actual break at 26 years was made to permit about equal numbers of plots in each class for each species. Additional age classes would have weakened the chi-square analyses by reducing the expected numbers of observations per cell.

Similarly, site was broken into two classes: good and poor. Site-index curves for these species do not go below 20 years (Curtis and Post, 1962), and such curves are less reliable for young stands.

Site class was computed by ratio regression, height of the five tallest trees on the plot over age, with the regression line extending from the origin, a = o. All plots that fell above the regression line were site class 1; those below were site class 2.

Three crown classes were used: dominant plus codominant, intermediate, and overtopped.

Analysis

For the chi-square analysis, each tree on a plot was treated as an independent count to be placed into a particular cell. First the plots were sorted by key species, age class, and site class. Then each stem of the key species was placed into its crown class-age class-site class-species cell. When this was done, the plot identity was lost. In other words, all plots of a given age class-site class-species cell were considered as samples of the same population whose data could be pooled. This assumption underlies all succeeding tests of hypotheses, in quality classes as well as crown classes.

It is important that when the plots were chosen there was no bias in selecting plots of a particular age class or site class. The fact that there were not equal numbers of plots in age and site classes had no effect upon the analysis.

For ease of analysis, the chi-square test was used to test hypotheses of independence in two-way contingency tables.

Results

Sugar maple. — When each site class was held constant in turn, the distribution of sugar maple into crown classes was independent of age classes (table 10). However, when age classes were pooled and the two site classes were compared, the null hypothesis of independence was rejected. On both sites less than 30 percent of the stems were in the dominant-plus-codominant crown class (table 3).

Table 3.—Distribution of sugar maple stems into crown classes by age class and site class, data pooled when possible

Item	Crown class			Basis
	Dominant and codominant	Intermediate	Overtopped	
	Percent	*Percent*	*Percent*	*No. stems*
Site class 1;				
age classes 1 & 2	29.2	20.3	50.5	216
Site class 2:				
age classes 1 & 2	23.1	14.7	62.2	251
Age class 1:				
Site class 1	31.8	22.0	46.2	132
Site class 2	22.6	12.8	64.6	164
Age class 2:				
Site class 1 & 2	24.6	18.1	57.3	171

When age class was held constant, the distribution into crown classes was independent of site in the older stands but not in younger stands (table 3). With one exception the tests showed fewer than 30 percent dominants and codominants when age was held constant; it was 32 percent in the excepted case.

Yellow birch. — When each site class was held constant, the distribution of yellow birch into crown classes was independent of age classes (table 10). When age classes were pooled and the two site classes were compared, the null hypothesis of independence was rejected. On the better sites, 42 percent of the stems were dominant or codominant; on poorer sites, only 28 percent (table 4).

At all age classes, there was interaction between crown class and site class (table 4). More than 40 percent of the stems were dominant or codominant on site class 1, and less than 30 percent on site class 2, regardless of age class.

Paper birch. — On site class 1, the distribution of paper birch into crown classes was independent of age classes (table 10). On site class 2, the null hypothesis of independence of crown class and age class was rejected. Here there was a larger proportion of overtopped trees in the younger stands.

Regardless of interaction, on site class 1 more than 70 percent of the paper birch stems were dominant and codominant in all instances. On site class 2, dominants were 31 percent in younger stands and 57 percent in older (table 5).

At neither age class was the distribution of paper birch into crown classes independent of site. In the younger stands, there was a much greater proportion of the stems in the dominant-plus-codominant class on the better sites. These made the largest contribution to chi-square. In stands over 27 years old, it was the large proportion of overtopped trees on poorer sites that made much of the contribution to chi-square.

Species comparisons. — Strictly speaking, valid tests of independence among species with pooled data are only for those cells where the null hypotheses of independence for comparable variables were accepted.

In these tests, we noted that on site class 1 the null hypotheses of independence of crown class and age class were accepted for each of the three species. Then the species were compared by pooling age class and testing species x crown class in a contingency table. The null hypothesis of independence was rejected (table 11). On site class 1, the species did differ significantly in differentiation into crown classes. On these better sites, 75 percent of the paper birch stems were dominant or codominant; so were 42 percent of the yellow birch and 29 percent of the sugar maple.

On most of the other individual tests among and between species, the null hypotheses of independence were rejected (table 11).

Table 4.—Distribution of yellow birch stems into crown classes by age class and site class, data pooled when possible

Item	Crown class			Basis
	Dominant and codominant	Intermediate	Overtopped	
	Percent	*Percent*	*Percent*	*No. stems*
Site class 1;				
age classes 1 & 2	42.5	12.3	45.2	212
Site class 2;				
age classes 1 & 2	27.9	15.8	56.3	222
Age class 1:				
Site class 1	46.9	10.9	42.2	64
Site class 2	29.3	14.7	56.0	150
Age class 2:				
Site class 1	40.5	12.8	46.7	148
Site class 2	25.0	18.1	56.9	72

Table 5.—Distribution of paper birch stems into crown classes by age class and site class, data pooled when possible

Item	Crown class: Dominant and codominant	Crown class: Intermediate	Crown class: Overtopped	Basis
	Percent	*Percent*	*Percent*	*No. stems*
Site class 1;				
age classes 1 & 2	75.3	14.5	10.2	117
Site class 2;				
Age class 1	31.1	22.2	46.7	167
Age class 2	57.3	20.7	22.0	82
Age class 1:				
Site class 1	78.3	8.7	13.0	23
Age class 2:				
Site class 1	74.5	15.9	9.6	94
Site class 2	57.3	20.7	22.0	82

From the results of all the individual tests we may infer that the three species differ in differentiation into crown classes.

Discussion

The data for the three species indicate the relative effects of site and age upon the distribution into crown classes.

Site appears to exert more influence than age in the distribution. The drop in the proportion of stems in the dominant and codominant class is much greater from site class 1 to site class 2 than from younger to older age classes.

For both sugar maple and yellow birch, when site class was held constant, age class had no significant effect upon crown class distribution. On the poorer sites of paper birch, there were relatively more stems in the main crown canopy in older stands than younger. Normally, the reverse trend would be expected.

These observations confirm those made in the previous chapter on differentiation in stand height. It may be recalled that sugar maple was the only species with a significant increase in standard deviation of height with increased age. For paper birch, the trend was reversed.

Köstler (1952) made similar observations concerning thickets in Germany. He found several patterns of crown differentiation, which he attributed partially to tolerance differences among species.

On the better sites, the three species differed much more than in any other class. Even though present site-index studies have been unable to measure site index in young hardwood stands, site differences are present. It is likely that the crown-differentiation pattern is a strongly inherited species characteristic and that it is modified by site.

The results of these tests of differentiation into crown classes suggest that if crown classes are to be used effectively in describing stands, their connotations should be revised. It now seems likely that different species, at different ages, on different sites, tend to follow different crown-class patterns. For example, if three out of four paper birch stems on site class 1 are dominant or codominant (table 5), knowledge that a stem is codominant in such a stand tells us little about its expected behavior in comparison with its neighbors. The same codominant stem on site class 2 and age class 1 would be among 3 out of 10 in such a position, and we might expect it to develop differently.

Differentiation into Quality Classes

Analysis

Each stem of the three species under study had been classified into one of five quality classes. The classes were composites of visible stem and crown characteristics. The hypotheses to be tested were:

1. The species differ in SDA for quality classes.
2. The differentiation into quality classes varies by site and age within and among species.

Here, as with crown classes, we were not interested in normal distributions, and the variables were discrete (counts). Hypotheses of independence in contingency tables were tested with the chi-square test.

Results

In the individual species tests, interaction between site or age and quality was prevalent.

Individual species tests. — For sugar maple, the only acceptance of the null hypothesis of independence was when age class 2 was held constant in the test of site quality x quality class (table 12).

The results showed a generally low proportion of prime-plus-good stems and a high percentage of culls for sugar maple (table 6).

For yellow birch, interaction between quality and age or site was the rule. Independence was accepted only in a test of age class x quality class, when site class 1 was held contant (table 12). Yellow birch, too, had a low proportion of stems in the prime and good quality classes the proportion of cull stems was high (table 7).

In general, the results of the quality-class tests were the same for paper birch as other species. Except when site class 1 was held constant, the null hypothesis of independence was rejected in the chi-square tests (table 12). Paper birch was found to have a high proportion of good growing stock: about one-third of its stems were prime-plus-good quality (table 8).

Species comparisons. — In the formal species comparisons, various combinations of age class and site class were held constant. Species x quality classes were tested in two-way tables. Whenever all three species were tested, the null hypothesis of independence was rejected (table 13). In several instances, however, sugar maple and yellow birch were found to have similar distributions into quality classes. Although the differences among species were too large to be random, they all showed the same trend on site class 2. That is, in the older stands there was a higher proportion of prime-plus-good trees than in stands under 26 years, on the poorer sites.

Table 6.—Percentage distribution of sugar maple stems into quality classes by age class and site class, data pooled when possible

Item	Quality class					Basis: No. stems
	Prime	Good	Fair	Poor	Cull	
Site class 1:						
Age class 1	8	17	24	19	32	114
Age class 2	0	13	31	18	38	84
Site class 2:						
Age class 1	2	5	19	28	46	156
Age class 2	4	14	30	27	25	83
Age class 2;						
Site class 1 & 2	2	14	30	22	32	167

Table 7.—Percentage distribution of yellow birch stems into quality classes by age class and site class, data pooled when possible

Item	Quality class					Basis: No. stems
	Prime	Good	Fair	Poor	Cull	
Site class 1;						
Age classes 1 & 2	4	11	38	22	25	211
Site class 2:						
Age class 1	3	8	18	21	50	151
Age class 2	6	18	22	20	34	71
Age class 1;						
Site class 1	3	6	38	22	31	64
Age class 2;						
Site class 1	4	13	38	23	22	147

In the crown-class comparisons, all species showed little or no downward trend in the proportions of dominants and codominants with increased age. Since almost all the better quality trees are found in the main crown canopy, one would expect that the proportion of higher quality trees would have remained almost as high in the older age class as the younger. But why should it increase? Apparently, the slower rate of growth on the poor sites favors some attributes of good quality — clean bole, fine branching.

Discussion

From the data analysis, we can make several deductions concerning the hypotheses on quality relationships. We can say that the SDA into quality classes varies by age and site classes within and among species.

Since there is so much interaction, a clear-cut comparison cannot be made among species as a whole. There is no statistical validity, but the quality-class distributions, in percent, can be tabulated:

Species	*Prime*	*Good*	*Fair*	*Poor*	*Cull*
Sugar maple	3	12	25	23	37
Yellow birch	4	11	29	21	35
Paper birch	7	25	37	23	8

Sugar maple and yellow birch are similar in differentiation into

Table 8.—Percentage distribution of paper birch stems into quality classes by age class and site class, data pooled when possible

Item	Quality class					Basis: No. stems
	Prime	Good	Fair	Poor	Cull	
Site class 1;						
Age classes 1 & 2	10	23	51	10	6	116
Site class 2:						
Age class 1	6	19	24	46	5	93
Age class 2	4	34	32	17	13	82
Age class 1;						
Site class 1	9	27	41	23	0	22
Age class 2;						
Site class 1	11	21	54	7	7	94

quality classes; they average about 15 percent prime-plus-good quality trees.

It may be recalled that paper birch had the largest proportion of stems in the dominant-codominant crown class. The two distributions — crown class and quality class —are undoubtedly related, but the relationship is not necessarily cause and effect.

To learn more about the relationship of the two distributions, a series of species-quality class tests of independence were run for dominant-plus-codominant stems only. Age class or site class was held constant in turn. In all but one test, the null hypothesis of independence was accepted. The general percentage distribution of dominants and codominants with age classes and site classes pooled was:

	Quality Class				
	Prime	*Good*	*Fair*	*Poor*	*Cull*
All species	11.5	30.0	44.3	10.8	3.4

This distribution of 42 percent of dominants and codominants as prime-and-good-quality trees represents almost a threefold increase for sugar maple and yellow birch over the distribution of quality among all crown classes.

Table 9.—Tests of hypotheses of equal variance of β

Item[1]	Differentiation of stand diameter analysis			Average diameter age analysis		
	SM	YB	PB	SM	YB	PB
Number of plots	15	13	15	15	13	15
b	0.052540	0.044046	0.049140	0.085933	0.103846	0.154666
S_b^2	$1.38(10^{-5})$	$1.03(10^{-5})$	$7.0(10^{-5})$	$4.97(10^{-5})$	$3.83(10^{-5})$	$1.468(10^{-4})$
$Sy.x^2$	$2.07(10^{-4})$	$1.339(10^{-4})$	$1.05(10^{-3})$	$7.455(10^{-4})$	$4.979(10^{-4})$	$2.202(10^{-3})$
F_{max}[1]		1.34	6.76**		1.30	3.83*
t^2		6.49**			7.16**	

[1]F_{max} applies to S_b^2. t applies to tests between regression coefficients.

Table 10.—Differentiation into crown classes, tests of independence

Species	Variables held constant	Variables tested	Chi-square value	Degrees of freedom	Sign fican
Sugar maple	Site class 1	Age class x crown class	2.5	2	NS
	Site class 2	Age class x crown class	1.7	2	NS
	Age class 1	Site class x crown class	10.4	2	S
	Age class 2	Site class x crown class	([1])	2	NS
Yellow birch	Site class 1	Age class x crown class	.7	2	NS
	Site class 2	Age class x crown class	.7	2	NS
	Age class 1	Site class x crown class	6.1	2	S
	Age class 2	Site class x crown class	5.2	2	S
Paper birch	Site class 1	Age class x crown class	.1	[2]1	NS
	Site class 2	Age class x crown class	8.3	2	S
	Age class 1	Site class x crown class	19.2	2	S
	Age class 2	Site class x crown class	6.8	2	S

[1]Less than 0.05.
[2]Classes combined.

Table 11.—Differentiation into crown classes, species comparisons, tests of independence

Variables held constant	Variables tested	Chi-square value	Degrees of freedom	Significance at 90% level
Age class 1, Site class 1	All species x crown classes	18.5	4	S
Age class 1, Site class 2	All species x crown classes	12.0	4	S
Age class 1, Site class 2	SM and YB x crown classes	2.6	2	NS
Age class 2, Site class 1	YB and PB x crown classes	37.2	2	S
Age class 2, Site class 2	YB and PB x crown classes	21.9	2	S
Site class 1, ages pooled	All species x crown classes	74.8	4	S
Site class 1, ages pooled	SM and YB x crown classes	10.2	2	S
Site class 2, ages pooled	SM and YB x crown classes	1.9	2	NS

Table 12.—Differentiation into quality classes, tests of independence

Species	Variables held constant	Variables tested	Chi-square value	Degrees of freedom	Significance at 90% level
	Site class 1	Age class x quality class	8.3	4	S
Sugar maple	Site class 2	Age class x quality class	15.2	4	S
	Age class 1	Site class x quality class	11.4	4	S
	Age class 2	Site class x quality class	4.4	4	NS
	Site class 1	Age class x quality class	3.1	[1]3	NS
Yellow birch	Site class 2	Age class x quality class	8.4	[1]3	S
	Age class 1	Site class x quality class	10.2	[1]3	S
	Age class 2	Site class x quality class	10.7	[1]3	S
	Site class 1	Age class x quality class	15.0	2	NS
Paper birch	Site class 2	Age class x quality class	7.1	[1]3	S
	Age class 1	Site class x quality class	1.3	2	S
	Age class 2	Site class x quality class	16.7	4	S

[1]Quality classes 1 and 2 were pooled to raise the expected number in each cell to at least 5. In some cases, classes 4 and 5 were also combined.

Table 13.—Species comparisons, differentiation into quality classes, tests of independence

Variables held constant	Variables tested	Chi-square value	Degrees of freedom	Significance at 90% level
Site class 1,			4	NS
ages pooled	YB and PB x quality classes	35.5	8	S
Site class 2,	All species x quality classes	63.4	[1]3	S
age class 1	SM and YB x quality classes	2.7		
Site class 2,	All species x quality classes	18.1	8	S
age class 2	SM and YB x quality classes	3.3		
Age class 1,	All species x quality classes	24.2	[1]3	S
site class 1	SM and YB x quality classes	7.5		
Age class 1,	All species x quality classes	63.4	4	S
site class 2	SM and YB x quality classes	6.5	8	S
Age class 2,			4	NS
site class 1	All species x quality classes	49.6	8	S
Age class 2,			4	NS
site class 2	YB and PB x quality classes	10.7	8	S

[1]Classes combined to raise expected number to at least 5.

Summary and Discussion

A mass of data has been collected, compiled, and analyzed to learn—

1. If sugar maple, yellow birch, and paper birch vary as to stand differentiation ability in diameter, in height, in crown-class distribution, and in qualty-class distribution.
2. If this differentiation, both within and among species, is related to age and site.

Differentiation in Stand Diameter

The data on standard deviation of stand diameter provide an example of the difference between significance and importance. Statistical significance has the support of mathematical theory, but it is still subject to the vagaries of chance. Importance is interpreted by people, and it is shaped by the particular values of the moment.

Stand differentiation ability varies among the three species. All species showed an increase in standard deviation of diameter of 0.04 to 0.05 inch a year. The greatest difference between species was 0.009 inch of diameter a year, certainly small; but the tests were sensitive enough to mark the differences as significant. But, the importance of these statistics to a practicing forester is another matter.

Differentiation in Stand Height

The results of the height analyses are unexpected. From the previous diameter analyses, it seemed possible to induce a more general theory concerning differentiation in stands: that as stands grow older the differentiation of many measurable attributes would increase. Other attributes might include diameter at 17 feet and total height.

But the height analyses preclude such a general theory, at least until we have more knowledge. The hypotheses were not supported by the statistical tests. The null hypotheses of no differences in differentiation in total height were accepted. Sugar maple was the only species of the three that showed a significant increase in differentiation of total height with increased age. The general tendency was the same for yellow birch, but the regression coefficient was not significant. However, for paper birch, the trend, although not significant, was the reverse — less differentiation with increased age.

The trend of paper birch is quite different from that of the other two species. The slope of the regression line was so slightly negative that it would take many more observations to detect statistical significance.

We can also infer that in these young natural stands the competition for light has a much greater effect upon height differentiation than upon diameter. The denseness of the stands tends to reduce the height differentiation. In stands of lesser density, more light would filter through the crown canopy, and we would expect increased differentiation in height. Thus it seems that the results of the height analyses are due not to a restricted range of ages, but to a restricted range in stand density. The reader may recall that the study was limited to well-stocked stands.

It must be noted that site was not included in the differentiation of stand-height analyses. Site does influence the rate of height growth and thus total height at given ages. But the height analyses showed homogeneous variance among the species (at the mean). Stratification by site would serve to reduce the variance and make the species even more homogeneous. Tests for the effects of site were left for the crown-class analyses.

Differentiation Into Crown Classes

The crown-class analyses included the effects of age and site upon the distribution of stems into crown class for each species and for comparisons among species.

Crown-class distributions are still somewhat concerned with the height parameter. So much of the theory of silviculture and of stand mensuration is related to crown-class strata that a study of stand development would be incomplete without an investigation of crown classes. Hauch's original hypothesis was that species differ in their distribution into crown classes.

In general, the results support Hauch's hypothesis: these three species do differ in their distribution into crown classes. Furthermore, these distributions are related to site, as Hauch had suggested. They also vary with age, within the limits of the ages studied, but to a lesser extent.

Presumably, if a species had a tendency to crowd many stems into the dominant and codominant crown classes, the stands would grow very slowly. This occurs commonly with lodgepole pine

(Pinus contorta Dougl.) and with other species on very poor sites. In other words, the tendency towards reduced crown differentiation is strongly inherited in some species and is due mostly to environment in others.

For paper birch, the distribution into the dominant and codominant crown class reached as high as 75 percent under certain age class-site class conditions, and averaged more than 50 percent. However, we have also seen that paper birch maintained the most rapid increase in average stand diameter within the age limits of the study material. Thus, we conclude that a proportion of dominants and codominates as high as 75 percent in itself is not a danger signal for reduced growth in paper birch.

The opposite situation, a low percentage of stems in the upper crown canopy, has implications for those interested in tree quality. This will be discussed later.

The question might arise as to why the species should differ in crown-class distribution and not in differentiation of total height. A partial answer is that the two distributions differ. Total height distribution may or may not be normal; crown class is binominal or multinomial.

Then, it appears that the crown class into which a species crowds its stems has little or no effect upon the variance of total height. Paper birch, with 51 percent of its stems in the dominant and codominant crown class, had a variance of 6.14; sugar maple, with 57 percent of its stems suppressed, had a variance of 14.89. Still the variances of all the species at their mean heights, were homogeneous.

There are also differences in total-height and crown-class observations. Total height is an absolute measurement and is subject to less error than an estimate of crown class, which is partially subjective. All in all, there are apparent inconsistencies for which proven explanations are lacking.

If the general objective of thinning were to shift the volume growth to selected trees in a stand, what, how much, and how often something should be done depends upon the species.

Differentiation Into Quality Classes

In young hardwood stands, many tree-quality characteristics — relative length of clear bole, branching habit, and fullness of crown — are closely related to a tree's crown classification. These help provide estimates of future stem quality. It follows, therefore, that interpretation of the quality-class data relies heavily upon the crown-class analyses.

Many aspects of the original hypotheses concerning quality classes were sustained. While all three species do not differ in differentiation of quality classes, paper birch differs significantly from sugar maple and yellow birch. It has twice as many prime and good quality stems, on the average, as sugar maple and yellow birch. And these differences between paper birch and the other two species are related to age and site.

There is consistent interaction between site or age and quality-class distributions, and few definite patterns were found. In each species, the cell that tended to have one of the highest concentrations of prime-plus-good stems was age class 2-site class 2. The only biological inference here is that the slower growth of stems in this cell favors some of the attributes of better quality stems.

A special analysis of the quality-class data for dominant and codominant stems only showed that here the species were remarkably alike in quality-class distribution. Thus most of the differences in quality-class distributions among species are directly related to the distributions into crown classes.

Both sugar maple and yellow birch had only 15 percent of their stems in the better quality classes. There was roughly a 1:2 ratio of better quality stems to dominant-plus-codominant stems:

Species	*Ratio of prime-plus-good stems to dominant-plus-codominant (percent)*
Sugar maple	58
Yellow birch	43
Paper birch	63

It is true that by merely removing poorer quality stems from the upper crown canopy, the ratio would be raised. But this would not provide any more better-quality stems. The problem for the silviculturist interested in growing high-quality hardwoods is to increase the actual numbers of better quality stems in the upper crown canopy.

There are several inferences from this study for those interested in growing hardwoods for quality in even-aged stands. Under natural conditions, there are not many prime and good quality stems of sugar maple and yellow birch to favor in silvicultural treatments. The better quality stems are found almost exclusively in the main crown canopy. This is where treatment would be centered; increases in the number of better quality stems are directly related to the maintenance of stems in the main crown canopy.

Sugar maple maintained the lowest percentage of dominants and codominants; yellow birch was next.

While the trend in main crown-class distribution with increased age was slightly downward for sugar maple and yellow birch, offhand this would not appear enough to justify intervention to improve quality distribution. But one must recognize that the youngest sugar maple and yellow birch stands sampled were 17 years old. By this age, the proportion of dominant and codominant stems may have already reached levels so low that it would be difficult to maintain enough better quality stems.

This sustains the earlier recommendations of Schädelin (1942) to start cleanings as soon as the stands close. However, for a species with a differentiation pattern like that of paper birch, the necessity for early intervention is greatly reduced (for the objective of altering crown-class distribution).

This situation has been discussed frequently in European literature in recent years (Köstler, 1952; Kunz, 1953; Kurth, 1946; Schädelin, 1942; Van Migroet, 1956). The general findings are that stems in the intermediate crown class cannot be brought up to codominant or dominant positions; but by early intervention, a greater percentage of trees can be held in the upper crown canopy.

Limitations

From the data gathered and analyzed in this study, we can allude to the complex inherited-environmental relationships. We now have a predictive ability, with no indication of the universality of the statistics, and little understanding of the underlying factors.

Our interest in measuring stand differentiation ability in young forests comes from inferences we may wish to draw concerning the future development of the stands. We are not concerned here with end use of the trees in the stand. Rather, we are concerned with prediction of the future differentiation in the stand, with estimates of how the stand might respond to treatments to meet various objectives, and with understanding of the factors at work.

The predictive ability may suffice for the present state of silvicultural practice. But, for the researcher, this predictive ability is insufficient. Questions are left unanswered, interactions are unexplained.

New knowledge will come from additional stand studies. In such studies there should be control of the variables — plots replicated in site, age, density, growing stock. The stand studies will be enhanced by individual stem analyses, by studies of the heritability of characteristics, and by physiological studies of tree and stand growth.

Bibilography

Bickford, C. Allen, et al.
1957. STOCKING, NORMALITY, AND MEASUREMENT OF STAND DENSITY. Jour. Forestry 55: 99-104.

Braun, E. Lucy.
1950. DECIDUOUS FORESTS OF EASTERN NORTH AMERICA. 596 pp., illus. Blakiston Co., Philadelphia.

Curtis, Robert O., and Boyd W. Post.
1962. SITE-INDEX CURVES FOR EVEN-AGED NORTHERN HARDWOODS IN THE GREEN MOUNTAINS OF VERMONT. Vt. Agr. Expt. Sta. Bull. 629, 11 pp.

Darwin, Charles Robert.
1859. THE ORIGIN OF SPECIES. 502 pp. J. Murray, London.

Deen, J. Lee
1933. SOME ASPECTS OF AN EARLY EXPRESSION OF DOMINANCE IN WHITE PINE (PINUS STROBUS L.). Yale Univ. School Forestry Bull. 36, 34 pp.

Gilbert, Adrian M.
1960. SILVICAL CHARACTERISTICS OF YELLOW BIRCH. U. S. Forest Serv. Northeast. Forest Expt. Sta., Sta. Paper 134, 18 pp., illus.

Gilbert, Adrian M., and
Victor S. Jensen.
1958. A MANAGEMENT GUIDE FOR NORTHERN HARDWOODS IN NEW ENGLAND. U. S. Forest Serv. Northeast. Forest Expt. Sta., Sta. Paper 112, 22 pp., illus.

Godman, R. M.
1957. SILVICAL CHARACTERISTICS OF SUGAR MAPLE. U. S. Forest Serv. Lake States Forest Expt. Sta., Sta. Paper 50, 24 pp.

Hauch, L. A.
1905. ÜBER DAS SOGENANNTE AUSBREITUNGSVERMÖGEN UNSERER HOLZARTEN. Allg. Forst. u. Jagd Ztg. 49: 41-45.

Hauch, L. A.
1910. ZUR VARIATION DES WACHSTUMS BEI UNSEREN WALDBÄUMEN MIT BESONDERER BERUCKSICHTIGUNG DES SOGENANNTEN AUSBREITUNGSVERMÖGENS. Forstwiss. Centbl. 54: 565-578.

Hauch, L. A., and A. Oppermann.
1898. HAANBOG I SKOVBRUG. 788 pp. Copenhagen, Denmark.

Hutnik, Russell J., and
Frank E. Cunningham.
1961. SILVICAL CHARACTERISTICS OF PAPER BIRCH. U. S. Forest Serv. Northeast. Forest Expt. Sta., Sta. Paper 141, 24 pp., illus.

Johannsen, W.
1909. ELEMENTE DER EXAKTEN ERBLICHKEITSLEHRE. Revised ed., 515 pp. Fischer; Jena, Germany.

Köstler, Josef N.
1952. ANSPRACHE UND PFLEGE VON DICKUNGEN. 80 pp., illus. Berlin, Germany.

Kunz, Robert.
1953. MORPHOLOGISCHE UNTERSUCHUNGEN IN NATÜRLICHEN FOHRENDICKUNGEN. Mitt. Schweiz. Anst. forstl. Versuchsw. 29 (2): 335-402.

Kurth, Alfred.
1946. UNTERSUCHUNGEN ÜBER AUFBAU UND QUALITÄT VON BUCHENDICKUNGEN. Mitt. Schweiz. Anst. forstl. Versuchsw. 24(2): 581-658.

Lönroth, Erik.
1926. UNTERSUCHUNGEN ÜBER DIE INNERE STRUKTUR UND ENTWICKLUNG GLEICHALTRIGER KIEFERNBESTÄNDE. Acta Forest. Fenn. 30: 1-268. Helsingfors.

Lutz, H. J.
1932. RELATION OF FOREST SITE QUALITY TO NUMBER OF PLANT INDIVIDUALS PER UNIT AREA. Journal of Forestry 30: 34-38.

Meyer, Walter H.
1930. DIAMETER DISTRIBUTION SERIES IN EVENAGED FOREST STANDS. Yale Univ. School Forestry Bull. 28, 105 pp.

Schädelin, Walter
1942. DIE AUSLESEDURCHFORSTUNG

Als Erziehungsbetrieb Höchster Wertleistung. Ed. 3, 147 pp., illus. Haupt; Berne, Switzerland.

Snedecor, George W.
1956. Statistical Methods Applied to Experiments in Agriculture and Biology. Ed. 5, 534 pp. Ames, Iowa.

Society of American Foresters.
1958. Forestry Terminology. Ed. 3, 97 pp. Washington, D. C.

Society of American Foresters.
1954. Forest Cover Types of North America (Exclusive of Mexico). 67 pp., illus. Washington, D. C.

Toumey, James W., and
Clarence F. Korstian.
1947. Foundations of Silviculture Upon an Ecological Basis. Ed. 2, 468 pp., illus. New York.

United States Department of Agriculture.
1938. Soils and Men. U. S. Dept. Agr. Yearbook. 1,232 pp., illus.

United States Department of Agriculture.
1941. Climate and Man. U. S. Dept. Agr. Yearbook. 1,248 pp., illus.

Van Migroet, Marcel.
1956. Untersuchungen Über Den Einfluss Der Waldbaulichen Behandlung und Der Umweltsfaktoren Auf Den Aufbau und Die Morphologischen Eigenschaften von Eschendickungen im Schweizerischen Mittelland. Mitt. Schweiz. Anst. forstl. Versuchsw. 32 (6): 229-370.

Walker, Helen M., and Joseph Lev.
1953. Statistical Inference. 510 pp. New York.