CALIBRATING A WATERSHED by using climatic data

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Contents

A Problem of Calibration

TO DETERMINE some of the interrelationships that affect two of our most vital natural resources—forests and water—the Northeastern Forest Experiment Station of the U. S. Forest Service, the Pennsylvania Department of Forests and Waters, and the U. S. Geological Survey began in 1948 a cooperative research project on a tract of fairly typical scrub-oak land in the Pocono Mountain region of eastern Pennsylvania.

The general purpose of this project was to convert the brushy vegetation of the area to forests of commercial timber, and then to determine what effect this change in vegetation has on the water yields from the area.

A 2,089-acre tract of land was made available for the study. Containing the watershed of Dilldown Creek, a tributary to the Lehigh River, this area was dedicated as the Dilldown Unit of the Delaware-Lehigh Experimental Forest.

The first requisite was to calibrate the watershed: to determine the water-yield regimen of the area as it was with its original cover of brushy vegetation. This would serve as a datum basis against which the water regimen could be compared after the land had been converted to commercial forest, thus providing a measure of the change in water yield effected by the change in vegetation.

Meanwhile other studies were designed to test species, site preparation and planting methods, and other treatments for producing the commercial timber stands. These studies will not be discussed here.

The Dilldown Watershed

The Dilldown Watershed (fig. 1) is an area of 1,529 acres lying on the Pocono Plateau in northeastern Pennsylvania. Except for a riparian zone along Dilldown Creek, and several other small isolated spots, the watershed is covered with typical scrub-oak (*Quercus ilicifolia*) vegetation: scrub oak with associations of pitch pine, gray birch, sassafras, red maple, and red oak.

The soils of the watershed are stony to bouldery sandy loams with a leached A_2 horizon, and they range in depth from 2 to 5 feet. The mineral soil is covered with a tough fibrous mor humus that prevents erosion almost completely. There has been no evidence of overland flow.

The parent rock material is made up of two types of sandstone and one type of shale that is found only in a small area. The watershed has been glaciated by one or more of the earlier glaciers and apparently was in the periglacial zone of the Wisconsin glacier. Several boulder fields are present, which, along with numerous boulder streams, make up about 1 percent of the area. Swamps comprise about 5 percent of the area.

The sandstone and shale parent rock material provides a groundwater aquifer of moderate capacity. Analyses by Storey (1951) indicated a capacity of more than 5 inches under the total watershed area. If groundwater were at a high level to start with, groundwater flow would recede to about 0.7 c.f.s in 100 days if there were no groundwater recharge during the period. The lowest daily streamflow during the calibration period was 0.54 c.f.s.

Precipitation at the Dilldown Watershed is well distributed throughout the year. In a 10-year period of measurements, annual precipitation averaged 55 inches, which occurred in an average of 94 storms of 0.02 inch and larger. The highest 12-month precipitation was 76.84 inches; the lowest was 39.73 inches.

The watershed is completely forested, but only about 8 percent of the area is in high forest: riparian hardwoods and conifers along the creek and in swampy areas. The remaining 92 percent is in brush or what is generally called scrub oak. Actually a large part of the brushy area is covered by sprouts of tree species that would form a poor-quality high forest of mixed hardwoods and pitch pine if it were protected from fire for a long time.

A field survey in 1951 and 1952 showed that about half the total area was in the better hardwood and pitch pine types. Stands in which bear oak *(Quercus ilicifolia,* the true scrub oak of this area) predominated covered only about 35 percent of the area, and only 3 percent was in pure scrub oak. Other brush types with little potential value, such as gray birch and aspen, covered the remainder of the area. Nearly all brushy types over-topped a dense ground cover of blueberry, sheep-laurel, and bracken fern.

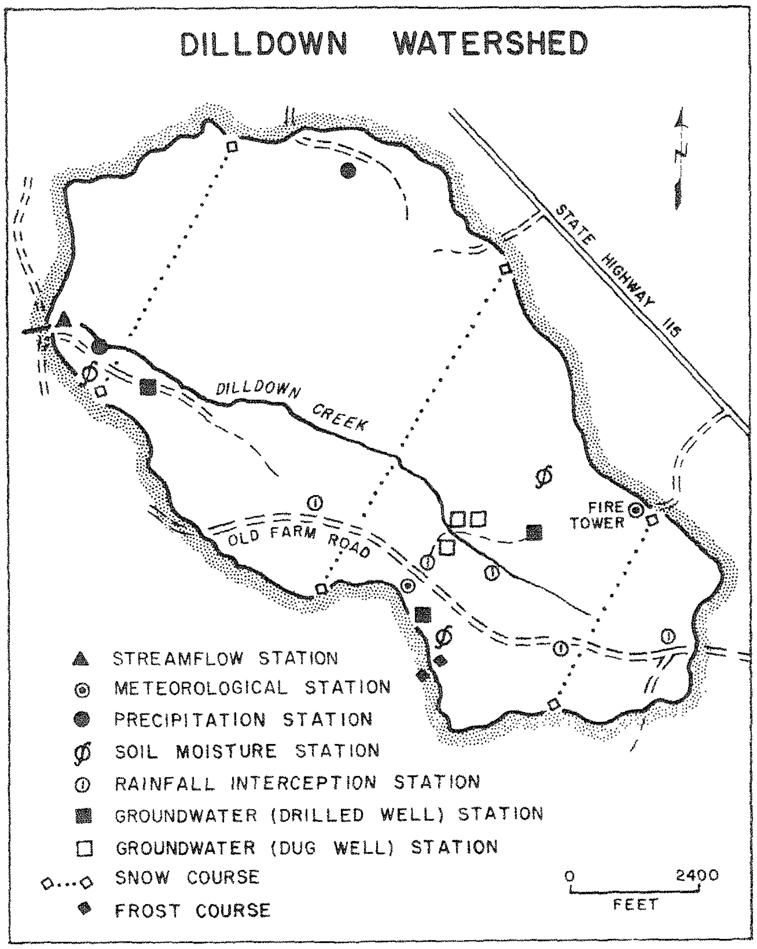


Figure 1. — The Dilldown Watershed, showing location of installations used to gather information used in the calibration.

Because the primary objective of the Dilldown Watershed study is to determine the effect on runoff of converting this brushy area to a high forest, the true scrub-oak areas have been planted to conifers wherever possible since the end of the calibration period. Areas too rocky to plant will be prepared for direct seeding by chemical treatment. Those hardwood and pine areas with sufficient density to form a closed canopy will be left untreated, while less dense stands will be reinforced with conifers. Every possible effort will be made to prevent wildfires from entering the watershed.

The Basic Approach

Choice of Method

There are two general methods of calibrating a watershed. One is to use a control watershed as a basis for comparison with a treated watershed. The other is to use a single watershed and calibrate it upon itself. This single-watershed calibration method was used in our studies.

In the control-watershed approach, streamflow of the untreated watershed is related to streamflow from the control watershed in a regression analysis. Once satisfactory correlation is achieved, the watershed is then treated and the regression can be used to detect significant changes.

To do this, improved analytic procedures, using statistical methods, have been developed by Wilm (1949), and elaborated on by Kovner and Evans (1954) and Reinhart (1956). These techniques have been used successfully to detect rather small changes in streamflow resulting from watershed treatment. They have been employed with good effect in almost all of the major watershed investigations.

The advantage of the control-watershed approach is its reliability: it is an established procedure with a background of experience and success. However, its disadvantages are several: a chance fire can completely change the character of the control; added costs are involved in installing and maintaining the control; it is difficult to locate two or more watersheds that are close together and physically similar; and the possibility exists that vegetation on the control can change after the calibration period.

The single-watershed approach is less costly because no control is involved, and the analysis is more informative because it relates streamflow to the factors that influence it rather than to streamflow from another watershed only. Its principal disadvantage is its complexity: considerably more tabulation and analysis of data are required than with the control-watershed system.

Disadvantages common to both systems are the uncertainty, at the beginning of the calibration period, that correlations will be high enough to achieve usefulness, and the possibility that populations produced during the post-treatment period may be different from the populations produced during the calibration period.

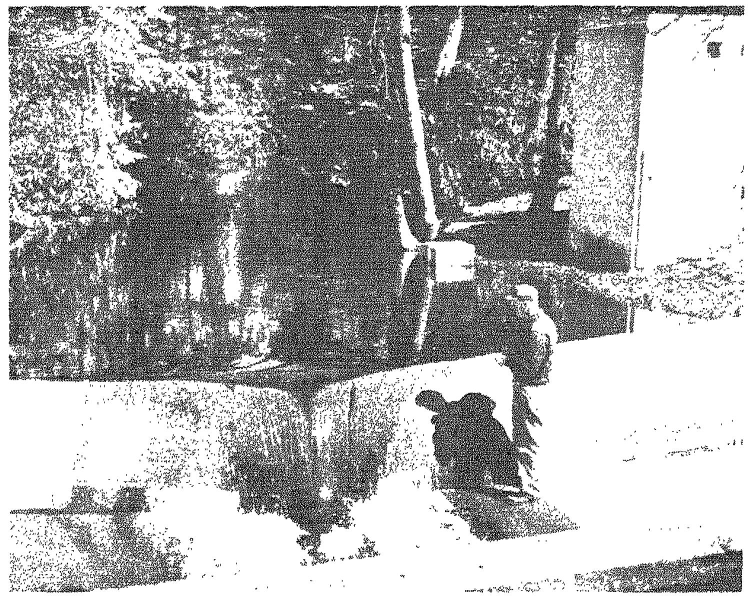
In the single-watershed approach, the watershed is calibrated on itself: streamflow, usually the dependent variable, is related to those factors that influence it. Though this type of calibration has never been fully utilized, theoretically there might be four methods of accomplishing it. First would be a direct comparison of tabulated annual, seasonal, or monthly streamflow adjusted for differences in precipitation. Second, this rather crude approach could be amplified by plotting the accumulated value of precipitation and runoff, giving mass curves whose trend may indicate land-use effects. Mass curves, however, provide no measurement of random error. A third method is to detect changes due to treatment in the water balance procedure in which every bit of water is accounted for on a daily basis—so far an impractical method.

The fourth method, which we employed, is to use streamflow and climatic data obtained during the calibration period, and develop regression equations to predict monthly and annual runoff in terms of climatic factors. These equations will be applied during the post-treatment period and the estimated runoff will be compared with actual runoff. The differences, if any, will be tested statistically for significance and will be a measure of the effect of treatment. Schneider and Ayer (1961) have used this method to relate annual and seasonal runoff to precipitation and time-trend variables for several watersheds. They used relatively long-time records, which ranged from 19 to 25 years.

Streamflow and Climatic Records

The data used in this calibration analysis were obtained between October 1, 1948, and September 30, 1954, at the Dilldown Watershed. They include measurements of daily streamflow, precipitation, air temperature, and humidity. Groundwater and soil-moisture storage data were also used. These data have been published by the Pennsylvania Department of Forests and Waters in a series of three reports describing the Dilldown studies

Figure 2. — The weir and instrument shelter, where the streamflow regimen of Dilldown Creek was recorded.



(*Storey* 1951; *Storey, McQuilkin,* and *McNamara* 1953; *Reigner, McQuilkin, McNamara,* and *Lull* 1955). All data used in this analysis were included in the published reports.

Streamflow was measured at an automatic recording station (fig. 2) built and maintained by the U. S. Geological Survey. It consists of a Columbus deep-notch weir and Stevens A-35 water-storage recorder. All streamflow data were computed and tabulated by the Hydrographic Service of the U. S. Geological Survey and the Pennsylvania Department of Forests and Waters.

Precipitation data were the simple averages of the amounts caught in four recording gages spaced fairly uniformly around the margins of the watershed (fig. 3). All gages were equipped with Alter-type windshields. Temperature and humidity were measured at two standard Weather Bureau type climatic shelters, each of which included maximum and minimum thermometers and a recording hygrothermograph.

Groundwater storage at the beginning and end of each month was calculated according to methods outlined by Storey (1951). In the first method, the average level of two groundwater wells was related to the groundwater yield of that level. The second method utilized the base-flow/groundwater-yield relationship. Results of the two methods checked closely. However, the first method was more accurate during summer periods, when riparian losses obscured base-flow observations; and the second method was more accurate during fall recharge periods, at which times one of the wells lagged behind actual recharge.

The soil-moisture storage values used in the analysis were the weighted averages of measurements at four stations located within the watershed. At each station, a stack of seven fiberglas Colman resistance units had been installed in a vertical series corresponding to the different soil horizons. These installations measured soil moisture to a depth of 27 inches, below which the soil was too rocky to insert the units. The measurements did not begin until October 18, 1949, about a year after the other measurements were started. They were continued, with some exceptions, five times per week throughout the calibration period. Some interpolations were required for the month-end values when they occurred on weekends.

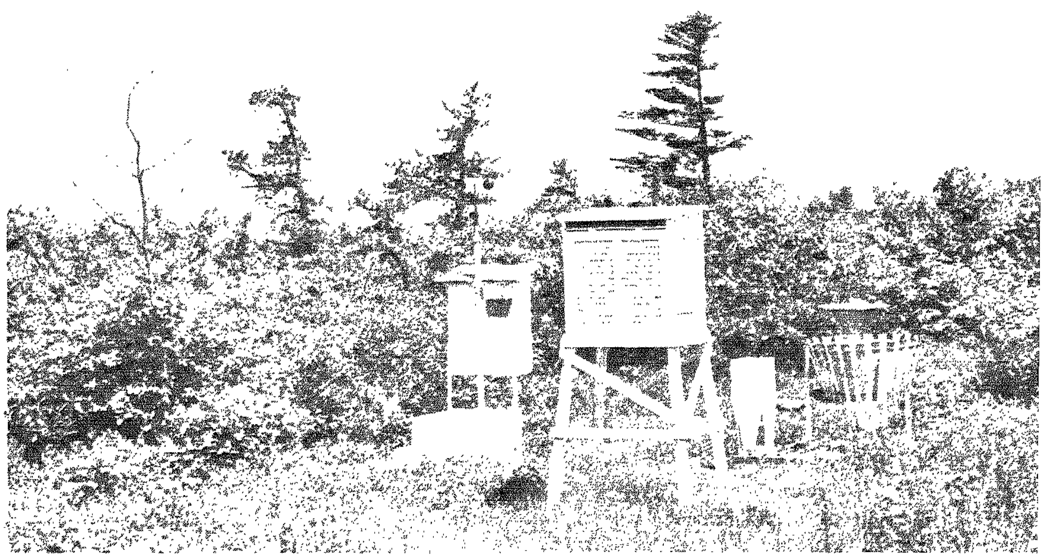


Figure 3. — One of the stations used to record climatic data on the Dilldown Watershed. The vegetation in the background is fairly typical of the scrub-oak cover on the area.

Limitations in Data

Storage values ranged considerably in accuracy. Groundwater storage estimates were fairly accurate. Soil-moisture values were weighted averages of the measurements at four index stations. Their accuracy on a watershed basis was only fair. Snow storage was measured periodically on snow-course lines at other permanent stations. On a watershed basis, snow-storage data were relatively inaccurate.

Measured storage values were not admissible because they would be subject to change by treatment. But they were useful in defining the relationships between runoff and streamflow. In a later analysis, they were in turn estimated from climatic data and the estimates were inserted in the equations.

For statistical purposes, annual and seasonal comparisons were limited by the 6 years of record which, by restricting degrees of freedom in multiple regression, sets up drastic limitations for statistical significance.

The use of monthly data, by providing a greater number of samples, overcomes the limitation of degrees of freedom. However, here the question of serial correlation is involved; i.e., the monthly values might be not independent but related through antecedent effects with each other. This objection was largely taken care of by using antecedent effects in the prediction equation.

The prediction of annual and monthly water yield was limited to those independent variables that would not be affected by treatment. This limitation set the pattern for the entire analysis, beginning with the basic equation:

Runoff = Precipitation − Evapotranspiration ± Storage Change

Obviously the evapotranspiration factor and storage change in groundwater levels, soil moisture, and snow could be affected by the treatment. Therefore it became necessary to predict these from climatic factors that would not be affected by treatment. This required the development of expressions of precipitation and air temperature for predicting evapotranspiration and storage change. These expressions could then be substituted in the basic equation, and runoff could be predicted. This procedure was used to develop prediction equations first for annual runoff and then for monthly runoff.

Annual Runoff

Annual runoff is often predicted from annual precipitation on the basis of the hydrologic year in which beginning and ending dates are selected that minimize storage change. To determine the most likely year, annual runoff-annual precipitation regressions were computed for hydrologic years beginning with each of the 12 months, and the standard error of estimates and correlation coefficients were calculated. Standard errors of estimate ranged from a December value of 0.20 inch to a September value of 3.77 inches (average annual runoff was 32.22 inches), with corresponding correlation coefficients of 0.9998 to 0.940. Apparently, there was no relation between degree of correlation and variation in groundwater and soil-moisture storage, nor any logical reason for the variability in correlation from month to month. The high correlations were judged to be happenstances and were not acceptable.

The Hydrologic Year

The U. S. Geological Survey uses a hydrologic year that begins October 1 and ends September 30, on the assumption that groundwater levels are lowest at that time of year and that storage variation is least at that time. Other watershed researchers have tested hydrologic years beginning on dates other than October 1. According to an unpublished report, an analysis was made at the Coweeta Hydrologic Laboratory in 1951 by the Southeastern Forest Experiment Station, using each of the months as the beginning of a year of record. Regression of runoff on precipitation showed the lowest standard deviation, 3.65 inches, for the May 1-April 30 year.

Brakensiek (1957) has propounded the use of an optimum water-year for agricultural watersheds, based on the uniformity of soil-moisture levels. The latter are at a maximum and are relatively uniform in the spring of the year. Thus the optimum water-year would begin with one of the spring months. As soil-moisture records were not available for most locations, however, a simple correlation between precipitation and runoff was used instead. Correlation coefficients between precipitation and runoff were computed for all 12 possible years. The year having the highest "r", the one chosen as the optimum water-year, coincided closely with maximum soil-moisture level.

The optimum water-year should coincide with those months having the least variation in groundwater and soil-moisture stor-

Table 1. — *Average groundwater and soil-moisture storage at beginning of each month, 1948-54, in inches*

At beginning of—	Groundwater storage		Soil-moisture storage		Groundwater plus soil-moisture storage: standard deviation
	Average	Standard deviation	Average	Standard deviation	
January	2.93	0.72	6.74	0.20	0.93
February	3.02	.90	6.92	.13	.89
March	2.80	.65	6.95	.23	.61
April	3.69	1.24	6.93	.18	1.35
May	3.30	.70	6.88	.30	.69
June	2.94	.81	6.97	.27	.89
July	1.51	.21	5.68	.52	.66
August	1.23	.47	5.54	1.13	1.31
September	.86	.45	5.61	1.19	1.32
October	.71	.48	6.28	.48	.67
November	.88	.35	7.02	.46	.85
December	2.34	1.00	6.99	.22	.87

Table 2. — *Precipitation-runoff regressions for hydrologic years beginning with different months*

Year beginning with—	Average annual precipitation	Average annual runoff	Regression coefficients		Standard error of estimate	Correlation coefficient
	X	Y	a	b	$s_{y.x}$	
January	57.74	34.73	−13.09	0.83	2.16	0.979
February	57.16	34.06	−13.45	.83	1.15	.995
March	57.35	33.86	−15.64	.86	3.14	.966
April	58.02	34.12	−18.23	.90	2.38	.977
May	57.86	34.02	−17.28	.89	2.61	.972
June	57.24	33.72	−14.59	.84	2.61	.975
July	57.44	33.68	−19.22	.92	1.19	.995
August	56.94	33.64	−16.74	.88	.92	.996
September	56.96	33.60	−13.29	.82	3.77	.940
October	56.82	33.58	−10.26	.77	2.82	.971
November	58.16	34.53	−12.18	.80	2.18	.977
December	57.75	34.52	−12.52	.81	.20	.9998
Average for all years		—	−14.71	.85	2.32	—
Average for years beginning during growing season		—	−15.23	.85	2.52	—

age. Table 1 shows the groundwater and soil-moisture storage pattern at the beginning of each month. Average values are lower during the summer and fall months because of high evapotranspiration rates. But the deviation of soil-moisture values is higher in the summer months than in other months, varying from field capacity after a long rainfall to near the wilting point after a long dry period. The standard deviations of groundwater-storage values show a less consistent pattern than those of soil-moisture values, although they are generally smaller in the summer months. The low standard deviation at the beginning of July is not a normal occurrence but is due to abnormally low rainfall in most of the June months during the calibration period. June rainfall averaged only 2.53 ± 1.16 inches in an area of well-distributed rainfall averaging 4.5 inches per month. In fact, long-term records at several Weather Bureau stations in the area show that June rainfall is somewhat above the average of all months.

Table 2 gives the correlation coefficients for hydrologic years beginning with each of the 12 months. A generally random pattern of the correlation coefficients and standard errors of estimate is evident. They are not even well related to the storage variation shown in table 1. The best correlation, by far, was obtained in the hydrologic year beginning December 1, at which time groundwater variation was great and snow storage was also a factor. Snow was measured on the watershed at the beginning and ending of each of the December 1 years, in greatly varying amounts. The August 1 years, which had the second best correlation, also had very large storage changes, as noted in table 1. These high correlations and the relatively poor correlation in the September 1 years are the result of other factors and will be explained in a later section. They are definitely not associated with time of year and therefore are not reproducible; that is, the high correlation observed in December 1 years probably will not be obtained in another group of December 1 years, such as the post-treatment period.

Water Losses by Hydrologic Years

Another method of relating precipitation (P) and runoff (RO) is to minimize the variation of annual water loss, which may be expressed as P — RO, or from the basic equation: Evapotranspiration ± Storage Changes. The hydrologic year with very low water-loss variation could also be used to predict streamflow. The results of such an analysis (table 3) show that the July 1

Table 3. — *Annual water-loss variation for hydrologic years beginning with different months*

(Water Loss = Precipitation — Runoff)

Year beginning with—	Average annual water loss	Standard deviation
January	23.01	2.63
February	23.10	2.23
March	23.49	3.17
April	23.89	2.33
May	23.91	2.62
June	23.54	2.92
July	23.75	1.31
August	23.30	1.49
September	23.34	3.82
October	23.44	3.40
November	23.63	2.85
December	23.23	1.89
Average for all years	—	2.56
Average for years beginning during growing season	—	2.59

year has the least variation. A similar analysis at the Coweeta Hydrologic Laboratory in 1953 showed the least variation in water loss within the April 1 year and the May 1 year.

The water-loss analysis has even less value than the precipitation-runoff relationship. The standard deviations of water loss are higher in all cases than the standard errors of estimate from the previous regressions. The lowest variation, in the July 1 years, is the result of uniform June rainfall, a happenstance, as discussed previously. The standard deviations in the water-loss analysis were higher than those in the precipitation-runoff analysis because the relationship of water loss to precipitation has not been taken into account. Water loss is affected by precipitation in that greater precipitation results in greater availability of water for evapotranspiration or other loss.

Using the average regression coefficients in table 2, an equation to predict runoff from precipitation might be expressed:

Annual Runoff = 0.85 Annual Precipitation — 14.71

The b coefficient (0.85) demonstrates that not all increase in precipitation goes into streamflow: part of the increase is lost in evapotranspiration or otherwise. For example, using the above equation:

40 inches of precipitation = 19.29 inches of runoff

and

70 inches of precipitation = 44.79 inches of runoff

In the first case, water loss would be 20.71 inches; but in the second case it would be 25.21 inches. In fact, if water loss were related to precipitation in a series of regressions, the standard deviations would be exactly the same as the standard errors of estimate shown in table 2.

Prediction of Annual Runoff from Precipitation, Measured Storage Change, and Estimated Evapotranspiration

To determine if storage changes were the only other factor besides precipitation that affects runoff, measured values of storage change were introduced as an independent variable in multiple regression equations.

First, annual runoff was related to precipitation and measured storage change (the sum of changes in groundwater storage, soil-moisture storage, and snow storage from one year to the next). Then the runoff was related to precipitation and evapotranspiration (estimated from mean annual temperatures and from potential evapotranspiration); and finally it was related to precipitation, measured storage change, and estimated evapotranspiration.

This final regression produced standard errors of estimates that ranged from 0.06 to 2.12 inches, and averaged 1.03 inches as against the 2.32-inch average for estimates based on precipitation-runoff only. These regressions were further improved (1) by dropping from the record those hydrologic-month years beginning in the winter months when snow had been an important but highly variable storage component, and (2) by utilizing groundwater storage data only in the storage-change estimates, dropping the less accurate soil-moisture data.

Relative to this point, storage changes were taken from the hydrograph or were actually measured. To meet the objectives of the calibration, storage changes estimated from antecedent climatic factors were required.

In relating annual runoff to precipitation and measured storage change, the regression equation took this form:

$$\text{Runoff} = a + b_1 \text{ Precipitation} + b_2 \text{ Measured Storage Change}$$

Table 4. — *Annual runoff regressions for hydrologic years beginning with different months*

Year beginning with—	Runoff on precipitation and measured storage gain			Runoff on precipitation and estimated evapotranspiration			Runoff on precipitation, measured storage gain, and estimated evapotranspiration			
	b_1	b_2	$s_{Y.12}$	b_1	b_3	$s_{Y.13}$	b_1	b_2	b_3	$s_{Y.123}$
January	0.81	—0.54	2.28	0.86	—0.84	2.40	0.85	—1.05	—1.85	1.39
February	.81	+ .72*	.36	.85	— .91	.69	.81	+ .66*	— .10	.50
March	.88	—2.31	2.50	.94	—2.86	.46	.94	+ .24*	—3.04	.60
April	.90	— .26	2.82	.96	—2.11	.82	.96	— .28	—2.12	.24
May	.90	—1.09	2.74	.91	—2.93	1.48	.92	— .93	—2.80	.63
June	.90	—1.06	2.20	.91	—1.48	2.89	.98	—1.13	—1.74	2.12
July	.96	+ .08*	1.21	1.02	—1.98	.63	1.03	+ .43*	—1.76	.70
August	.88	.00	1.10	.70	+2.19*	.86	.70	— .06	+2.21*	1.21
September	.81	—1.29	.71	1.16	—5.07	1.68	.82	—1.28	— .05	1.00
October	.82	—2.13	1.12	.93	—2.55	2.14	.85	—1.81	— .62	1.45
November	.77	—1.42	.37	.92	—1.94	.40	.84	— .76	— .93	.33
December	.81	— .08	.20	.81	+ .08*	.25	.82	— .20	— .26	.06
12-year average	0.85	—0.78	1.74	0.91	—1.70	1.49	0.87	—0.51	—1.09	1.03
Average for years beginning during growing season	.88	— .91	1.67	.94	—1.97	1.79	.87	— .80	— .79	1.29
Composite regression for growing season	.86	—1.03	1.51	.97	—2.29	1.59	.95	— .80	—1.69	1.16

b_1 = Regression coefficient for precipitation.
b_2 = Regression coefficient for measured storage gain.
b_3 = Regression coefficient for estimated evapotranspiration.
$s_{Y.12}$ = Standard error of estimate in multiple regression.
* = Coefficient has wrong sign; probably impossible.

Equations were calculated for all hydrologic years. The results are tabulated in the first section of table 4; and though the average error is lower than that shown in table 2, the individual errors are still relatively large. The two marked with asterisks show a positive relationship: they say, in effect, that runoff would be increased by a gain in storage. This was obviously incorrect, and the low standard error of estimate in February was a chance occurrence.

Even though the storage values were thought to be inexact, the errors of estimate were greater than might be expected. Apparently the other component of the basic equation—evapotranspiration—had enough variability to cause a part of these errors. Thus another variable had to be added to the multiple regression to remove this source of variation.

Evapotranspiration

In the search for a suitable variable that would be well related to evapotranspiration, the moisture-storage data available in this study were utilized. From the basic equation, a measurement of annual evapotranspiration was obtained:

Evapotranspiration = Precipitation — Runoff ± Storage Change

A simple and readily available value that might be expected to be related to annual evapotranspiration was mean annual temperature, the mean of daily average temperature computed from the maximum and minimum temperature. This value was well related to annual evapotranspiration under certain conditions in which other factors were constant. For example, in consecutive hydrologic years beginning in 1949, there was a decrease in evapotranspiration that was highly correlated to a decrease in annual temperature:

Year beginning 1949	*Measured evapotranspiration (inches)*	*Mean annual temperature, (°F.)*	*Sums of temperature over 30°F./100*
May	22.2	46.2	140
June	21.2	45.9	137
July	21.5	45.3	134
August	22.1	45.0	131
September	20.8	44.7	129
October	19.3	44.7	128

Another expression of annual temperature was also investigated: the summation of temperatures over a designated level. Budyko (1956) found a very close correlation between the sums of temperature higher than 10° C. and radiation balance. Budyko (p. 170, Weather Bureau translation) explained his procedure as follows: "The sums of temperature higher than 10° is obtained by adding up mean daily temperatures in the Centigrade scale for the period when these temperatures are higher than 10°."

The procedure used in our analysis was slightly different but followed the same principle. The value 30 was subtracted from each daily maximum and minimum temperature in the Fahrenheit scale and the remainders were summed. No minus values were used, and all temperatures of 30° F. and below were therefore ignored. Perhaps a better system would entail the summation, or average, of 2-hour samples, as taken from a thermograph, instead of only two samples per day. But this would be a much longer process, and a thermograph might not be available in all watershed studies.

The summation of temperatures over a limit is a more logical approach than the use of daily means, because it is the higher temperatures that relate to evapotranspiration. Low temperatures —say 30° F. and lower—probably have very little relationship to evapotranspiration. But when averaged with higher temperatures, the means are considerably affected. It is hard to say what the correct level may be, whether 30° F. or 10° C. (50° F.) or some other temperature; but the summations over 30° F. gave good results in most cases. It should be noted that in monthly relationships the summation of temperatures over 30° F. would be perfectly related to mean monthly temperature in the summer months, when minimum temperatures do not fall to 30° F.

A serious weakness in the use of an expression of temperature to estimate evapotranspiration is that temperature is related to potential evapotranspiration and not necessarily to actual evapotranspiration. If an adequate supply of moisture were always available to satisfy the demands of potential evapotranspiration, the temperature variable would be well related to actual evapotranspiration. But moisture deficits occur frequently and should be recognized if possible.

Thornthwaite and Mather (1955) have devised a procedure for estimating monthly evapotranspiration: mean monthly temperature is used to estimate potential evapotranspiration; then this value is adjusted by the amounts of precipitation and stored moisture available for consumption.

As these procedures used climatic data rather than hydrograph data, they were useful in the calibration analysis. Monthly evapotranspiration was estimated according to Thornthwaite and

Mather's method, and the monthly figures were summed to give an estimated evapotranspiration value. The improvement over temperature in its correlation with measured evapotranspiration is shown in the following example:

Years beginning September	*Measured evapotranspiration (inches)*	*Sums of temperature over 30°F./100*	*Estimated evapotranspiration (inches)*	*Annual precipitation (inches)*
1949	20.8	129	20.0	46.0
1950	23.4	134	21.8	60.3
1951	26.9	136	22.5	72.5
1952	23.2	139	20.9	59.0
1953	22.9	142	21.5	47.0

It is also evident that the measured evapotranspiration, which may include other water losses besides evapotranspiration, was very well related to annual precipitation values. This suggests that annual precipitation, already a variable in the runoff equation, would take care of the moisture availability factor. But this is not always the case. Records are available for several years following the calibration period, in which annual precipitation was high but poorly distributed during the growing season. In these years, evapotranspiration was restricted by moisture deficits and lower than expected.

The estimated evapotranspiration values were not exact enough for adjusting the dependent variable in this analysis, but were considered an index of evapotranspiration; thus they were used as an independent variable in multiple regression.

The results of regressions in which estimated evapotranspiration was used are given in table 4. In the middle section it is used with precipitation only, while in the right-hand section it is used with the two other independent variables. While there is still considerable variation between the hydrologic years, the average errors of estimate show an improvement due to the inclusion of estimated evapotranspiration.

The causes of the peculiar correlations discussed earlier were evident at this point. An examination of the extremely high correlation between precipitation and runoff in the December 1 years revealed that there was a high negative covariance between the other two independent variables in the multiple regression. They cancelled each other; i.e., those years with a high storage gain had low evapotranspiration and vice versa. An almost perfect precipitation-runoff relationship remained.

On the other hand, the September years showed a particularly poor correlation between precipitation and runoff. In these years, the two other independent variables were practically unrelated, and the variation in the storage change variable was extremely high — higher than in any other hydrologic year. Thus the deviations in the simple precipitation-runoff regression were related to the changes in storage and were minimized by the addition of this variable.

Deletion of Years Affected by Snow Storage

Some of the regressions listed in the right-hand section of table 4 were still obviously inaccurate; those for February, July, and August years had regression coefficients with sign opposite to what might be expected. Without much doubt, this was caused by inaccurate data in the storage-change variable. One known source of inaccuracy in the storage-change variable was the snow data. These were known to be relatively inexact on a watershed basis, thereby causing the storage-change variable to have large inaccuracies. For this reason, those years in which snow storage was a factor were deleted from the subsequent analysis. Remaining were those years beginning with the months May through October.

Inaccuracy of Soil-Moisture Storage Data

Even in these six hydrologic years, the storage-change data were obviously inaccurate, as shown in the third section of table 4. Almost certainly this inaccuracy was the result of the relatively inexact measurement of soil moisture on a watershed basis. Even though the soil moisture may have been measured accurately at the measuring stations, the great variation of soil depth, texture, rock content, and so on throughout the watershed made an exact estimate of watershed soil moisture an impossibility.

To test the relative accuracy of the two storage variables, regressions were computed for the six hydrologic years, May through October, using groundwater storage change instead of total storage change. The results (table 5) indicated an improved standard error of estimate in three of the six hydrologic years. More important, a greater consistency of the regression coefficients developed. The positive b_2 coefficient in the July years which say, in effect, that a gain in storage would result in increased runoff, became a reasonable negative coefficient. The high positive b_3 coefficient, which would indicate an increase in

Table 5. — *Annual runoff regressions for hydrologic years, beginning with different months*

Year beginning with—	Runoff on precipitation, estimated evapotranspiration, and total measured storage change	Runoff on precipitation, estimated evapotranspiration, and groundwater storage change			
	$s_{Y.123}$	b_1	b_3	b_4	$s_{Y.134}$
May	0.63	0.94	—3.35	—1.10	0.30
June	2.12	.97	—1.90	—1.38	1.14
July	.70	1.02	—2.14	— .88	.78
August	1.21	.87	+ .02*	—1.26	.54
September	1.00	1.15	—4.45	—1.43	1.68
October	1.45	.94	—2.15	— .86	2.87
Average	1.29	.98	—2.33	—1.15	1.49

b_1 = Regression coefficient for precipitation.
b_3 = Regression coefficient for estimated evapotranspiration.
b_4 = Regression coefficient for groundwater storage change.
$s_{Y.134}$ = Standard error of estimate in multiple regression.
* = Coefficient has wrong sign; probably impossible.

runoff with high evapotranspiration, became a small and nonsignificant coefficient when groundwater change was substituted for total storage change.

It would have been desirable to treat each storage change as a separate variable and to consider each value as an index of the true storage change. However, such a regression would have required four predictor variables. As there were only five samples in each hydrologic year group, four variables would allow no error measurement and, therefore, could not be used.

Combining Hydrologic Years

Dropping the hydrologic-month years influenced by snow storage left six hydrologic-month years, May through October. To develop prediction variables for storage, these years were combined into 30 years (6 hydrologic-month years x 5 years of record). Although the estimate of error from such a regression may not be valid, the relative value of each variable is assessed quite accurately because there are many more combinations of variables available for testing.

The procedure invited serial correlation. Though this limitation was recognized, the procedure was nevertheless adopted initially to gain information on independent variables. Further analysis suggested that much or all of the serial correlation was removed by the use of antecedent variables.

Economists commonly use time-series data that may or may not be serially correlated. They have devised tests to show evidence of serial correlation as well as procedures to adjust for it. In applying one of these tests (Durbin and Watson 1951), it was shown that residuals from the simple regression of runoff on precipitation were significantly serially correlated, as expected. The addition of the two independent variables—estimated evapotranspiration and total measured storage change—did not change the situation, although the residuals became smaller with the addition of each successive variable. In the tabulation of the individual residuals, the serial correlation, or lack of randomness in adjacent samples, was apparent. However, when groundwater storage gain and soil-moisture storage gain were treated as separate independent variables, the test was inconclusive at the 5-percent level of significance, but showed that the residuals were not significantly correlated at the 10-percent level.

Later analyses, using climatic variables in place of measured storage variables, showed a definite lack of serial correlation: the residuals were not seriously correlated at the 5-percent level. Thus the effect of the additional variables, two of which dealt with antecedent events, was to remove the influence of antecedent conditions.

Prediction of Storage Change

A simple variable related to annual storage change was the difference in precipitation between the last month of the runoff year and the same month of the preceding year. Using data taken from table 8, two examples of the relationship were devised:

		Precipitation (inches)	*Annual storage value (groundwater plus soil-moisture storage at end of month) (inches)*
(1)	August 1953	1.37	4.32
	August 1952	6.58	7.75
		−5.21	−3.43
(2)	August 1954	6.04	7.46
	August 1953	1.37	4.32
		+4.67	+3.14

The storage loss in the first example is related to the decrease in precipitation, while the storage gain in the second example is in accordance with the higher rainfall in the last month of the year.

Obviously watershed storage on any one day (say September 30) would be more intimately affected by rainfall on September 29 than by rainfall on September 1. Therefore a weighting system was required to give the later rainfall more weight than earlier rainfall.

Furthermore, storage on September 30 might also have been affected by rainfall in August or July admittedly to a lesser extent than rainfall during September. A longer antecedent period would be expected for the summer months when the groundwater-depletion curve is relatively flat (Storey 1951) and the effect of a storm that recharges groundwater will be long-lasting. On the other hand, during the winter and spring months, when the depletion curve is steep, the effect of a storm of similar size will last a much shorter time. Groundwater levels between the beginning of December and the beginning of June are generally much higher than in summer and fall months (table 1), and base flows are considerable greater. An accretion added to a high groundwater level will run off in a shorter time than if added to a low groundwater level.

A number of different antecedent periods were tested. In the system finally selected, the length of the period varied from month to month. The antecedent period for the year beginning with July was 80 days, for August 100 days, and for September and October 120 days. In years beginning with May and June, a 40-day antecedent period was used. Weighting, or transforming, was applied linearly; i.e., rainfall on the last day of the shorter period was given a weight of 40, the preceding day a weight of 39, and so on, decreasing 1 unit each day. The longer periods were weighted similarly: the last day's rainfall was given a weight equal to the total length of the period; then the weight was decreased 1 unit for each preceding day.

Antecedent Evapotranspiration

Storage was also affected by evapotranspiration during an antecedent period that created soil-moisture deficiencies. Groundwater recharge did not take place until these deficiencies were satisfied.

Records show that monthly potential evapotranspiration varies considerably from year to year. For example (table 6), potential

Table 6. —*Monthly adjusted potential evapotranspiration, in inches*

Year	April	May	June	July	August	September
1949	1.34	2.95	4.72	5.24	4.68	2.44
1950	.67	2.72	3.70	4.49	3.98	2.40
1951	1.30	2.95	3.98	4.88	4.25	2.91
1952	1.57	2.60	4.41	5.16	4.29	2.99
1953	1.18	3.31	4.45	5.00	4.41	2.91
1954	1.65	2.68	4.33	4.76	4.17	2.87

evapotranspiration for June 1950 was more than an inch less than that in June 1949. Thus the storage at the end of June 1950 might have been as much as 1 inch higher than at the end of June 1949, if rainfall had been adequate and similar for the 2 months.

A simple temperature variable, similar to the preceding precipitation variable, was also inserted in the regression. The mean monthly temperature for the last month of the antecedent year was subtracted from the mean monthly temperature for the last month of the runoff year. The difference obtained was also related to the storage change from the beginning to the end of the runoff year. A positive difference would result in a lower storage gain (or a storage loss), which would increase runoff for the year.

The temperature variable was expressed in a number of ways. For the years beginning on May 1, mean April temperatures were used; for years beginning June 1, the averages of April and May temperatures were used. Average monthly temperatures for the preceding 3 months were used in all other years; i.e., July through October years.

Correction for Direct Runoff

Direct or storm runoff, (the sum of overland flow, channel interception, and storm seepage) was produced in disproportionate amounts by large storms. The small storms produced little or no direct runoff, but almost 3 inches of the large 10-inch storm of July 1952 ran off quickly and had no effect on storage. Therefore a correction had to be made to reduce the effect of large storms on the storage relationship. As high monthly precipitation values are usually the reflection of large storms, the square root of monthly precipitation had the effect of adjusting for large storms.

Though the square root of monthly precipitation is a useful and rather simple expression of the precipitation entering storage, a more accurate but more complex method of correcting for large storms was devised. This method is presented in a later section that deals with the estimation of various storage factors. Estimated storage precipitation, as it is called in this paper, is an estimate of monthly precipitation minus monthly direct runoff. In all cases, the differences between these monthly values, from year to year, are the values used as one of the two storage-change variables.

Regressions Based on Combined Years

A series of multiple regressions were run with combined data, the best predictions employing annual precipitation (X_1), annual estimated evaporation (X_2), and the differences between weighted storage precipitation values and sums of temperature over 30° F. ($X_6 - X_7$). The equation took this form:

$$\text{Annual Runoff} = a + b_1X_1 + b_2X_2 + b_3(X_6 - X_7) + E$$

These three variables were then used with uncombined data, and a multiple regression was calculated for each of the six hydrologic years as follows:

Year beginning —	a	b_1	b_2	b_3	$S_{Y.123}$
May	55.60	0.96	−3.41	−1.90	0.45
June	40.71	.98	−2.71	−3.03	.72
July	26.50	1.02	−2.29	−1.13	.79
August	35.52	.77	+1.24	−0.93	.85
September	40.23	1.02	−2.84	−2.50	1.55
October	14.48	.90	−1.25	−3.48	2.21
Average	35.51	0.94	−1.88	−2.16	1.25

The following equations were calculated from regressions in which the six hydrologic years were combined to provide a population of 30 samples.

[1] $$Y = 18.99 + 0.95\,X_1 - 1.69\,X_2 - 0.80\,X_3$$

in which:

Y = annual runoff, in inches.

X_1 = annual precipitation, in inches.

X_2 = annual estimated evapotranspiration, in inches.

X_3 = total measured storage gain (sum of groundwater plus soil-moisture storage changes plus 4.00), in inches.

[2] $Y = 24.22 + 0.98 X_1 - 2.05 X_2 - 1.36 X_{3A} - 0.38 X_{3B}$

in which:

X_{3A} = measured groundwater gain (change in groundwater storage + 2.00), in inches.

X_{3B} = measured soil-moisture gain (change in soil-moisture storage + 3.00), in inches.

[3] $Y = 27.86 + 0.97 X_1 - 2.24 X_2 - 1.16 X_4 + 10.55 X_5$

in which:

X_4 = precipitation in last month of runoff year, in inches, plus 2.00 minus precipitation in last month of preceding year.

X_5 = sum of temperatures over 30°F./1000 for last month of runoff year + 0.40 minus sum of temperatures over 30° F./1000 for last month of preceding year.

[4] $Y = 31.08 + 0.98 X_1 - 2.36 X_2 - 2.37 X_6 + 10.94 X_7$

in which:

X_6 = differences between weighted storage precipitation for the end of each year + 2.00.

X_7 = differences between the sums of temperature over 30° F. for the end of each year + 0.40.

[5] $Y = 30.28 + 0.98 X_1 - 2.28 X_2 - 2.47 (X_6 - X_7)$

The mean squared residuals (the square of the standard error of estimate) of the preceding equations are as follows: (1) 1.35 inches, (2) 1.07 inches, (3) 0.98 inch, (4) 0.87 inch, and (5) 0.90 inch.

In equation 5, the two variables related to storage change have been combined into one variable, which may be considered the storage-change variable.

Equation 2, in which the two measured storage values were inserted separately, confirms the statement made earlier: soil-moisture storage values were rather inaccurate on a watershed basis. When combined with the more accurate groundwater data, the accuracy of the estimates decreases. Furthermore, the X_1 and X_2 coefficients in equation 2 are similar to those in equations 3, 4, and 5, whereas they are rather dissimilar in equation 1.

The reliability of the estimating equations was questionable because only 1 degree of freedom remained in each small population to measure error. Nor were the coefficients very accurate, as indicated by this lack of uniformity between the various equations. No reason can be given for the lack of uniformity, nor for this curious increase in the standard errors of estimate as the seasons progressed. However, it is suspected that the low standard errors were the result of chance happenings of over-manipulation of the data.

However, all standard errors of estimate were well below the minimum practical change in yield we considered significant; namely, about 10 percent of the annual runoff, or about 3.2 inches.

One way to provide additional samples for multiple regression is to pool data from the calibration period and the post-calibration period. A dummy variable included in the regression would show the difference, if any, between periods. Such an analysis is similar to covariance analysis, but is probably more efficient than the latter in instances where the number of samples is limited, as in this case.

An analysis including a dummy variable was used with some success with a 4-year period of record following the calibration years. However, this was an unusual period in which occurred drought, hurricane rainfalls, and a wildfire that burned over about one-third of the watershed.

Another way to increase the reliability of the predicting equations is to provide more accurate estimates of evapotranspiration and storage changes. Research is continuing on methods to estimate evapotranspiration more accurately, and a large group of samples of storage changes would provide opportunity to estimate them more accurately.

Table 7. — ***Basic data for annual runoff analysis***

Hydrologic year beginning with	Annual precipi-tation	Annual runoff	Estimated evapo-transpiration	Ground-water storage change	Total storage change	Precipitation: last month minus antecedent month	X_3	X_4	X_3-X_4+1.60
May 1									
1949	48.97	26.86	21.47	—0.60	0.14	—2.67	—0.49	—0.26	1.37
1950	59.22	38.54	20.59	.44	— .26	1.29	.43	.22	1.81
1951	62.57	35.54	22.08	1.46	1.88	6.42	.97	.12	2.45
1952	72.36	46.95	22.24	— .80	— .79	—4.08	— .54	— .19	1.25
1953	46.11	22.30	22.25	— .92	— .65	—1.80	— .48	.28	.84
June 1									
1949	46.10	24.59	21.24	— .18	.28	—2.88	— .32	— .22	1.50
1950	58.04	37.73	20.82	—1.47	—1.67	—1.18	— .23	.21	1.16
1951	65.73	38.17	21.73	2.30	2.50	3.16	.86	— .25	2.71
1952	71.39	46.08	22.95	— .90	— .44	— .96	— .43	.33	.84
1953	44.96	21.96	21.62	— .80	—1.48	—1.15	— .59	— .30	1.31
July 1									
1949	48.26	25.47	21.12	.38	1.34	2.16	.14	— .26	2.00
1950	59.24	36.99	21.14	— .18	.32	1.20	.15	.18	1.57
1951	63.99	39.25	21.92	.10	—1.04	—1.74	.03	— .01	1.64
1952	70.75	45.37	22.80	— .22	— .34	— .63	— .27	.04	1.29
1953	44.91	21.34	21.54	— .28	— .01	— .06	— .16	— .02	1.46

CONTINUED

Table 7.—*Continued*

Hydrologic year beginning with	Annual precipi-tation	Annual runoff	Estimated evapo-transpiration	Ground-water storage change	Total storage change	Precipitation: last month minus antecedent month	X_3	X_4	X_3-X_4+1.60
August 1									
1949	49.76	26.66	20.92	0.56	0.97	1.49	0.31	—0.29	2.20
1950	60.03	36.51	21.53	.16	1.24	.81	.39	.15	1.84
1951	68.41	42.89	22.20	.31	—1.44	4.40	.04	.00	1.64
1952	64.22	41.20	22.40	— .99	— .57	—6.51	— .64	.09	.87
1953	42.37	21.02	20.13	— .17	—1.95	—2.54	— .57	— .15	1.18
September 1									
1949	45.98	26.73	19.97	.08	—1.50	—3.75	— .47	— .29	1.42
1950	60.28	36.63	21.81	.08	.33	.25	.25	.09	1.76
1951	72.49	44.47	22.47	.68	1.13	4.06	.51	.09	2.02
1952	59.01	39.21	20.87	—1.16	—3.43	—5.21	—1.04	.00	.56
1953	47.04	21.04	21.54	— .12	3.14	4.67	.51	— .09	2.20
October 1									
1949	45.12	26.75	19.93	— .39	— .98	— .87	— .50	— .20	1.30
1950	59.96	36.62	22.32	.01	— .34	— .30	.11	.14	1.57
1951	75.86	47.22	22.55	.96	.91	3.36	.63	.10	2.13
1952	58.06	36.50	20.79	—1.30	—1.49	— .96	— .65	— .02	.97
1953	45.05	20.81	21.50	— .18	1.01	—1.98	.14	— .10	1.84

X_3 = Weighted storage precipitation: last period minus antecedent period.
X_4 = Sum of temperatures over 30°F.: last period minus antecedent period.

Monthly Runoff

The greater number of samples available for monthly prediction provided more opportunities for exploring pertinent relationships. Most of these involved the immediate disposition of rainfall into soil-moisture storage, groundwater recharge, or streamflow. Earlier analyses showed that snowfalls and snow accumulation upset the rainfall-derived relationships. Accordingly, all months were eliminated in which a snow cover had been recorded at the beginning or end. This left 49 months for analysis.

Monthly Water Loss as the Dependent Variable

The monthly water-loss value (precipitation minus runoff) was adapted as the dependent variable for a number of reasons:

1. Except in the December-April season, precipitation is much more closely related to water loss than it is to runoff. During the summer months, when there is low beginning storage, practically none of the precipitation becomes runoff. Even in the December-April season, precipitation is almost as well related to water loss as it is to runoff.

2. The weighting systems that are applied to daily precipitation were developed for storage estimation, as described in previous sections. They are weighted heavily at the end of the period and are applicable to the precipitation expressions for water-loss estimation. The weighting systems for runoff estimation, on the other hand, would be weighted heavily at the beginning of the period.

3. Storage precipitation, the expression that has been corrected for direct runoff, is equally useful for water-loss estimation because water loss is essentiallly a storage factor. The first general equation:

Runoff = Precipitation — Evapotranspiration — Storage gain

now becomes:

Precipitation — Runoff = Water loss = Evapotranspiration + Storage gain

Storage precipitation would not relate well with runoff, for it is total precipitation minus direct runoff. Direct runoff, of course, is a part of total runoff. If runoff were used as the dependent variable, direct runoff would have to be a separate independent variable.

The water-loss concept is confusing to some workers in this field who have considered consumptive use, evapotranspiration, and water loss as synonymous terms. Water loss, as used here, is more than just evapotranspiration; it is all rainfall during the period that does not show up at the streamgaging station during the period. Thus, storage gain during the month is a water loss for that month. Evapotranspiration is almost entirely a loss from soil-moisture storage (interception loss is a loss of potential soil-moisture storage), and therefore the two terms are complementary. If evapotranspiration is high in a particular month, storage gain will be low in comparison with another month having the same precipitation but low evapotranspiration. Ignoring evapotranspiration, the estimation of water loss is an estimation of storage gain; and therefore the expressions developed for estimating storage applied to the water-loss estimates as well.

Therefore an estimate of evapotranspiration was not required in this analysis. Except for a small amount of riparian water loss, evapotranspiration was not considered. Disregarding riparian water loss temporarily, monthly water loss could be related to just two factors: storage at the beginning of the month and rainfall during the month:

$$\text{Monthly Water Loss} = a + b_1 \text{ Precipitation} + b_2 \text{ Beginning Storage}$$

In the development of basic relationships, groundwater values and precipitation corrections were derived from hydrograph analysis. The beginning storage variable, for instance, was specifically the groundwater storage variable. Initial soil-moisture storage has no direct effect on water loss or runoff; it affects only groundwater recharge.

Expression of monthly precipitation required these steps: first, the straight monthly precipitation was determined; then calculations were made for the transformed values in which late-in-the-month precipitation was given greater weight than earlier rainfall because the one affected storage gain more than the other. Several weighting systems were tested and a system was developed based on linear weighting.

Weighting Precipitation

Several weighting systems were tested: logarithmic, reciprocal, and linear, as well as numerous levels within each system. In the linear system finally adopted, weights were varied according to groundwater level. In months with high beginning groundwater level, precipitation was weighted heavily; i.e., the last day's rainfall was given a weight of 40, decreasing one unit each day until the first day of the month, which had a weight of 10 or 11. Months with lower groundwater levels were weighted less heavily, on an 80-basis: the last day's rainfall was given a weight of 80 and the first day's a weight of 50 or 51.

Next, months were grouped to obtain uniform weightings based on groundwater levels. Four groups were formed: the two groups of lower groundwater were the summer and fall months, and the two high groundwater groups were the cooler months. In group I were months in which the total amount of water in underground storage could provide a runoff yield of less than 1 inch. Group II included all the other summer and fall months in which beginning groundwater was 1 inch or higher. Group III contained all May and June samples, while December through April months comprised group IV. In both groups III and IV groundwater at the beginning of all months was at relatively high levels—higher than any months in group II.

In group I, monthly precipitation was weighted very lightly, the first day of the month receiving only 4 percent less weight than the last day. Group II required a weighting on an 80-basis; and both cool-season groups were given a 40-basis weighting.

Finally, the relationship of precipitation to water loss was corrected for the amount of direct runoff from large storms. As direct runoff, or storm runoff, leaves the watershed immediately after the storm, it has no effect on groundwater or soil-moisture recharge. Measured on the hydrograph, daily direct runoff was subtracted from daily precipitation to give the storage precipitation value that was used in the prediction of annual runoff. This correction method was found to be much more accurate than the use of an exponential form of monthly weighted precipitation.

Relatively minor differences between months within groups were apparently the effect of riparian water loss. They were accounted for by an average length-of-day variable.

Estimates for May and June were improved by the addition of: (1) an interaction between precipitation and the average length of day, and (2) the sums of temperatures over 30° F.

Groundwater storage and weighted storage precipitation proved to be well related to water loss. But they could not be used to predict water loss after watershed treatment, for both were subject to change due to treatment. Substitutes for these values had to be derived that would not be affected by treatment.

Prediction of Groundwater Storage

Groundwater storage at the end of a month (GWS-E) was first estimated from antecedent groundwater and soil-moisture storage, measured at the end of the previous month, and storage precipitation (precipitation minus direct runoff) for the month. All three independent variables proved to be positive and significant.

The next step was to develop precipitation and temperature variables related to antecedent storage. The variables were aimed at expressing total storage, the sum of both groundwater and soil-moisture storages. Accordingly, daily weighted rainfall for varying periods was related to total storage, using the system described earlier. These values became the antecedent precipitation variable. Its period ended 1 month prior to the time of groundwater measurement.

Monthly analyses indicated that precipitation for the month in which groundwater was measured should not be weighted. Apparently, during the summer much of the rainfall in the month immediately preceding the time of measurement had little effect on groundwater storage; it was held by the soil, satisfying soil-moisture deficiencies, and therefore was not available for groundwater recharge. An estimate of soil-moisture storage would require a 30-day precipitation variable with increasing weight toward the end of the period. Thus an unweighted precipitation variable would be more closely related to groundwater storage.

Data for all the summer months—groundwater storage at the end of the months of June through August (GWS-E)—were combined in one regression of this form:

$$GWS\text{-}E = a + b_1P + b_2P_a + b_3T_a$$

in which:

P =Storage precipitation (unweighted) for the month of measurement.

P_a=Daily weighted storage precipitation for a variable antecedent period ending a month prior to groundwater measurement. The periods were 40, 80, and 100 days, respectively, for June, July, and August.

T_a=Sums of temperatures over 30° F. for the antecedent period.

As P_a is in part a measure of antecedent soil moisture, an interaction between P and P_a was indicated, the effect of P should increase as P_a increases. A product variable (interaction), *viz*

PP_a, was tested and was found to be consistent and highly significant in various monthly groupings, and was included as an additional variable.

An equation was next derived for months ending in September through November. It was similar to the one for the summer months, except that a 120-day antecedent period was used for all months.

The T_a variables (temperature) varied between monthly groups. For the June, July, and August groups, the sums of temperatures over 30° F. for the second and third months preceding the month in question were averaged. For example, in the estimation of GWS-E for June, sums of temperatures over 30° F. for April and May were averaged.

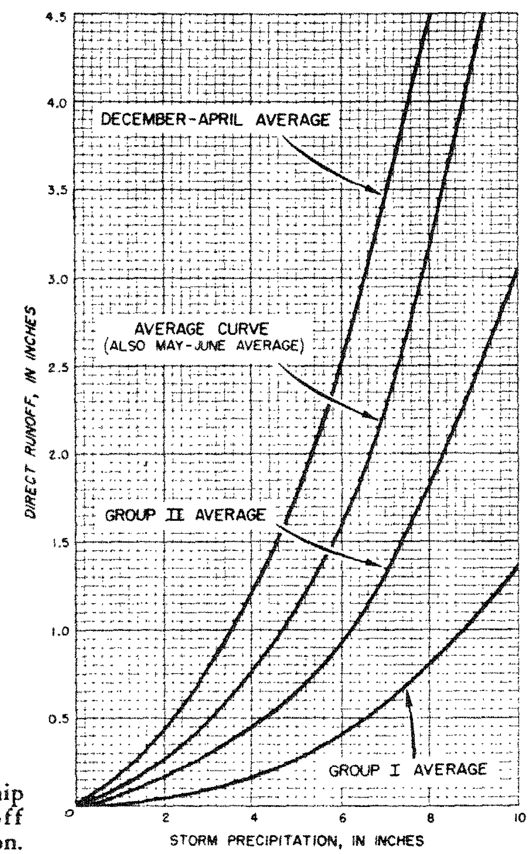


Figure 4. — Relationship between direct runoff and storm precipitation.

For the September, October, and November groups, sums of temperatures for the first and second months preceding the time of estimate were averaged; that is, for an estimate of groundwater storage at the end of October, the average sums of temperatures over 30° F. for September and October were used. The antecedent periods were determined by regression analysis, and the reason for the two different periods is rather obscure. It may be that during the summer months soil-moisture depletions were so great that the temperature during the measurement month had no effect on storage; whereas during the cooler fall months, soil moisture had begun to recharge and its level was affected by recent temperature events.

Fairly accurate estimates for April and May were obtained with an equation using daily weighted storage precipitation for a 40-day period and average monthly sum of temperatures over 30° F. since April 1. For April, only the sums of April temperatures over 30° F. were used. For May, averages of April and May sums were used.

Groundwater storage for the other 4 months of the year was estimated fairly well by use of a simple relationship with daily weighted storage precipitation. Temperature had no effect in these colder months; apparently there was little or no evapotranspiration or there was no difference between months.

Storage precipitation was originally calculated by means of hydrographic analysis. As the precipitation-direct runoff relationship may be affected by watershed treatment, an estimate of storage precipitation was substituted in the above equations.

Estimation of Storage Precipitation

Storage precipitation, that portion of total precipitation that remains on the watershed to evaporate or to go into storage, was mentioned briefly in the section on annual runoff and was discussed in some detail in the previous section on groundwater storage estimation. This section will deal with estimating storage precipitation from climatic factors.

The first attempt to approximate storage precipitation was by use of the square root of monthly precipitation in place of the linear expression of precipitation. The amount of direct runoff is closely related to size of storm, and large storms generally result in high values of monthly precipitation. Thus the square root of monthly precipitation almost always estimated storage better than the linear expression. But, although the square root of monthly precipitation is easy to calculate, its relationship to storage precipitation is fairly rough. The following methods described, while rather complex, are highly accurate.

Relationship between Precipitation and Direct Runoff

First, direct runoff was plotted over total storm precipitation for a large number of storms during the calibration period. The scatter of points appeared to be well related to differences in antecedent conditions; so an estimate of total storage at the

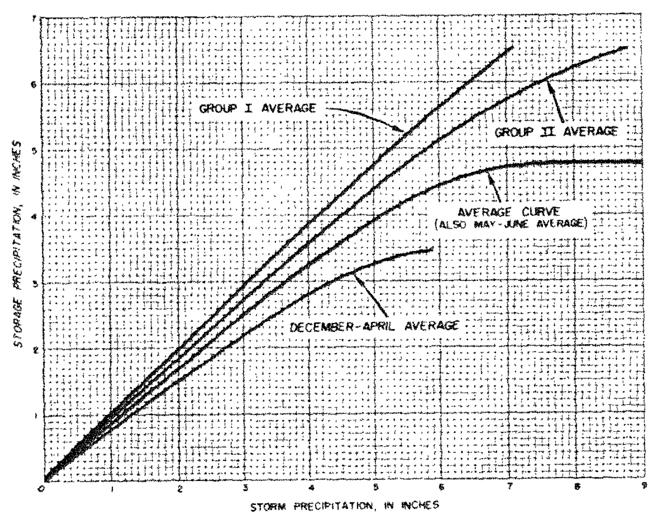


Figure 5. — Relationship between storage precipitation and storm precipitation.

beginning of the storm was assigned to each point and a curve was drawn through points representing average storage (fig. 4). The average curve was J-shaped, with a limitation in the higher values.

The curve of direct runoff over storm precipitation was converted to a curve of storage precipitation over storm precipitation. Direct runoff from the average curve on figure 4 was subtracted from its respective storm precipitation value. The remainder, storage precipitation, was plotted over storm precipitation and the average curve was drawn on figure 5. This curve begins at a slope of 1.0, indicating no direct runoff up to this point; then the curve gradually becomes parallel to the abscissa, indicating that all precipitation over a certain amount becomes direct runoff.

Converting Storm Precipitation to Average Storage Precipitation

A table was prepared to convert storm precipitation to average storage precipitation, and the table was applied to the daily precipitation values for the entire period of calibration. Although storm precipitation was used to prepare the curves and table, it was applied to daily values without much loss of accuracy. There seems to be no way to tabulate precipitation on a storm basis, instead of a daily basis, and then to apply a uniform system of weights. Perhaps such a method could be devised, but it would complicate the procedures still further.

Tables of average storage precipitation were prepared by using daily precipitation in place of storm precipitation; monthly totals and weighted sums were also calculated. Naturally these average values would not be accurate for all months. The average curve would tend to overadjust in the summer months and underadjust in the winter months. At this stage in the analysis, the following tables of precipitation expressions were available:

a. Daily unadjusted precipitation.
b. Daily storage precipitation (unadjusted daily precipitations minus measured direct runoff).
c. Average daily storage precipitation (taken from fig. 5).

Correcting Average Storage Precipitation by Groups of Months

Obviously, expression (b) is a more accurate expression of storage precipitation than (c), and methods had to be devised to interpolate between and extrapolate from (a) and (c) to approximate (b). The 49 months used in the monthly analysis had been divided into four groups:

1. Twelve summer and fall months with low beginning groundwater storage.
2. Thirteen summer and fall months with high beginning groundwater storage.
3. Twelve May and June months.
4. Twelve winter months, December through April.

For each group, the totals of each expression were computed and a correction factor was determined by the following equation:

$$\text{c.f.} = \frac{(a) - (b)}{(a) - (c)}$$

	Precipitation expressions			
	Unadjusted	*Storage*	*From curve*	
Group	*a*	*b*	*c*	*c.f.*
I	15.13	14.63	13.39	0.3
II	18.56	16.44	15.99	.8
III	14.94	13.38	13.53	1.1
IV	22.26	18.29	19.89	1.7

The previous equation was solved as follows:

$$(b) = \text{c.f.} \times (c) + (1 - \text{c.f.}) \times (a)$$

in which (b) was then an estimated value. One of the correction factors was applied to each month in its respective group and an estimate of monthly storage precipitation was calculated.

These estimated values were tested in the monthly runoff analysis. Though they gave better results than unadjusted values of precipitation, and better results than the square roots of precipitation, they still were not as good as the measured values of storage precipitation. Apparently the use of the group correction factor was still not precise enough. And the range of storage values within groups suggested that the correction factor should be varied month by month, depending on some storage variable.

Monthly Correction Factors

Going back to the three different expressions of precipitation, we computed correction factors for each month, as we did previously for the four groups. These monthly correction factors were then used as a dependent variable, to be estimated by some storage or climatic variable. A substantial portion of the variation had already been removed by the delineation into the four groups. As noted, the variation between groups was consistent and followed a pattern related to groundwater level.

Within groups, regressions were computed by using the following data as predictor variables:

Groundwater storage at the beginning of the month.
Groundwater storage at the end of the month.
Average groundwater storage.

Precipitation for the month.
Soil-moisture storage at the beginning of the month.
Sums of temperature over 30° F. for the month.

Groundwater storage at the end of the month was found to be more closely related to the correction factor than any other variable. The relationship is logical, as GWS-E is a reflection of at least three factors: groundwater storage at the beginning of the month, soil-moisture storage at the beginning of the month, and rainfall for the month, as brought out in the section on groundwater storage estimation.

Groups II and IV were both linearly related to GWS-E, but they had quite different regression coefficients. May and June were troublesome, and it became evident that June should be added to the group II months, while May's relationship was close to that of group IV, but at a lower level.

Group I months, those with low beginning groundwater storage, were not well related to GWS-E. The variation of correction factors within this group was not great, but a multiple regression showed that they were related to monthly rainfall and to monthly temperature. The equation took this form:

$$c.f. = a + b_1P - b_2T$$

in which:

P=10 x precipitation for month/number of days in month.
T=sum of temperatures over 30° F./number of days in month.

The relationships between the correction factor and storage values were thus established. However, it should be stressed that these storage values must be those estimated from climatic factors and not those determined from hydrographic analysis, which created an apparent impasse: groundwater storage estimates were needed to estimate storage precipitation, but the latter value was used in the estimation of groundwater storage. The problem was resolved by using a preliminary estimate of groundwater storage based on total precipitation values to determine estimated storage precipitation. Estimated storage precipitation was then inserted into all subsequent regressions. These estimates are given in table 8. While the preliminary groundwater estimates were not quite as accurate as their counterparts in which estimated storage precipitation was used, they are accurate enough for the purposes described here.

Actually, the estimation of the correction coefficients does not require great accuracy. Accordingly, graphs were prepared from which the correction factors could be read quickly and without computation (fig. 6).

Figure 7 shows the development of the estimation of storage precipitation in group II months. The abscissa in all four relationships is measured storage precipitation. In figure 7 (A) the relationship is curvilinear because of the excessively high direct

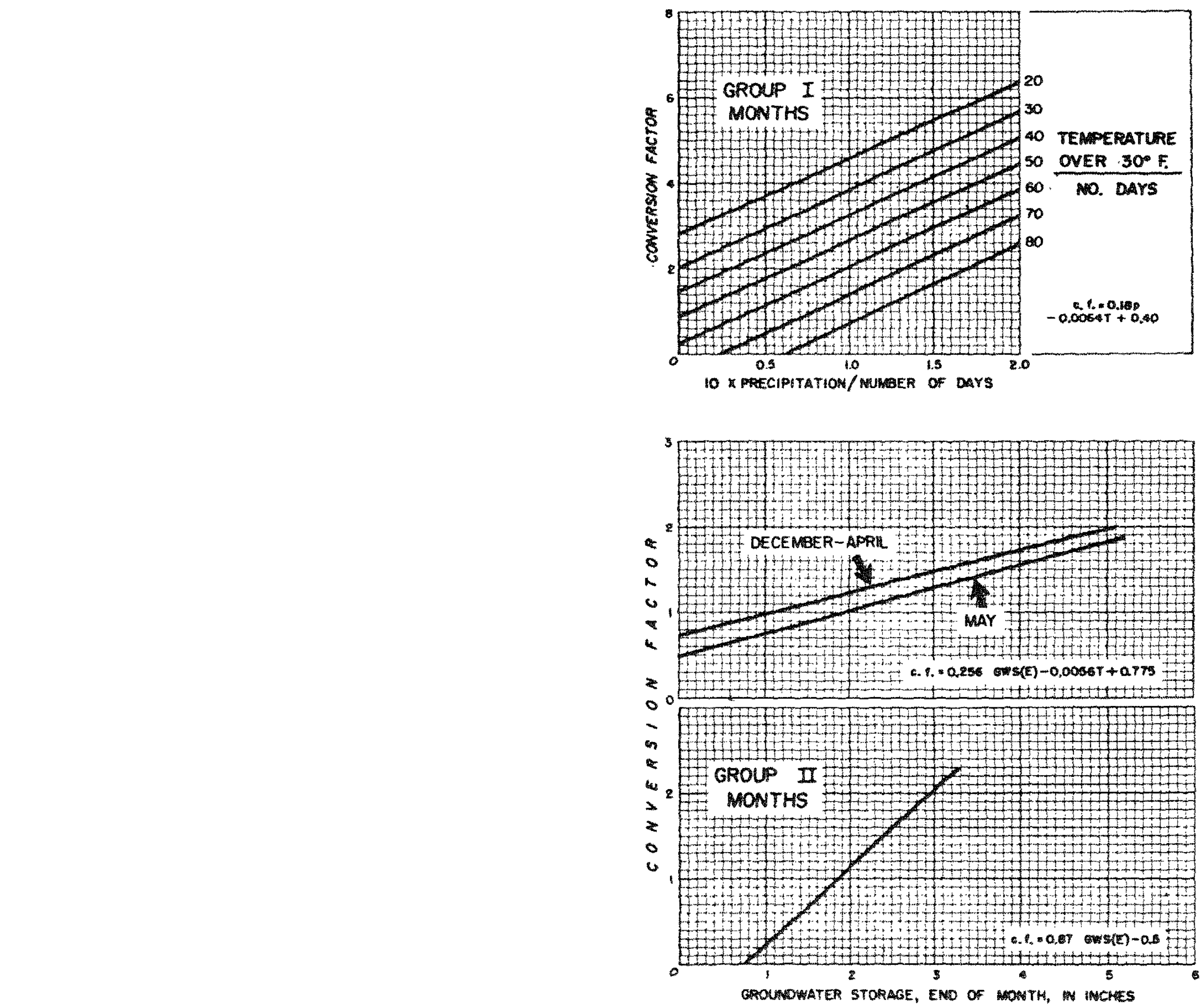


Figure 6. — Monographs for obtaining conversion factors for storage precipitation.

runoff in the months of high rainfall. In figure 7 (B) a linear relationship is obtained by substituting the square root of precipitation, but the scatter is excessive. Figure 7 (C), in which one correction factor was used for all months in the group, still shows an excessive scatter of points, most of which was removed in figure 7 (D).

Basic data for monthly prediction equations are given in table 8.

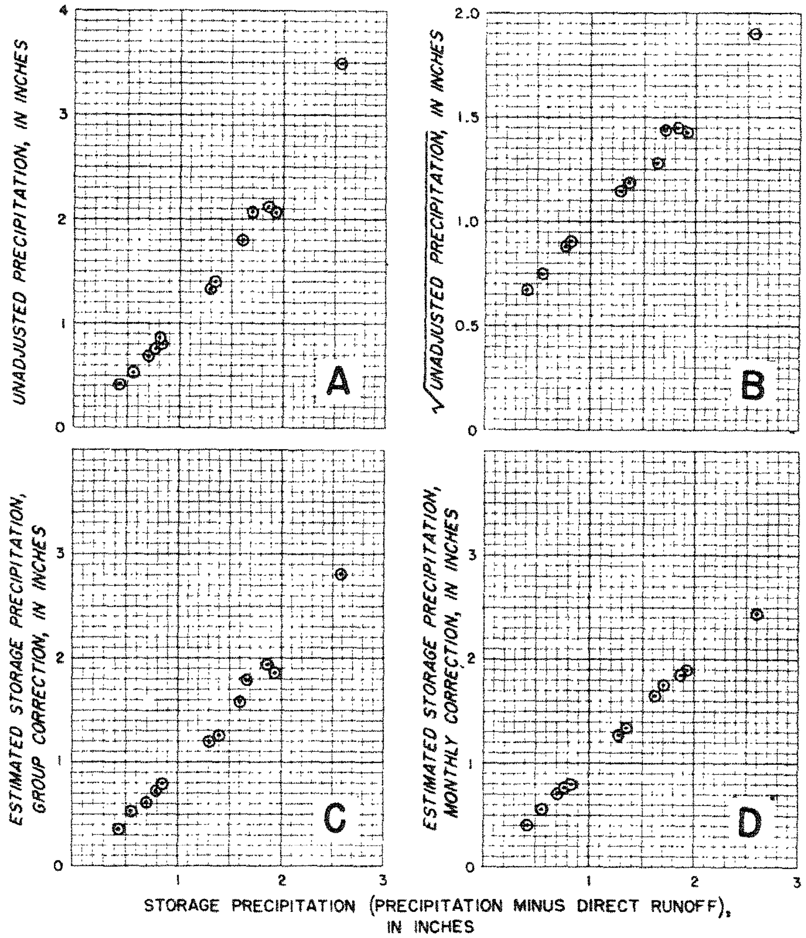


Figure 7. — The evolution of estimated storage precipitation for group II months.

Table 8. — *Basic data and estimates for monthly runoff analysis*

Year and month	Days in month, No.	Precipitation	Runoff	GWS (B)	TS (B)	GWS (E)	TS (E)	Average temp.	Sum of temp. over 30°F./10	Direct runoff	Est. GWS (B)[1]	Est. GWS (B)[2]	Est. GWS (E)[1]	Est. runoff[3]
1949														
April	30	5.23	3.47	2.01	8.96*	3.20	9.80*	44.6	90	0.84	1.82	1.80	3.17	3.13
May	31	7.22	4.91	3.20	9.80*	3.35	9.85*	56.6	165	1.13	3.17	3.09	3.27	4.87
June	30	1.02	1.26	3.35	9.85*	1.38	6.38*	67.1	222	.02	3.27	3.32	1.36	1.27
July	31	4.12	.69	1.38	6.38*	.86	6.36*	71.2	255	.10	1.36	1.27	.88	.75
August	31	6.03	.69	.86	6.36*	.79	7.79*	68.6	240	.25	.88	.85	.77	.61
September	30	4.02	.55	.79	7.79*	1.04	7.96	56.6	159	.13	.77	.86	.85	.58
October	31	2.63	.80	1.04	7.96	1.23	8.54	54.2	150	.10	.85	1.00	1.09	.78
November	30	2.59	1.17	1.23	8.54	1.39	8.46	35.6	51	.15	1.09	1.19	1.47	1.25
December	31	4.04	2.07	1.39	8.46	2.92	9.78	28.0	24	.50	1.47	1.50	2.85	1.99
1950														
May	31	4.35	2.64	2.60	9.74	3.17	10.13	53.0	143	.51	2.63	2.61	2.92	2.66
June	30	3.18	2.14	3.17	10.13	1.76	7.72	62.0	192	.34	2.92	2.99	1.69	2.13
July	31	5.62	1.88	1.76	7.72	1.42	7.33	65.6	220	.57	1.69	1.74	1.28	1.91
August	31	2.25	.76	1.42	7.33	.87	6.29	65.2	218	.10	1.28	1.33	.78	.63
September	30	3.16	.57	.87	6.29	.65	6.98	55.8	155	.06	.78	.81	.65	.57
October	31	2.66	.59	.65	6.98	.79	7.52	51.7	135	.09	.65	.62	.92	.58
1951														
March	31	8.50	5.94	3.46	10.36	5.50	12.64	31.6	29	2.03	3.28	3.37	5.35	5.79
April	30	3.83	5.10	5.50	12.64	3.04	9.46	44.3	87	.70	5.35	5.45	3.09	4.71
May	31	3.17	1.83	3.04	9.46	1.70	8.46	56.4	164	.23	3.09	3.03	2.16	1.82
June	30	4.38	1.40	1.70	8.46	1.58	8.04	62.6	195	.29	2.16	2.11	1.53	1.42
July	31	6.41	1.40	1.58	8.04	1.58	8.57	67.7	234	.45	1.53	1.59	1.37	1.48
August	31	2.50	.88	1.58	8.57	.98	6.62	65.9	222	.07	1.37	1.50	.89	.89
September	30	2.84	.56	.98	6.62	.66	6.64	59.8	178	.06	.89	.93	.62	.59
October	31	4.06	.68	.66	6.64	1.04	8.22	51.0	130	.17	.62	.63	1.00	.71

1952														
April	30	10.26	7.71	3.26	10.05	4.50	11.34	46.3	99	2.55	3.34	3.41	4.80	7.78
May	31	6.33	4.46	4.50	11.34	4.00	10.96	52.4	139	.84	4.80	4.63	3.82	4.47
June	30	2.64	2.48	4.00	10.96	1.68	7.00	65.1	211	.19	3.82	3.86	1.85	2.44
July	31	10.83	5.04	1.68	7.00	1.89	7.13	70.2	250	2.89	1.85	1.85	1.94	5.08
August	31	6.58	2.46	1.89	7.13	1.66	7.75	66.2	224	.76	1.94	1.83	1.71	2.57
September	30	6.21	3.31	1.66	7.75	1.62	7.55	61.5	189	1.11	1.71	1.72	1.69	2.98
October	31	1.41	1.01	1.62	7.55	1.12	7.49	45.2	98	.06	1.69	1.67	1.22	1.13
1953														
January	31	6.14	4.33	2.79	9.47	3.75	10.53	27.4	18	1.26	3.62	3.66	3.79	4.82
February	28	4.06	3.51	3.75	10.53	3.24	9.99	28.2	22	.63	3.79	3.98	3.15	3.67
March	31	5.74	4.45	3.24	9.99	3.80	10.54	33.6	44	.89	3.15	3.21	3.56	4.26
April	30	6.17	4.96	3.80	10.54	3.70	10.55	42.8	80	.84	3.56	3.78	3.65	4.90
May	31	5.36	3.59	3.70	10.55	3.10	10.52	57.8	172	.50	3.65	3.91	3.03	3.63
June	30	2.00	1.77	3.10	10.52	1.96	6.66	65.4	212	.10	3.03	3.08	1.47	1.81
July	31	4.30	.87	1.96	6.66	.90	6.56	69.2	243	.12	1.47	1.48	.92	.90
August	31	1.37	.47	.90	6.56	.50	4.32	67.0	230	.01	.92	.94	.48	.45
September	30	5.26	.60	.50	4.32	.32	6.06	60.8	185	.26	.48	.46	.41	.61
October	31	3.70	.48	.32	6.06	.86	8.36	53.0	143	.14	.41	.40	.88	.46
November	30	3.78	.97	.86	8.36	1.55	8.82	41.3	76	.16	.88	.85	1.45	.94
1954														
February	28	3.93	2.01	1.58	8.43	2.83	9.90	30.0	32	.47	1.78	1.74	3.02	2.03
March	31	4.91	3.71	2.83	9.90	2.92	9.81	31.8	38	.71	3.02	3.15	2.84	4.09
April	30	4.38	3.05	2.92	9.81	2.78	9.92	46.5	108	.59	2.84	2.90	2.65	3.17
May	31	4.21	3.25	2.78	9.92	2.30	9.04	52.8	142	.52	2.65	2.55	2.32	3.26
June	30	1.95	1.15	2.30	9.04	1.18	6.65	64.8	209	.11	2.32	2.27	1.25	1.13
July	31	1.76	.55	1.18	6.65	.73	4.61	67.2	230	.03	1.25	1.23	.77	.55
August	31	6.04	.49	.73	4.61	.38	7.46	65.2	218	.16	.77	.76	.62	.54
September	30	3.27	.37	.38	7.46	.31	6.90	59.6	177	.08	.62	.47	—	.36

[1]Estimated by equations using total precipitation.
[2]Estimated by equations using estimated storage precipitation.
[3]Estimated by equations 7, 8, 9, and 10.
[4]The soil-moisture portion of this total was estimated.

GWS (B) = Groundwater storage at the beginning of the month.
GWS (E) = Groundwater storage at the end of the month.
TS (B) = Total storage at the beginning of the month.
TS (E) = Total storage at the end of the month.

Water-Loss Equations

Using estimated values of groundwater storage and storage precipitation, water-loss equations were recomputed as follows: December through April—group IV:

[7] $$\text{Water loss} = 1.273 + 0.484_{P_{11}} - 0.668_{S_{11}}$$

May and June—group III:

[8] $$\text{Water loss} = 1.757 - 0.634_{P_{11}} - 5.944_{T_1} + 2.487_{P_{11}T_1} + 3.017_{T_2} - 0.424_{S_{11}}$$

July through November—group II (in which all S_{11} values were 1.00 and above).

[9] $$\text{Water loss} = 0.881_{P_{12}} + 1.894_{T_1} - 0.492_{S_{11}} - 1.256$$

July through November—group I (in which all S_{11} values were below 1.00).

[10] $$\text{Water loss} = 0.996_{P_{13}} + 1.390_{T_1} - 0.512_{S_{11}} - 1.076$$

in which—

P_{11} = 30 x daily storage precipitation weighted on 40-basis/sum of weights.

P_{12} = 30 x daily storage precipitation weighted on 80-basis/sum of weights.

P_{13} = 30 x daily storage precipitation weighted on 4-percent decrease basis/sum of weights.

T_1 = Average monthly length of day, sunrise to sunset, in minutes/1000.

T_2 = Sum of temperatures over 30°F. for month/number of days in month x 100.

S_{11} = Estimated groundwater storage at the beginning of the month (GWS-B), based on storage precipitation data.

As to precision: figure 8 (A) illustrates the relationship between precipitation and runoff for May and June months. Nearly all of the scatter of the points is due to the variation in temperature between individual months and is accounted for by an additional variable in the 5-variable equation. The divergent curves reflect the interaction in the equation.

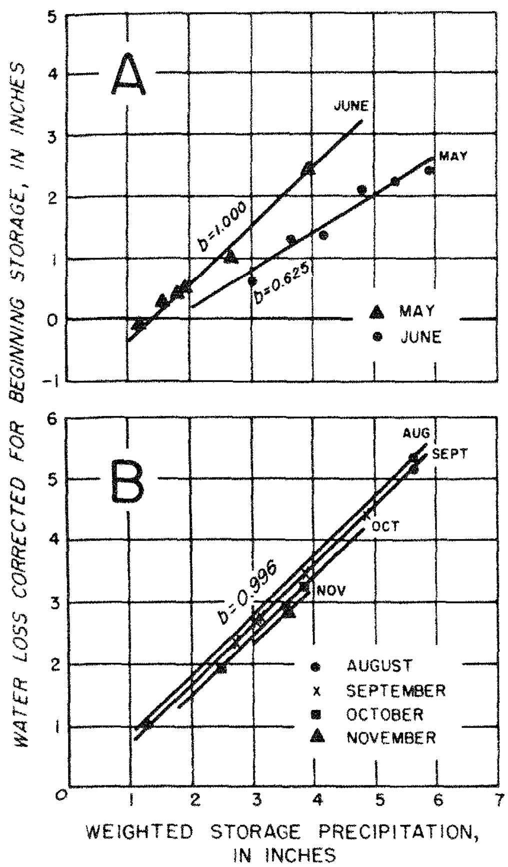


Figure 8.—The relationships between precipitation and runoff for (A) May and June months and (B) group I months — July, August, September, October, November.

Figure 8 (B) shows the relationship between precipitation and water loss for group I months. The family of curves shows the variation due to months. In both figures, actual water loss has been corrected by the calculated effect of the estimated beginning groundwater storage level. Table 9 presents the statistics related to the preceding equations.

Standard errors of estimate and correlation coefficients shown in table 9 indicate the accuracy of the estimating equations; the standard errors are well under 10 percent of mean monthly runoff so that reasonably small changes in water loss or runoff might be detected in a post-calibration period. However, there is no way of knowing if the high correlations are valid, if they are the results of chance, or if they stem from overmanipulation

Table 9. — *Statistics for estimating equations*

Item	Group IV December through April: *Equation 7*	Group III May and June: *Equation 8*	Group II July through November (high GWS-B): *Equation 9*	Group I July through November (low GWS-B): *Equation 10*
Number of samples	12	12	13	12
Degrees of freedom for error	9	6	9	8
$S_{y.x}$ (standard error of estimate)	0.291	0.037	0.136	0.041
Mean water loss	1.41	1.24	2.80	3.26
5 percent of WL	0.070	0.062	0.140	0.163
Correlation coefficient	0.966	0.9996	0.997	0.9996
Mean runoff	4.19	2.57	1.60	0.58
10 percent of runoff	0.419	0.257	0.160	0.058
Correlation coefficient	0.987	0.9998	0.996	0.972

of the data. Tested on a 24-month record of the 4-year period that followed the calibration period, average deviations were 0.34 inch or 16 percent of the average monthly runoff. This, however, as noted before, was an unusual period in which occurred drought, hurricane rainfalls, and a wildfire that burned over about one-third of the watershed.

There has been some question as to the importance of the water-loss estimate. Even though it is a natural phenomenon and may legitimately be used to detect the effects of treatment, most hydrologists and watershed managers are interested in the runoff value. Runoff during the summer months with low groundwater storage (group I months) is particularly important to watershed managers.

If the object of a future analysis is merely to detect differences due to treatment, water-loss analysis should be used because of its greater sensitivity. In any event, water-loss estimates can easily be converted to runoff by subtraction from precipitation. If it were desired to estimate runoff directly, precipitation would have to be weighted oppositely to the weighting procedure used in the storage estimates—another lengthy operation—and direct runoff would have to be a separate variable.

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