An Approach to YELLOW - POPLAR Tree Valuation

by Edwin L. Lucas

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RICHARD D. LANE, DIRECTOR

The Author

EDWIN L. LUCAS received his bachelor of science degree in forests products from Colorado State University in 1961, and his master's degree in forest economics from Iowa State University in 1967. The research for this report was conducted while he was assigned to the Northeastern Forest Experiment Station's hardwood log and tree grade project in Columbus, Ohio. At present he is an associate wood technologist in the Station's hardwood product improvement project at the Forest Products Marketing and Utilization Laboratory at Princeton, West Virginia.

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INTRODUCTION

HARDWOOD log grades are used today to classify logs or trees with similar characteristics into groups with stated average percentages of grade lumber so that an average value may be determined for the group. This discontinuous grading system introduces a *grouping error* in estimating value because there is really no distinct difference in value between the poorest logs of one grade and the best logs of the next lower grade. This grouping error could be eliminated by using a *continuous* grading system—a continuous predictor.

This paper describes the technique used to develop, and explore the feasibility of, a continuous predictor for the grade lumber yield of standing yellow-poplar trees (*Liriodendron tulipifera L.*).[1]

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[1] Two previously-developed yellow-poplar grading systems (*5*) each have three grades of trees, and hence the grouping error is built into them. The equations listed later in this paper still contain the grouping error, because current log grade yield tables were used to compute lumber recoveries by grade. When actual sawed yields are used to establish the equation coefficients, however, the grouping error will be eliminated.

Our results show that a continuous predictor for the lumber recovery of standing yellow-poplar trees is feasible. Equations were developed that, when applied to a sample of 126 yellow-poplar trees in Illinois, Kentucky, and Ohio, predicted a value of lumber yield within 2 percent of the value computed from individual log yields based on Forest Service log grades (*6*).

An important feature of the equations is that they require only three tree characteristics to be measured: diameter breast height, merchantable height, and the number of 5-foot clear cuttings on the four faces of the 16-foot butt log. Thus the equations require only one characteristic to be measured in addition to those now needed for volume estimation, and they eliminate the need for further grading in the field. Another feature is that the calculated yields are divided into each of the seven yellow-poplar standard lumber grades (*3*).

SAMPLE DATA

The data used came from yellow-poplar tree diagrams originally collected as part of an overall study developed at the Carbondale, Illinois, field office of the Central States Forest Experiment Station to establish hardwood tree grades. The data sheets for 226 yellow-poplar trees collected during the Carbondale tree-grade study were used in developing the prediction equations reported in this paper. Merchantable tree heights ranged from 13 to 96 feet, and d.b.h. from 12 to 36 inches.

PROCEDURES

Variables

Seven dependent variables were used. The value for each was expressed in terms of the board-foot volume of 4/4 green lumber recoverable from the tree in each of the seven standard yellow-poplar grades (*3*): FAS, Selects, Saps, No. 1 Common, No. 2A Common, No. 2B Common, and No. 3 Common.

The board-foot yield for each log in the tree was calculated by multiplying the International 1/4-inch log rule gross scale by the appropriate percent lumber grade yield.[2] The volume in each lumber grade was summed over all logs in the tree to arrive at total tree lumber volume.

Two sources of error are apparent in computing tree lumber yields in this manner. They are the variances associated with the lumber grade yield tables, and the log-scale rule (International 1/4-inch log rule) used in computing gross log volume. As there is no practical method for evaluating these two sources of error, the computed lumber yields were accepted as an estimate of the actual yield.

Twenty-four independent variables (15 tree characteristics and 9 transformations) were examined (table 1). Except for merchantable tree height, only those characteristics occurring in the first 16 feet in the tree above a 12-inch stump were evaluated. Space limitations preclude a detailed description of the independent variables. Those that were significant in the predictions will be described later.

Developing the Model

The data were analyzed by using the least-squares principle for fitting a multiple linear[3] regression model. The regression models used were of the general form:

$$Y_i = B_0 + B_1 X_{1i} + B_2 X_{2i} - - - + B_k X_{ki} + e_i$$

Where:

n = Number of sample units.

K = Number of independent variables.

Y_i = Observed value of the dependent variable for the i^{th} unit in the sample, i = 1,2, - - -, n.

X_{ji} = The value of the j^{th} independent variable on the i^{th} sample unit, j = 1,2, - - -, k.

B_j = The regression coefficient of the j^{th} independent variable.

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[2]The percent lumber yields for the logs grade 1 through 3 were taken from the Forest Products Laboratory's curved percent yield tables. Yields for logs that did not meet the requirements for grade 3 logs, but were merchantable under the standards set for this study, were taken from tables developed by Campbell (*2*).

[3]Linear refers to the linearity of the model's coefficients and not to the independent variables.

However, to get a reliable estimate of the confidence interval and its associated probability, the specific terms in the model, and their form, must be selected before the analysis is started. Not enough was known about the relationship between the dependent and independent variables to construct such a model. To overcome this, the initial sample of 226 trees was divided into two parts. One hundred tree diagrams were randomly selected and used to derive the models. These models were then used with the remaining sample data to develop the final prediction equations.

The data for all 31 variables, 7 dependent and 24 independent, were compiled; and a simple correlation matrix was computed. Using the correlation matrix as a guide, we selected 10 sets of independent variables for model searching (table 1).

A step-wise fitting procedure was used to select those independent variables within each set that had a significant effect on the dependent variable. For example, in set 1 there were six independent variables: X_1, X_2, X_{20}, X_{21}, X_{22}, and X_{23}; the step-wise procedure was used to fit

$$
\begin{array}{l}
X_1, X_2, X_{20}, X_{21}, X_{22}, X_{23} \text{ on } Y_1 \\
X_1, X_2, X_{20}, X_{21}, X_{22}, X_{23} \text{ on } Y_2 \\
- \qquad\qquad\qquad\qquad\qquad - \\
- \qquad\qquad\qquad\qquad\qquad - \\
- \qquad\qquad\qquad\qquad\qquad - \\
- \qquad\qquad\qquad\qquad\qquad - \\
X_1, X_2, X_{20}, X_{21}, X_{22}, X_{23} \text{ on } Y_7,
\end{array}
$$

retaining only those variables significant at the 0.05 probability level. This procedure was repeated for the other 9 sets of independent variables.

When the coefficient of determination, R^2, was used as an index for ranking, the results showed that no single set of variables was best for predicting all seven grades of yellow-poplar lumber. For example: in predicting FAS grade lumber, set 9 had the highest R^2; in predicting No. 1 Common, set 2; and in predicting No. 3 Common, sets 3 and 8.

This required some scheme for selecting which set or sets to

Table 1.—Independent variables and the selected sets used for model searching

Symbol	Independent variable	Set number 1	2	3	4	5	6	7	8	9	10
X1	Diameter breast height (DBH)	x[a]	x	x	x	x	x	x	x	x	x
X2	Tree merchantable height (HT)	x	x	x	x	x	x	x	x	x	x
X3	2-foot cuttings, BPK[b] a defect	—	—	—	—	—	—	—	—	—	—
X4	3-foot cuttings, BPK a defect	—	—	—	—	—	—	—	—	x	—
X5	5-foot cuttings, BPK a defect	—	—	—	—	—	—	—	—	—	x
X6	Select quarter sections, BPK a defect	—	—	—	—	x	—	—	—	—	—
X7	Select 4-foot sections, BPK a defect	—	—	x	—	x	—	—	—	—	—
X8	Total defect count, BPK a defect	—	—	x	—	—	—	—	x	—	—
X9	Total defect count, BPK no defect	—	—	—	x	—	x	—	—	—	—
X10	Total knot count	—	—	x	x	x	x	x	—	x	x
X11	2-foot cuttings, BPK no defect	—	—	—	—	—	—	—	—	—	—
X12	3-foot cuttings, BPK no defect	—	—	—	—	—	—	—	—	—	—
X13	5-foot cuttings, BPK no defect	—	—	—	—	—	—	—	—	—	—
X14	Select quarter sections, BPK no defect	—	—	—	—	—	—	x	—	—	—
X15	Select 4-foot sections, BPK no defect	—	—	—	x	—	—	—	—	—	—
X16	X_1 squared	—	—	x	x	—	—	—	—	—	—
X17	X_{11} squared	—	x	—	—	—	—	—	—	—	—
X18	X_{12} squared	—	x	—	—	—	—	—	—	—	—
X19	X_{13} squared	—	x	—	—	—	—	—	—	—	—
X20	X_3 squared	x	—	—	—	—	—	—	—	—	—
X21	X_4 squared	x	—	—	—	—	—	—	—	x	—
X22	X_5 squared	x	—	—	—	—	—	—	—	—	x
X23	DBH^2 times HT	x	x	x	x	x	x	x	x	x	x
X24	1/DBH times HT	—	—	—	—	—	—	—	—	—	—

[a]X indicates the variable was selected for the set; i.e., set one contains the independent variables X_1, X_2, X_{20}, X_{21}, X_{22}, and X_{23}.

[b]BPK indicates birdpeck.

Table 2.—Board-foot volume and lumber value by grade for the 100 sample trees

Lumber grade	Symbol	Price per M board feet	Lumber volume		Lumber value	
		Dollars	*Board-feet*	*Percent*	*Dollars*	*Percent*
FAS	Y_1	220	1,291	2.7	284	4.7
Selects	Y_2	210	437	.9	92	1.5
Saps	Y_3	200	1,528	3.2	306	5.1
1C	Y_4	160	17,952	37.0	2,872	48.0
2A	Y_5	107	14,182	29.2	1,517	25.4
2B	Y_6	73	8,505	17.5	621	10.4
3C	Y_7	63	4,597	9.5	290	4.9
Total	—	—	48,492	100.0	5,982	100.0

use in fitting the final prediction equations with the remaining sample data. An average R^2 for each set did not seem advisable because of the large differential between the yields of the seven grades of lumber and their respective prices (table 2). To account for differences in lumber grade prices and yields, a weighted R^2 value was used to evaluate the predictive properties of individual sets.

Three terms were used in calculating the weighted R^2: (1) a price relative for each grade, calculated by dividing the price per M board feet for each grade by the price of No. 1 Common lumber; (2) a volume relative for each grade, determined by dividing the total volume in each lumber grade by the volume in the Select grade; and (3) the coefficient of determination, R^2, for each grade. The cross-product of the three terms was calculated for each lumber grade and then was summed for all grades by sets. The assumption of the weighting scheme was that the set with the highest sum would be the best overall predictor of tree value.

In set 10, knot count was significant only in predicting the No. 2B Common grade of lumber (table 3). It was deleted from the set, and the step-wise analysis was rerun for the No. 2B

Table 3.—Variables that were significant in predicting lumber grade yields, and the associated equation coefficients of determination (R^2)

Independent variable	Independent variables found to be significant in predicting the dependent variables						
	Y_1 FAS	Y_2 Sel	Y_3 Saps	Y_4 1C	Y_5 2A	Y_6 2B	Y_7 3C
			SET 1				
X_1	x[a]	x	—	—	—	—	x
X_2	—	—	—	x	—	x	x
X_{20}	—	—	—	—	—	—	—
X_{21}	x	x	x	x	—	—	x
X_{22}	—	—	—	—	x	x	—
X_{23}	—	—	x	x	x	x	x
R^2	0.47	0.48	0.51	0.89	0.79	0.45	0.62
			SET 9				
X_1	—	—	—	—	—	—	x
X_2	x	x	—	x	—	x	x
X_4	—	—	—	x	—	—	—
X_{10}	—	—	—	—	—	x	—
X_{21}	x	x	x	—	x	x	x
X_{23}	—	—	x	x	x	x	x
R^2	0.47	0.48	0.51	0.89	0.78	0.46	0.62
			SET 10				
X_1	x	x	—	—	—	—	x
X_2	—	—	—	x	x	x	x
X_5	x	x	—	x	x	x	—
X_{10}	—	—	—	—	—	x	—
X_{22}	—	—	x	—	—	—	x
X_{23}	—	—	x	x	x	x	x
R^2	0.46	0.47	0.52	0.89	0.79	0.48	0.61

[a]Indicates the variable was significant at the 0.05 probability level.

equation. There was only a slight reduction (0.48 to 0.46) in the R^2 value when knot count was omitted. Knot count was thus dropped from set 10 and the weighted R^2 was recalculated.

The results of the weighted R^2 calculation for each set were as follows:

Set	*Weighted R^2*
1	40.56
9	40.55
10	40.53
5	40.39

Set	*Weighted R^2*
3	40.29
6	39.83
4	39.30
2	39.26
8	38.84
7	37.82

The weighted R^2s were interpreted as showing no practical differences in the predictive properties of the three top-ranking sets: 1, 9, and 10. These sets were the only ones considered for further investigation.

Since knot count had been dropped from set 10 without an appreciable reduction in R^2, it was assumed that the same could be done in set 9. This left sets 9 and 10 with three variables requiring field measurements, and set 1 with four. Set 1 was eliminated because it had a greater number of variables requiring measurements. Set 10 was selected for developing the prediction equations because, in field application, it would be the least expensive to use. The equations were developed for this set, using the remaining 126 trees from the initial sample.

Prediction Equations

In set 10, five variables (excluding knot count) were significant in predicting at least one of the seven dependent variables. Each of the seven prediction equations contains all five of these variables. Using the independent variables in set 10 and the remaining 126 sample trees, the following prediction equations were developed:

$$Y_{FAS} = 116.7101 - 5.6021\,X_1 - 1.8947\,X_2 - 0.3431\,X_5 + 0.1987\,X_{22} + 0.004478\,X_{23}$$

$$Y_{SEL} = 25.5130 - 1.2318\,X_1 - 0.4754\,X_2 + 0.1487\,X_5 + 0.05897\,X_{22} + 0.001116\,X_{23}$$

$$Y_{Saps} = -46.4927 + 2.0630\,X_1 + 0.2903\,X_2 + 2.4959\,X_5 + 0.03040\,X_{22} - 0.0003053\,X_{23}$$

$$Y_{1C} = 213.9343 - 9.3608\,X_1 - 5.9098\,X_2 + 2.6055\,X_5 + 0.4121\,X_{22} + 0.01898\,X_{23}$$

$$Y_{2A} = -112.1970 + 6.8718\,X_1 + 1.4716\,X_2 - 3.2297\,X_5 + 0.06368\,X_{22} + 0.002520\,X_{23}$$

$$Y_{2B} = -39.0000 + 2.5340\,X_1 + 1.3392\,X_2 - 1.5769\,X_5 - 0.01917\,X_{22} + 0.0008661\,X_{23}$$

$$Y_{3C} = -31.8421 + 1.9420\,X_1 + 1.5599\,X_2 - 2.1895\,X_5 + 0.06318\,X_{22} - 0.0008192\,X_{23}$$

Where:

X_1 = D.b.h. rounded to the nearest inch.

X_2 = Tree merchantable height, rounded to the nearest foot.

X_5 = The number of 5-foot clear cuttings. Clear cuttings are defined as the portions of the face that lie between defects (*4*), or between log ends and defects. The value recorded was the total number of 5-foot clear cuttings on all four faces of the indicator section.

X_{22} = The X_5 count squared.

X_{23} = D.b.h. squared times merchantable height.

The actual volume of the 126 sample trees was 65 M board feet. The predicted volume of these trees was 63 M board feet, a difference of only 3 percent. When the value of the recoverable lumber from the trees was considered, the difference between actual and predicted value was only 2 percent (table 4).

Table 4.—Comparison of actual and equation-predicted lumber volume and values for 126 yellow-poplar trees

Lumber grade	Price per M board feet	Actual volume	Predicted volume	Actual value	Predicted value
	Dollars	*Board feet*	*Board feet*	*Dollars*	*Dollars*
FAS	220	2,159	2,227	474.98	489.94
Sel	210	709	1,135	148.89	238.35
Saps	200	2,208	2,312	441.60	462.40
1C	160	25,383	25,161	4,061.28	4,025.76
2A	107	17,875	15,627	1,912.62	1,672.09
2B	73	10,513	10,674	767.45	779.20
3C	63	6,299	6,208	396.84	391.10
Total		65,146	63,344	8,203.66	8,058.84
Difference		—	- 1,802	—	-144.82
Percent difference		—	2.77	—	1.76

SUMMARY AND CONCLUSIONS

Advantages of the System

The tree-valuation system reported has several advantages over valuation systems based on discrete tree grades.

First, it is less expensive to use. Of the three variables requiring field measurement, two—d.b.h. and merchantable height—are measured anyway when estimating tree volume. By measuring one additional variable—the number of 5-foot clear cuttings—the user can get lumber volume by grade, and this can be converted easily to dollar value. Since volume estimation is a necessary requisite for timber appraisal, the added cost of field application would be small. The forester can examine the indicator section of the selected trees and record the values for the independent variables on field punch cards. With the prediction equations, the data could then be processed directly by computers.

Second, the predictions are given directly in board-foot volume by lumber grade and can easily be converted to current gross dollar value. Since the predictions are given directly in board-feet, volume tables are not necessary; the equations have their own built-in volume tables. As with any statistical procedure of this nature, the equations may not show the value of an individual tree accurately; they should be used to estimate the total value of a number of trees.

The statistics for making an evaluation of the precision of the estimate are readily available (tables 5 and 6). The user not only knows what the estimated lumber recovery is, but also the variation he might expect in his estimate.

Research Implications

The feasibility of a continuous predictor of tree value has been demonstrated. The design of the study insures, for statistical purposes, the applicability of the equations to field use. However, the use of the equations is limited by the restrictions imposed on their development. The lumber recovery volumes by grade were calculated volumes and are subject to

Table 5.—The relevant statistics and the associated confidence intervals for the seven lumber-grade predictions, using the set 10 independent variables

Dependent variables		R^2	$S_{Y\cdot\cdot}$	S_Y[a]	$\bar{Y}$	Confidence interval[a] for study sample			
						0.05 probability level		0.01 probability level	
Symbol	*Grade*		*Board feet*	*Board feet*	*Board feet*	*Board feet*	*%*	*Board feet*	*%*
Y_1	FAS	0.74	18.43	1.64	17.1	3.3	19.0	4.3	25.1
Y_2	Sel	.74	5.22	.46	5.6	.9	16.4	1.2	21.6
Y_3	Saps	.43	14.59	1.30	17.5	2.6	14.7	3.4	19.4
Y_4	1C	.93	51.95	4.63	201.4	9.2	4.6	12.1	6.0
Y_5	2A	.82	34.94	3.11	141.9	6.2	4.3	8.1	5.7
Y_6	2B	.66	24.98	2.23	83.4	4.4	5.3	5.8	7.0
Y_7	3C	.65	11.02	.98	50.0	1.9	3.9	2.6	5.1

[a]At the mean value of all independent variables. n=126

Table 6.—Mean and inverse matrix of independent variables for use with prediction equations developed from set 10

Independent variables	Mean	Independent variables				
		X_1	X_2	X_5	X_{22}	X_{23}
X_1	21.9	2.310×10^{-3}				
X_2	42.0	4.604×10^{-4}	2.044×10^{-4}			
X_5	4.8	-2.167×10^{-4}	5.111×10^{-5}	5.787×10^{-3}		
X_{22}	34.7	1.696×10^{-5}	-5.897×10^{-6}	4.796×10^{-4}	4.569×10^{-5}	
X_{23}	21822	-8.600×10^{-7}	-2.495×10^{-7}	2.195×10^{-8}	2.146×10^{-10}	4.156×10^{-10}

the errors associated with the log scale and yield tables used. Also, the sample was not selected over the entire range of yellow-poplar, but was taken from commercial logging operations in Illinois, Kentucky, and Ohio. Priority in future work should be given to testing the equations over the geographic range of the species against actual sawed volumes.

The prediction equations are based on an examination of the butt 16-foot log. No other indicator section was tested. By the same decision rules, other indicator section lengths should be tested to see if the lumber recovery estimates are improved. Bulgrin and Schroeder (*1,5*) tried various lengths of indicator sections in the butt 16-foot log and found that the grade of the best 10-foot section was the best predictor of tree quality.

In determining net tree volume, each predicted lumber grade yield must be reduced by the percent of scalable defect contained in the tree. Under the assumption that most scalable defect comes from the core of the log or tree and represents mostly lower grade lumber, errors arise when the yield in each lumber grade is reduced by the same percent of scalable defect. This can result in underestimating the yields of higher grades of lumber and overestimating the yields of the lower grades. In future studies, a variable expressing cull deduction should be added to the equations and tested for significance.

This method gives a continuous predictor of tree value based on lumber as the end product. Future research will be needed to develop methods for valuation of trees for other products—veneer for example—and for multiproduct tree valuation.

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