Analyzing Uncertain Timber Investments

by
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U. S. FOREST SERVICE RESEARCH PAPER NE-23
1964

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FOREST SERVICE, U. S. DEPARTMENT OF AGRICULTURE
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THIS paper is addressed to those who are interested in analyzing timber acquisition, protection, and management projects from a financial point of view. Financial appraisals of proposed forestry projects are undertaken to assign a profit index to the project, based on estimates of project costs and returns. Profit estimates are used increasingly by both government and industry as one important basis for choosing among competing projects.

Unfortunately, analysis does not always provide an accurate estimate of a project's financial outcome. Available information, particularly about future yields and prices, is often too unreliable to provide anything more than a rather uncertain prediction.

The uncertainty problem is compounded by the way that most analyses are prepared. Often the analyst presents only a single estimate of profit — a best guess. This practice tends to obscure

[1] This paper is based in part on a dissertation presented to the faculty of the Graduate School of Yale University in candidacy for the degree of Doctor of Philosophy in 1961.

the degree of uncertainty associated with the project, and the investor is left without any way of appraising the reliability of the profit estimate.

Presented here is a method of analysis that makes use of multiple estimates of project profitability stemming from different assumptions about the true values of uncertain factors like yields and prices. The purpose of the analysis is to screen competing projects to eliminate those that can be proved to be unprofitable, or surely less profitable than some other, regardless of the assumptions made about the values of uncertain factors.

When there is relatively little uncertainty about the true values of profit determinants, this screening identifies a single, most profitable project by discarding all others. When uncertainty is more pervasive, the screening ends with several competing projects remaining, no one of which can be shown to be more profitable than the others in all possible circumstances. Only in this last case does uncertainty have any operative significance for choice.

The purpose of this screening is not to reduce uncertainty—that can be done only by obtaining better information—but rather to identify just how much operative uncertainty exists. When more than one project does remain, the final choice will still be uncertain, and a matter of judgment, based on the investor's attitude toward risk-taking and his personal prognosis of relevant future events. Sometimes a formal decision rule embodying these personal judgments is useful in making a final choice, and some of these are discussed briefly.

Taking Uncertainty Into Account

RECOGNIZING UNCERTAINTY IN ANALYSIS

Three Types of Investments

A timber-acquisition or management project eventually results in one particular financial outcome. But it is what can be determined about this outcome beforehand, when the investment is made, that is important in decision-making. Three classes of preknowledge about outcomes have been distinguished: certainty, risk, and uncertainty (Knight 1921).

An investment is classed as certain if it is known to lead to one specified outcome without any serious doubt, as many persons feel is the case with a U. S. Government savings bond for example. An investment is classed as a risk if it leads to one of several outcomes, each of which occurs with a known probability. Lotteries, games of chance, and insurance are based on probabilistic outcomes of this sort, but investments in production do not often fall in the risk class.

Uncertain investments may lead to any one of many different outcomes also, but in this case outcome probabilities are unknown. Uncertainty means not only that the investor is unsure of what the outcome of an investment will be, but also that he cannot objectively rank the possible outcomes on the basis of their likelihood. Most business investments tend to be uncertain, at least to some degree; and investments in timber are no exception.

An Example

A somewhat different method of analysis has been suggested for risky or uncertain investments than is usual for investments that fall into the certainty class (e.g., Chernoff and Moses 1959). A simple example will help to illustrate the differences in these two systems.

Perhaps the most frequent investment decision in timber management is whether to delay the harvest of a merchantable stand for an additional period of growth. If the stand is harvested, the owner in effect liquidates an investment in growing stock, receives its value in the market, and then must find new uses for this capital. If harvest is delayed, the growing-stock investment is maintained and earns a return related to the increase in stand value that occurs during the ensuing period. Suppose that a stand of timber has a present stumpage value of $3,000, and that 5 years hence it will have a value of $3,400. The owner will buy U. S. Series E Savings Bonds with his stumpage income whenever harvest occurs. Should harvest be delayed 5 years?

If the owner of this stand wishes to maximize his dollar income, if future rotations are not of interest to him, and if the land is not to be sold, then the answer depends on whether a $400 increase in stumpage value is more than the increase in bond value he might receive as an alternative. This problem of choice can be summarized as follows:

Alternate courses of action	*Outcome as measured by market value 5 years hence*
1. Delay harvest	$3,400
2. Harvest and buy bonds	3,585

Investments like this, with outcomes that are quite certain, can be analyzed in a straightforward manner that leads to unequivocal decisions. This stand should be harvested now because the bond alternative will produce close to $600 of income during the period as compared with the $400 that can be earned by delaying harvest.

Of course there is usually some question about future growth rates and market prices in a problem of this sort, and it is apparent that the outcome of delaying harvest will be contingent upon the growth rate and stumpage price that actually materialize. Also, delaying harvest means that the stand will be subject to fire, pest, and weather losses for another 5 years, which could endanger present volume as well as future growth.

On the bad side then, it is conceivable that the stand might be destroyed by fire, or that there might be a depressed market for stumpage 5 years hence. On the good side, it is possible that growth will be better than expected, or that stumpage prices will increase to such an extent, say, that the stand doubles in value during the period. So delaying harvest might result in a stand value in 5 years of anywhere between, say, $1,000 and $6,000. Thus, when the analyst wishes to recognize that outcomes are risky or uncertain, a different form of analysis is called for:

Alternate courses of action	Outcome as measured by market value 5 years hence		
	A	B	C
	Poor prices or high losses (dollars)	Expected growth and prices (dollars)	Exceptional prices (dollars)
1. Delay harvest	1,000-3,000	3,000-4,000	4,000-6,000
2. Harvest and buy bonds	3,585	3,585	3,585

The different sets of circumstances listed under A, B, and C embody different assumptions about losses, growth rates, and prices; and it is clear that the best choice between harvesting now and delaying harvest depends on which of these sets of circumstances most nearly mirrors the future. If A comes about, it would be best to harvest now; if C occurs, harvest should be delayed; and if B happens, the analysis does not indicate which alternative will be most profitable.

This analysis differs from the first one in that it considers several different assumptions about underlying circumstances. This is the way uncertainties about these determinants of profitability are introduced into the analysis. Notice that the analysis shifts the uncertainty from outcome to these underlying factors. The question is no longer what outcome will materialize, but which particular set of circumstances will come into being. The unique contribution of this form of analysis is that it shows exactly how outcomes are affected by changes in circumstance, relationships that the first analysis cannot disclose.

Another important difference is that this analysis will indicate when uncertainties are serious enough so that one cannot be sure which alternative will be best. In some cases one alternative will have a better outcome than others in all circumstances that it seems relevant to consider, while in other cases the alternative with the best outcome will change from one set of circumstances to the next. When the latter happens, as it does in the harvest-delay example, it is a signal that available information is too unreliable to allow any analysis to demonstrate that one alternative will surely be better than all the others. The decision-maker then has the choice of: (1) delaying the decision in hopes of obtaining better information; (2) obtaining a contract or agreement that reduces the uncertainty (for example, a forward pricing agreement); or (3) making an immediate judgment decision as best he can.

CHOOSING AMONG UNCERTAIN ALTERNATIVES

Some Criteria for Choice

Since a comparison of costs and returns does not always lead to a clear preference among alternatives when uncertainty is recognized, other decision tactics have been suggested. Most of these take the form of simple rules.

Several analysts have suggested decision tactics which insure against unwanted outcomes in choosing among investments with uncertain outcomes. Wald (1950) suggests choosing the alternative with the highest minimum outcome over all sets of circumstances considered. Savage (1951) has proposed that outcomes be replaced by "regret" values, which substitute for each outcome the amount that would have to be added to make each equal to the largest outcome of any alternative in that particular set of circumstances. Thus the highest outcome for alternatives in a particular set of circumstances is replaced by zero, and an outcome of 10 less than the highest is replaced by 10. These substituted values are taken as a measure of the loss or regret one would experience by choosing the wrong alternative; and Savage suggests choosing the alternative with the lowest maximum regret over all sets of circumstances considered.

These approaches are very conservative, because they both assume that the worst will happen. Hurwicz (1951) has developed a means of adjusting the choice criteria to reflect a more optimistic outlook. He asks the decision-maker to choose a number between zero (entirely optimistic) and unity (entirely pessimistic), which he terms a *pessimism index.* A weighted value is then computed for each alternative, made up of the product of the pessimism index and the minimum outcome for the alternative, plus the product of one minus the pessimism index and the maximum outcome. The alternative with the highest weighted value is then chosen. This criteria becomes equivalent to Wald's when the pessimism index is unity, but does take into account the best outcome as well for smaller index values. However, it still takes no account of sets of circumstances that give rise to intermediate outcome values.

The principle of insufficient reason, attributed to Laplace, can be used to take all circumstances and outcomes into consideration. This principle says that since there is no objective information about the relative likelihood of the various sets of circumstances, all should be treated as equally likely. One applies this criteria simply by averaging the outcomes for each alternative and choosing the one with the best average.

These four decision rules all assume that absolutely nothing is known about the relative probability of the various sets of circumstances considered. Savage (1954) has suggested a subjective probability theory to treat situations where, intuitively at least, some assumptions about uncertain factors seem more plausible than others. He suggests a method of transforming these opinions into a consistent subjective probability distribution for the sets of circumstances considered. The outcomes for each action are then weighted by the subjective probabilities associated with the corresponding circumstance sets, and these are summed to give a subjective expected value for the action or alternative. The alternative with the highest weighted value is selected.

When there are risk outcomes to deal with — that is, when the various possible outcomes and their underlying sets of circumstances have a known and objective probability distribution—then the most commonly suggested decision rule is to choose the project with the highest weighted average outcome — expected value, as it is termed. Since risk probabilities are objective, or what have been termed "actuarial certainties," the alternative with the highest expected value can really be said to have the best chance of being most profitable. This is in contrast to Savage's subjective probability approach, where no such interpretation would be valid.

All Choice Criteria Require or Assume That Outcome Probabilities Are Known

When the outcome of investment alternatives are certain, analysis leads to an unequivocal choice on the basis of outcome comparisons. When investments have risk outcomes, analysis provides an objective estimate of the alternative that is most likely to be most profitable. Highest expected value is not a perfect guide to choice, and it is not the only characteristic of importance in choosing among risk alternatives; but this criterion will lead to the right choice more frequently than any other.

When there is absolute uncertainty about outcomes — that is, when as far as is known one underlying set of circumstances is just as likely as any other — no particular choice criteria can be singled out as best. However, most of the criteria are logical only under a particular assumption about the likelihood of the various circumstances and their outcomes. It is logical for a decision-maker to use Wald's "maximin" criteria or Savage's "minimax regret" criteria only if he really believes that the worst outcome is the only one that has much chance of occurring. It is logical for him to use Laplace's "equal likelihood" criteria only if he really believes that all outcomes are equally likely, and so forth.

The various approaches to the problem of absolute uncertainty all point to the idea that the decision-maker must have some information or belief about the likelihood of various underlying circumstances and their outcomes if he is to have a logical preference for one decision rule over another. Savage has developed a system of making explicit an individual's subjective estimates of likelihood. Of course, when a complete likelihood distribution for the various sets of circumstances has been developed, one can dispense with special decision rules and use the more general notion of weighted average outcome as the basis for choice.

Perhaps the major point to be made here is that although one can develop a consistent, subjective probability distribution as the basis for applying the expected-value choice criteria, this simply throws the subjectivity back one more stage. Intstead of selecting a special choice criteria subjectively, one chooses likelihoods for the various outcomes subjectively and applies a general choice criterion. The expected value criterion is no more likely to lead to the best choice than any other when expected values are based on subjectively derived probability estimates.

USING ANALYSIS TO DISCARD ALTERNATIVES

All the foregoing decision rules attempt to single out the one best alternative. When there is a significant degree of uncertainty, this cannot be done without making assumptions about the probabilities of various circumstances and outcomes that may be untrue and so misleading. However, analysis can begin with the more limited objective of excluding clearly non-optional alternatives. It is possible to exclude alternatives that are not sufficiently profitable under any set of circumstances to interest the investor, to exclude alternatives that are less profitable than some other alternative in every circumstance, and to do this without reference to the probabilities of these circumstance sets. This initial screening may leave no acceptable alternatives, or only a single acceptable alternative, or several. In the first two cases analysis has shown the uncertainty to be negligible or unimportant. In the last case there is still a decision problem to be resolved, but analysis has simplified and clarified it as much as the available information allows.

When uncertainty is recognized, then, analysis may not always be able to identify the best alternative. Rather, analysis must be thought of as a means of excluding the worst alternatives. Thus, the analysis defines the amount of operative uncertainty in any decision situation by the number of alternatives that it cannot exclude. The analogy to the general scientific method is clear. The analyst hypothesizes that each alternative considered is both profitable and better than any of the others, and in the analysis he attempts to disprove this hypothesis for as many alternatives as he can.

The first task in any investment analysis is to formulate the investment problem. This is done by determining the investor's primary goal or objective, selecting a set of alternative investment possibilities that the investor can undertake in the near future with the investment resources at his command and that are relevant to his primary investment objective, and choosing a measure of economic efficiency, such as internal rate of return, benefit/cost, or present worth, as a basis for comparing alternatives. Further steps in the analysis of uncertain investments include (1) developing a system for predicting the outcome of each investment alternative under a variety of circumstances, (2) setting up formal comparisons of the investment alternatives and discarding those that are not sufficiently profitable or are non-optimal, and (3) posing a question of choice for the investor among the alternatives that are left. The remaining sections of this paper consider these analytical tasks in detail.

Analyzing Uncertain Investments

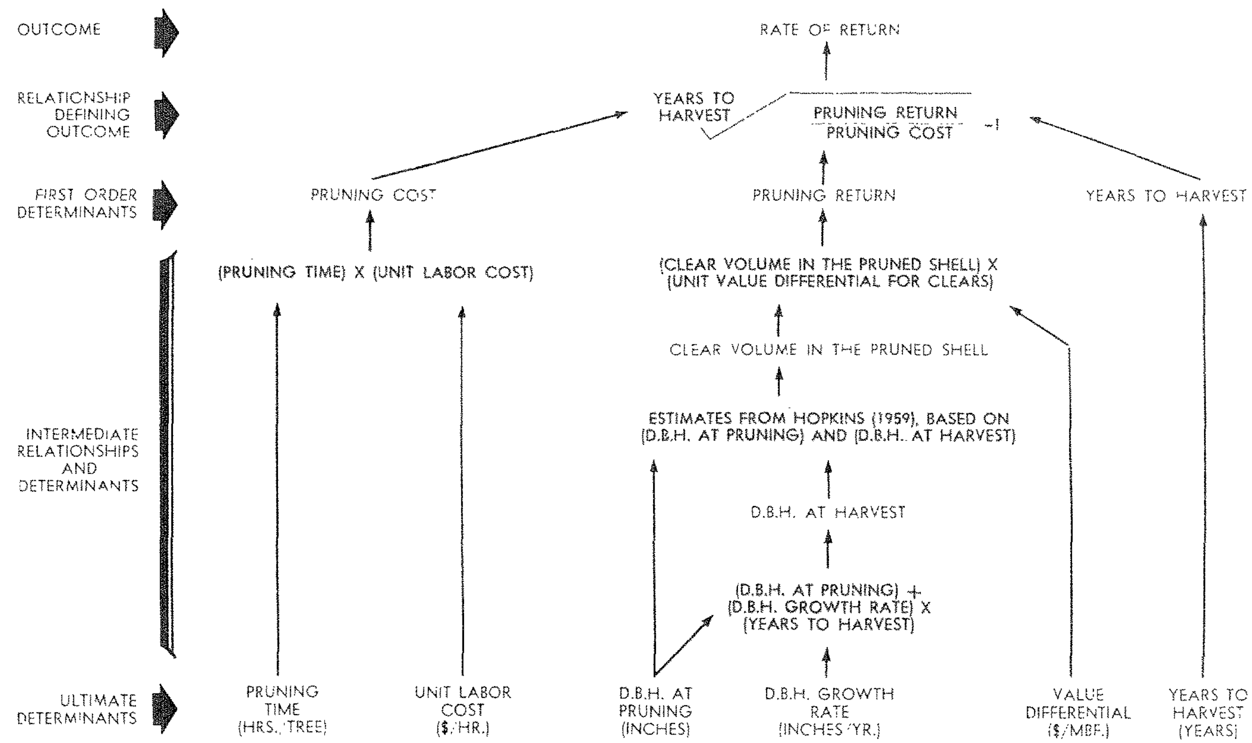
PREDICTING OUTCOMES FOR PROJECTS

Outcome Predictions Are Based on a System of Relationships

The economic outcome of an investment is determined by the costs it occasions and returns it generates. These costs and returns occur at different dates, but all can be brought for comparison to a common point in time with a compound-interest formula. The size of each cost or return is determined by a physical input or output and its unit value. Physical inputs indicate the cost of undertaking the investment in question in terms of man- and machine-hours and quantities of materials, while physical outputs indicate the yield response that is anticipated. These physical quantities and their unit values may be estimated directly or with the aid of empirical relationships. When a relationship is used, the physical quantity or unit value is itself determined by the values of the "independent variables" called for in the relationship. Outcome estimates, then, are built up from a series of interconnected relations. Once such a set of relations is constructed, the analyst can compute outcome by assigning values to each of the ultimate determinants of the system.

As an example, consider the outcome of pruning an eastern white pine crop tree. If butt-lot pruning is done in one stage there will be one cost at the time of pruning and a single return when the tree is harvested. When rate of return is chosen to measure outcome, then a system of relations for predicting the rate of return might be constructed as shown in figure 1. Notice that the system begins with a mathematical definition of rate of return

Figure 1. — The return to pruning white pine: an example of a system of relations defining investment outcome.


OUTCOME
RELATIONSHIP DEFINING OUTCOME
FIRST ORDER DETERMINANTS
INTERMEDIATE RELATIONSHIPS AND DETERMINANTS
ULTIMATE DETERMINANTS
RATE OF RETURN
YEARS TO HARVEST
PRUNING RETURN
PRUNING COST
-1
PRUNING COST
PRUNING RETURN
YEARS TO HARVEST
(PRUNING TIME) X (UNIT LABOR COST)
(CLEAR VOLUME IN THE PRUNED SHELL) X (UNIT VALUE DIFFERENTIAL FOR CLEARS)
CLEAR VOLUME IN THE PRUNED SHELL
ESTIMATES FROM HOPKINS (1959), BASED ON (D.B.H. AT PRUNING) AND (D.B.H. AT HARVEST)
D.B.H. AT HARVEST
(D.B.H. AT PRUNING) + (D.B.H. GROWTH RATE) X (YEARS TO HARVEST)
PRUNING TIME (HRS./TREE)
UNIT LABOR COST ($/HR.)
D.B.H. AT PRUNING (INCHES)
D.B.H. GROWTH RATE (INCHES/YR.)
VALUE DIFFERENTIAL ($/MBF.)
YEARS TO HARVEST (YEARS)

that identifies its determinants: cost, return, and investment period. Supporting empirical relationships are used to estimate these "first order" determinants, and so on until ultimate determinants are identified that must be measured or estimated directly.

In this example, if one uses $100 per 1,000 board feet for the added value of the clear volume in the pruned shell, 8 inches for the d.b.h. at pruning, 0.2 inches for annual rate of d.b.h. growth, 40 years to harvest, 1/6 hour for pruning time, and $1.50 per hour for labor cost, then the system estimates a rate of return to the investment of 7.9 percent. Other determinant values will produce other outcomes.

These estimates are subject to error if any costs or returns are left out of account, or if the empirical relationships included in the system are not exact; but this "model" error is usually small in comparison with the error associated with choosing correct values for determinants. The complexity and accuracy of the model can vary greatly, and they depend on the availability of information and on the analyst's skill in constructing models.

Ranges of Values for Uncertain Determinants

In the pruning example, it is apparent that current knowledge is not sufficient to be sure of what value every determinant should be assigned. The labor wage rate and tree d.b.h. at pruning would presumably be known without significant error. The other four elements are uncertain, at least to some degree. It is possible that pruning of the first log might take anywhere from 5 to 20 minutes (Ralston and Lemmien 1956; Meyer 1940; and Cline and Fletcher 1928). The d.b.h. growth rate might range anywhere from 1/20 to 1/5 inch per year, depending on site quality and competitive conditions. Years to harvest might be anything between 10 and 100; and the added value of clear volume, which at the mill level is now about $100 (Fedkiw et al. 1960) might range from zero, if clear volume is not valued in tomorrow's markets, to $200 if grade/price differentials broaden and most of this value is passed on to the producer in the form of increased stumpage prices.

After the outcome-predicting system is developed, the analyst must decide on the range in values he will consider for uncertain determinants. He can, more or less arbitrarily, drop from consideration some possible values for uncertain determinants. It seems most reasonable to drop the least likely values if any are to be dropped, and there is often an objective basis for believing that some values are less likely than others. To this extent the analyst does not often have to deal with absolute uncertainty. For example, a 5- to 20-minute time range for butt-log pruning has been observed in various cases, but in most studies butt logs took less than 10 minutes to prune. Times in excess of 10 minutes are less likely, perhaps considerably so. Ownership characteristics may allow the analyst to judge some rotation ages as quite unlikely in particular cases, and so forth.

The analyst may wish to reduce the range of determinant values he considers in this way, or he may feel that he must retain all possible values. The advantage of disregarding some of the least likely values is that this narrows the range of outcome for an investment alternative without a commensurate narrowing in the probability of having included the actual outcome. The danger in disregarding some values is simply that the outcome that will actually materialize may not be included in the predicted range. Sometimes analysts disregard all but one value for uncertain determinants and, in effect, analyze the investment as if its outcome were certain. This might be viewed as one extreme in treating uncertainty, while dealing with all possible determinant values is the other. It seems reasonable and useful to pursue a middle course by disregarding some of the least likely determinant values, when there is an objective and reasoned basis for identifying the values that are indeed least likely. However, it must be stressed that the discarding of determinant values is to some degree subjective, and the truncated ranges of outcome which result cannot be assigned an objective probability density.

COMPARING PROJECTS

Sensitivity Testing

After an outcome-predicting system is developed for each alternative, the next step is to test the system by determining how outcome changes when determinant values are changed. The analyst can judge the effect of each determinant in this way, and he may decide to simplify his prediction system by using a single arbitrary value for an uncertain determinant that changes outcome very little regardless of what value it is assigned within its range. This "sensitivity testing" should be an initial step in analyses of this sort. It requires that a sequence of outcome values be generated for each alternative.

Such a sequence is shown in table 1 for the pruning example. Notice that two or more determinant values are used to represent the range of each uncertain determinant. In this case outcome, measured by rate of return, varies significantly from one set of determinant values to another, indicating that none of the uncertain determinants have a negligible affect within their ranges; and so all determinants are retained. The essential test for each determinant is whether it changes outcome significantly when other determinants are held constant.

Table 1.—Rates of return to pruning[1] under various assumptions about the values of determining factors

Growth rate	Pruning time	Time to harvest	Additional value of clear volume, in dollars per M board feet— 50	100	150
Inches per year	*Minutes per tree*	*Years*	*Percent*	*Percent*	*Percent*
0.10	10	60	2.6	3.8	4.5
		50	1.4	2.8	3.9
	5	60	3.8	5.0	5.7
		50	2.8	4.2	5.1
0.15	10	60	4.4	5.7	6.4
		50	4.5	6.0	6.8
		40	3.9	5.7	6.8
	5	60	5.7	6.9	7.6
		50	6.0	7.4	8.3
		40	5.7	7.6	8.6
0.20	10	60	5.5	6.7	7.4
		50	5.8	7.3	8.2
		40	6.0	7.9	9.0
		30	5.2	7.7	9.1
	5	60	6.7	7.9	8.2
		50	7.3	8.8	9.7
		40	7.9	9.8	10.9
		30	7.7	10.2	11.7

[1] Assumes pruning of an 8-inch d.b.h. white pine when the labor wage rate is $1.50 per hour.

Independent Determinants and Project Comparisons

Although all uncertain determinants can be used to define sets of circumstances for explicit consideration — or "states" as they are sometimes termed — this is not always desirable. The purpose of considering individual sets of circumstances, or states, separately is to facilitate comparisons among the outcomes of the various alternatives. These within-state comparisons are most revealing when the determinant values defining each state must be the same regardless of what alternative is chosen.

This is true only for determinants with values that are not significantly influenced by the choice that is made among alternatives. Determinants reflecting market conditions, over which many stumpage producers have little or no control, are good examples of these "independent" determinants. For most producers, a particular system of stumpage prices will obtain at a given date regardless of what immediate choice among management programs he makes. Therefore it is not necessary to compare one program assuming a low unit stumpage price with a second alternative assuming that a high unit price will be paid. Whatever unit price is paid, it is likely to be the same regardless of the intermediate management alternative chosen.

Decisions about stand management that will be made in the future may also fall into this category of independent determinants. For example, although optimum economic rotation age differs according to how intensively a stand is managed (i.e., economic maturity is not independent of intermediate management), in many cases rotation ages are set without reference to value growth rates or other effects of past management. Determinants that relate to future management, then, may be independent enough of immediate management decisions so that comparisons assuming independence are likely to be valid.

A third class of determinants with values that are independent of the alternative selected are those that are unique to a single alternative. Not all investment alternatives have the same outcome determinants. In the example that compared harvest delay with bond purchase there were no determinants that were common to both alternatives, for instance. These unique determinants may also be used to define states of nature since their values cannot be influenced by the choice among alternatives.

When independent determinants are used to define states, the "dependent" determinants are left, and are free to assume any values within their ranges. So the outcome estimate for each set of circumstances will be a range of values rather than a point estimate if there are any dependent determinants. This is because the underlying circumstances are not completely defined in these cases. Some determining factors are left undefined because their values change from alternative to alternative.

An Example of Defining States and Comparing Projects

Consider the problem of deciding which of two neighboring pine trees is to be pruned. The first tree is now 10 inches d.b.h., while the second is an 8-inch tree. The unit cost of labor is $1.50 per hour. The four uncertain determinants are, again, the subsequent growth rate, the time required for pruning, years to harvest, and the added value of clear volume at harvest. In this case both alternatives have exactly the same outcome determinants. Two of these might be considered independent of the choice: years to harvest and the value differential. In even-aged management, each of these determinants will have the same value regardless of which tree is pruned.

The other two determinants, growth rate and pruning time, might have different values for each of the trees. Suppose that the analyst chooses to consider the following value ranges for these four determinants: years to harvest, 40 to 60; value differential, $50 to $150; growth rate, 0.15 to 0.20 inches per year; and pruning time, 5 to 7 minutes per tree. He sets up nine different states made up of different combinations of values for the two independent determinants, years to harvest and the value differential. He then computes a range of outcomes for each state determined by the extreme values for the dependent factors. The results are shown in table 2. Notice that regardless of the state that comes into existence, the return ranges for the two trees are very similar.

Consider a contrasting example where, again, two trees are compared for their return to pruning. In this case both trees are the same size, but one is expected to grow more rapidly than the other. The first tree is expected to grow on the average between 0.15 and 0.17 inches per year, while the second tree is expected to grow between 0.10 and 0.12 inches per year. Table 3 shows the results of this second comparison.

The analysis shows that the faster-growing tree will return more to pruning regardless of the state that obtains, and despite the considerable range in outcome that exists within states. Notice

Table 2.—Rate-of-return ranges for pruning two different sized white pine trees, in percent

Alternate courses of action considered	Added value of clear volume, dollars per M board feet —								
	$50 Years to harvest			$100 Years to harvest			$150 Years to harvest		
	40	50	60	40	50	60	40	50	60
Prune the 10-inch pine	4.9-8.1	5.4-7.5	5.2-7.2	6.8-10.0	6.8-9.0	6.5-8.5	7.8-11.1	7.7-9.9	7.2-9.2
Prune the 8-inch pine	4.8-7.9	5.2-7.3	5.1-6.7	6.7-9.8	6.7-8.8	6.3-7.9	7.7-10.9	7.6-9.7	7.0-8.2

Table 3.—Rate-of-return ranges for pruning two pine trees of different growth rates, in percent

Alternate courses of action considered	Value differential, dollars per M board feet —								
	$50 Years to harvest			$100 Years to harvest			$150 Years to harvest		
	40	50	60	40	50	60	40	50	60
Prune the fast-growing pine	4.3-6.3	4.9-6.3	4.8-5.9	6.2-8.2	6.4-7.7	6.0-7.1	7.2-9.3	7.2-8.6	6.7-7.9
Prune the slow-growing pine	0-3.4	2.3-4.5	3.8-4.7	0-5.2	3.9-6.9	4.4-6.9	0-6.7	4.8-6.2	5.1-6.7

that the entire range in outcome — the range over all states — is 0 to 6.7 percent for the slow-growing tree and 4.3 to 9.3 percent for the fast-growing tree. So if the analyst computed only these total ranges he could not assert that the faster-growing tree would surely have the better outcome, since these total ranges overlap. It is only because states defined by the independent determinants are used that comparisons show the operative uncertainty to be negligible.

Discarding Strongly Dominated and Unprofitable Alternatives

This last example demonstrates one extreme in outcome comparisons. If one alternative has an outcome range that is entirely below the outcome range of some other alternative in every set of circumstances considered, then it is said to be strongly dominated. Strongly dominated alternatives can be dropped from consideration by the analyst because they are certain to be less than optimal, within the range of conditions examined.

Unprofitable alternatives can also be weeded out at this point. In most situations there is a definite minimum rate of return that is required by the investor. The analyst can discard alternatives that do not reach this minimum in any state, as being not sufficiently profitable. For example, if the investor wished to make at least 7 percent on his pruning investments, the analyst could discard the slow-growing pine in table 3 as an alternative that was not sufficiently profitable, even if it had not been strongly dominated.

POSING THE QUESTION OF CHOICE

More than one alternative may remain after those that are not sufficiently profitable or that are strongly dominated by others have been eliminated. In these cases there is a problem of choice that the analyst cannot resolve, because he does not possess sufficient information.

These remaining alternatives may be characterized by outcomes of various sorts. At one extreme are the more certain alternatives whose outcomes do not change much from state to state. Other alternatives may have outcomes that vary markedly from one state to the next, moving from heavy losses to large gains. The harvest-delay example contrasted a very certain alternative with one that was rather uncertain, and the contrast is sharp. The first pruning example (table 2) portrays another type of comparison. Here both alternatives, pruning the 8-inch tree and pruning the 10-inch tree, had nearly identical outcomes in all sets of circumstances considered.

When alternatives have overlapping outcome ranges in all states considered, the analysis provides little basis for choosing among them. When the contrasts among outcomes are sharp enough so that one alternative strongly dominates others in at least some states, the analysis does provide some basis for preferring one above the others.

The investor's preference in these cases will depend basically on how he views a rather uncertain alternative as opposed to one that is less uncertain, or on whether or not he has any strong convictions about the likelihood of various states. If he believes that one or more of the states are more likely than the rest, he will probably prefer the alternative that has the better outcome range in these states. If he has no particular feeling about the probability of various states, he may want to make the choice on the basis of performance over all states. Quite commonly this comes down to choosing between a safe alternative with a rather modest return and an uncertain alternative that combines lower returns or even losses in some states with the chance of exceptional gains in others. His choice in such cases will depend on his reactions toward risk-taking.

The analyst's final task is to present the results of his analysis so that these aspects are brought forward and made as clear as possible. In making the final choice among the remaining alternatives, it may sometimes be appropriate to suggest one of the formal criteria for choice discussed on pages 6-8.

The system of analysis presented here is designed to simplify the choice among investments without ignoring the uncertainties involved in predicting their outcomes. The analysis proceeds by discarding investments that are clearly less rewarding than others despite outcome uncertainty, and those that are not sufficiently profitable under any of the sets of circumstances considered to interest the investor.

When outcome uncertainty is slight, the analysis will discard all alternatives but one. When uncertainty is more pervasive, several alternatives will remain; and the analysis will have defined an area for subjective judgment by the investor.

There is no analytical technique that can validly remove the uncertainty about which of the remaining alternatives is best. That can be done only by obtaining better and more complete information. One advantage of this system of analysis is that it goes only as far toward a final solution as available information justifies.

This system of analysis is not easy to apply. It requires repeated estimates of outcome, assuming many differing sets of conditions; and it calls for comparisons among alternatives, which are not always easy to construct. Therefore, uncertainty analysis should not be undertaken casually or applied to problems of minor importance. It does seem to provide a useful middle ground between perfect-knowledge analyses based on a single set of assumptions about determinant values, and completely subjective choices, when the problem of choice is important enough to warrant the necessary analytical costs.

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