Corrections for Cluster-Plot Slop

Harry T. Valentine, Mark J. Ducey, Jeffrey H. Gove, Adrian Lanz, and David L.R. Affleck

Abstract: Cluster-plot designs, including the design used by the Forest Inventory and Analysis program of the USDA Forest Service (FIA), are attended by a complicated boundary slopover problem. Slopover occurs where inclusion zones of objects of interest cross the boundary of the area of interest. The dispersed nature of inclusion zones that arise from the use of cluster plots precludes the use of most of the slopover-correction methods that apply to solitary plots. One exception is the walkthrough method, which corrects for slopover bias in radially symmetric cluster-plot designs. In this article, we provide a modification of the walkthrough method, the "walkabout method," which is applicable to some asymmetric cluster-plot designs, such as the one used by FIA. We also present two general correction methods, the "vectorwalk method" and the "reflection method," both of which are applicable to cluster-plot designs with satellite subplots arranged in a regular or irregular pattern. The reflection method has two sets of protocols, one that applies to radially symmetric designs and to asymmetric designs with random orientation; the other set applies to asymmetric designs with fixed orientation. Both the vectorwalk and reflection methods incorporate the walkthrough method on a subplot-by-subplot basis. All four methods correct for slopover bias for straight or curved boundaries, and where work outside the tract is impossible. FOR. SCI. 52(1):55–66.

Key Words: Boundary overlap, edge effect, Monte Carlo integration, reflection method, slopover, vectorwalk method, walkabout method, walkthrough method.

CLUSTER-PLOT DESIGNS, which are widely used to estimate forest inventories, tend to complicate the problem known variously as slopover, boundary overlap, or edge effect. Slopover occurs where inclusion zones of objects of interest cross the boundary of the area of interest. Slopover can lead to bias in estimates unless corrective measures are taken. In this article we describe four methods that solve the slopover problem for cluster plots. The walkthrough method (Ducey et al. 2004) solves the problem for radially symmetric cluster-plot designs, and a new variant of the walkthrough method—the walkabout method—solves the problem for some asymmetric cluster-plot designs, including the design used by the USDA Forest Service in its nationwide Forest Inventory and Analysis (FIA) program. We also present two general methods, the vectorwalk and reflection methods, that solve the slopover problem for both symmetric and asymmetric cluster-plot designs, including the FIA design. Finally, we discuss and compare the practical implications of the four correction methods, and we discuss the choice of symmetric versus asymmetric designs.

Sampling sans Slopover

We consider a tract of land with a closed boundary. The boundary may be piecewise linear or the boundary may be curved, in whole or in part. The tract may or may not be contiguous. The tract may be the sole focus of an inventory, or it may be a stratum within a stratified inventory. In either case, we desire estimates from the tract that reflect the contents of the tract; we do not wish to make inferences based on a chimeric mixture with the contents of adjoining areas. Let $\mathscr{A}$ identify the tract and let A be the horizontally projected land area of $\mathscr{A}$.

We use an approach based on Monte Carlo integration to motivate our solution to the slopover problem; a "Horvitz-Thompson approach" (Horvitz and Thompson 1952, et seq.) leads to the same conclusions by a more tortured path. From the Monte Carlo perspective, the population of interest comprises the infinitely many location points in $\mathscr{A}$ (see, e.g., Valentine et al. 2001, Barabesi 2003). The target parameter is the total amount of some attribute, which is distributed discretely among N objects of interest in $\mathscr{A}$. N ordinarily is unknown.

We assume that m location points with coordinates $(x_j, z_j), j = 1, 2, \ldots, m$, serve as sample points. Each sample point is selected at random with uniform probability density $f(x_j, z_j) = 1/A$. We restrict our interest to cluster-plot designs that comprise a central subplot centered at a sample point and $n - 1$ nonoverlapping satellite subplots in a fixed arrangement. This type of subplot arrangement defines a spatially disjoint inclusion zone about the center point of any object of interest.

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We may use "direction vectors" to define the center points of the satellite subplots relative to the sample point (see Figure 1a,b), in which case inverse vectors—the direction vectors rotated 180°—define the center points of "satellite subzones" of an object's inclusion zone relative to the center point of the object (see Figure 1c,d). The object is centered in the "central subzone" of its inclusion zone. Thus, a 180° rotation of the cluster plot about the sample point provides the shape of the inclusion zone. A cluster plot includes an object of interest if the sample point falls anywhere in the object's inclusion zone (Figure 1e,f).

Let a be the horizontal land area of each fixed-radius subplot within a cluster plot. The cluster plot diagrammed in Figure 1 has a central subplot and three satellite subplots, so the total horizontal land area of an inclusion zone is $4a$. Let y be an attribute of an object of interest, e.g., the volume of a tree. Although y is an attribute of a discrete object, we can transform the attribute into a continuous attribute density (ρ) by dividing y by the horizontal land area of the object's inclusion zone; to wit, $\rho = y/4a$. If, for example, the object of interest is a tree and the attribute is volume, then the resultant attribute density is tree volume per unit land area.

More generally, let the subscript k identify the kth of the N objects in $\mathcal{A}$, let I_k symbolize the kth object's inclusion zone, and let na_k be the horizontal area of I_k. Naturally, if all clusters comprise n nonoverlapping, fixed-area subplots of equal size, then a_k equals a constant a for all k. The attribute density—the amount of attribute per unit land area—is defined to be $\rho_k = y_k/na_k$ at every location point in I_k.

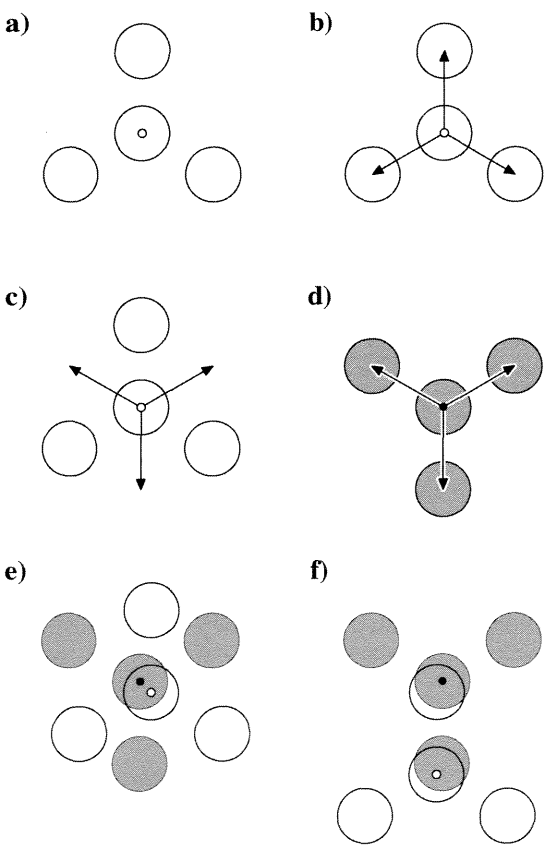

Figure 1. (a) A cluster plot comprising a central subplot and satellite subplots is fixed about a sample point (○). (b) Direction vectors indicate distances and directions from the sample point to centers of the satellite subplots. (c) Inverse vectors—the direction vectors rotated 180°—indicate the distances and directions from the center point (●) of an object to the center points of the satellite subzones of the object's inclusion zone (d). An object occupies a cluster plot if the sample point falls anywhere in the object's inclusion zone (e and f).

Inclusion zones of two or more objects may overlap a given location point, in which case the total attribute density at the location point equals the sum of the individual attribute densities. For example, if several inclusion zones overlap a sample point at (x_j, z_j), then the total attribute density at (x_j, z_j) is

$$\rho(x_j, z_j) = \sum_{(x_j,z_j)\in I_k} \rho_k = \sum_{(x_j,z_j)\in I_k} \frac{y_k}{na_k},$$

where $(x_j, z_j) \in I_k$ indicates that the summation is over all objects in $\mathcal{A}$ whose inclusion zones include the sample point. Such objects are identified by their presence within any of the subplots of the fixed-area cluster plot.

If every inclusion zone of every object of interest is wholly contained within the boundary of $\mathcal{A}$, then the total amount of attribute τ in $\mathcal{A}$ is

$$\tau = \iint_{\mathcal{A}} \rho(x, z)\, \mathrm{d}x\, \mathrm{d}z = \sum_{k}^{N} y_k$$

τ is unbiasedly estimated by

$$\hat{\tau}_j = \frac{\rho(x_j, z_j)}{f(x_j, z_j)} = A \sum_{(x_j,z_j)\in I_k} \frac{y_k}{na_k}. \tag{1}$$

Or, combining the results from m sample points, τ is unbiasedly estimated by

$$\hat{\tau} = \frac{1}{m} \sum_{j}^{m} \hat{\tau}_j. \tag{2}$$

The estimators, (1) and (2), are expected to underestimate τ if part of any inclusion zone slops over a boundary—unless corrective actions are taken.

The Slopover Problem

Slopover (or boundary overlap) occurs where the inclusion zone of any object of interest crosses a tract boundary (Figure 2). Slopover is a problem because we assume that the attributes of an object extend across the object's inclusion zone with uniform density. If some proportion of the area of the inclusion zone is outside the tract, then an equal proportion of the attribute is also outside the tract. Hence, if sample points are constrained to fall only within the tract, a portion of the attribute is ignored and our estimate of the total amount of attribute for the tract is expected to be biased downward.

By contrast, that whole subplots or parts of subplots fall outside the tract is not a problem (Figure 2). Subplots merely serve to indicate which objects in $\mathcal{A}$ have inclusion

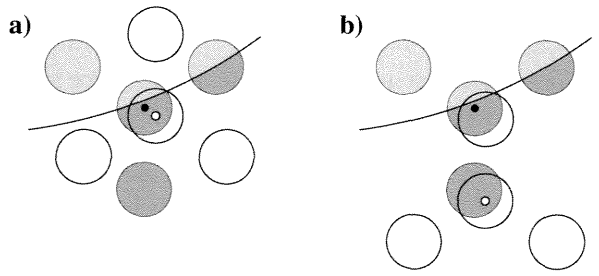

Figure 2. Slopover (light gray) occurs where inclusion zones extend outside the boundary of the tract.

zones that include the sample point. Subplots and parts of subplots that fall outside the tract may be ignored, since attributes of objects outside the tract are not of interest.

Single-Plot Correction Methods

Schreuder et al. (1993), Ducey et al. (2004), and Iles (2004) reviewed some existing slopover correction methods as they apply to conventional single-plot, rather than cluster-plot, inventories. These methods entail (1) allowing sample points to fall outside the tract; (2) protocols for reflecting, rotating, or translating the slopover back into the tract; (3) using half-plots or quarter-plots near the boundary; or (4) measuring the horizontal area of the in-tract portion of the inclusion zone.

The protocols of these methods are formulated to apply to the compact inclusion zones of independent solitary plots. Applying these protocols in a cluster-plot inventory, on a subplot-by-subplot basis, ordinarily will not correct for cluster-plot slop. However, as we shall see, independent subplot-by-subplot corrections can be part of a comprehensive solution, as provided by the vectorwalk and reflection methods, and by Mandallaz's (1991) method.

Several single-plot methods require work outside the boundary, which precludes general applicability. Tract boundaries may include natural features such as edges of lakes, rivers, and cliffs, which make sampling or working outside the tract infeasible. Natural features and roads may also preclude location points within a tract from serving as sample points.

Subplot-by-Subplot Failures

It is easy to show by example that protocols of single-plot methods can completely fail to correct for boundary slopover bias that results from the use of cluster plots. Here we show the shortcomings of three different methods: the buffering method of Masuyama (1954), the mirage method (Schmid-Haas 1969), and the walkthrough method (Ducey et al. 2004). We do not delve into the details of each method; interested readers may wish to consult the reviews in Schreuder et al. (1993), Ducey et al. (2004), or Iles (2004).

In the buffering method (Masuyama 1954), or the related toss-back method (Iles, 2004, Flewelling and Iles 2004), sample points are allowed to fall outside the tract, but only those objects within the tract are tallied. To constrain the scope of sampling, points often are allowed only within a buffer, defined by the maximum distance between a sample point and an included object. Where independent fixed-radius plots are used, that distance is the plot radius. Now, consider a cluster-plot design, where objects of interest have inclusion zones as depicted in Figure 3. If the buffer width is set as the subplot radius, it is clear that sample points still cannot fall in the slopover of the inclusion zone (the subzone labeled **d**). Rather, the buffer must be much larger, equal in width to the distance from the true plot center to the center of a satellite subplot, *plus* the plot radius. Large buffers can add substantially to both the sampled area and the operational burden of sampling, but large buffers are required for slopover correction when cluster plots are used.

Next, suppose that the mirage method, as proposed by Schmid-Haas (1969) and further developed by Gregoire (1982), is used on a subplot-by-subplot basis. In the mirage method, the plot center is reflected through the boundary, and objects within the tract are tallied if they occur in the plot centered at the mirage point. When inclusion zones are circular, only those objects tallied from the original point are eligible for re-tally (Gregoire 1982, Ducey et al. 2001). Again, consider Figure 3. If a sample point falls in the central subzone of the inclusion zone (the subzone labeled **a**), the true cluster-plot center will be reflected through the boundary, but the object would not be re-tallied from any mirage point in the reflection of **a**, because the object is inside the boundary by a distance greater than the subplot radius. If the sample point were to fall in the satellite subzone labeled **b**, then the object would be tallied in a satellite subplot. However, the object, which occurs in **a**, would not be tallied from any mirage point of any sample point that occurs in **b**. The same observation holds if the sample point falls in **c**. So, if the sample point were to fall in **a**, **b**, or **c**, y_k would receive its usual weight. However, the sample point cannot fall in **d**, because **d** is outside the tract. The boundary slopover bias remains wholly uncorrected.

Finally, consider the walkthrough method of Ducey et al. (2004), which is described in detail in the next section. Like the mirage method, the walkthrough method relies on reflecting the slopover back into the tract, and giving extra

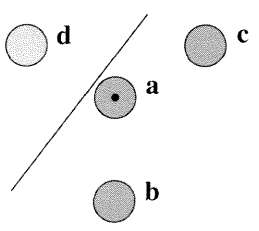

Figure 3. Inclusion zone for an object in a cluster-plot design with a center subplot and three satellite subplots. The central subzone of the inclusion zone (a) and two satellite subzones (b and c) fall completely within the tract. One satellite subzone (d) falls completely outside the tract. If either the mirage method or conventional walkthrough are applied, treating subplot centers as sample points, slopover bias will remain completely uncorrected in this case. If sampling is conducted in a buffer around the tract, but the buffer width is set by the size of a subplot, the bias will also remain completely uncorrected.

weight to y_k if the sample point falls in the reflection. With circular inclusion zones, the object must fall closer to the boundary than to the sample point to be double-tallied. However, in Figure 3, the distance from the object to the boundary is greater than the length of the subplot radius. Therefore, if the center of a subplot is treated as the plot center for walkthrough purposes, the object can never be double-tallied, and once again the slopover bias arising from **d** remains wholly uncorrected.

Corrections for Cluster-Plot Slop

The new correction methods described in the following sections alter the way the attribute density maps across the area of the inclusion zone, I_k, when the kth object is near a boundary. A walkthrough correction for slopover reduces the attribute density to zero in a section of I_k that falls outside the tract and doubles the density in another section of equal area within the tract. With an FIA cluster-plot design, a walkabout correction either (1) reduces the density of a section of I_k to zero and doubles the density in another, or (2) reduces the attribute density to zero in two sections of I_k and triples the density in a third section, all three sections having the same area. If the cluster plot contains a central subplot and $n - 1$ satellite subplots, the walkabout method may lead to a density multiplier of up to $n - 1$. Hence, let $t_k(x, z)$ indicate the factor $(0, 1, 2, \ldots, n - 1)$ by which the attribute density of the kth object is multiplied to effect a walkthrough or walkabout correction at any point $(x, z) \in I_k$.

The vectorwalk method translates the portions of attribute in I_k that fall outside the tract into the central subzone of I_k. These translated portions of attribute are heaped one on another in the central subzone and it turns out that $t_k(x, z)$ may take values 0, $1i$, or $2i$, where $i = 1, 2, \ldots, n$. The reflection method folds out-of-tract portions of attribute about the boundary into the tract, effectively redefining the shape of the inclusion zone, I_k. Because reflected sections may overlap other parts of I_k, $t_k(x, z)$ may take values 0, 1, 2, . . . , up to a maximum that depends on the configuration of the cluster plot and the boundary in the neighborhood of the object. Like the walkthrough and walkabout methods, the vectorwalk and reflection methods conserve area and attribute. Thus, after correction for slopover by any of the methods, the total attribute density at the jth sample point is

$$\rho(x_j, z_j) = \sum_{(x_j, z_j) \in I_k} \frac{t_k(x_j, z_j)\, y_k}{n a_k}.$$

For convenience, we define $t_k \equiv t_k(x_j, z_j)$. Hence, t_k corresponds to the number of times the kth object is tallied at the sample point.

The Walkthrough Method

As was noted, the walkthrough method (Ducey et al. 2004) does not solve the boundary slopover problem if it is applied naively and incorrectly. However, it does solve the problem for certain cluster configurations if performed correctly, and it forms the basis for the walkabout solution, so we review it here.

The walkthrough method for cluster plots may be applied to solve the slopover problem if inclusion zones are radially symmetric about the center points of the objects of interest (Figure 4). By "radially symmetric" we mean that each satellite subplot is matched with another satellite subplot that is equidistant from the sample point in the opposite direction. This does not mean that all the subplots must be the same distance from the sample point (see Figure 4c).

The method effectively deals with both defined and natural boundaries. Moreover, the boundaries may be straight or curved, though some small amount of bias may accrue with tightly curved boundaries that wrap around objects in the tract. The operation of the walkthrough method is illustrated in Figure 5.

When the inclusion zone is symmetric about the center point, any location point may be reflected to another location point in the inclusion zone. For example, if we start at any location point in an inclusion zone and walk, say, 4 m to the center point of the object, and then continue on this line "through the object" another 4 m, we will end at a location point in the inclusion zone that is 4 m from the starting point and 4 m from the center point. The starting point is a reflection point of the ending point, and vice versa.

For most boundary configurations, each of the infinitely many location points in any slopover section of an inclusion zone will have a refection point in the "in-tract" portion of the inclusion zone (Figure 5). Thus, we can imagine reflecting the entire portion of attribute in a slopover section of the inclusion zone into an in-tract section of the inclusion zone, doubling the attribute density at each point in the in-tract section.

Operationally, we need only concern ourselves with just one reflection point in each inclusion zone, i.e., the reflection point of the sample point. Suppose that a sample point falls within the inclusion zone of the kth object, which is near a boundary. If the reflection point falls in slopover, then the sample point falls within the reflection of the slopover, in which case the attribute density attributable to the kth object is doubled at the sample point, i.e., $t_k = 2$.

To ascertain whether the reflection point is outside the tract, we apply the walkthrough method, i.e., we "walk" from the sample point to the object of interest, recording the

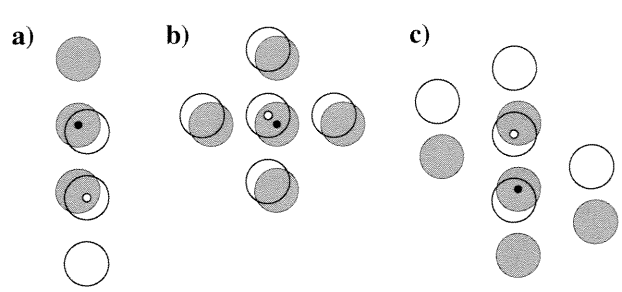

Figure 4. Radially symmetric cluster plots centered at a sample point (○) define inclusion zones that are radially symmetric about the center point (●) of an object of interest.

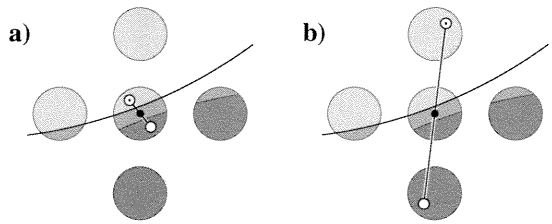

Figure 5. The walkthrough method: slopover (light gray) is reflected back into the tract, doubling the attribute density where the reflection occurs (dark gray). Operationally, the sampler walks from the sample point (○) to the center point (●) of an object and through the object an equal distance to the reflection point, ⊙. If the reflection point is outside the tract (a, b), the object's attribute density at the sample point is doubled, i.e., $t_k = 2$. Usually this can be determined if the boundary is reached before the reflection point is reached; thus, little or no travel outside the boundary is required.

distance, and then "through" and beyond the object an equal distance to the reflection point. If we encounter the boundary before we reach the reflection point, we double the object's contribution to the attribute density at the sample point.

The method is unbiased if every location point in the slopover has a reflection point inside the tract. This constraint is not met if more than half of the area of an inclusion zone is outside the tract and it may not be met if the boundary configuration is unusually convoluted (Ducey et al. 2004). Even when these conditions are not met, the walkthrough method reduces the bias due to boundary slopover.

The Walkabout Method

The original walkthrough method imposes a strict requirement that inclusion zones be radially symmetric about each object. This requirement is not met for many cluster-plot designs. We have formulated a "walkabout method" to deal with the slopover problem that results when the cluster-plot design takes the radially asymmetric form used by FIA (Figure 2), where satellite subplots are equidistant from the sample point and the angles formed by the direction vectors are also equal. The method is best described with the aid of diagrams (Figure 6). Although we present the method with angles specific to the situation shown in Figure 2, it generalizes readily to other radial cluster designs.

How one proceeds depends on whether the object of interest falls within the central subplot (Figure 6a) or a satellite subplot (Figure 6c). If the object falls within the central subplot, then the standard walkthrough method is applied to determine whether the reflection point of the sample point is inside or outside the tract (Figure 6b).

Walkabout is used when the object of interest falls within a satellite subplot. We distinguish two walkabout directions: "right-hand walkabout" and "left-hand walkabout." To apply right-hand walkabout, we walk from the sample point to the object of interest, recording the distance. On reaching the object, we turn two-thirds right—i.e., clockwise 60°—and continue an equal distance to a "right-hand reflection point." If the reflection point is reached without first

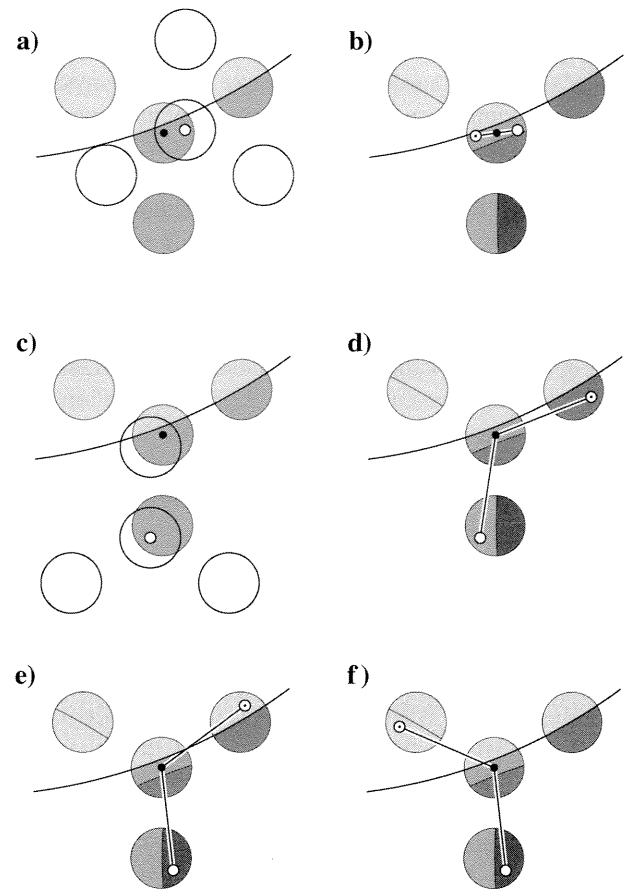

Figure 6. The walkabout method: If an object of interest occurs in the central subplot of the cluster (a), then conventional walkthrough is conducted (b). In b, the reflection point is inside the tract, so the value of the object's attribute density at the sample point is not adjusted ($t_k = 1$). If the object of interest falls on a satellite subplot (c), then right-hand walkabout is conducted (d). In d, the right-hand reflection point is inside the tract, so the object's attribute density at the sample point is not adjusted ($t_k = 1$). In e, however, the sample point is positioned in the inclusion zone where the right-hand reflection point is outside the tract, so the object's attribute density at the sample point is doubled ($t_k = 2$). Because this reflection point can never be a sample point, left-hand walkabout is also conducted (f). The left-hand reflection point is outside the tract, so the value of the object's attribute density at the sample point is tripled ($t_k = 3$).

encountering the boundary, then the right-hand reflection point is in the tract (Figure 6d). The object's attribute density at the sample point needs no correction, i.e., $t_k = 1$.

By contrast, if the boundary is encountered before the right-hand reflection point, then the reflection point is outside the tract and the object's attribute density at the sample point is doubled, i.e., $t_k = 2$ (Figure 6e). In addition, left-hand walkabout is also conducted because (1) the right-hand reflection point, being outside the tract, could never be a sample point where right-hand walkabout is conducted, and (2) the right-hand reflection point of the sample point's right-hand reflection point is also the sample point's left-hand reflection point. Hence, left-hand walkabout ensures that any additional slopover section of the inclusion zone is accounted for. Starting from the sample point, left-hand walkabout is the same as right-hand walkabout, except that we turn two-thirds left, counter-clockwise 60°, on reaching

the object of interest (Figure 6f). If the left-hand reflection point is outside the tract, then the measured value of the object's attribute density at the sample point is tripled, i.e., $t_k = 3$. We emphasize that left-hand walkabout is conducted only if the sample point's right-hand reflection point is outside the tract.

There are obvious shortcuts to the method. If it were possible to work outside the tract, we could conduct right-hand walkabout from the out-of-tract, right-hand reflection point instead of returning to the sample point to conduct left-hand walkabout. Or, we could just return to the object to take up left-hand walkabout from there. Indeed, right-hand walkabout from the out-of-tract, right-hand reflection point effectively does just that. Of course, the 60° turns in walkabout are specific for the arrangement of the subplots in Figure 6. Different cluster-plot configurations would require different angles.

Walkabout could be performed in the office with a good map detailing the positions of the objects of interest in the different subplots and the positions of the plot centers relative to points on the boundary line. The USDA Forest Service, for example, maps the locations of trees in their FIA cluster plots. With the coordinates of a sufficient number of boundary points to spline a boundary line, walkabout could be conducted graphically or mathematically. Obviously, walkthrough also could be conducted in this way.

Fundamentally, both the walkthrough and walkabout methods may be considered rotation methods. The walkthrough method rotates the attribute density at an out-of-tract reflection point 180° about the center point of the object of interest. This rotation, of course, moves the attribute density at the reflection point to the sample point, doubling the attribute density of the object at the sample point. In expectation, any location point in the tract can be a sample point with equal probability density. Hence, in expectation, the walkthrough method rotates the attribute density at every out-of-tract reflection point of every possible sample point into the tract (Figure 7a). Some small amount of bias may result from the fact that some out-of-tract location points in some inclusion zones may not have in-tract reflection points.

In the case with three satellite subplots, the right-hand walkabout method rotates the attribute densities at out-of-tract reflection points clockwise 120° into the tract. Left-hand walkabout rotates counter-clockwise (Figure 7b). Unlike the walkthrough method, unbiasedness may be maintained with the walkabout method if more than half the inclusion zone is outside the tract. This is because the attribute in half of the central subzone and two-thirds of the satellite subzones may be rotated into the tract.

The Vectorwalk Method

The vectorwalk correction method is more generally applicable than either the walkthrough or walkabout method. It may be applied as an alternative to either the walkthrough or walkabout method and it also may be applied where cluster plots have irregular shapes, i.e., where

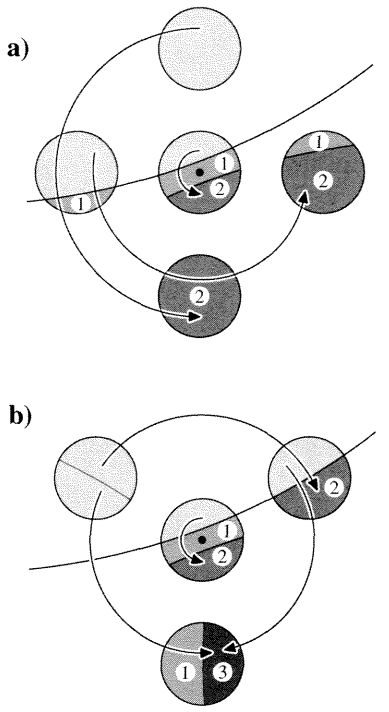

Figure 7. In expectation, the walkthrough method (a) and walkabout method (b) rotate portions of attribute in slopover (light gray) into the tract. These maps pertain to the examples in Figures 5 and 6, respectively. Each circled number is the value of t_k when the sample point falls into a particular section of the inclusion zone.

neither the walkthrough nor walkabout method is applicable.

Although the vectorwalk method has wider applicability than the walkthrough or walkabout method, it nevertheless incorporates the walkthrough method on a subplot-by-subplot basis. Figure 8a depicts a cluster plot, its direction vectors, and the inclusion zone of an object that occurs in the central subplot. In expectation, walkthrough effectively re-maps the attribute density in the central subzone of the inclusion zone, rotating the attribute in any slopover section back into the tract (Figure 8b). Moreover, because walkthrough is conducted subplot by subplot, the identical re-mapping of all the satellite subzones is implicit (Figure 8c). Under our Monte Carlo theory, we actually "measure" an object's attribute density only at the sample point. If an object occurs in a satellite subplot, as in Figure 8e, then the object's walkthrough-corrected attribute density at the center point (see Figure 8f) must transfer to the sample point, and this is accomplished by the identical re-mapping of each subzone.

In expectation, subplot-by-subplot walkthrough does not rotate all the attribute in slopover back into tract. At best, walkthrough is only expected to rotate all the attribute in each object's central subzone back into the tract. Residual attribute may remain in slopover sections of satellite subzones, as depicted in Figure 8c,f. The vectorwalk method moves the remaining out-of-tract attribute, point by point, from the sections of slopover to the central subzone.

An object occurs in a cluster's central subplot if the

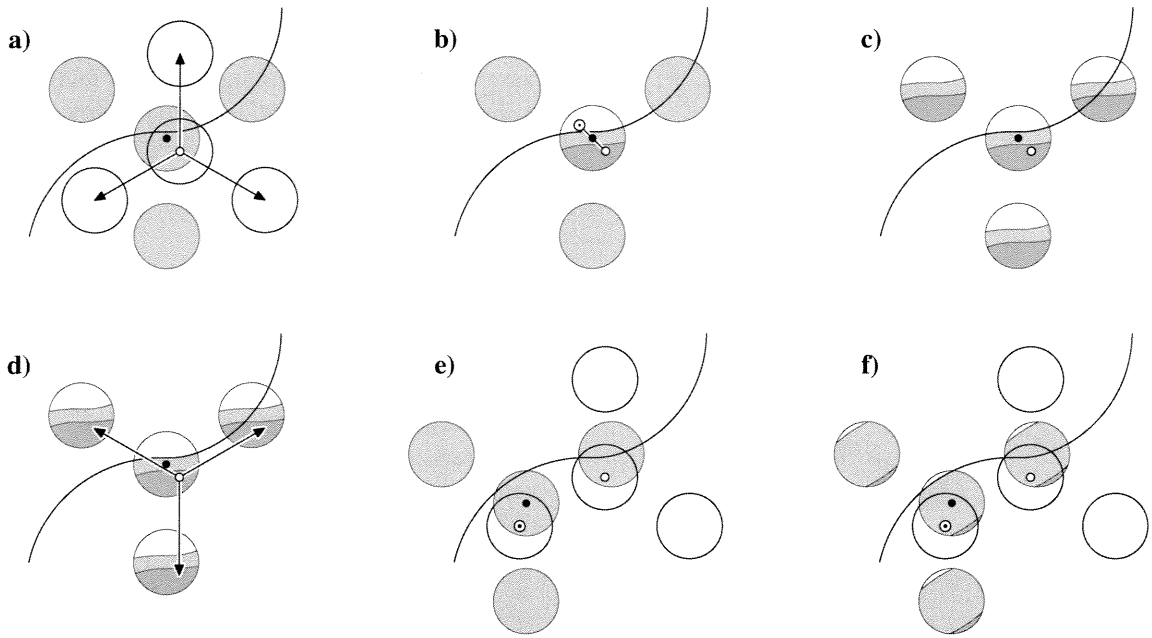

Figure 8. The vectorwalk method. (a) An object, which is centered in its inclusion zone, occurs in the central subplot of a cluster plot, which is shown with its direction vectors. (b) In expectation, walkthrough re-maps the attribute density in the object's central subzone (white, zero density; gray, uncorrected density; dark gray, double density). (c) Because walkthrough is conducted subplot by subplot, attribute densities in the satellite subzones are implicitly remapped in a pattern identical to the central subzone. (d) Inverse vectors terminate at matching points of the sample points. The attribute densities at out-of-tract matching points are shifted to the central subzone and added to the density at the sample point. In d, the object's attribute density was doubled by walkthrough at the sample point and implicitly doubled at the northwest matching point. Hence, overall, the object's attribute density at the sample point is quadrupled, i.e., $t_k = 4$. Walkthrough is conducted from the center point of each satellite subplot (e), which re-maps the attribute density in the central subzone and implicitly re-maps the satellite subzones in an identical pattern (f). In f, the object's attribute density is unchanged at the center point and the sample point, so $t_k = 1$.

sample point falls in the object's central subzone. For such objects, each of the cluster plot's inverse vectors terminates at a "matching point" in one of the object's satellite subzones. As shown in Figure 8d, each matching point has the same remapped attribute density as the sample point. The vectorwalk method shifts the attribute density at each out-of-tract matching point to the central subzone and adds this density to the total attribute density at the sample point. In Figure 8d, for example, the double density at the matching point in the northwest subzone is added to the double density at the sample point, yielding a quadruple density. Hence the object is tallied four times ($t_k = 4$).

The vectorwalk method derives its name from the fact that, in the field, we walk along each inverse vector to determine whether it terminates at a matching point inside or outside the tract. In expectation, the vectorwalks pull whole portions of attribute from the sections of slopover in the satellite subzones and place them in matching sections in the central subzone. In effect, different portions of attribute from different sections of slopover are heaped one on another. Because slopover sections may differ in shape, the resultant attribute map of the central subzone may resemble a patchwork quilt, with the attribute density uniform within each patch, but varying among them. Figure 9a shows—for the example in Figure 8—the expected maps of the attribute densities in the subzones after walkthrough, and Figure 9b shows the densities after the slopover is moved into the central subzone. Depending on where the sample point falls in the central subzone, the object may be tallied two, three, four, or six times.

Note: there is no need to undertake vectorwalks from the center points of satellite subplots. If an object occurs in a satellite subplot, then its attribute density at the sample point equals its walkthrough-corrected density at the center point of the satellite subplot (Figure 8f). A sampler's interest in an object properly begins and ends with the object's attribute density at the sample point.

The field protocols of the vectorwalk method are easy and straightforward. The cluster plot is centered at the sample point, though satellite subplots whose center points fall outside the tract are not installed. Walkthrough is applied in each individual subplot from the subplot's center point. The walkthroughs from the center point provide a single or double tally for each object in a satellite subplot, i.e., for the kth object, $t_k = 1$ or 2. The walkthroughs from the sample point in the central subplot provide a preliminary tally for each object in the central subplot, i.e., $t'_k = 1$ or 2. We also walk along each of the $n - 1$ inverse vectors from the sample point and count the number, s, that terminate outside the tract. We use this count to calculate the final tally value for each object in the central subplot, i.e., $t_k = (1 + s)t'_k$. It turns out that the vectorwalks are unnecessary in radially symmetric designs, because the terminal points of the inverse vectors are the center points of the satellite

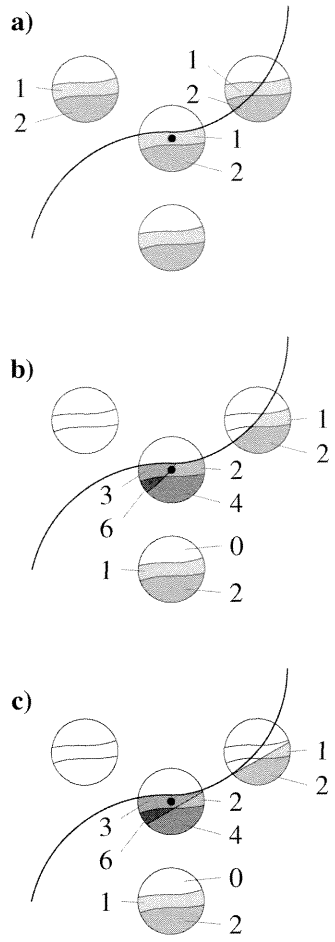

Figure 9. (a) Map of expected attribute densities in the subzones after walkthrough for the example in Figure 8 (white, zero density; 1, original single density; 2, double density). (b) In expectation, the vectorwalk method pulls the remapped portion an object's attribute from each slopover section and puts it in a matching section in the central subzone. In effect, portions of attribute from different sections of differing size are heaped in the central subzone, which results in an attribute-density map that resembles a patchwork quilt. In this example, the label of each section indicates the factor by which the original attribute density is multiplied to effect the correction for slopover and, thus, the number of times the object is tallied when a sample point falls into that particular section of the inclusion zone. (c) Under a modified protocol, a vectorwalk ends at the first encounter with the boundary, even if the inverse vector re-enters, and terminates at a matching point within the tract. An inclusion zone's attribute density map, under this protocol, may differ somewhat from the map that results from the use of the matching points (compare c with b).

subplots. Hence, s is the count of satellite subplots that were not installed.

In asymmetric designs, walking along an inverse vector from the sample point to the matching point could cause you to first leave and then re-enter the tract. If leaving the tract is potentially problematic, the protocols may be modified so that leaving the tract is unnecessary: Simply stop each vectorwalk on first encountering the boundary, count the number, s, of such stoppages, and use this count in exactly the same way as before to calculate the final tally. The resultant attribute density maps may differ between the two protocols (compare Figures 9b and 9c), but everything works out fine.

If boundaries are straight with square corners, the mirage method can substitute for walkthrough in the subplots because, in expectation, the mirage method folds the attribute in slopover sections of inclusion zones back into the tract, accomplishing the requisite re mapping of the attribute density.

The Reflection Method

The reflection method provides another viable alternative to correct for slopover in radially symmetric cluster-plot designs. Although our primary focus is on symmetric designs in this section, the reflection method may also be applied—under specific constraints or with some modification—to asymmetric designs. Application of the method to asymmetric designs is described briefly at the end of this section.

The reflection method for cluster plots can be viewed as a generalization of the reflection method described by Gregoire and Monkevich (1994) for line intersect sampling. We begin with the establishment of the central subplot around the sample point. If the direction vectors terminate within the tract, the satellite subplots are established as prescribed by the design (Figure 10). However, if a direction vector intersects the boundary, then it is folded back over itself at the boundary and the center point of the reflected satellite plot is established within the tract at the terminus of this folded direction vector (Figure 10b,c).

The reflection method incorporates the use of walkthrough on a subplot-by-subplot basis. In expectation, the walkthrough re-maps the attribute density in any central subzone with slopover and, in so doing, implicitly re-maps the attribute density in the associated satellite subzones

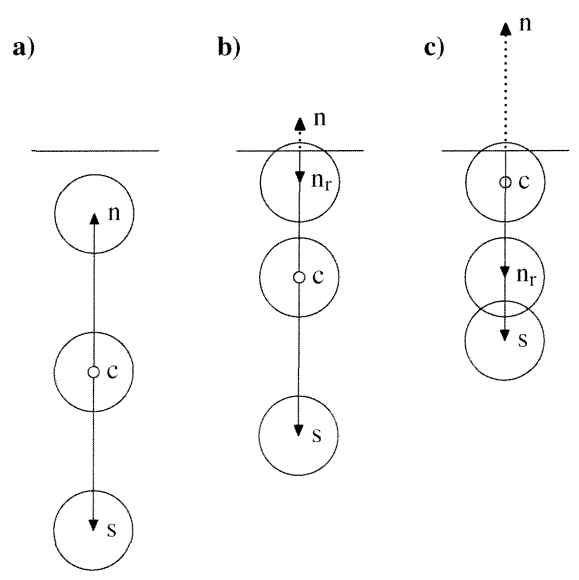

Figure 10. a) The reflection method for a cluster plot with a north (n), central (c), and a South (s) subplot. If a direction vector intersects the boundary of the tract (b and c), then it is folded back on itself and the center point of a reflected subplot—in this case a reflected northern subplot n_r—is established where the folded vector terminates within the tract.

(Figure 11a,b). Under the protocols of the reflection method, each point in a slopover section of a satellite subzone has a reflection point across the boundary and inside the tract, provided the tract is wide enough. The slopover point and its reflection point are equidistant from the boundary on a line parallel to the vector that marks the center point of the satellite subzone (Figure 11a,c). In aggregate, the reflection points coalesce into a reflection of the slopover (Figure 11c). If a sample point falls within a reflected section of an object's inclusion zone, then the object is tallied by a reflected subplot. In the example shown in Figure 11d, the object is actually tallied twice ($t_k = 2$), because walkthrough indicates that the center point falls in a section of the subzone where the attribute density is doubled.

It is worth noting that any location point in the slopover could serve as a sample point, if we allowed sample points to fall outside the tract. Were we to establish a cluster plot centered at a sample point, (○), in the slopover, and reflect the cluster plot about the boundary—hinging any direction vector at its intersection with the boundary—then this reflected cluster plot would coincide with a cluster plot centered at the reflection point of (○). Thus, in effect, the reflection method not only reflects the slopover into the tract, but also the points that sample its attribute density.

The sections of reflection and the in-tract sections of the original inclusion zone define a "reflected inclusion zone." The shape of a reflected inclusion zone and the map of its attribute density depend on the proximity of the object to the boundary, the shape of the boundary line, and the orientation of the object's original inclusion zone relative to the boundary line. For pedagogical purposes, most of our diagrams show inclusion zones oriented perpendicular to a straight border (Figures 11 and 12). In Figure 12, we use labels to indicate which subplots would tally an object if the sample point were to fall into a particular section of a reflected inclusion zone. For example, if the sample point falls into a section labeled n_r, then the object is tallied by the reflected north subplot.

The shapes of reflected inclusion zones and their attribute density maps become quite complicated where boundary lines are curved (Figure 13). Indeed, the reflected portion of an inclusion zone may look somewhat smeared or distorted. Appearances notwithstanding, curved boundary lines do not vitiate the method; each point in a slopover section of a satellite subzone ordinarily has a reflection point inside the tract. The smearing is caused by the obliquity and curvature of the boundary. Two points that are close to each other in slopover may differ considerably in their respective distances to a curved or oblique boundary and, therefore, their respective reflection points inside the tract may be relatively far from each other. Nevertheless, area and attribute is conserved with curved and/or oblique boundaries, since the reflection of the infinitely many points in the slopover is point by point. In expectation, estimation under the reflection method is unbiased if (1) walkthrough rotates all the slopover in central subzones back into the tract, and (2) every point with nonzero attribute density in satellite slopover has a boundary reflection point inside the tract.

The reflection method is straightforward to implement in the field. Simply establish the cluster plot as you would in the interior of the tract, but when walking to a satellite center point—if the boundary is encountered—reflect off

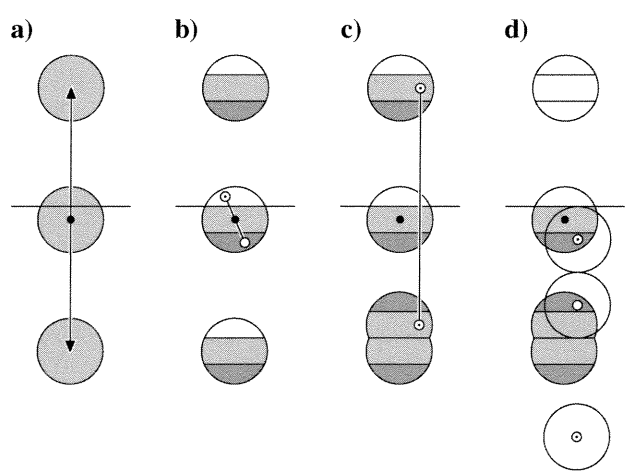

Figure 11. a) A radially symmetric inclusion zone, shown with north and south vectors, slops over the boundary in the northern direction. (b) Walkthrough re-maps the attribute density in the central subzone and implicitly re-maps the satellite subzones in an identical pattern (white, zero density; light gray, single density; dark gray, double density). (c) Each point in the north subzone has a reflection point inside the tract. The slopover point and its reflection point are equidistant from the boundary on a line parallel to the north vector. The reflection method ignores the portion of an inclusion zone that lies outside the tract; instead, the method utilizes the reflection of the slopover, which is inside the tract. If a sample point falls in the reflected section of the inclusion zone, then the object is tallied by a reflected subplot. In d the object is tallied twice, because walkthrough indicates that the center point of the reflected north subplot (⊙) has double attribute density.

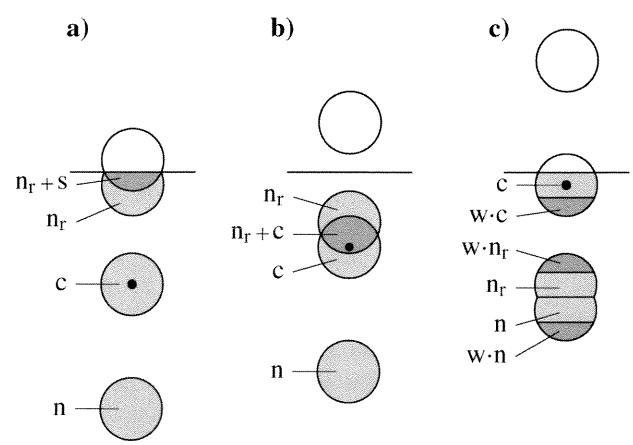

Figure 12. The shape of a reflected inclusion zone depends on the proximity of the object to the boundary line, among other things. The label for a particular section of reflected inclusion zone indicates which subplot or subplots would tally the object if the sample point were to fall in that section (c, central; n, north; n_r, reflected north; s, south; w · c, central with walkthrough; w · n, north with walkthrough; w · n_r, reflected north with walkthrough). For example, n_r + s indicates that the object is tallied by the reflected north subplot and the south subplot, and w · n_r indicates that the object is double-tallied by walkthrough in the reflected north subplot.

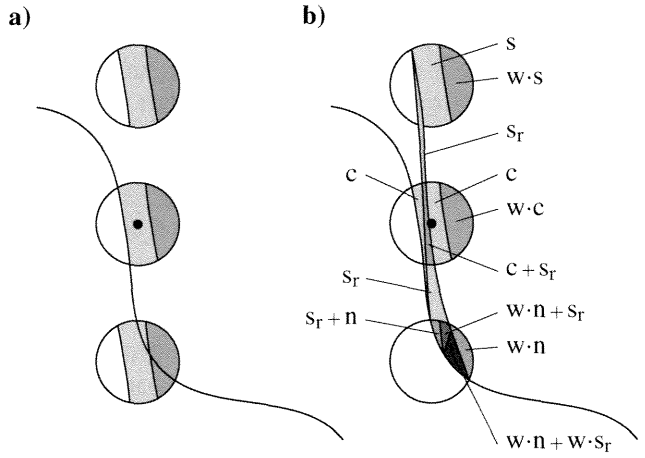

Figure 13. The reflection method works for curved boundaries, but reflections of slopover can look quite distorted. (a) Expected attribute densities after walkthrough. (b) The reflected inclusion zone. Labels indicate which subplots tally the object when the sample point falls in a particular section of the reflected inclusion zone (see Figure 12 for labeling conventions).

the boundary and install the satellite subplot within the tract rather than beyond the boundary. In addition, one must implement the walkthrough method within the individual subplots. Of course, these protocols (and the theory behind them) hold for radially symmetric cluster plots with four or more satellite subplots.

The reflection method can also correct for slopover in asymmetric cluster designs if suitable precautions or modifications are made at the design or sampling stages. If the directional orientations of cluster plots are selected uniformly at random, then even radially asymmetric clusters can be viewed as symmetric in expectation. Thus, in expectation, the reflection method, as described above, will eliminate slopover bias for asymmetric, randomly oriented cluster designs.

Alternatively, the reflection method itself can be generalized to handle asymmetric clusters with fixed orientation, e.g., the FIA design. Specifically, if the reflection procedures are defined in terms of the inverse vectors, rather than the direction vectors, then the method can be applied to asymmetric clusters. Under this procedure, the cluster plot is installed in the usual manner. However, if a direction vector intersects the boundary, the vector is abandoned without installing either a subplot or a reflected subplot. Whether or not n subplots are already installed, we must also undertake vectorwalks along each of the inverse vectors. If, and only if, an inverse vector intersects the boundary, then it is folded at the boundary and a reflected subplot is centered at the point where the folded inverse vector terminates inside the tract. Of course, walkthrough is conducted in all the resultant subplots, if need be. In practice, this reflection procedure may result in the installation of a cluster of n, more than n, or fewer than n subplots. While intuitively unappealing, more than or fewer than n subplots is not problematic: Recall that subplots merely serve to identify those objects whose inclusion zones—or, in this case, whose reflected inclusion zones—include the sample point.

Residual Bias

All four slopover correction methods conserve area and attribute within the inclusion zones of the N objects in a tract, $\mathscr{A}$. Thus, integration of the attribute density across the land area of an inclusion zone, I_k, provides the attribute, y_k, i.e.,

$$\iint_{I_k} \frac{t_k(x, z)\, y_k}{n a_k}\, \mathrm{d}x\, \mathrm{d}z = y_k.$$

A total attribute, τ, in $\mathscr{A}$ is estimated by

$$\hat{\tau}_j = A \sum_{(x_j, z_j) \in I_k} \frac{t_k y_k}{n a_k},$$

where the n in the estimator is the number of subzones in an inclusion zone, not the number of installed subplots. The value of n is not altered by any of the correction methods.

In expectation,

$$\mathrm{E}[\hat{\tau}_j] = \iint_{\mathscr{A}} \left[\sum_{(x,z) \in I_k} \frac{t_k(x, z)\, y_k}{n a_k} \right] \mathrm{d}x\, \mathrm{d}z \leq \tau\,.$$

The estimator is unbiased, i.e., $\mathrm{E}[\hat{\tau}_j] = \tau$, only if integration across the in-tract land area of each object's inclusion zone provides the object's attribute, i.e.,

$$\iint_{I_k \cap \mathscr{A}} \frac{t_k(x, z)\, y_k}{n a_k}\, \mathrm{d}x\, \mathrm{d}z = y_k \quad k = 1, 2, \ldots, N.$$

In other words, the estimator is unbiased only if the slopover correction method can successfully move all of the attribute in the slopover sections of every inclusion zone back into $\mathscr{A}$. Generally, however, some residual amount of attribute will remain in slopover somewhere around the boundary of a tract, but this residual may be quite small. For example, Ducey et al. (2004) reported that, in simulations, the walkthrough method reduced under-estimation of tree volume of a 46.5-ha tract in New Hampshire from 6.7% to 0.11% when the tract was sampled with fixed-radius circular plots. To put it another way, walkthrough rotated all but 1.64% of the attribute in slopover into the tract.

Discussion

In this article we have restricted our interest to cluster-plot designs that comprise a central subplot centered at a sample point and constant number of satellite subplots in a fixed arrangement, all subplots having the same fixed radius. Each of our four correction methods for slopover works with curved boundaries and corrections usually can be carried out without leaving the tract.

The walkthrough method largely solves the slopover problem for radially symmetric cluster-plot designs. The

advantages of the walkthrough method include quick identification of objects that do not require correction, and the requirement to check only a single point for those that do. However, a large dispersed inclusion zone with large distances between satellite subzones is a disadvantage and may require an unreasonable amount of walking in the field. As was noted, however, walkthrough can be conducted graphically in the office with a map detailing the positions of the objects of interest in the different subplots and the positions of the plot centers relative to points on the boundary line.

The walkabout method largely solves the slopover problem for cluster-plot designs where satellite plots are a constant distance from the central subplot and where the angle between adjacent direction vectors is constant. Walkabout as illustrated above, with a 120° rotation, applies to the FIA cluster-plot design. However, the method generalizes readily, at least in theory.

For example, one could perform walkabout given the radially symmetric cluster-plot design shown in Figure 5. This design would require a 90° right-hand turn, no turn (straight) if the right-hand reflection point were in slopover, and a 90° left-hand turn if the no-turn reflection point were in slopover. This walkabout approach would require more walking than walkthrough, but it would be more robust to boundary wrapping, because three of the four satellite subzones of an object's inclusion zone could slop over the boundary. This same walkabout procedure could substitute for walkthrough on an individual subplot. The mirage method may use as many as three mirage points outside the boundary, when sample points fall near square corners. Walkabout with 90° angles is not only a surrogate for mirage at square corners, but it is also applies more generally and does not require leaving the tract.

As a practical matter, field implementation of the walkabout method for cluster plots involves a large amount of walking, unless the cluster-plot design is very compact or the objects sampled are very sparse. Even for the three-satellite design emphasized here, it is likely to be somewhat difficult to judge a priori which objects require walkabout. Finally, it may be quite difficult to turn and maintain odd rotations (such as 120°) using only a hand compass. For all these reasons, mapping the boundary and implementing walkabout in the office may prove quite attractive. Of course, mapping the boundary entails mapping the entire section of boundary that slices through the inclusion zones of any objects in the cluster plot.

The vectorwalk method largely solves the slopover problem for both radially symmetric and asymmetric cluster-plot designs, and also for irregular cluster-plot designs, where the direction vectors differ in length and/or where the angles between adjacent direction vectors differ. The method works even if all the satellite subzones of an object's inclusion zone slop over the boundary. Only half of the central subzone needs to be inside the tract. This method also is easy to implement in the field, requiring relatively quick and easy subplot-by-subplot walkthrough corrections, augmented by at most $n - 1$ vectorwalks from the sample point.

One minor concern regarding the vectorwalk method relates to the fact that all portions of an object's attribute in slopover are translated to, and heaped in, the central subzone of the inclusion zone. This concentrates a lot of attribute density in a small area, which means that some objects in the central subplot have substantial influence in the estimation process. Another minor concern relates to the re-mapping of the attribute densities in the satellite subplots. If more than half of the central subzone is in slopover, or the object is tightly wrapped by the boundary, some residual slopover is inevitable. In expectation, the residual slopover in the central subzone may magnify in satellite subzones.

Like the vectorwalk method, the reflection method is easy to apply. Moreover, if a symmetric cluster-plot design is used, application of the reflection method always results in the installation of the prescribed number of subplots in each cluster. With the exception of the walking required for the subplot-by-subplot walkthrough method, the reflection method requires only as much walking as a cluster plot installed in the interior of the tract. Compared to the vectorwalk method, which heaps the slopover density in the central subzone, the reflection method tends to smear the slopover density over a larger area inside the tract, which may yield smaller sampling variances. And, as was noted, the protocols of the reflection method for symmetric radial designs also apply to asymmetric or irregular designs with random orientation. The protocols for asymmetric or irregular designs with fixed orientation are also easy to apply in practice, though the number of installed subplots may vary among clusters near the boundary.

All four of our corrections methods generalize to clustered point designs involving Bitterlich sampling or point relascope sampling (Gove et al. 1999), where satellite points, rather than fixed-radius plots, are arranged radially about a sample point. In these "point methods" an object's size and shape determines the area and shape of its dispersed inclusion zone, and an angle-gauge is used to determine whether the inclusion zone includes the sample point. However, the walkthrough, walkabout, vectorwalk, and reflection protocols are the same as for fixed-radius cluster plots.

When viewed from the perspective of slopover correction, radial symmetric cluster-plot designs seem a better choice than asymmetric designs. The walkthrough method for cluster plots is a viable correction alternative for symmetric designs, especially if the locations of objects and boundary points are mapped. The vectorwalk method benefits from the fact that direction vectors and inverse vectors are coincident. Thus, the center points of the satellite subplots are the matching points of the sample point, and so the inverse-vectorwalks are unnecessary. And, as was noted, the reflection method always yields the prescribed number of subplots inside the tract. In large-area cluster designs where subplots may be 100 m or even 1 km apart from each other, any method that involves extra walking may be viewed with a jaundiced eye. Under symmetry, however, the vectorwalk and reflection methods remain viable and appealing alternatives, since these methods require no extra walking apart from the subplot-by-subplot walkthrough corrections.

Unlike the methods proposed in this article, Mandallaz (1991) considered cluster sampling where sample points are allowed to fall outside the tract. The number of subplots in a cluster plot is defined by the number of center points that fall within the tract, so cluster size is a random variable. Slopover bias is corrected on a subplot-by-subplot basis, for example, with the walkthrough method. Estimates of τ that obtain from individual cluster plots are not unbiased. However, if we average the estimates of τ that obtain from the individual subplots of all the clusters, we obtain an asymptotically unbiased estimate of τ, where the bias diminishes as the number of subplots increases. In effect, Mandallaz's point estimator formally ignores the cluster design and treats all the subplots of all clusters as independent plots. Variance estimation, however, is still based on cluster means weighted with the cluster size. Of course, the estimator of τ is asymptotically unbiased only if the slopover correction method is completely successful at the subplot level. By contrast, if completely successful, each of the four correction methods presented in this article provides for an unbiased estimate of τ from the measurement at a single sample point. And, of course, with the measurements from two or more sample points, we can calculate an unbiased estimate of the sampling variance.

Maps of objects and boundary points offer another possibility for slopover correction: the calculation of the area of the in-tract portion of each inclusion zone that slops over the boundary. Let c_k be the area of the in-tract portion of the kth object's inclusion zone, then

$$\hat{\tau}_j = A \sum_{(x_j,\, z_j) \in I_k} \frac{y_k}{c_k}$$

unbiasedly estimates τ, provided the object locations and boundary spline are accurate. It is likely that this method would require more detailed mapping compared to that required for a graphical application of walkthrough or walkabout. And, it is important that field crews realize that boundary mapping may be necessary even where none of the subplots overlaps the boundary.

Yet another way to deal with cluster-plot slop would be to abandon the use of cluster plots, but that is not likely to happen because large scale inventories that use cluster-plot designs have become institutionalized. Besides, all things considered, cluster-plot designs may be more efficient than single-plot designs, as sampling with a cluster of small subplots around a sample point may provide a more "representative sample" with lower variance if the dispersed coverage of each cluster plot results in more uniform attribute densities among sample points.

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